NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS

2017 HSC Course Assessment Task 4 (Trial Examination)
Thursday, 3 August 2017

General instructions

- Working time – 3 hours. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt all questions.
- A NESA Reference Sheet is provided.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT NUMBER:... **# BOOKLETS USED:** ....

Class: (please ✓)

- ○ 12MAT3 – Mr Wall
- ○ 12MAT4 – Mr Sekaran
- ○ 12MAT5 – Mrs Gan
- ○ 12MAT6 – Ms Park
- ○ 12MAT7 – Mr Tan

Marker's use only

QUESTION	1-10	11	12	13	14	15	16	Total
MARKS	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks
Attempt Question 1 to 10

Mark your answers on the answer grid provided

1. A deck of cards consists of 5 white and 5 black cards. Two cards are selected at random without replacement. What is the probability of choosing the same colour? **1**

(A) $\frac{1}{25}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$

2. What is 4.29564 correct to three significant figures? **1**

(A) 4.29 (B) 4.295 (C) 4.296 (D) 4.30

3. Which of the following is the derivative of $y = \frac{e^{7x}}{e^{3x}}$? **1**

(A) $4e^{4x}$ (B) e^{4x}

(C) $\frac{7e^{3x}e^{7x}+3e^{3x}e^{7x}}{e^{9x}}$ (D) $\frac{3e^{3x}e^{7x}-7e^{7x}e^{3x}}{e^{9x}}$

4. For what values of a does $ax^2 + 5x + a = 0$ have no solution? **1**

(A) $a > 0$ (B) $a = \frac{5}{2}$

(C) $-\frac{5}{2} < a < \frac{5}{2}$ (D) $a > \frac{5}{2},\ a < -\frac{5}{2}$

5. The graph of $y = f(x)$ passes through the point $(1,4)$ and $f'(x) = 3x^2 - 2$. Which of the following expressions is $f(x)$? **1**

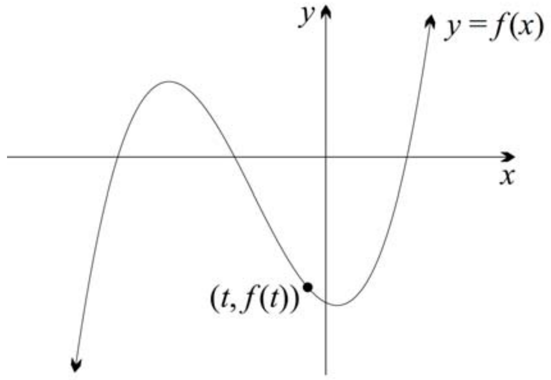
(A) $x^2 - 2x$ (B) $2x - 1$

(C) $x^3 - 2x + 3$ (D) $x^3 - 2x + 5$

6. Which of these is the perpendicular distance of the point $(2, -3)$ to the line $y = 4x - 1$? **1**

(A) $\frac{4}{\sqrt{13}}$ (B) $\frac{10}{\sqrt{17}}$ (C) $\frac{10}{\sqrt{13}}$ (D) $\frac{4}{\sqrt{17}}$

7. The diagram below shows the graph of $y = f(x)$. Which of the following statements is true? **1**



(A) $f'(t) > 0$ and $f''(t) < 0$ (B) $f'(t) > 0$ and $f''(t) > 0$

(C) $f'(t) < 0$ and $f''(t) < 0$ (D) $f'(t) < 0$ and $f''(t) > 0$

8. How many solutions does the equation $|2\sin 3x| = 1$ have for $0 \leq x \leq \pi$? **1**

(A) 4 (B) 3 (C) 6 (D) 2

9. Which expression is a term of the geometric series $4y - 8y^2 + 16y^3 - \ ...$? **1**

(A) $-4096y^{10}$ (B) $-4096y^{11}$

(C) $4096y^{10}$ (D) $4096y^{11}$

10. Which of the following would be the result if $y = \log_e(-x + 5)$ was reflected about the line $y = x$? **1**

(A) $y = -e^x - 5$ (B) $y = e^x + 5$

(C) $y = e^x - 5$ (D) $y = -e^x + 5$

Section II

90 marks
Attempt Question 11 to 16

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Commence a new booklet

(a) Solve $2y^2 - 4y = 0$. **1**

(b) Express $\frac{\sqrt{2}}{2-3\sqrt{2}}$ with a rational denominator. **1**

(c) Solve the equation $\log_2(x-1) - \log_2(x-2) = 2$. **2**

(d) i. Find $\int e^{-5y}\, dy$. **1**

ii. Evaluate $\int_0^{\frac{\pi}{4}} (\sec^2 x - x)\ dx$. **2**

(e) Differentiate with respect to x:

i. $(3\text{x}^2 + 4)^5$ **1**

ii. $\frac{\tan x}{x}$ **2**

iii. $\log_e \sqrt{x^2 - 1}$ **2**

(f) Find the value of m if $y = 2\sin 3x + 4\cos 2x$ and $\frac{d^2y}{dx^2} = m\sin 3x - 4y$. **3**

Question 12 (15 Marks) Commence a new booklet

(a) A game is played in which two coloured dice are thrown once.
The six faces of the red die are numbered 3, 5, 7, 8, 9 and 11.
The six faces of the white die are numbered 1, 2, 4, 6, 10 and 12.
The player wins if the number on the white die is larger than the number on the red die.

