NORMANHURST BOYS HIGH SCHOOL

NEW SOUTH WALES

Student Number.......................

Class Teacher...........................

2011
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) Expand and simplify $(2x-3y)^2 - 5x(x-2y)$. **2**

b) Express $2.4\dot{0}\dot{5}$ as a mixed number in simplest form. **2**

c) Express $6^{-\frac{2}{5}}$ as a surd, with no fractional or negative indices. **1**

d) Find **2**

$$\lim_{x\to 3}\frac{x^2+5x-84}{x-7}$$

e) Find the exact solutions of $2x^2 - x - 9 = 0$. **2**

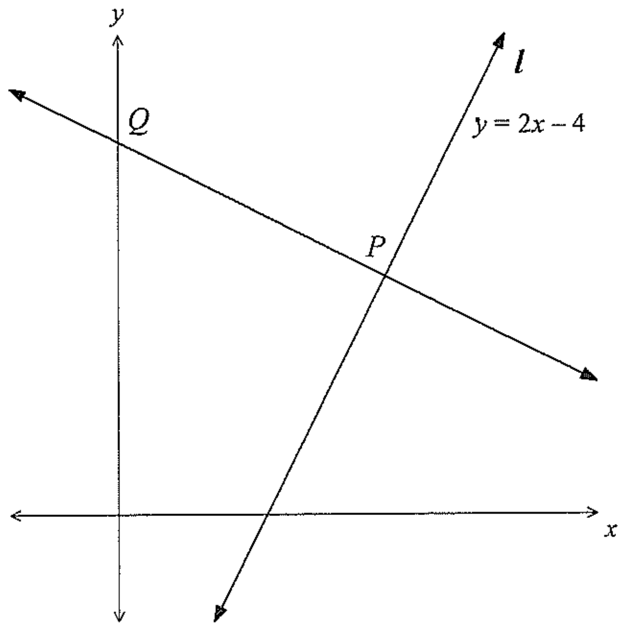
f) Factorise $6x^2 - 3xy - 4xz + 2yz$. **2**

g) Evaluate $\log_5\left(\frac{1}{25}\right)$. **1**

End of Question 1

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) The sketch below shows the line PQ and the line (l) $y = 2x - 4$, which is perpendicular to PQ.



(i) Show that the point R (3, 2) lies on the line l. **1**

(ii) Q is the point (0, 5). Find the midpoint of QR. **1**

(iii) Find the equation of the line PQ. **2**

(iv) Find the gradient of QR. **1**

(v) Find the distance QR in simplest surd form. **2**

(vi) Find the distance PQ. **2**

b) Find

(i) $$\int \frac{x^2}{x^3 + 1}\, dx$$ **1**

(ii) $$\int_1^2 \frac{x^3 + 1}{x^2}\, dx$$ **2**

End of Question 2

Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) Find :

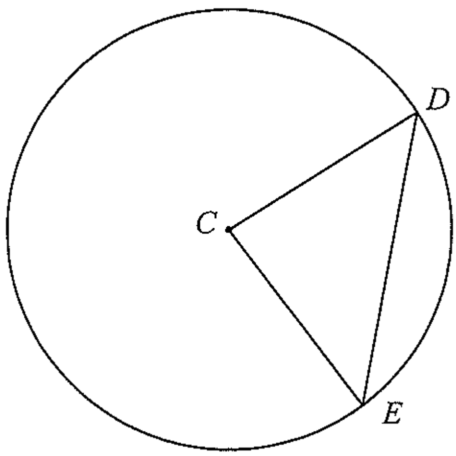
(i) $\frac{dy}{dx}$ when $y = \frac{2}{3\sqrt{x^3}}$. **1**

(ii) $\frac{d}{dx}(\cos^2 x)$ **1**

(iii) the derivative of $\frac{e^{2x}+4}{x^3}$. **2**

(iv) $f'(x)$ if $f(x) = (x^3 - 2x) \cdot \ln(x)$. **2**

b) A circle has centre C and radius 12 cm. The length of the arc DE is 2π cm.



NOT TO SCALE

(i) Find $\angle DCE$ (in radians). **1**

(ii) Find the area of the minor segment cut off by the chord DE. **2**

c) Given that $\frac{d^2y}{dx^2} = \frac{2}{x^2} + 2e^{2x}$ and that, when $x = 1$, $\frac{dy}{dx} = e^2$ and $y = \frac{e^2}{2}$, **3**
find an expression for y in terms of x, with no other unknown variables.

End of Question 3

Question 4 **(12 Marks)** Use a Separate Sheet of paper **Marks**

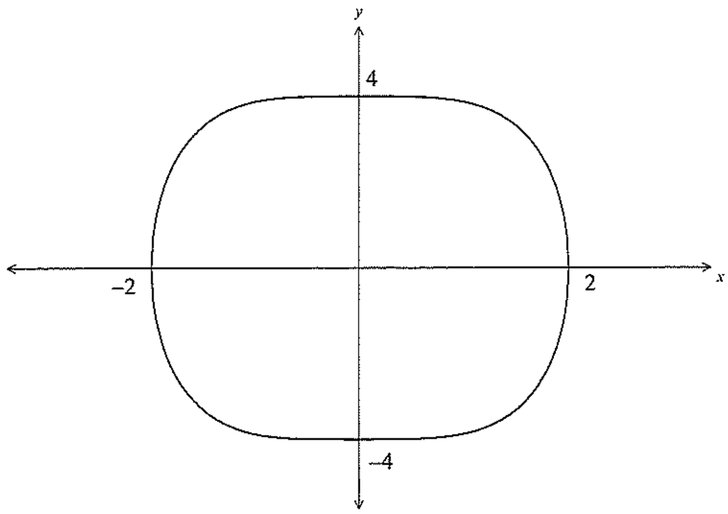
a) Damien runs a marathon, which is a distance of 42 km. He runs the first kilometre in 4 minutes, and from there on, he takes 5 seconds longer to run each successive kilometre. (i.e the 2nd kilometre takes 4 minutes and 5 seconds.)

(i) How long does he take to run the 16th kilometre? **1**

(ii) How long does it take to run the entire marathon? **1**

(iii) How far had he run after 2 hours 36 minutes and 15 seconds? **2**

b) The sketch shows the curve $y^2 + x^4 = 16$. **2**



The area enclosed within the curve is rotated about the x axis. Find the volume of the solid of revolution so formed.

