Total marks (120)
Attempt questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE Writing Booklet. Extra Writing Booklets are available.

Question 1 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) Solve $1-2x<9$. **2**

(b) Factorise $6x^2-x-1$. **2**

(c) Solve $|2x-5|=8$. **2**

(d) Sketch the graph of $3y+x=6$, showing the intercepts on both axes. **2**

(e) Rationalise the denominator of $\dfrac{5}{\sqrt{3}+3}$. **2**

(f) Sketch $f(x)=\sqrt{9-x^2}$. **2**

Question 2 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) Differentiate with respect to x:

(i) $(3+x^2)^{12}$. **2**

(ii) $\dfrac{\ln x}{e^x}$. **2**

(iii) $x^2 \cos\dfrac{x}{2}$. **2**

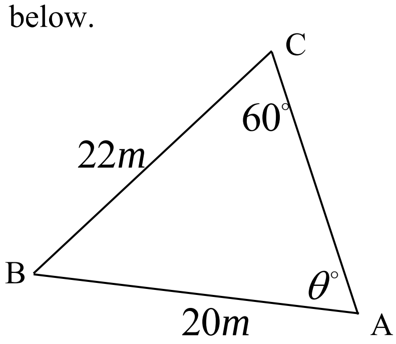
(b) (i) Find $\displaystyle\int \frac{x}{x^2-4}\,dx$. **1**

(ii) Evaluate $\displaystyle\int_1^4 \left(\frac{1}{x^2}-\sqrt{x}\right)dx$. **3**

(iii) Evaluate $\displaystyle\int_0^{\frac{\pi}{8}} \sec^2 2x\ dx$. **2**

Question 3 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

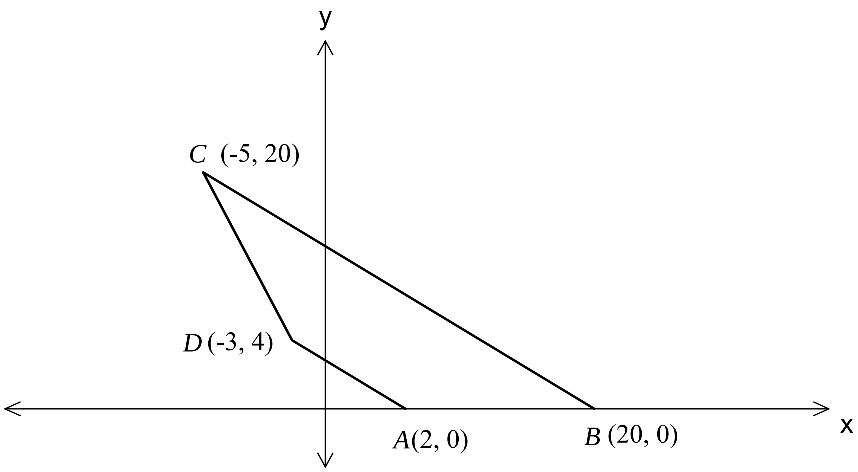
(a) Consider the triangle below.



(i) Find the size of the angle θ , correct to the nearest degree. **2**

(ii) Find the area of the triangle, using your approximation for the angle found in (i). (correct to the nearest cm^2 .) **1**

(b)



(i) Show that *ABCD* is a trapezium, by showing that *AD* is parallel to *BC*. **2**

(ii) Show that the equation of the line *BC* is $4x + 5y – 80 = 0$. **2**

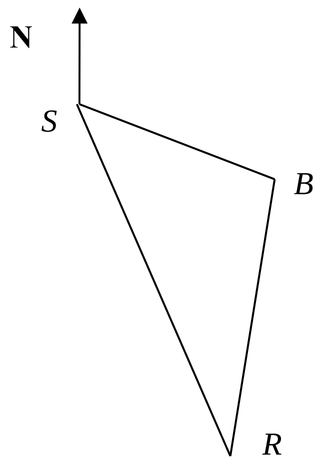
(iii) Find the length of *BC*. (Leave in exact form) **1**

(iv) Find the perpendicular distance of the point D from the line *BC*. **2**

(v) Hence, or otherwise, find the area of the trapezium *ABCD*. **2**

Question 4 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) A triathlon course begins with a 500 m swim on a bearing of 110° from the start S. This is followed by a 1800 m cycling leg on a bearing of 185°. The triathlon is completed with a run back to S.



(i) Copy and complete this diagram. Find the size of $\angle SBR$. **1**

(ii) How far was the run home? (nearest m) **2**

(iii) Find the bearing of S from R. (nearest degree) **2**

(b) Sarah plays computer games competitively. From past experience, she has a 0.9 chance of winning a game of *Staplestory* and a 0.6 chance of winning a game of *Bota*. In one afternoon of competition she plays two games of *Staplestory* and one of *Bota*.