i. What is the probability of the player winning a game? **1**

ii. What is the probability that the player wins once in two successive games? **2**

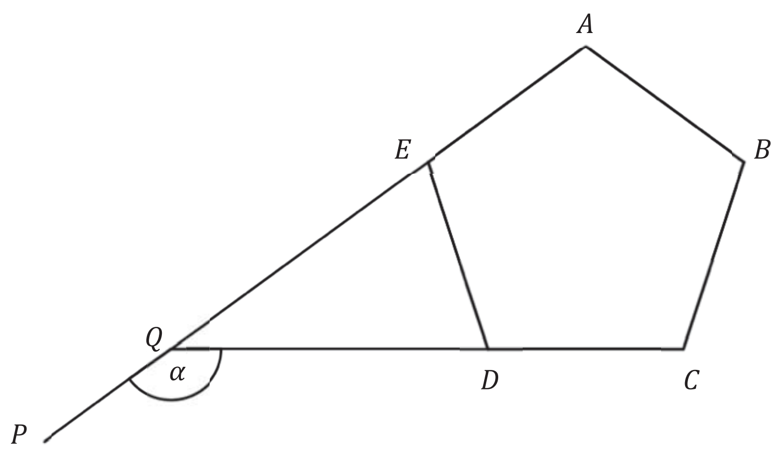
iii. What is the probability that the player wins at least once in two successive games? **1**

(b) The table below shows the values of a function $y = f(x)$ for five values of x. **2**

x	1	1.5	2	2.5	3
$f(x)$	11.2	17.8	9.3	4.1	11.6

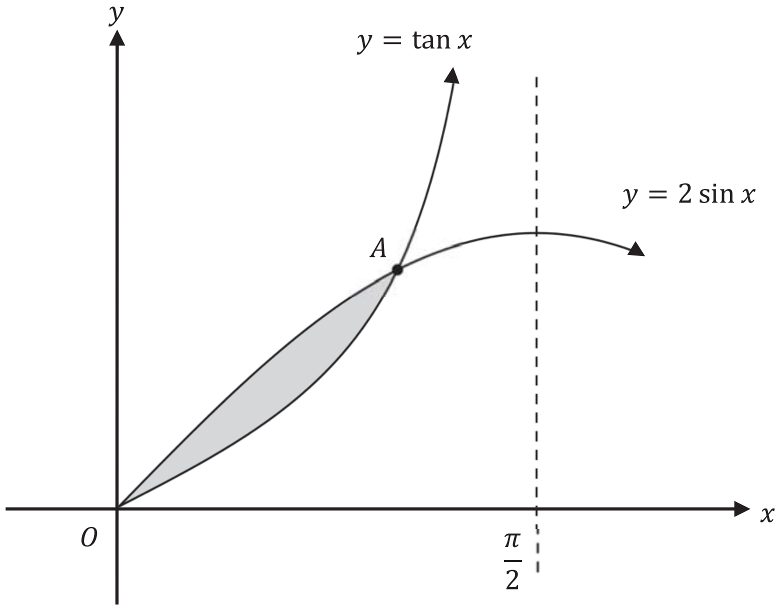
Use the Trapezoidal Rule with these five function values to estimate $\int_1^3 f(x)\ dx$.

(c) $ABCDE$ is a regular pentagon. The points P, Q, E and A are collinear. **2**
The points Q, D and C are also collinear. Find the size of angle α, giving reasons.



Not to scale

(d) The diagram below shows the curves $y = \tan x$ and $y = 2\sin x$.

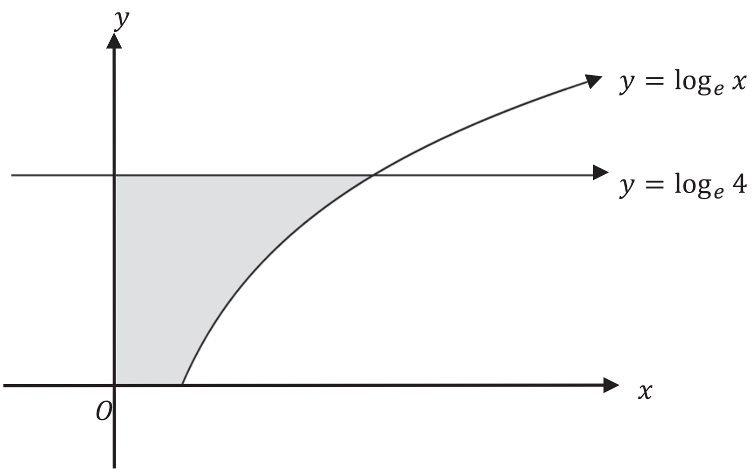


i. Show that the coordinates of the point A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$. **1**

ii. Show that $\frac{d}{dx}\left[\log_e(\cos x)\right] = -\tan x$. **1**

iii. Hence find the area of the shaded region in the diagram. **2**

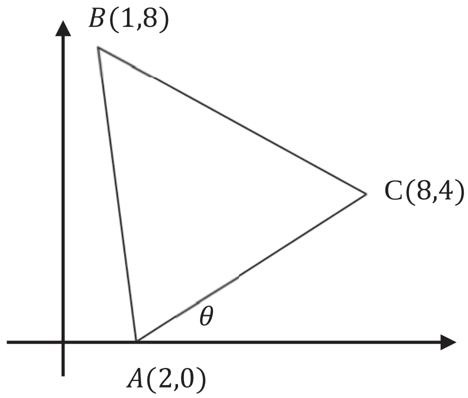
(e) The shaded region bounded by the curve $y = \log_e x$, the x and y-axes and the line $y = \log_e 4$, is rotated about the y-axis. Find the exact volume of the solid of revolution formed. **3**



Question 13 (15 Marks) Commence a new booklet

(a) The points A, B and C have coordinates $(2,0)$, $(1,8)$ and $(8,4)$ respectively.
The angle between the line AC and the x-axis is θ.