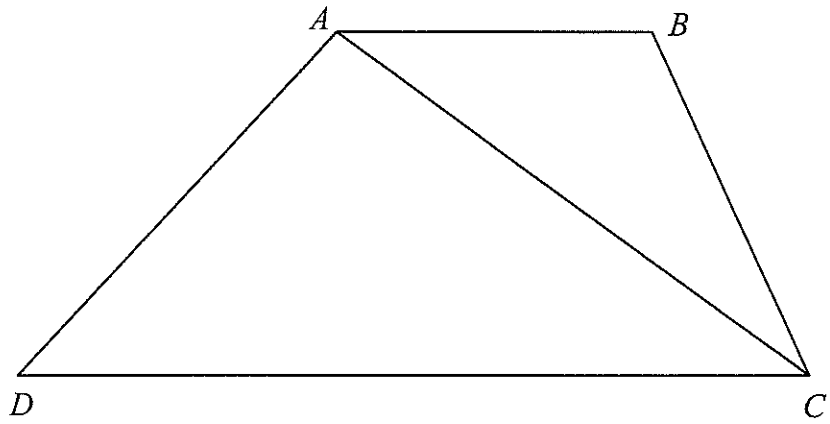
c) Find the equation of the normal to the curve $y = x \ln x$ at the point on the curve where $x = 1$. **3**

d) Draw a sketch showing the graph of $y = 2 + 2\sin 3x$ for $0 \leq x \leq 2\pi$ **3**

End of Question 4

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) The figure $ABCD$ is a trapezium. $\angle ABC = \angle DAC$. AB =12 cm, AD = 20 cm and AC = 24 cm.



(i) Prove that $\Delta ABC \parallel\!| \Delta ADC$. **2**

(ii) Calculate the length of BC. **2**

b) On a single set of axes, show the region where the following inequalities hold simultaneously. **3**

$$x + 2y - 8 < 0$$
$$x^2 + y^2 \leq 16$$
$$y \geq x^2$$

(c) Find the values of m for which the expression below is always positive. **2**
$x^2 + 2mx + (3m - 2)$

(d) The sum of the first 10 terms of the series **3**

$\log_2(\frac{1}{x}) + \log_2(\frac{1}{x^2}) + \log_2(\frac{1}{x^3}) + ...(x > 0)$ is 440. Find the value of x.

End of Question 5

Question 6 **(12 Marks)** Use a Separate Sheet of paper **Marks**

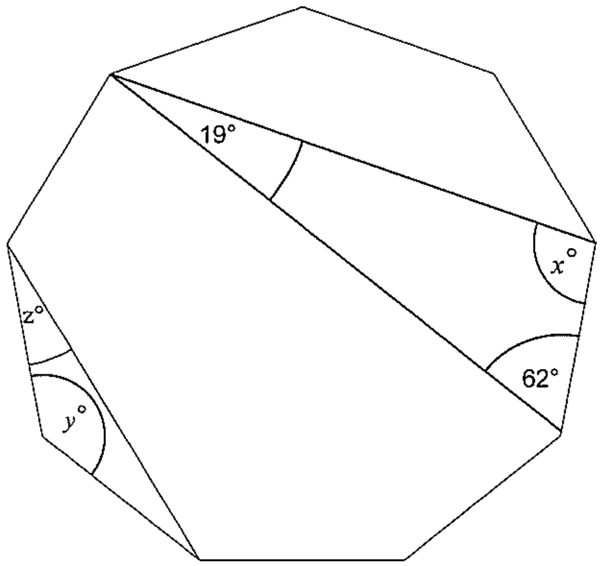
a) Draw a sketch graph of the function $y = 2x^3 - 10x^2 + 6x$, for the domain $-1 \le x \le 4$, showing any turning points and inflections and the end points for the domain. **5**

b) For the curves $y = x^3 + x^2 - 5x$ and $y = x^3 + 4x - 18$

(i) Show that the curves intersect at points whose x values are $x = 3$ and $x = 6$. **2**

(ii) Find the area enclosed between the two curves between these points of intersection. **2**

c) The diagram below shows a regular nonagon (nine sided figure).



(i) Find the value of x. **1**

(ii) Find the value of y (the internal angle of the nonagon). **1**

(iii) Find the value of z. **1**

End of Question 6

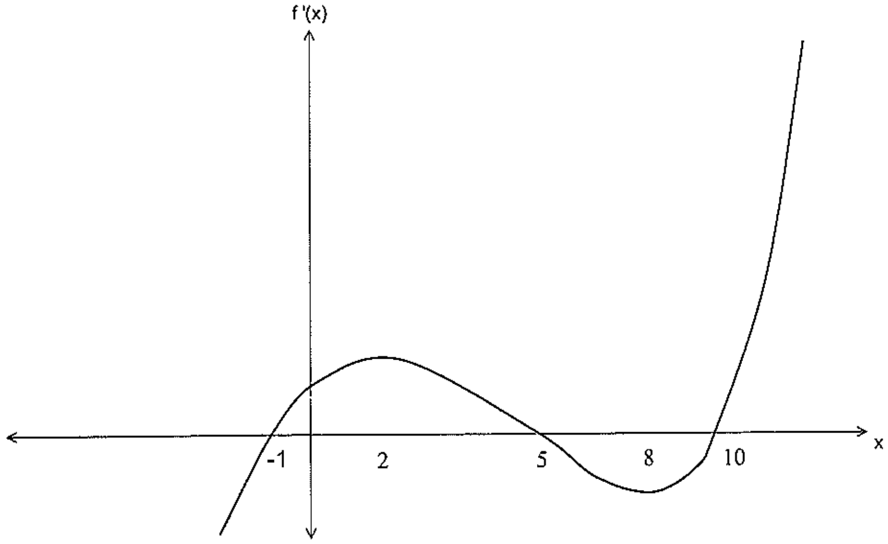
Question 7 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Use the trapezoidal rule with two function values to find an approximation for $\int_1^3 \ln(x+1)\,dx$, correct to 1 decimal place **2**

(b) Find **2**

$$\frac{d(10^x)}{dx}$$

(c) The graph of $y = f'(x)$ is shown below.