(i) What is the probability that she will win all three games? **1**

(ii) What is the probability that she wins at least one game of *Staplestory* and loses the game of *Bota*.? **2**

(c) Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

(i) On which day does she run a distance of 10 km in training? **2**

(ii) What is the total distance she will have run in training by the end of that day? **2**

Question 5 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) Solve the equation $\sin\theta = -\cos\theta$ for $0^\circ \le \theta \le 360^\circ$. ($\cos\theta \ne 0$) **3**

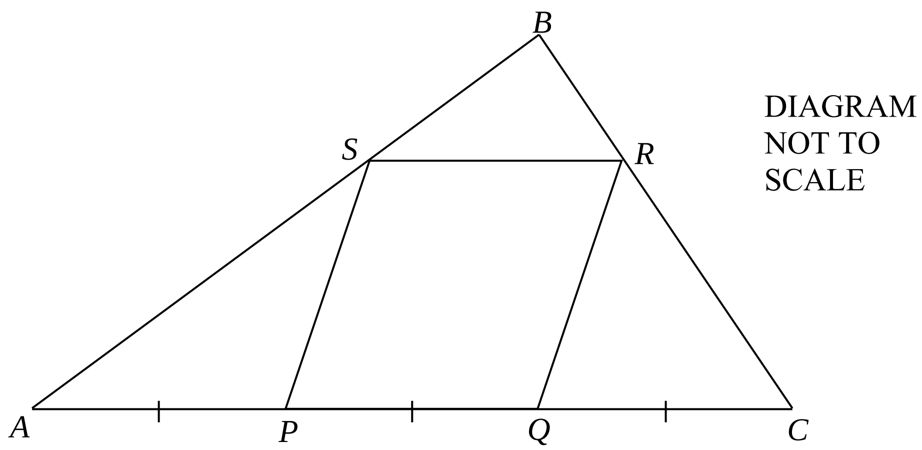
(b) If α and β are the roots of the equation $5x^2 - 3x - 2 = 0$, find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(c)



The diagram above shows ΔABC, where $AP = PQ = QC$ and $PQRS$ is a rhombus.

(i) If $\angle SAP = x^\circ$ prove that $\angle SPQ = 2x^\circ$ **2**

(ii) Prove that $\angle ABC = 90^\circ$ **3**

Question 6 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the equation $4x^2 + kx + 1 = 0$. For what values of k does this equation have two real and distinct roots? **3**

(b) One model for the number of individuals connected to the internet worldwide is the exponential growth model.

$$N = Ae^{kt}$$

where N is the estimate for the number of individuals connected to the internet (in millions), and t is the time in years after 1 January 2011. It is estimated that at the start of 2012, when $t = 1$, there will be 1800 million individuals connected to the internet, while at the start of 2013, when $t = 2$, there will be 2400 million individuals connected to the internet.

(i) Show that $A = 1350$ and $k = \ln\left(\frac{4}{3}\right)$. **2**

(ii) According to the model, during which month and year will the number of individuals connected to the internet first exceed 4000 million. **2**

(iii) At what rate will the number of individuals be increasing in 2015? **2**

(c) Use Simpson's Rule with five function values to estimate the area under the curve $y = \ln(x+1)$ and the x-axis between $x = 1$ and $x = 5$. (correct to two decimal places) **3**

Question 7 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

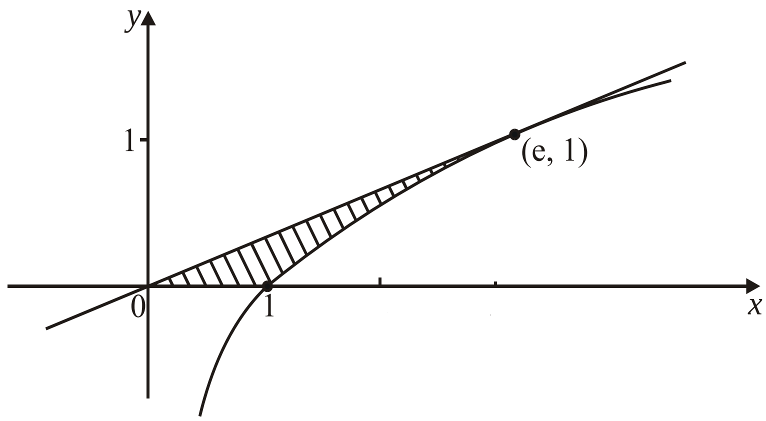
(a) Evaluate $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^n$. **2**

(b) Let $f(x) = \frac{x^3}{3} + x - 3$

(i) Show that the graph of $y = f(x)$ has no stationary points. **2**

(ii) For what values of x is the graph of $y = f(x)$ concave up? **1**

(c)



(i) Find the equation of the tangent line to the curve $y = \ln x$ at the point *(e, 1)*, and verify that the origin is on this line. **2**

(ii) Show that $\frac{d}{dx}\left(x\ln x - x\right) = \ln x$. **2**

(iii) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line and the line $y = 0$. Use the result of part (ii) to show that the area of this region is $\frac{e}{2} - 1$. **3**