Copy or trace the diagram below into your writing booklet.



i. Find the gradient of the line AC. **1**

ii. Calculate the size of the angle θ to the nearest minute. **1**

iii. Find the equation of the line AC. **1**

iv. Find the coordinates of D, the midpoint of AC. **1**

v. Show that AC is perpendicular to BD. **1**

(b) Prove that **3**

$$\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}=2\operatorname{cosec}\theta$$

(c) The quadratic equation $2x^2-3x+8=0$ has roots α and β.

i. Write down the values of $\alpha+\beta$ and $\alpha\beta$. **1**

ii. Hence, find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$. **2**

(d) For the parabola $x^2-4x-8y=20$:

i. Find the coordinates of the vertex by completing the square. **2**

ii. Draw a neat sketch of the parabola, showing the coordinates of the focus and the directrix. **2**

Question 14 (15 Marks) Commence a new booklet

(a) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44.

i. Find the value of the common difference and the value of the first term. **2**

ii. Find the sum of the first 75 terms. **2**

(b) Find the common ratio of a geometric series with a first term of $\frac{1}{2}$ and a limiting sum of 1.5 **2**

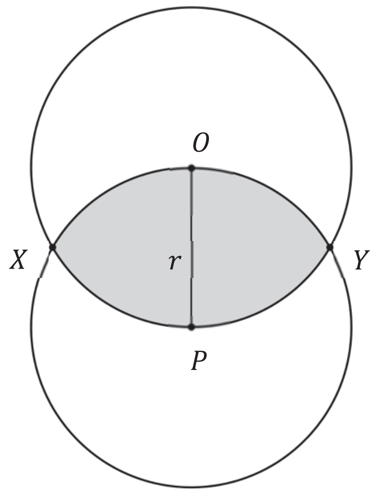
(c) Ellie and Mark worked out that they would save $400 000 in five years by depositing all their combined monthly salary of $\$x$ at the beginning of each month into a savings account and withdrawing $5000 at the end of each month for living expenses. The savings account paid interest at the rate of 3% p.a. compounding monthly.

i. Show that at the end of the second month they have $\$\,[(1.0025^2 + 1.0025)x - 5000(1.0025 + 1)]$ in their savings account. **2**

ii. Write down an expression for the balance in their account at the end of the five years. **1**

iii. What is their combined monthly salary? **2**

(d) Two circles with equal radii and centres O and P intersect at X and Y as shown below. The centres of each circle lie on the circumference of the other circle.



Not to scale

i. Find the exact area of the shaded region $XOYP$. **2**

ii. What fraction of the circle with centre O lies outside of the shaded region $XOYP$? **2**

Question 15 (15 Marks) Commence a new booklet

(a) Find the equation of the tangent on $y = \log_e(2x^2 + 1)$ at the point $(2, \log_e 9)$. **3**
Express your answer in general form.

(b) For the function $f(x) = x^3 - 3x^2 - 9x + 18$:

i. Find the stationary points and determine their nature. **2**

ii. Find the coordinates of the point of inflexion. **1**

iii. Sketch the graph of $y = f(x)$, showing the turning points and point of inflexion. **2**

iv. For what values of k does the equation $x^3 - 3x^2 - 9x + 18 - k = 0$ have 3 solutions? **1**

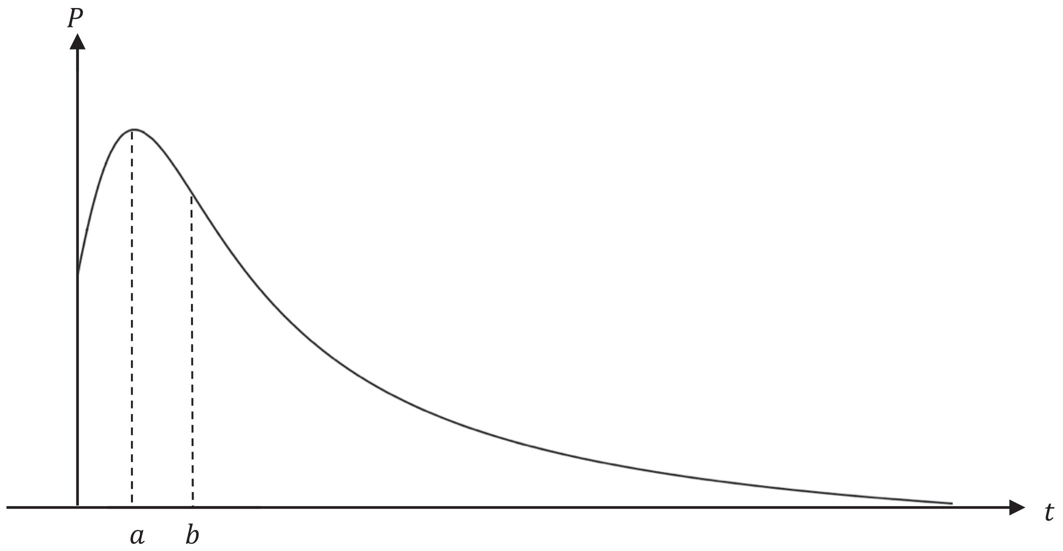
(c) The change in the population of an organism is proportional to the time t, measured in years, that is