(i) Give the x values for the turning points of $y = f(x)$. **1**

(ii) Draw a sketch of $y = f(x)$. **2**

(iii) Draw a sketch of $y = f''(x)$. **2**

(d) (i) If $sin\ (\frac{\pi}{2} - x) = cos\ A$, find A. **1**

(ii) Hence show that $\frac{d(cosx)}{dx} = -sinx$ **2**

End of Question 7

Question 8 **(12 Marks)** Use a Separate Sheet of paper **Marks**

a) For the parabola $x^2 = -16y + 32$.

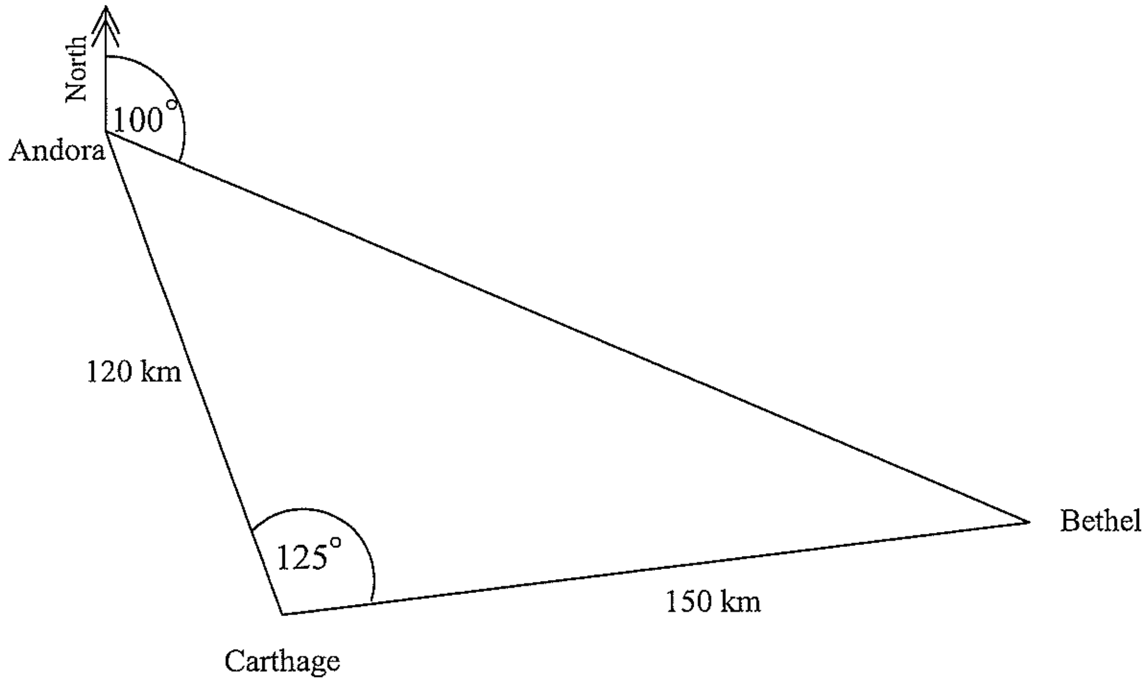
(i) Give the vertex and focus of the parabola. **2**

(ii) Find the equation of the tangent to the parabola at the point (8, -2). **2**

(iii) Show that the focal chord which is parallel to the tangent above has endpoints whose x values are $x = 8 \pm 8\sqrt{2}$. **2**

b) Given that $y = \tan(e^x)$ is continuous for $x \le 0.5$, find an approximation to the area bounded by the curve $y = \tan(e^x)$, the lines $x = -0.4$ and $x = 0.4$ and the x axis, using Simpsons rule with 3 function values. **2**

c) The diagram below shows the relative positions of three towns called Andora, Bethel and Carthage.



(i) Calculate the distance from Andora to Bethel. **2**

(ii) Given the bearing of Bethel from Andora is 100°, find the bearing of Carthage from Andora. **2**

End of Question 8

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

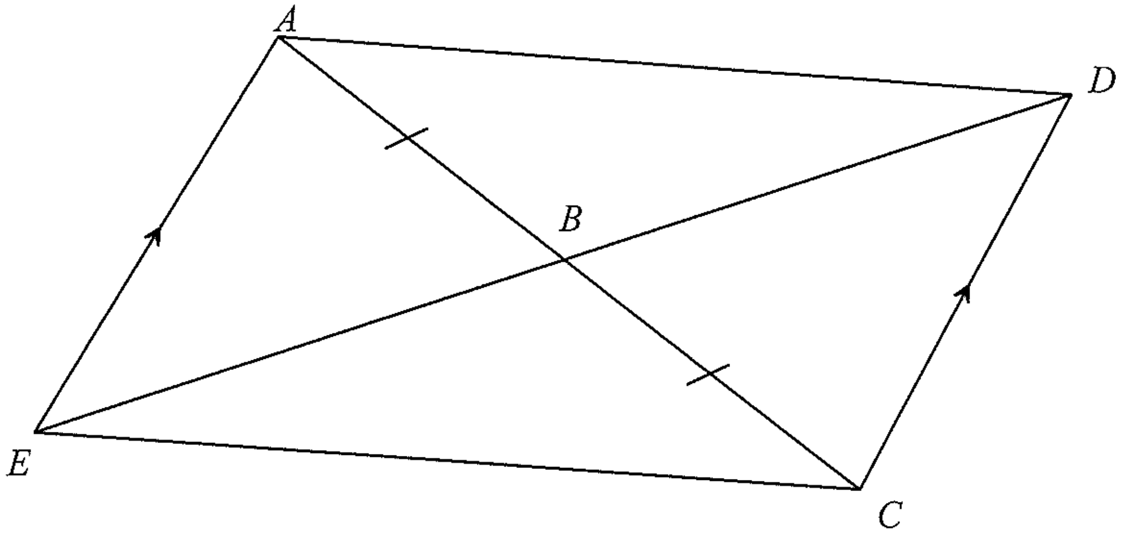
a) Rhonda takes out a loan for \$48 000 on terms where no repayments are made for a year and then equal monthly repayments are made for a further 3 years. Interest is compounded at 1.5% per month on the current monthly balance for the full duration of the loan, (including the time when no repayments are made). The repayment ($\$N$) is deducted each month before the interest is calculated.