Question 8 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

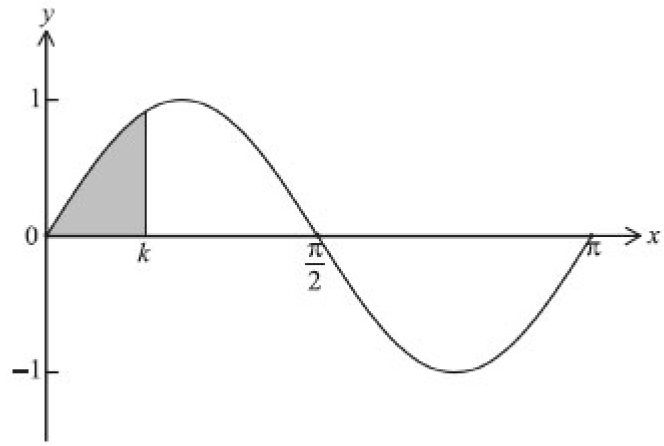
(a) The velocity, v, in m s^{-1} of a particle moving in a straight line is given by $v = e^{3t-2}$, where t is the time in seconds.

(i) Find the acceleration of the particle at $t = 1$. **1**

(ii) At what value of t does the particle have a velocity of 22.3 m s^{-1}? (correct to one decimal place) **2**

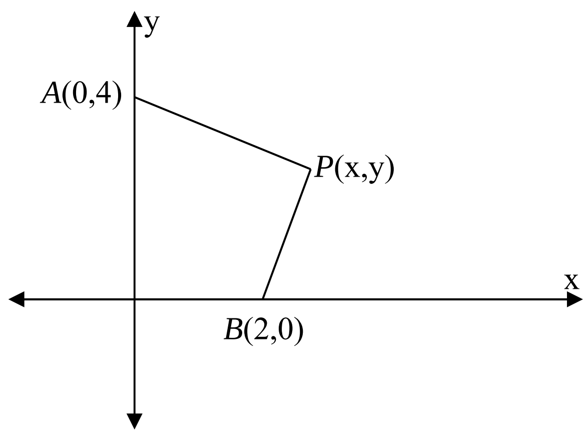
(iii) Find the distance travelled in the first second .(correct to three decimal places) **2**

(b) The graph of $y = \sin 2x$ from $0 \le x \le \pi$ is shown below.



The area of the shaded region is 0.85. Find the value of k. (correct to two decimal places) **3**

(c) A point $P(x,y)$ moves in such a way that PA is perpendicular to PB.



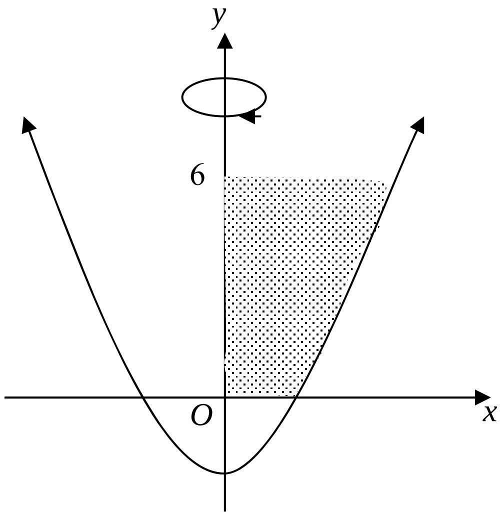
(i) Show that the equation of the locus is given by
$$x^2 - 2x + y^2 - 4y = 0.$$ **2**

(ii) Find the centre and radius of this circle. **2**

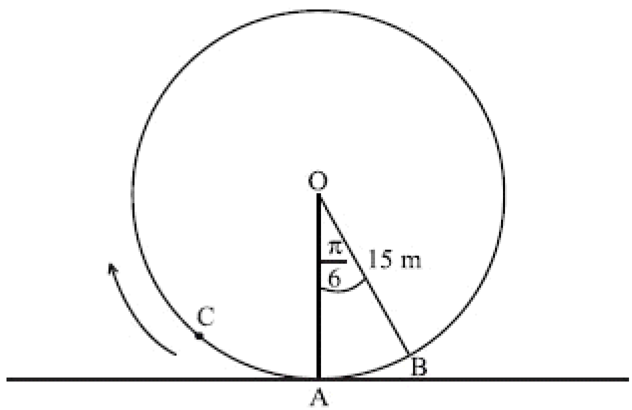
Question 9 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) The diagram shows the region bounded by the curve $y = 3x^2 - 12$, the line $y = 6$, and the x and y axes. **4**

Find the exact volume of the solid of revolution formed when the shaded region is rotated about the y axis.



(b) A Ferris wheel with centre O and a radius of 15 metres is represented in the diagram below. Initially seat A is at ground level. The next seat is B, where $A\hat{O}B = \frac{\pi}{6}$.