$$\frac{dP}{dt} = kt$$

i. Given that the initial population is 400, write an expression for the size of the population P in terms of k and t. **2**

ii. Calculate the size of the population after 3 years given that it is 448 after 2 years. **2**

(d) The graph below shows the estimated population P of an organism after t months.



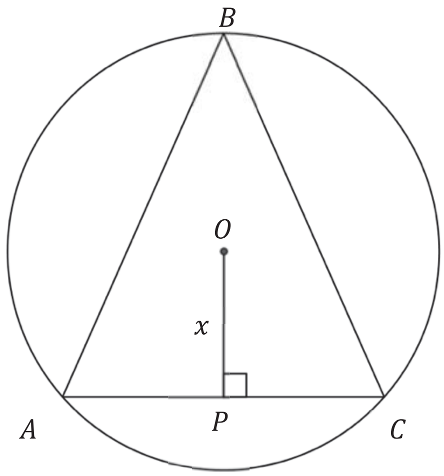
i. For what value(s) of t is the population increasing most rapidly? **1**

ii. For what value(s) of t is the population decreasing at an increasing rate? **1**

Question 16 (15 Marks) Commence a new booklet

(a) An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10cm. $OP = x$ and OP bisects AC, such that $AC \perp OP$.

Copy or trace the diagram into your writing booklet.



Not to scale

i. Show that the area, A, of ΔABC is given by $A = (10 + x)\sqrt{100 - x^2}$. **2**

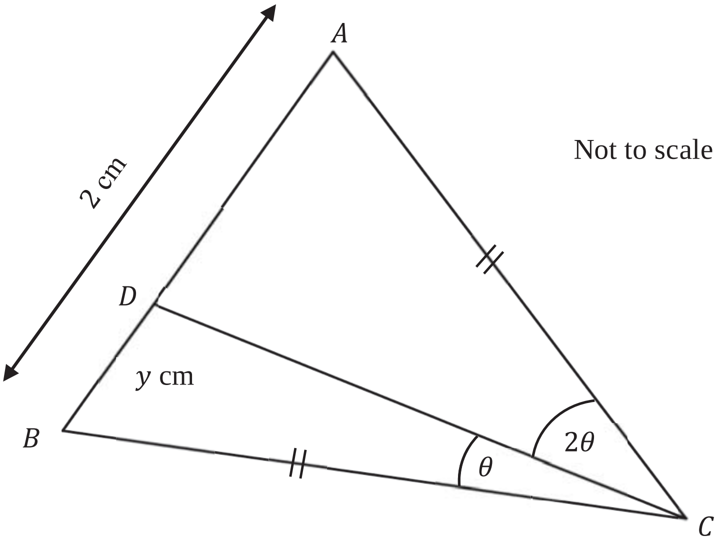
ii. Show that $\frac{dA}{dx} = \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$. **2**

iii. Hence prove that the triangle with maximum area is equilateral. **3**

Examination continues overleaf…

(b) The diagram below shows an isosceles triangle ABC with $AC = BC$ and $AB = 2$ cm.
D is a point on AB so that $\angle ACD = 2\theta$, $\angle BCD = \theta$ and $BD = y$ cm.

Copy or trace the diagram into your writing booklet.



i. Show that $\angle BAC = \frac{\pi}{2} - \frac{3\theta}{2}$. **1**

ii. Use the Sine rule in ΔACD to show $CD = \frac{(2-y)\cos\left(\frac{3\theta}{2}\right)}{\sin 2\theta}$. **2**

iii. Similarly, find another expression for the length of CD in ΔBCD. **1**

iv. Given the formula $\sin 2\theta = 2\sin\theta\cos\theta$, show that $y = \frac{2}{(1+2\cos\theta)}$. **2**

v. Hence prove that as θ varies, $\frac{2}{3} < BD < 1$ for $0 < \theta < \frac{\pi}{3}$. **2**

End of paper.

2017 Mathematics Extension 1 HSC Course Assessment Task 1 Student Self Reflection

1. In hindsight, did I do the best I can? Why or why not?

   ....

   ....

   ....

2. Which topics did I need more help with, and what parts specifically?

   - Q1 - Locus

     ....

     ....

     ....

   - Q2 - Locus

     ....

     ....

     ....

   - Q3 – Series

     ....

     ....

     ....

   - Q4 – Euclidean Geometry

     ....

     ....

     ....

   - Q5 -

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3. What other parts from the feedback session can I take away to refine my solutions for future reference?

   ....

   ....

   ....

   ....

   ....