(i) Show that the amount still owing, 15 months after the loan is taken out is given by: **2**

$$48000(\cancel{1.105}\ 1.015)^{15} - N(1.015 + 1.015^2 + 1.015^3).$$

(ii) What will be the amount of each repayment ($\$N$)? **3**

b) In the figure below, $AB = BC$ and $AE \parallel DC$. **4**



Prove that $\Delta ABE \equiv \Delta CBD$ and hence that $AD = CE$.

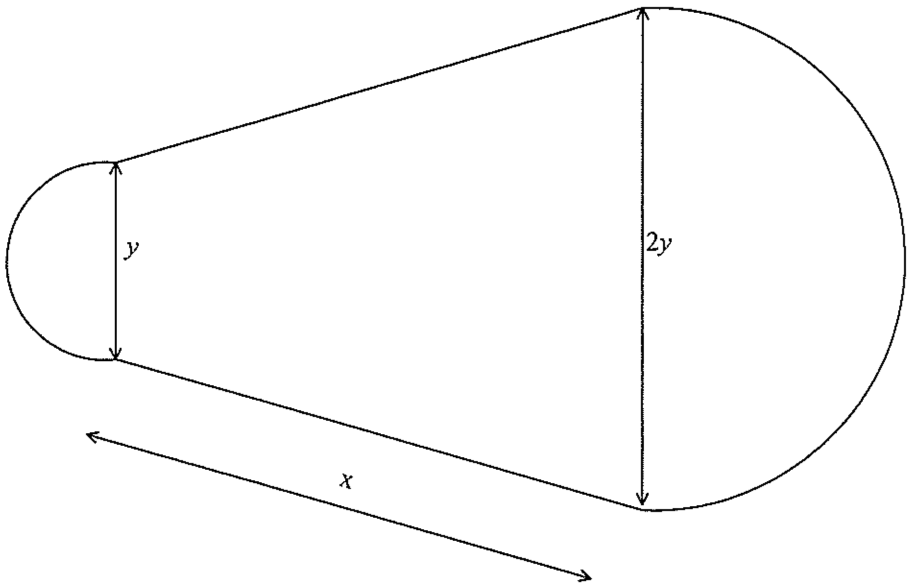
c) Prove that $\sin\theta + 1 + \cos\theta \cot\theta - \operatorname{cosec}\theta = 1$ **3**

End of Question 9

Question 10 (12 Marks) Use a Separate Sheet of paper **Marks**

a) Solve $3x^2 + 14x - 24 \leq 0$. **2**

b) The track for a race is in the shape shown, with two semicircular curves, one whose diameter is twice that of the other. It also has two straights which are equal in length. The total length of the track is 400 km.



(i) Using x km to represent the length of the straight and y km for the diameter of the smaller semicircle, show that $y = \dfrac{800 - 4x}{3\pi}$. **2**

(ii) The average speed that a vehicle can attain on a lap of the track depends on the length of the straights. Given that the average speed that a certain vehicle can attain on the track is given by: **3**

$$\text{Speed} = 100 - \left(\left(\frac{x}{30}\right)^3 + \frac{\pi}{6}(y) \right)$$

find the length of straight which maximizes the speed of this vehicle.

Question 10 continues.

Question 10 continued. **Marks**

(c) On Joshua's 20th birthday he decides to start investing money for his retirement. He plans to invest \$30 at the end of each month for the next 40 years so that he can retire on his 60th birthday. The interest rate remains at 8% per annum over the 40 years and he will be paid interest monthly.

(i) Show that the amount he can expect his investment to grow to is given by the expression **2**

$30 + 30(1.2) + 30(1.2)^2 + 30(1.2)^3 + \ldots\ldots\ldots\ldots\ldots 30(1.2)^{479}$

(ii) After 20 years he decides to increase his monthly investment to \$60. Find the total value of his investments after 40 years. **3**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

NBHS

2011
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

Question 1	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
a)	$(2x-3y)^2-5x(x-2y)=4x^2-12xy+9y^2-5x^2+10xy$ $=9y^2-x^2-2xy$	2	1 mark for expanding 1 mark for collecting like terms
b)	$x=2.4\dot{0}\dot{5}$ $100x=240.5\dot{0}\dot{5}$ $99x=238.1$ $990x=2381$ $x=\frac{2381}{990}$ $x=2\ \frac{401}{990}$	2	1 for working either as shown or using [illegible] 1 for evaluating answer
c)	$6^{-\frac{2}{3}}=\frac{1}{6^{\frac{2}{3}}}$ $=\frac{1}{\sqrt[3]{6}^{\,2}}$ $=\frac{1}{\sqrt[3]{36}}$	1	1 mark for either of last two lines
d)	$\lim_{x\to 3}\frac{x^2+5x-84}{x-7}=\lim_{x\to 3}\frac{(x+12)(x-7)}{(x-7)}$ $=\lim_{x\to 3}(x+12)$ $=3+12$ $=15$ OR Substitution	2	1 for factorisation 1 for answer
e)	$2x^2-x=9$ $2x^2-x-9=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{1\pm\sqrt{1-4(2)(-9)}}{2(2)}$ $=\frac{1\pm\sqrt{73}}{4}$	2	1 for formula or completion of square 1 for result Exact Solution

Question 1	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
f)	$6x^2-3xy-4xz+2yz=3x(2x-y)-2z(2x-y)$ $=(2x-y)(3x-2z)$	2	1 for partial factors 1 for completing result
g)	$\log_5\left(\frac{1}{25}\right)=\log_5(5^{-2})$ $=-2\log_5(5)$ $=-2$	1	1 for answer
		/12	

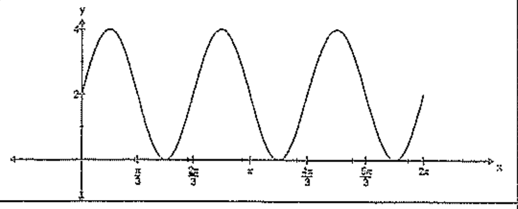
Question 2	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
a) (i)	$y=2x-4$ Substitute the point $R(3,2)$ RHS $=2(3)-4$ $=6-4$ $=2=$ LHS $\therefore R$ lies on $y=2x-4$	1	
a) (ii)	Midpoint of the interval joining $Q(0,5)$ and $R(3,2)$ Midpoint $=\left(\frac{0+3}{2},\ \frac{5+2}{2}\right)$ $=\left(\frac{3}{2},\frac{7}{2}\right)$ $=\left(1\frac{1}{2},\ 3\frac{1}{2}\right)$	1	
a) (iii)	PQ is perpendicular to $y=2x-4$ which has gradient $=2$ $\therefore$ gradient of $PQ=-\frac{1}{2}$ y Intercept $=5$ $\therefore$ Equation is $y=-\frac{1}{2}x+5$ [$x+2y-10=0$]	2	1 for gradient correct 1 for equation using gradient
a) (iv)	Gradient of the interval joining $Q(0,5)$ and $R(3,2)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{2-5}{3-0}$ $=-\frac{3}{3}$ $=-1$	1	
a) (v)	Length of the interval joining $Q(0,5)$ and $R(3,2)$ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(3-0)^2+(2-5)^2}$ $=\sqrt{9+9}$ $=\sqrt{18}$ $=3\sqrt{2}$ Simplest form	2	1 for distance 1 for simplifying surd