(i) Find the length of the arc AB. **1**

(ii) Find the area of the sector AOB. **1**

(iii) The wheel turns clockwise through an angle of $\frac{2\pi}{3}$. Find the height of A above the ground. **3**

(iv) The height, h metres, of seat C above the ground after t minutes, can be modelled by the function

$$h(t) = 15 - 15\cos\left(2t + \frac{\pi}{4}\right).$$ **3**

Find the time at which the height is changing most rapidly.

Question 10 *(12 marks)* Use a SEPARATE Writing Booklet. **Marks**

(a) A car dealership has a car for sale for a cash price of \$35 000. It can also be bought on terms over five years. The first six months are interest free and after that interest is charged at the rate of 1.5% per month on the balance owing for that month. Repayments are to be made in equal monthly installments of $\$M$ with the first repayment applied at the end of the first month. A customer agrees to buy the car on these terms.

Let $\$A_n$ be the amount owing at the end of the nth month.

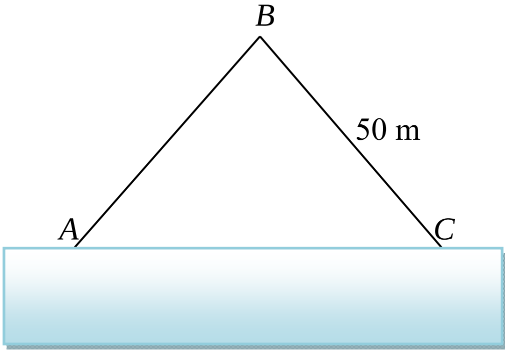
(i) Find an expression for A_6. **1**

(ii) Show that $A_8 = (35\ 000 - 6M)1.015^2 - M(1 + 1.015)$. **2**

(iii) Find an expression for A_{60}. **2**

(iv) Find the value of M. **2**

(b) An isosceles triangular pen is enclosed by two fences AB and BC each of length 50 m, and a river is the third side.



(i) If $AC = 2x\ m$, show that the area of the triangle is given by:

$$A(x) = x\sqrt{2500 - x^2}.$$ **2**

(ii) Hence find the length of AC when the area is a maximum. (correct to the nearest m) **3**

END OF PAPER

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1

(*a*)

$$1 - 2x < 9$$
$$-2x < 8 \quad ✓$$
$$x > -4 \quad ✓$$

(*b*)

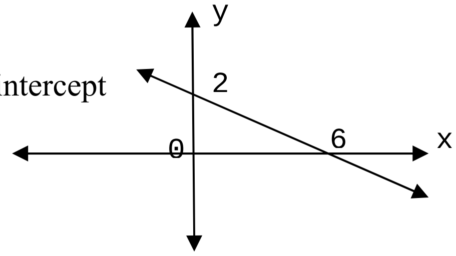
$$6x^2 - x - 1$$
$$= (2x-1)(3x+1) \quad ✓✓$$

(*c*) $|2x-5| = 8$

$2x - 5 = 8$ ✓ *or* $-2x + 5 = 8$ ✓

$x = \dfrac{13}{2}$ $\qquad$ $x = -\dfrac{3}{2}$

(d) ✓ ✓ One for each intercept



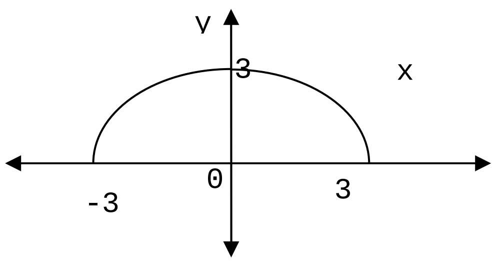
(*e*)

$$\frac{5}{\sqrt{3}+3} \times \frac{\sqrt{3}-3}{\sqrt{3}-3} \quad ✓$$
$$= \frac{5\sqrt{3}-15}{3-9}$$
$$= \frac{15-5\sqrt{3}}{6} \quad ✓$$

(f)

✓ -A semi circle!!

✓ -Intercepts



Question 2

(a) (i) $Let\ y=(3+x^2)^{12}$

$$\therefore \frac{dy}{dx}=12(3+x^2)^{11}\times 2x \quad \boxed{\checkmark}$$

$$\therefore \frac{dy}{dx}=24x(3+x^2)^{11} \quad \boxed{\checkmark}$$

(ii) $Let\ y=\frac{\ln x}{e^x}$

$$\therefore \frac{dy}{dx}=\frac{e^x.\frac{1}{x}-e^x\ln x}{e^{2x}} \quad \boxed{\checkmark}$$

$$\therefore \frac{dy}{dx}=\frac{1-x\ln x}{xe^x} \quad \boxed{\checkmark}$$

(iii) $Let\ y=x^2\cos\frac{x}{2}$

$$\therefore \frac{dy}{dx}=-x^2\sin(\frac{x}{2})\times\frac{1}{2}+2x\cos\frac{x}{2} \quad \boxed{\checkmark}$$

$$\therefore \frac{dy}{dx}=\frac{-x^2}{2}\sin(\frac{x}{2})+2x\cos\frac{x}{2} \quad \boxed{\checkmark}$$