2017 Mathematics T4 (Trial) – Suggested Solutions

Multiple choice

Q1) C
Q2) D
Q3) A
Q4) D
Q5) D
Q6) B
Q7) D
Q8) C
Q9) D
Q10) D

Q11) a)
$$2y^2 - 4y = 0$$
$$2y(y-2) = 0$$
$$y = 0, 2 \quad ✓$$

b)
$$\frac{\sqrt{2}}{2-3\sqrt{2}} = \frac{\sqrt{2}(2+3\sqrt{2})}{4-9\cdot 2}$$
$$= \frac{2\sqrt{2}+3\cdot 2}{-14} \quad ✓$$
$$= \frac{\sqrt{2}+3}{-7}$$

c)
$$\log_2(x-1) - \log_2(x-2) = 2$$
$$\log_2\left(\frac{x-1}{x-2}\right) = 2 \quad ✓$$
$$2^2 = \frac{x-1}{x-2}$$
$$4(x-2) = x-1$$
$$4x - 8 = x - 1$$
$$3x = 7$$
$$x = \frac{7}{3} \quad ✓$$

d) i) $\int e^{-5y}\,dy = \frac{1}{-5}e^{-5y} + c$ ✓

ii) $\int_0^{\frac{\pi}{4}} \sec^2 x - x \; dx = \left[\tan x - \frac{1}{2}x^2\right]_0^{\frac{\pi}{4}}$ ✓

$= \left(1 - \frac{1}{2}\left(\frac{\pi}{4}\right)^2\right) - (0-0)$

$= 1 - \frac{\pi^2}{32}$ ✓

e) i) $\frac{d}{dx}(3x^2+4)^5 = 5(3x^2+4)^4 \cdot 6x$

$= 30x\,(3x^2+4)^4$ ✓

ii) $\frac{d}{dx}\left(\frac{\tan x}{x}\right) = \frac{\sec^2 x \cdot x - 1 \cdot \tan x}{x^2}$ ✓✓

iii) $\frac{d}{dx}\log_e(x^2-1)^{\frac{1}{2}} = \frac{\frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x}{(x^2-1)^{\frac{1}{2}}}$

$= x \cdot (x^2-1)^{-\frac{1}{2}-\frac{1}{2}}$

$= x \cdot (x^2-1)^{-1}$ ✓✓

Alternatively,

$\frac{d}{dx}\frac{1}{2}\left[\log_e(x^2-1)\right]$

$= \frac{1}{2} \cdot \frac{2x}{x^2-1}$

$= \frac{x}{x^2-1}$

f) $y = 2\sin 3x + 4\cos 2x$

$\frac{dy}{dx} = 2 \cdot 3\cos 3x + 4(-2)\sin 2x$

$= 6\cos 3x - 8\sin 2x$ ✓

$\frac{d^2y}{dx^2} = 6(-3)\sin 3x - 8 \cdot 2\cos 2x$

$= -18\sin 3x - 16\cos 2x$ ✓

$-18\sin 3x - \cancel{16\cos 2x} = m\sin 3x - 4(2\sin 3x + 4\cos 2x)$

$= m\sin 3x - 8\sin 3x - \cancel{16\cos 2x}$

$-10\sin 3x = m\sin 3x$

$m = -10$ ✓

Q12) a)

white \ red	3	5	7	8	9	11
1						
2						
4	X					
6	X	X				
10	X	X	X	X	X	
12	X	X	X	X	X	X

i) $P(\text{win}) = P(\text{white} > \text{red})$

$= \frac{14}{36}$ ✓

ii) $P(\text{win, lose}) + P(\text{lose, win})$

$= P(\text{win, lose}) \times 2$ ✓

$= \left(\frac{14}{36} \times \frac{22}{36}\right) \times 2$

$= \frac{77}{162}$ ✓

iii) $P(\text{at least 1 win in 2 games})$

$= 1 - P(\text{no wins in 2 games})$

$= 1 - \frac{22}{36} \times \frac{22}{36}$

$= \frac{203}{324}$ ✓

b) $\int_1^3 f(x)\,dx \approx \frac{0.5}{2}\left[11.2 + 2(17.8) + 2(9.3) + 2(4.1) + 11.6\right]$ ✓

$= \frac{1}{4}[85.2]$

$= 21.3$ ✓

c) $\angle AED = \angle CDE = \frac{(5-2)\times 180°}{5}$ (interior angles of a regular pentagon)

$= 108°$

$\angle DEQ = \angle EDQ = 180 - 108°$ (adjacent supplementary angles) ✓

$= 72°$

$\alpha = 72° + 72°$ (exterior angle equals sum of 2 opposite interior angles)

$= 144°$ ✓

Alternatively,

$\angle DEQ = \frac{360°}{5}$ (exterior angle of a regular polygon)

$= 72°$

$\alpha = 72° + 72°$ (exterior angle equals sum of 2 opposite interior angles)