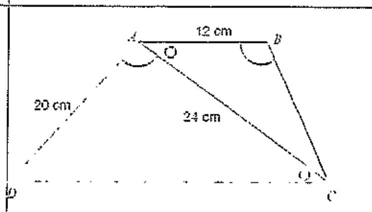
Question 2	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (vi)	Perpendicular distance from (0, 5) to $y = 2x - 4$ i.e. $2x - y - 4 = 0$ Perp dist = $\left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{2(0) - (5) - 4}{\sqrt{2^2 + (-1)^2}} \right\rvert$ $= \left\lvert \frac{-9}{\sqrt{5}} \right\rvert$ $= \frac{9}{\sqrt{5}} = \frac{9\sqrt{5}}{5}$ or $\sqrt{\frac{81}{5}}$ or [illegible]	2	1 for substitution in formula 1 for answer in either surd form
b) (i)	$\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx$ $= \frac{1}{3}\ln(x^3+1) + C$	1	
b) (ii)	$\int_1^2 \frac{x^3+1}{x^2}\,dx = \int_1^2 x + x^{-2}\,dx$ $= \left[\frac{x^2}{2} + \frac{x^{-1}}{-1}\right]_1^2$ $= \left(\frac{4}{2} - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right)$ $= 1\frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 2$ correct answer	2	1 mark for integral 1 mark for evaluation of integral
		/12	

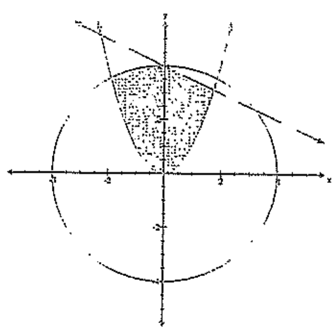
Question 3	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	$y = \frac{2}{3\sqrt{x^3}} = \frac{2}{3}x^{-\frac{3}{2}}$ $\frac{dy}{dx} = \frac{2}{3} \times \left(-\frac{3}{2}\right)x^{-\frac{5}{2}}$ $= -x^{-\frac{5}{2}}$ $= \frac{-1}{\sqrt{x^5}}$	1	Accept either of the final 2 answers shown for 1 mark
a) (ii)	$\frac{d}{dx}(\cos^2 x) = 2\cos x(-\sin x)$ $= -2\sin x\cos x$ $= -\sin 2x$	1	Accept either answer
a) (iii)	$\frac{d}{dx}\left(\frac{e^{2x}+4}{x^3}\right) = \frac{(x^3)(2e^{2x}) - (e^{2x}+4)(3x^2)}{x^6}$ $= \frac{2xe^{2x} - 3e^{2x} - 12}{x^4}$	2	1 mark for correctly applying product rule or quotient rule or equivalent 1 mark for simplifying
a) (iv)	$f(x) = (x^3 - 2x).\ln(x)$ $f'(x) = (x^3 - 2x).\frac{1}{x} + (3x^2 - 2)\ln(x)$ $= x^2 - 2 + (3x^2 - 2)\ln(x)$	2 04/6	
b) (i)	Arc length $= r\theta = 2\pi$ cm $12\theta = 2\pi$ $\theta = \frac{2\pi}{12} = \frac{\pi}{6}$	1	accept 30° reluctantly

Question 3	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
b) (ii)	Area of sector $= \frac{1}{2}r^2\theta$ $= \frac{1}{2}(12^2)\left(\frac{\pi}{6}\right)$ $= 12\pi\ \text{cm}^2$ Area of $\Delta DCE = \frac{1}{2}e.d.\sin C$ $= \frac{1}{2}(12^2)\sin\left(\frac{\pi}{6}\right)$ $= \frac{1}{2}.\ 144.\ \frac{1}{2}\ \text{cm}^2$ $= 36\ \text{cm}^2$ Area of segment $= 12\pi - 36\ \text{cm}^2$	2 02/3	1 mark if either sector or triangle area calculated correctly $A = \frac{1}{2}r^2(\theta - \sin\theta)$ $= \frac{1}{2}.12^2(\frac{\pi}{6} - \sin\frac{\pi}{6})$ $= 72\left(\frac{\pi}{6} - \frac{1}{2}\right)$ $= 12\pi - 36$ 2 marks if the area of the segment is found correctly
c)	$\frac{d^2y}{dx^2} = \frac{2}{x^2} + 2e^{2x}$ $= 2x^{-2} + 2e^{2x}$ $\frac{dy}{dx} = -2x^{-1} + e^{2x} + C_1$ When $x = 1, \frac{dy}{dx} = e^2$ $e^2 = -2 + e^2 + C_1$ $C_1 = 2$ $\frac{dy}{dx} = -2x^{-1} + e^{2x} + 2$ $y = -2\ln x + \frac{e^{2x}}{2} + 2x + C_2$ When $x = 1, y = \frac{e^2}{2}$ $\frac{e^2}{2} = -2\ln(1) + \frac{e^2}{2} + 2(1) + C_2$ $\frac{e^2}{2} = 0 + \frac{e^2}{2} + 2 + C_2$ $C_2 = -2$ $y = -2\ln x + \frac{e^{2x}}{2} + 2x - 2$	3 04/3	1 mark for integration to get $\frac{dy}{dx}$, disregarding constants. 1 mark for C_1 1 mark for correct y, including constant.
		/12	