(b) (i) $$\int\frac{x}{x^2-4}dx=\frac{1}{2}\int\frac{2x}{x^2-4}dx$$

$$=\frac{1}{2}\ln(x^2-4)+c \quad \boxed{\checkmark}$$

(ii) $$\int_1^4\left(\frac{1}{x^2}-\sqrt{x}\right)dx=\int_1^4\left(x^{-2}-x^{\frac{1}{2}}\right)dx \quad \boxed{\checkmark}$$

$$=\left[\frac{1}{-x}-\frac{2x^{\frac{3}{2}}}{3}\right]_1^4 \quad \boxed{\checkmark}$$

$$=\left(\frac{1}{-4}-\frac{2(4)^{\frac{3}{2}}}{3}\right)-\left(\frac{1}{-1}-\frac{2(1)^{\frac{3}{2}}}{3}\right)$$

$$=\quad -3\frac{11}{12} \quad \boxed{\checkmark}$$

(iii) $$\int_0^{\frac{\pi}{8}}\sec^2 2x dx=\frac{1}{2}\left[\tan 2x\right]_0^{\frac{\pi}{8}} \quad \boxed{\checkmark}$$

$$=\frac{1}{2}\tan\frac{\pi}{4}-\frac{1}{2}\tan 0$$

$$=\frac{1}{2} \quad \boxed{\checkmark}$$

Question 3

(a) (i) $\dfrac{\sin\theta}{22} = \dfrac{\sin 60}{20}$ $\boxed{\checkmark}$

$$\sin\theta = \frac{22\sin 60}{20}$$

$$\therefore \theta = 72 \ (nearest\ \deg ree) \boxed{\checkmark}$$

(ii) $Area = \dfrac{1}{2}\times 20\times 22\times \sin 48$

$$= 163.49186\ m^2 \ \ (nearest\ cm^2) \quad \boxed{\checkmark}$$

(b) (i) $Gradient\ AD = \dfrac{4}{-3-2} = \dfrac{-4}{5}$

$$Gradient\ BC = \frac{20}{-5-20} = \frac{-4}{5}$$

$$\therefore AD \parallel BC \ \boxed{\checkmark}\boxed{\checkmark}$$

(ii) $Equation\ of\ BC:$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-4}{5}(x - 20) \quad \boxed{\checkmark}$$

$$\therefore 5y = -4x + 80 \quad \boxed{\checkmark}$$

$$\therefore 4x + 5y - 80 = 0$$

(iii) use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(20+5)^2 + (0-20)^2} \quad \boxed{\checkmark}$$

$$\therefore d = \sqrt{1025} = 5\sqrt{41} \quad \boxed{\checkmark}$$

(iv) use $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

use $d = \dfrac{|4(-3) + 5(4) - 80|}{\sqrt{(4)^2 + (5)^2}}$ $\boxed{\checkmark}$

$$\therefore d = \frac{72}{\sqrt{41}} \quad \boxed{\checkmark}$$

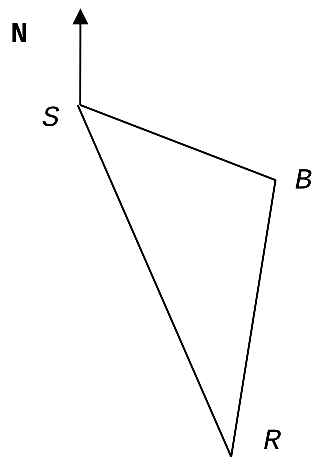
(v) $AD = \sqrt{(-4)^2 + 5^2}$

$$AD = \sqrt{41}$$

$Area\ \ of\ \ ABCD = \dfrac{1}{2}\times\dfrac{72}{\sqrt{41}}\left[\sqrt{41} + 5\sqrt{41}\right]$ $\boxed{\checkmark}$

$$= 196\ u^2 \quad \boxed{\checkmark}$$

Question 4



(a) (i) $\angle SBR = 105^\circ$. ☑ *From diagram.*

(ii) use $d^2 = 1800^2 + 500^2 - 2\times1800\times500\times\cos105^\circ$

$\therefore d = \sqrt{1800^2 + 500^2 - 2\times1800\times500\times\cos105^\circ}$ ☑

$d = 1989$ (nearest m) ☑

(iii) now $\angle SRB = \cos^{-1}\left(\dfrac{1989^2 + 1800^2 - 500^2}{2\times1989\times1800}\right)$

$\angle SRB = 14^\circ$ ☑

$\therefore$ Bearing of S from R is

$(360 - 14^\circ) + 5^\circ = 351^\circ T$ (nearest degree) ☑

(b) (i) $P(\text{Sarah wins all 3 games}) = 0.9\times0.9\times0.6$

$= 0.486$ ☑

(ii) $P(\text{win 1}S, \text{lose1}S \text{ and loose}B) + P(\text{win 2}S, \text{lose1}S \text{ and loose}B)$