$= 144$

d) i) $\tan x = 2\sin x$

$$\frac{\sin x}{\cos x} = 2\sin x$$

$$0 = 2\sin x - \frac{\sin x}{\cos x}$$

$$= \sin x\left(2 - \frac{1}{\cos x}\right)$$

$\sin x = 0$ $\quad$ $2 - \frac{1}{\cos x} = 0$

~~$x = 0$~~ other pt of intersection

$$2 = \frac{1}{\cos x}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} \rightarrow y = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$A = \left(\frac{\pi}{3}, \sqrt{3}\right)$ ✓

ii) $\frac{d}{dx}\left[\log_e \cos x\right] = \frac{-\sin x}{\cos x}$

$$= -\tan x$$ ✓

iii) $\text{Area} = \int_0^{\frac{\pi}{3}} 2\sin x - \tan x \; dx$

$$= \left[-2\cos x + \log_e \cos x\right]_0^{\frac{\pi}{3}}$$ ✓

$$= -2 \cdot \frac{1}{2} + \log_e \frac{1}{2} - \left(-2 \cdot 1 + \log_e 1\right)$$

$$= 1 + \log_e \frac{1}{2} \text{ units}^2$$ ✓

e) $y = \log_e x$

$e^y = x$

$$V = \pi \int_0^{\log_e 4} \left(e^y\right)^2 dy$$ ✓

$$= \pi \left[\frac{1}{2} e^{2y}\right]_0^{\log_e 4}$$ ✓

$$= \frac{\pi}{2}\left[e^{2(\log_e 4)} - e^0\right]$$

$$= \frac{\pi}{2}\left[e^{\log_e 4^2} - 1\right]$$

$$= \frac{\pi}{2}\left[16 - 1\right]$$

$$= \frac{15\pi}{2} \text{ units}^3$$ ✓

Q13) a) i) $m_{AC} = \frac{4-0}{8-2}$
$= \frac{2}{3}$ ✓

ii) $\tan\theta = m_{AC}$
$= \frac{2}{3}$
$\theta = \tan^{-1}\frac{2}{3}$
$= 33°41'$ ✓

iii) $y - 0 = \frac{2}{3}(x-2)$
$y = \frac{2}{3}x - \frac{4}{3}$ ✓

iv) $D = \left(\frac{8+2}{2}, \frac{4+0}{2}\right)$
$= (5, 2)$ ✓

v) $m_{BD} = \frac{8-2}{1-5}$
$= -\frac{3}{2}$
$= -m_{AC}$ ✓
$\therefore AC \perp BD$

b) $LHS = \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}$
$= \frac{\sin\theta(1+\cos\theta) + \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$ ✓
$= \frac{2\sin\theta}{\sin^2\theta}$ ✓ ✓
$= \frac{2}{\sin\theta}$
$= 2\csc\theta$
$= RHS$

c) i) $\alpha + \beta = \frac{3}{2}$
$\alpha\beta = 4$ ✓

ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$
$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ ✓
$= \frac{(\frac{3}{2})^2 - 2(4)}{4^2}$
$= -\frac{23}{64}$ ✓

d) i) $x^2 - 4x - 8y = 20$

$x^2 - 4x + 4 = 20 + 4 + 8y$

$(x-2)^2 = 24 + 8y$ ✓

$= 4(2)(y+3)$

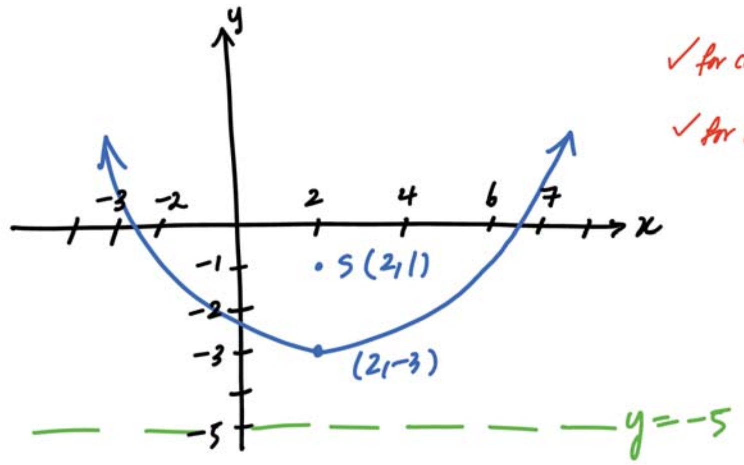
vertex $= (2, -3)$ ✓

ii) x-int: $y = 0 \rightarrow (x-2)^2 = 24$

$x - 2 = \pm\sqrt{24}$

$x = 2 \pm \sqrt{24}$

$= 6.89\ldots, -2.89\ldots$



✓ for correct focus, directrix

✓ for correct shape/orientation

i.e. must show intercepts

$-3 < x_1 < -2$, $6 < x_2 < 7$

or calculate intercepts and label on diagram.

Q14) a) i) $T_{10} = 29$

$T_{15} = 44$

$T_{10} = a + 9d$

$T_{15} = a + 14d$

$a + 9d = 29$ — ①

$a + 14d = 44$ — ② ✓

② − ①: $5d = 15$

$d = 3$

$a = 29 - 9(3)$

$= 2$ ✓

ii) $S_{75} = \frac{75}{2}[2(2) + (75-1)(3)]$ ✓

$= 8475$ ✓

b) $a = \frac{1}{2}$, $S_\infty = 1.5$

$\frac{a}{1-r} = \frac{3}{2}$

$a = \frac{3}{2}(1-r)$ ✓

$\frac{1}{2} = \frac{3}{2}(1-r)$

$r = \frac{2}{3}$ ✓

c) i) Let A_n = amount in savings account at end of n months

$$A_1 = x\left(1+\frac{0.03}{12}\right)^1 - 5000$$
$$= x(1.0025) - 5000 \checkmark$$

$$A_2 = [A_1 + x]((1.0025)^1 - 5000 \checkmark$$
$$= [x(1.0025) - 5000 + x](1.0025)^1 - 5000$$
$$= [(1.0025)^2 + 1.0025]x - 5000(1.0025+1)$$

ii) 5 years = 60 months.