Question 4	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	This is an Arithmetic series with $a = 4 \times 60 = 240$, $d = 5$ Want the 16th term $u_n = a + (n-1)d$ $= 240 + 15 \times 5$ $= 315$ sec $= 5$ min and 15 sec	1	
a) (ii)	Want the sum of the 42 km. $s_n = \frac{n}{2}(2a + (n-1)d)$ $= 21(480 + 41 \times 5)$ $= 14\,385$ sec $= 3$ hrs 59 min 45 sec	1	
a) (iii)	2 hours 36 minutes and 15 seconds = 9375 sec $s_n = \frac{n}{2}(2a + (n-1)d)$ $9375 = \frac{n}{2}(480 + 5(n-1))$ $18750 = n(475 + 5n)$ $5n^2 + 475n - 18750 = 0$ $n^2 + 95n - 3750 = 0$ $n = \frac{-95 \pm \sqrt{9025 + 15000}}{2}$ $= \frac{-95 \pm 155}{2}$ $= 30$ or -125 Ignore the negative, so he has travelled 30 km.	2 01/4	1 for sub into the sum formula 1 for solving the equation

Question 4	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
b)	$y^2 + x^2 = 16.$ Limits are from -2 to 2 $V = \pi \int_{-2}^{2} y^2 dx$ $= \pi \int_{-2}^{2} 16 - x^2\, dx$ or $2\pi \int_{0}^{2} (16 - x^2)\, dx$ $= \pi \left[16x - \frac{x^3}{3} \right]_{-2}^{2}$ $= \pi \left[\left(32 - \frac{32}{3} \right) - \left(-32 + \frac{32}{3} \right) \right]$ $= \frac{256\pi}{3} \approx 160.8\ units^3$	2	1 for integral 1 for numerical value
c)	$y = x \ln x$ $\frac{dy}{dx} = x.\ \frac{1}{x} + \ln x .1$ $= 1 + \ln x$ when $x = 1$ $y = 1.\ \ln 1 = 1 \times 0 = 0$ $\frac{dy}{dx} = 1 + \ln 1$ $= 1$ Gradient of tangent $= 1$ Gradient of normal $= -1$ Normal through (1, 0) with gradient -1 $y - 0 = (-1)(x - 1)$ $y = -x + 1$	3 05/5	1 for derivative 1 for gradient of normal 1 for equation
d)	$y = 2 + 2\sin 3x$ 	3 02/3	3 marks with 1 deducted if period, amplitude or position is incorrect.
		/12	

Question 5	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	 In ΔABC and ΔDAC $\angle ABC = \angle DAC$ (given) $\angle BAC = \angle ACD$ (alternate angles on ‖ lines) $\therefore \angle ACB = \angle ADC$ (angle sum of triangle) $\therefore \Delta ABC \parallel\!\mid \Delta DAC$ (corresponding angles equal)	2	1 mark for including the correct angle equivalences 1 mark for reasons for the above
a) (ii)	$\frac{BC}{AD} = \frac{AB}{AC}$ (corresponding sides of similar triangles are in the same ratio) $\frac{BC}{20} = \frac{12}{24} = \frac{1}{2}$ $BC = 20 \times \frac{1}{2}$ $BC = 10$ cm	2	1 for correct initial proportion 1 for answer

Question 5	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
b)		3	1 for each of the inequalities with 1 mark taken off if wrong border used on any of the regions
c)	Coefficient of $x = 1 > 0$ Expression positive if $\Delta = b^2 - 4ac < 0$ ie $4m^2 - 4.1.(3m-2) < 0$ $4m^2 - 12m + 8 < 0$ $m^2 - 3m + 2 < 0$ $(m-2)(m-1) < 0$ Ref. points are m = 1 and m = 2 $\therefore\ 1 < m < 2$	2	1 for correctly substituting into Δ 1 for correct solution
d)	$-\log_2 x - 2\log_2 x - 3\log_2 x - \ldots\ldots = 440$ AP with $a = -\log_2 x$, $d = -\log_2 x$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $440 = \frac{10}{2}[-2.\log_2 x + (10-1)(-\log_2 x)]$ $440 = 5(-11 \log_2 x)$ $88 = -11 \log_2 x$ $8 = -\log_2 x$ $-8 = \log_2 x$ $x = 2^{-8}$	3	1 for recognising AP with correct a and d values 1 for correctly substituting into Sn formula 1 for correct solution
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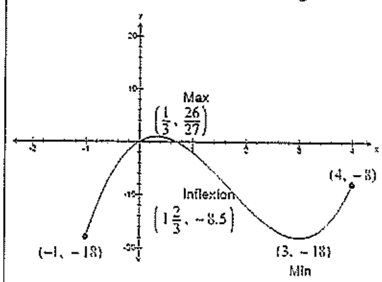
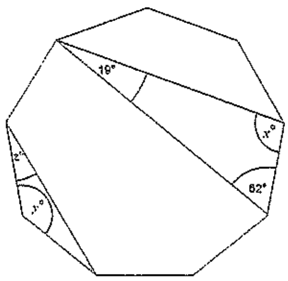
Question 6	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (ii)	$y = 2x^3 - 10x^2 + 6x$ $y' = 6x^2 - 20x + 6$ $= 6x^2 - 18x - 2x + 6$ $= 6x(x-3) - 2(x-3)$ $= 2(3x-1)(x-3)$ $y' = 0$ when $x = \frac{1}{3}$ and $x = 3$ $y'' = 12x - 20$ Test turning points $x = \frac{1}{3},\ y'' = -16$ Local Max $x = 3,\ y'' = 16$ Local Min $y'' = 0$ when $x = 1\frac{2}{3}$ $y = -8.5$ Inflexion since change in concavity. 	5	1 for turning points 1 for classifying 1 for inflexion 1 for diagram 1 for end points
b) (i)	Solve simultaneously $y = x^3 + x^2 - 5x$ and $y = x^3 + 4x - 18$ $x^3 + x^2 - 5x = x^3 + 4x - 18$ $(x^2 - 9x + 18) = 0$ $(x-3)(x-6) = 0$ $x = 3$ or $x = 6$	2	1 for equation 1 for x values