$+P(\text{lose 1}S, \text{win1}S \text{ and loose}B)$ ☑

$= 2\times0.9\times0.1\times0.4 + 0.9\times0.9\times0.4$

$= 0.396$ ☑

(c) $1000, 1250, 1500, ..., 10000$

(i) use $T_n = a + (n-1)d$

$1000 + (n-1)250 = 10000$ ☑

$(n-1)250 = 9000$

$(n-1) = \dfrac{9000}{250}$

$\therefore n = 37^{th}$ day ☑

(ii) use $S_n = \dfrac{n}{2}(a + l)$

$\therefore S_{37} = \dfrac{37}{2}(1000 + 10000)$ ☑

$\therefore S_{37} = 203500m$ ☑

Question 5

(*a*) $\sin\theta = -\cos\theta$

$\therefore \tan\theta = -1 \quad (\cos\theta \neq 0)$ ☑

$\therefore \theta = 180 - 45, 360 - 45$

$\therefore \theta = 135^\circ, 115^\circ$ ☑☑

(*b*) $5x^2 - 3x - 2 = 0$

(*i*) $\alpha + \beta = \dfrac{-(-3)}{5}$

$= \dfrac{3}{5}$ ☑

(*ii*) $\alpha\beta = \dfrac{-2}{5}$ ☑

(*iii*) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ☑

$= \left(\dfrac{3}{5}\right)^2 - 2\left(\dfrac{-2}{5}\right)$

$= \dfrac{29}{25}$ ☑

(*c*) (*i*) since $PQRS$ is a rhombus then $PQ = AP = SP \quad \therefore \Delta APS$ *is isosceles* ☑

if $\angle SAP = x^\circ$ *then* $\angle ASP = x^\circ$

$\therefore \angle SPQ = 2x$ (*exterior angle theorem of a triangle*) ☑

(*ii*) $\angle SPQ = 2x^\circ$

$\angle RQP = 180 - 2x$ (*co – interior angles as* $SP \parallel RQ$) ☑

since $PQ = QR = QC$ *then* ΔQRC *is an isoceles* Δ

$\therefore \angle QRC = \angle RCQ = 90 - x$ ☑

since $\angle SAP = x^\circ$ *and* $\angle RCQ = 90 - x$

then $\Delta ABC = 180 - (90 - x) - x = 90^\circ$ (*angle sum of a* Δ) ☑

Question 6

(a) *Roots are real and distinct if*

$\Delta > 0$ ☑

$k^2 - 16 > 0$ ☑

$\therefore (k-4)(k+4) > 0$

$\therefore k > 0 \text{ or } k < -4$ ☑

(b) (i) *given* $1800 = Ae^{k}$ ....(1)

and $2400 = Ae^{2k}$ ....(2)

$\therefore (2) \div (1)$ *gives*

$e^{k} = \frac{4}{3} \;\therefore k = \ln\left(\frac{4}{3}\right)$ ☑

$\therefore Ae^{\ln\left(\frac{4}{3}\right)} = 1800$

$\therefore A = \frac{1800}{4/3} = 1350$ ☑

(ii) *let* $N = 4000$

$\therefore 1350e^{\ln\left(\frac{4}{3}\right)t} = 4000$

$\ln\left(\frac{4}{3}\right)t = \ln\left(\frac{4000}{1350}\right)$

$\therefore t \approx 3.776$ *years* ☑

October 2014 ☑

(iii) *Now* $N = 1350e^{\ln\left(\frac{4}{3}\right)t}$

$\therefore \frac{dN}{dt} = 1350\ln\left(\frac{4}{3}\right)e^{\ln\left(\frac{4}{3}\right)t}$ ☑

let $t = 4$ $\therefore \frac{dN}{dt} \approx 1227$ *million individuals / year* ☑

(c) Using Simpson's Rule with 5 function values

x	**1**	**2**	**3**	**4**	**5**
ln(x+1)	**ln2**	**ln3**	**ln4**	**ln5**	**ln6**

☑

$$\therefore\ Area \approx \frac{1}{3}\big((\ln 2 + \ln 6) + 4(\ln 3 + \ln 5) + 2\ln 4\big) = 5.36$$

$$= 5.36\ u^2$$ ☑

$$or\ Area \approx \frac{3-1}{6}\big[\ln 2 + 4\ln 3 + \ln 4\big] + \frac{5-3}{6}\big[\ln 4 + 4\ln 5 + \ln 6\big]$$

$$= 5.36\ u^2$$ ☑

Question 7

(a) $\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^n = 5\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right)^3 + \ldots\ldots\ldots\ldots\ldots$ ☑

as $S_\infty = \frac{a}{1-r}$

$$S_\infty = \frac{\frac{5}{2}}{1-\frac{1}{2}} = \frac{5}{2-1} = 5$$ ☑

(b) (i) $f(x) = \frac{x^3}{3} + x - 3$

$f'(x) = x^2 + 1$ ☑

let $f'(x) = 0$

as $x^2 \neq -1$ ☑

$f'(x) \neq 0$ $\therefore$ *no stationary points.*

(ii) $f''(x) = 2x$

concave up when $f''(x) > 0$

$\therefore$ *when* $x > 0$ ☑

(c) (i) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$ $\quad \therefore Gradient = \frac{1}{e}$ $\quad when\ x = e.$