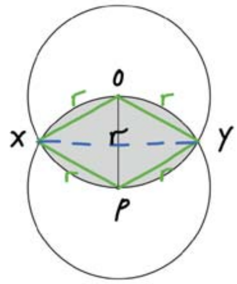
$$A_{60} = [\underbrace{(1.0025)^{60} + \dots + 1.0025^1}_{\text{GP with 60 terms, } r=1.0025,\ a=1.0025}]x - 5000(\underbrace{1.0025^{59} + \dots + 1}_{\text{GP with 60 terms, } r=1.0025,\ a=1}) \checkmark$$

iii)
$$A_{60} = \frac{1.0025(1.0025^{60}-1)}{1.0025-1}\cdot x - 5000\cdot\frac{1(1.0025^{60}-1)}{1.0025-1} \checkmark$$
$$= 401(1.0025^{60}-1)x - 2000000(1.0025^{60}-1)$$
$$= (1.0025^{60}-1)(401x - 2000000)$$

But $A_5 = 400000$

$$\therefore (1.0025^{60}-1)(401x - 2000000) = 400000$$
$$x = \$11159.58 \checkmark$$

Q14) d) i)



ΔOXP, ΔOYP equilateral

$\therefore \angle XOP = \angle YOP = 60°$

$\Rightarrow \angle XOY = 120°$

$= \frac{2\pi}{3}$ ✓

Area of $XOYP = 2 \cdot$ Area of segment XYO

$= 2 \cdot \frac{1}{2} r^2 (\theta - \sin\theta)$

$= r^2 \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= r^2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ square units ✓

ii) Area of circle with centre $O = \pi r^2$

$$\therefore \frac{\text{area unshaded}}{\text{area circle}} = \frac{\pi r^2 - r^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)}{\pi r^2}$$ ✓

$$= \frac{\pi - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)}{\pi}$$

$$= \frac{3\pi - 2\pi}{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}{\pi}$$ ✓

$$= \frac{1}{3} + \frac{\sqrt{3}}{2\pi}$$

Q15) a) $y = \log_e (2x^2 + 1)$

$\frac{dy}{dx} = \frac{4x}{2x^2+1}$ ✓

$x = 2 \rightarrow \frac{dy}{dx} = \frac{4(2)}{2(2)^2+1}$

$= \frac{8}{9}$ ✓

$y - \log_e 9 = \frac{8}{9}(x - 2)$

$9y - 9\log_e 9 = 8(x-2)$

$0 = 8x - 9y + 9\log 9 - 16$ ✓

b) i) $f(x) = x^3 - 3x^2 - 9x + 18$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$

$x = -1 \rightarrow f''(x) = 6(-1) - 6 < 0 \quad \therefore (-1, 23)$ local max ✓

$x = 3 \rightarrow f''(x) = 6(3) - 6 > 0 \quad \therefore (3, -9)$ local min ✓

must classify using $f''(x)$ value or slope table with $f'(x)$ values

ii) $6x - 6 = 0$

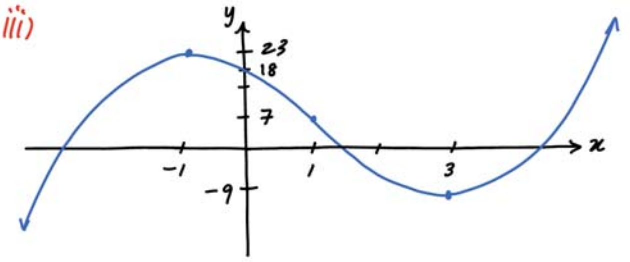
$x = 1$

x	0	1	2
$f''(x)$	−6	0	6
concavity	∩		∪

must show change in concavity usin table with $f''(x)$ values

$\therefore (1, 7)$ a pt of inflexion ✓

iii)



✓ shows max, min

✓ shows pt of inflexion (vertical)

iv) $x^3 - 3x^2 - 9x + 18 - k = 0$

$x^3 - 3x^2 - 9x + 18 = k$

$y = f(x)$ has 3 intersections with $y = k$ when $-9 < k < 23$ ✓

c) i) $\frac{dP}{dt} = kt$

$P = \frac{kt^2}{2} + c$ ✓

$P = 400$ when $t = 0$: $\quad 400 = \frac{k(0)^2}{2} + c$

$c = 400$

$\therefore P = \frac{k}{2}t^2 + 400$ ✓

ii) $P = 448$ when $t = 2$: $\quad 448 = \frac{k}{2}(2)^2 + 400$ ✓

$k = 24$

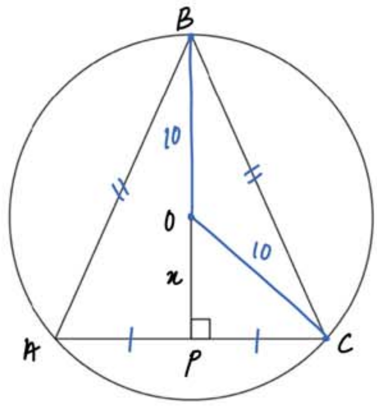
$\therefore$ when $t = 3 \quad P = \frac{24}{2}(3)^2 + 400$