Question 6	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
b) (ii)	Area $= \left\| \int_3^6 x^3 + x^2 - 5x - (x^3 + 4x - 18)\, dx \right\|$ $= \left\| \int_3^6 x^2 - 9x + 18 \; dx \right\|$ $= \left\| \frac{x^3}{3} - \frac{9x^2}{2} + 18x \right]_3^6$ $= \left\| (18) - \left(22\frac{1}{2}\right) \right\|$ $= 4\frac{1}{2}$ sq units	2	1 for correct integration 1 for area
c) (i)	 $x = 180 - (62 + 19)$ $= 99°$	1	
c) (ii)	Angle sum $= (9 - 2) \times 180$ $= 1260$ Internal angle $= \frac{1260}{9}$ $y = 140°$	1	
c) (iii)	$z = \frac{180 - 140}{2}$ $= 20°$	1	
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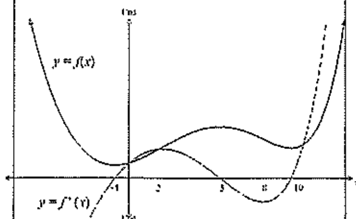
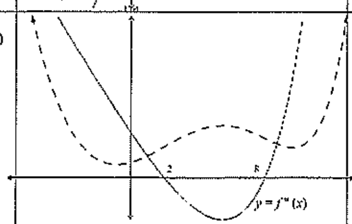
Question 7	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	Let $f(x) = ln(x + 1)$ $f(1) = ln(2)$ $f(3) = ln(4)$ $\int_1^3 \ln(x+1)\, dx = \frac{2}{2}(ln2 + ln4)$ $= 2.079$	2	1 correct substitution into trapezoidal rule 1 for correct solution.
a) (ii)	Let $y = 10^x$ $= e^{x ln10}$ $y' = ln10 e^{x ln10}$ $= ln10(10^x)$	2	1 mark for $y = e^{x ln10}$ 1 mark for correct answer
b) (i)	Turning points occur where $x = -1$, $x = 5$ and $x = 10$	1	

Question 7	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
b) (ii)		2	2 marks total for including turning points and inflexions 1 mark for a sketch with one or two minor errors Position on y axis and x intercepts is not important
b) (iii)		2	2 marks total x intercepts are important at 2 and 8 1 mark for a sketch with one or two minor errors
c) (i)	$A = x$		
c) (ii)	Let $y = \cos x$ $= \sin(\frac{\pi}{2} - x)$ $\frac{dy}{dx} = -\cos(\frac{\pi}{2} - x)$ $= -\sin x$		
		/12	

Question 8	Trial HSC Examination - Mathematics		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	$x^2 = -16y + 32$ $x^2 = -4.(4)(y - 2)$ Vertex (0, 2) focal length 4 and concave down Focus (0, -2)	2	1 each for vertex and focus
a) (ii)	$x^2 = -16y + 32$ $-16y = x^2 - 32$ $y = \frac{x^2}{-16} + 2$ $\frac{dy}{dx} = -\frac{x}{8}$ When $x = 8$, $\frac{dy}{dx} = -1$ Equation is $y + 2 = -1(x - 8)$ $y + 2 = -x + 8$ $y = -x + 6$	2	1 for gradient 1 for equation
a) (iii)	$y + 2 = -1(x - 0)$ $y = -x - 2$ Sub into parabola $x^2 = -16(-x - 2) + 32$ $x^2 - 16x - 64 = 0$ $x = \frac{16 \pm \sqrt{256 + 256}}{2}$ $x = \frac{16 \pm \sqrt{512}}{2}$ $x = \frac{16 \pm 16\sqrt{2}}{2}$ $x = 8 \pm 8\sqrt{2}$	2	1 for solving simultaneo usly 1 for x values
b)	$h = 0.4$ A $= \frac{h}{3}$[(sum of ends) + 4(middle)] $x_1 = -0.4$ $= \frac{0.4}{3}$[($f(-0.4) + f(0.4)$) + $4f(0)$] $= \frac{0.4}{3}$[19.66] $x_2 = 0$ = 2.62 (2 decimal places) $x_3 = 0.4$	2	1 for using correct function values 1 for sub in to formula correctly

Question 8	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
c) (i)	$AB^2 = 120^2 + 150^2 - 2 \cdot 120 \cdot 150 \cdot \cos 125^\circ$ $= 57\,549$ $AB = 239.89$ $= 240\ m$ (nearest metre)	2	1 for using cosine rule 1 for correct calculation
c) (ii)	$\frac{\sin A}{150} = \frac{\sin 125^\circ}{240}$ $\sin A = \frac{150 \sin 125^\circ}{240}$ $= 0.51219$ $A = 31^\circ$ (nearest degree) Bearing $= 100^\circ + 31^\circ$ $= 131^\circ$	2	1 for using Sine rule (or cosine rule) to find A 1 for bearing from that answer
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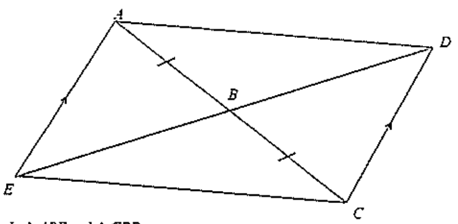
Question 9	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
a) (i)	For the first year interest accumulates with no payments so new balance after 12 is given by: $A_{12} = 48000(1.015)^{12}$ $= 57\ 389.68$ After 13 months $A_{13} = (48000(1.015)^{12} - N) \times 1.015$ $= 48000(1.015)^{13} - 1.015N$ After 14 months $A_{14} = (48000(1.015)^{13} - 1.015N - N) \times 1.015$ $= 48000(1.015)^{14} - (1.015)^2N - 1.015N$ After 15 months $A_{15} = ((48000(1.015)^{14} - (1.015)^2N - 1.015N) - N) \times 1.015$ $= 48000(1.015)^{15} - (1.015)^3N - (1.015)^2N - 1.015N$ $= 48000(1.105)^{15} - N(1.015 + 1.015^2 + 1.015^3)$	2	1 mark for attempt to obtain series by writing previous months 2 marks if completed to required result
a) (ii)	After 48 months $A_{48} = 48000(1.105)^{48} - N(1.015 + 1.015^2 +$ $+ \ldots\ldots 1.015^{47-12} + 1.015^{48-12})$ After 48 months $A_{48} = 48000(1.105)^{48} - N(1.015 + 1.015^2 +$ $+ \ldots\ldots 1.015^{35} + 1.015^{36})$ Loan is repaid after 48 months so $A_{48} = 0$ $48000(1.015)^{48} = N(1.015 + 1.015^2 +$ $+ \ldots\ldots 1.015^{35} + 1.015^{36})$ $1.015 + 1.015^2 + \ldots\ldots 1.015^{35} + 1.015^{36}$ is a geometric series with $a = 1.015$, $r = 1.015$ and $n = 36$ Sum of series $= \frac{a(r^n - 1)}{r - 1}$ $= \frac{1.015(1.015^{36} - 1)}{1.015 - 1}$ $= \frac{1.015(1.015^{36} - 1)}{.015}$ $48000(1.015)^{48} = N\left(\frac{1.015(1.015^{36} - 1)}{.015}\right)$ $N = 48000(1.015)^{48} \times \frac{0.015}{1.015(1.015^{36} - 1)}$ $= \$2044$ (to nearest dollar.)	3	1 mark for obtaining expression for 48 months 1 for obtaining the expression for sum of series 1 for final result