$\therefore \tan gent\ is\ y - 1 = \frac{1}{e}(x - e) \quad \therefore y = \frac{1}{e}x$ ☑

$(0,0)$ *satisfies this* ☑

(ii) $\frac{d}{dx}\left(x \ln x - x\right) = \ln x + x \times \frac{1}{x} - 1$ ☑

$= \ln x + 1 - 1$ ☑

$= \ln x$

(iii) $Area = Area\ of\ triangle \ - \int_1^e \ln x\, dx$

$= \frac{1}{2} \times e \times 1 - \left[x \ln x - x\right]_1^e$ ☑

$= \frac{e}{2} - (e \times 1 - e + 1)$ ☑

$= \frac{e}{2} - 1$ ☑

Question 8

(*a*) (*i*) *let* $\frac{dx}{dt}=e^{3t-2}$

$\therefore \frac{d^2x}{dt^2}=3e^{3t-2}$

when $t=1$ $\frac{d^2x}{dt^2}=3e$ ☑

(*ii*) *let* $e^{3t-2}=22.3$

$\therefore 3t-2=\ln 22.3$ ☑

$t=\frac{2+\ln 22.3}{3}$

$\therefore t=1.7\sec$ ☑

(*iii*) *dist in* 1*st* sec $=\int_0^1 e^{3t-2}dt$

$=\frac{1}{3}\left[e^{3t-2}\right]_0^1$ ☑

$=\frac{1}{3}\left[e-e^{-2}\right]$

$=0.861m$ ☑

b) The graph of $y = \sin 2x$ from $0\le x\le\pi$ is shown below.

let $\int_0^k \sin 2x\,dx=0.85$

$-\frac{1}{2}\left[\cos 2x\right]_0^k=0.85$ ☑

$\cos 2k-1=-1.7$

$\cos 2k=-0.7$ ☑

$2k=\pi-\cos^{-1}0.7$

$k=1.17$ ☑

(correct to two decimal places)

(*c*) (*i*) $P(x,y)$ moves in such a way that PA is perpendicular to PB,

$\therefore \frac{y-4}{x} \times\frac{y}{x-2}=-1$ ☑

$\therefore y^2-4y=-x^2-2x$

$\therefore x^2+2x+y^2-4y=0$ ☑

(*ii*) $(x+1)^2+(y-2)^2=5$

centre $(-1,2)$ *and radius* $=\sqrt{5}$ ☑☑

Question 9

(a) $y = 3x^2 - 12$

$\therefore x^2 = \frac{1}{3}(y+12)$ ☑

$\therefore Volume = \pi\int_0^6 x^2 dy$

$V = \pi\int_0^6 \frac{1}{3}(y+12)dy$ ☑

$V = \frac{\pi}{3}\left[\frac{y^2}{2} + 12y\right]_0^6$ ☑

$V = \frac{\pi}{3}(18+72)$

$V = 30\pi$ *cubic units* ☑

(b) (i) $Arc\ length = r\theta = 15 \times \frac{\pi}{6} = \frac{5\pi}{2}\ m$ ☑

(ii) $Area\ of\ sector = \frac{1}{2}r^2\theta = \frac{225\pi}{12}\ m^2$ ☑

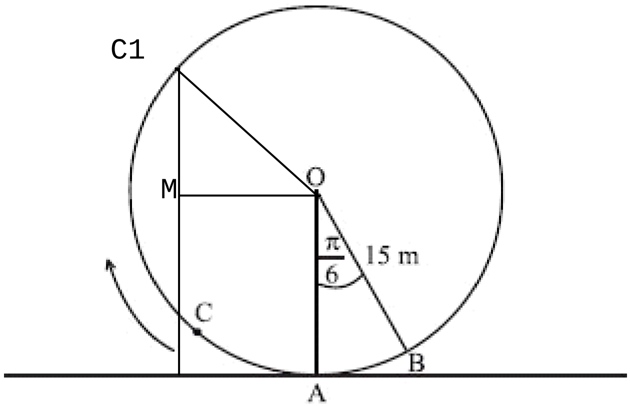
(iii)

$In\ \Delta C1OM$

$\sin\frac{\pi}{6} = \frac{MC1}{15}$

$\therefore MC1 = 15 \times \frac{1}{2} = \frac{15}{2}$ ☑

$\therefore Height = 15 + \frac{15}{2} = 22.5m$ ☑



(iv) $h(t) = 15 - 15\cos(2t + \frac{\pi}{4})$

$h'(t) = 30\sin(2t + \frac{\pi}{4})$ ☑

$h''(t) = 60\cos(2t + \frac{\pi}{4})$

let $h''(t) = 0$, *height changes most rapidly when acceleration is zero.*

$60\cos(2t + \frac{\pi}{4}) = 0$ ☑

$2t + \frac{\pi}{4} = \frac{\pi}{2}$

$\therefore\quad t = \frac{\pi}{8}$ *seconds.* ☑

Question 10

(a) *Let* $r = 1.015$ *and* $\$A_n$ *be the amount owing at the end of the nth month.*