$= 508$ ✓

d) i) when $t = 0$ ✓

ii) when $a < t < b$ ✓

Q16) a) i)



$PC^2 = OC^2 - OP^2$

$PC = \sqrt{100 - x^2}$ ✓

Area of $\Delta ABC = \frac{1}{2} \cdot AC \cdot BP$

$= \frac{1}{2}(2 \cdot PC) \cdot (10 + x)$ ✓

$= \sqrt{100 - x^2}\,(10 + x)$

ii) $A = (10 + x)\sqrt{100 - x^2}$

$= (10 + x)(100 - x^2)^{\frac{1}{2}}$

$\frac{dA}{dx} = 1 \cdot (100 - x^2)^{\frac{1}{2}} + \frac{1}{2}(100 - x^2)^{-\frac{1}{2}}(-2x)(10 + x)$ ✓

$= \sqrt{100 - x^2} - \frac{x(10 + x)}{\sqrt{100 - x^2}}$

$= \frac{100 - x^2 - 10x - x^2}{\sqrt{100 - x^2}}$

$= \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$ ✓

iii) max A occurs when $\frac{dA}{dx} = 0$ and $A = (10 + x)\sqrt{100 - x^2}$ concave down

$\frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} = 0$

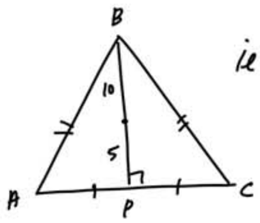
$x^2 + 5x - 50 = 0$

$(x - 5)(x + 10) = 0$

$x = 5, \cancel{-10}$ ✓

x	0	5	6
$\frac{dA}{dx}$	10	0	-4
slope	/	—	\

$\therefore$ max A when $x = 5$



ie. when $BP = 15$

$PC = \sqrt{100 - 5^2}$

$= \sqrt{75}$

and $BC = \sqrt{15^2 + (\sqrt{75})^2}$

$= \sqrt{300}$

$= 2\sqrt{75}$

$= 2 \cdot PC$

$= AC$ ✓

ie. ΔABC equilateral

Q16) b) i) $\angle BAC + \angle ABC + 3\theta = \pi$

$$2\angle BAC + 3\theta = \pi$$

$$= \pi - 3\theta$$

$$\angle BAC = \frac{\pi}{2} - \frac{3\theta}{2}$$

✓

ii) $$\frac{CD}{\sin \angle BAC} = \frac{AD}{\sin 2\theta}$$

$$\frac{CD}{\sin\left(\frac{\pi}{2} - \frac{3\theta}{2}\right)} = \frac{2-y}{\sin 2\theta} \quad ✓$$

$$CD = \frac{2-y}{\sin 2\theta} \cdot \sin\left(\frac{\pi}{2} - \frac{3\theta}{2}\right)$$

$$= \frac{(2-y)\cos\frac{3\theta}{2}}{\sin 2\theta} \quad ✓$$

iii) $$\frac{CD}{\sin \angle DBC} = \frac{BD}{\sin\theta}$$

$$CD = \frac{y}{\sin\theta} \cdot \sin\left(\frac{\pi}{2} - \frac{3\theta}{2}\right)$$

$$= y\,\frac{\cos\frac{3\theta}{2}}{\sin\theta} \quad ✓$$

iv) $$\frac{(2-y)\cos\frac{3\theta}{2}}{\sin 2\theta} = y\,\frac{\cos\frac{3\theta}{2}}{\sin\theta}$$

$$(2-y)\sin\theta = y\sin 2\theta \quad ✓$$

$$= y\,(2\sin\theta\cos\theta)$$

$$2 = 2\cos\theta\, y + y$$

$$= y\,(1 + 2\cos\theta)$$

$$y = \frac{2}{1+2\cos\theta} \quad ✓$$

v) $0 < \theta < \frac{\pi}{3}$

$$\theta = 0 \rightarrow y = \frac{2}{1+2\cos 0}$$

$$= \frac{2}{3}$$

$$\theta = \frac{\pi}{3} \rightarrow y = \frac{2}{1+2\cos\frac{\pi}{3}}$$

$$= 1$$

✓

$$\therefore \frac{2}{3} < y < 1$$

$$\frac{2}{3} < BD < 1$$

✓

Alternatively,

$$0 < \theta < \frac{\pi}{3}$$

$$\cos\frac{\pi}{3} < \cos\theta < \cos 0$$

$$\frac{1}{2} < \cos\theta < 1$$

$$1 < 2\cos\theta < 2$$

$$2 < 1 + 2\cos\theta < 3$$

$$\frac{1}{3} < \frac{1}{1+2\cos\theta} < \frac{1}{2}$$

$$\frac{2}{3} < \frac{2}{1+2\cos\theta} < 1$$

i.e. $\frac{2}{3} < y < 1$