Question 9	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
b)	 In ΔABE and ΔCBD $\angle ABE = \angle CBD$ (verticallyu opposite angles) $\angle AEB = \angle BDC$ (Alternate angles on ∥ lines) $AB = BC$ (given) $\therefore \Delta ABE \equiv \Delta CBD$ (AAS) $\therefore$ $AE = CD$ $\therefore ADCE$ is a paralleogram (A pair of opposite sides parallel and equal) $\therefore$ $AD = CE$ (Other pair of opposite sides of \\gram are equal)	4	3 marks for congruence 1 mark deducted if any of the three steps needed (including reasons), or the conclusion are missing. 1 mark for statement as to why $AD = CE$
c)	$\sin\theta + 1 + \cos\theta \cot\theta - \text{cosec}\theta = \sin\theta + 1 + \cos\theta \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$ $= \frac{\sin^2\theta + \sin\theta + \cos^2\theta - 1}{\sin\theta}$ $= \frac{\sin^2\theta + \cos^2\theta + \sin\theta - 1}{\sin\theta}$ $= \frac{1 + \sin\theta - 1}{\sin\theta}$ $= \frac{\sin\theta}{\sin\theta}$ $= 1$	3	3 marks for full proof 2 marks for partial proof 1 mark if one or two relevant equivalences are stated
		/12	

Question 10	Trial HSC Examination - Mathematics		2011
Part	Solution	Marks	Comment
a)	$3x^2 + 14x - 24 \leq 0$ $3x^2 + 18x - 4x - 24 \leq 0$ $3x(x + 6) - 4(x + 6) \leq 0$ $(3x - 4)(x + 6) \leq 0$ Test between $x = \frac{4}{3}$ and $x = -6$ $x = 0 \quad -24 \leq 0$ True $-6 \leq x \leq \frac{4}{3}$	2	1 for factorising 1 for correct solution

b) (i)	$P = 2x + \frac{\pi y}{2} + \pi y$ $P = 400$ $400 = 2x + \frac{\pi y}{2} + \pi y$ $800 = 4x + \pi y + 2\pi y$ $800 = 4x + 3\pi y$ $y = \frac{800 - 4x}{3\pi}$	2	1 for equation for P 1 for making y the subject
b) (ii)	Speed $= 100 - \left(\left(\frac{x}{30} \right)^3 + \frac{\pi}{6}(y) \right)$ $S = 100 - \left(\frac{x}{30} \right)^3 - \frac{\pi}{6}\left(\frac{800 - 4x}{3\pi} \right)$ $S = 100 - \frac{x^3}{27000} - \frac{800 - 4x}{18}$ $\frac{dS}{dt} = -\frac{3x^2}{27000} + \frac{4}{18}$ $-\frac{3x^2}{27000} + \frac{4}{18} = 0$ $x^2 = \frac{4}{18} \times \frac{27000}{3}$ $= 2000$ $x = \sqrt{2000}$ $x = 44.7$ $\frac{d^2S}{dt^2} = -\frac{6x}{27000} = -\frac{x}{4500}$ when $x = 44.7$ $\frac{d^2S}{dt^2} = -\frac{44.7}{4500} = -0.009777$ $\therefore$ Local Max when $x = 44.7$	3	1 for sub y into Speed equation 1 for finding derivative and setting =0 1 for finding x and showing max

c) (i)	Let A_n = the amount of the investment after n months 40 years = 480 months $A_{480} = 30(1 + 0.007)^{479}$ $= 30(1.007)^{479}$ *1* $A_{479} = 30(1.007)^{478}$ *2* $A_1 = 30$ *3* Total $= 30 + 30(1.007)^1 + 30(1.007)^2 + \ldots\ldots\ldots\ldots 30(1.007)^{479}$ S_1	2	1 mark for correct expression for A_{480} 1 for steps 1,2,3 and S_1
c) (ii)	Total if additional \$30 added $= 30 + 30(1.007) + \ldots\ldots 30(1.007)^{239}$ S_2 Adjusted total $= S_1 + S_2$ $= \frac{30(1.007^{480}-1)}{1.007-1} + \frac{30(1.007^{240}-1)}{1.007-1}$ $= 4285.7(1.007^{480} - 1) + 1500(1.007^{240} - 1)$ $= 4285.7(1.007^{480} - 1 + 1.007^{240} - 1)$ $= 4285.7(1.007^{480} + 1.007^{240} - 2)$ = \$136 236 Or $S_{240} = 30 + 30(1.007)^1 + 30(1.007)^2 + \ldots\ldots\ldots\ldots 30(1.007)^{239}$ $S_{240} = \frac{30(1.007^{240}-1)}{1.007-1}$ $S_{241} = S_{240}(1.007) + 60$ $S_{242} = (S_{240}(1.007) + 60)1.007 + 60$ $= (S_{240}(1.007)^2 + 60(1.007) + 60$ $S_{243} = ((S_{240}(1.007)^2 + 60(1.007) + 60)1.007 + 60$ $= S_{240}(1.007)^3 + 60(1.007)^2 + 60)1.007 + 60$ $S_{480} = (S_{240}(1.007)^{240} + 60(1.007)^{239} \ldots\ldots + 60)1.007 + 60$ $= S_{240}(1.007)^{240} + \frac{60(1.007^{240}-1)}{1.007-1}$ $= (\frac{30(1.007^{240}-1)}{1.007-1})1.007^{240} + \frac{60(1.007^{240}-1)}{1.007-1}$ = \$136 235	3	1 for correct S_2 or equivalent 1 for correct substitution into S_1 or S_2 1 for correct solution
		/12	