(i)

$$\therefore A_1 = 35000 - M$$
$$A_2 = 35000 - 2M$$
$$A3 = 35000 - 3M$$
$$\therefore A_6 = 35000 - 6M. \quad \checkmark$$

(ii) $A_7 = (35000 - 6M)1.015 - M \quad \checkmark$

$$A_8 = \left((35000 - 6M)1.015 - M\right)1.015 - M$$
$$\therefore A_8 = (35000 - 6M)1.015^2 - M1.015 - M \quad \checkmark$$
$$\therefore \quad A_8 = (35000 - 6M)1.015^2 - M(1.015 + 1).$$

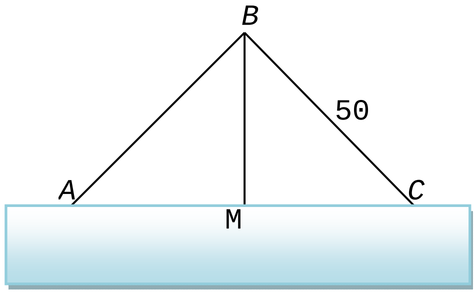
(iii) $A_9 = (35000 - 6M)1.015^3 - M(1.015 + 1)1.015 - M$

$$A_9 = (35000 - 6M)1.015^3 - M(1.015^2 + 1.015 + 1)$$
$$\therefore A_{60} = (35000 - 6M)1.015^{54} - M(1 + 1.015 + 1.015^2 + \dots\dots\dots + 1.015^{53}) \quad \checkmark$$
$$\therefore A_{60} = (35000 - 6M)1.015^{54} - M\left(\frac{1.015^{54} - 1}{1.015 - 1}\right)$$
$$A_{60} = (35000 - 6M)1.015^{54} - M\left(\frac{1.015^{54} - 1}{0.015}\right). \quad \checkmark$$

(iv) *let* $A_{60} = 0$

$$(35000 - 6M)1.015^{54} - M\left(\frac{1.015^{54} - 1}{0.015}\right) = 0$$
$$M\left(\frac{1.015^{54} - 1}{0.015}\right) = 35000 \times 1.015^{54} - 6 \times 1.015^{54} M$$
$$M\left(\frac{1.015^{54} - 1}{0.015}\right) + 6 \times 1.015^{54} M = 35000 \times 1.015^{54}$$
$$M\left(\frac{1.015^{54} - 1}{0.015} + 6 \times 1.015^{54}\right) = 35000 \times 1.015^{54}$$
$$M = 35000 \times 1.015^{54} \div \left(\frac{1.015^{54} - 1}{0.015} + 6 \times 1.015^{54}\right) \quad \checkmark$$
$$M = \$817.17 \quad (\textit{nearest cent}) \quad \checkmark$$

(b)



(i) $as \triangle ABC\ is\ isosceles.(AB = BC)$

$MC = x(\triangle BMC \equiv \triangle BMA)$

$U\sin g\ Pythagoras$

$BM^2 = 2500 - x^2$

$\therefore BM = \sqrt{2500 - x^2}$ ☑

$\therefore area\ of\ \triangle ABC = \frac{1}{2} \times AC \times BM$

$\therefore area\ of\ \triangle ABC = \frac{1}{2} \times 2x \times \sqrt{2500 - x^2}$ ☑

$\therefore area\ of\ \triangle ABC = x\sqrt{2500 - x^2}$

(ii) $Now\ A(x) = x\left(2500 - x^2\right)^{\frac{1}{2}}$

$$A'(x) = x \times \frac{1}{2}\left(2500 - x^2\right)^{-\frac{1}{2}} \times (-2x) + \left(2500 - x^2\right)^{\frac{1}{2}} \times 1$$

$$A'(x) = \frac{-x^2}{\sqrt{2500 - x^2}} + \sqrt{2500 - x^2}$$ ☑

$$A'(x) = \frac{-x^2 + 2500 - x^2}{\sqrt{2500 - x^2}}$$

$$A'(x) = \frac{2500 - 2x^2}{\sqrt{2500 - x^2}}$$

$let\ A'(x) = 0 \quad \therefore 2500 - 2x^2 = 0$

$\therefore x = \sqrt{1250}\ (x > 0)$

$\therefore x = 25\sqrt{2}$ ☑

$First\ derivative\ test: A'(25\sqrt{2} - 0.2) > 0$

$A'(25\sqrt{2} + 0.2) < 0 \quad \therefore Max\ when\ x = 25\sqrt{2}.$ ☑

$\therefore Max\ area\ occurs\ when\ AC = 50\sqrt{2}$

$AC = 71m\ (nearest\ m)$