NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS

2018 HSC Course Assessment Task 4 (Trial Examination)
Monday August 13, 2018

General instructions

- Working time – 3 hours. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided (on page 11)

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: .... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12MAT.3 – Mr Tan
◯ 12MAT.4 – Mrs Gan
◯ 12MAT.5 – Mr Lam
◯ 12MAT.6 – Ms Park
◯ 12MAT.7 – Mrs Dupuche

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	%
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{100}$

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 10 minutes for this section

Mark your answers on the answer grid provided (labelled as page 11).

Questions **Marks**

1. The quadratic equation $x^2 - 5x + 1 = 0$ has roots α and β. **1**

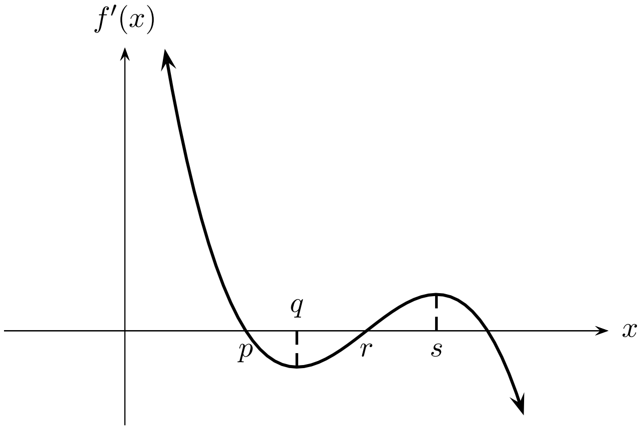
What is the value of $(\alpha + 1)(\beta + 1)$?

(A) -3 (B) -5 (C) 5 (D) 7

2. Which of the following is the value of $\displaystyle\int_0^4 |2x - 4|\ dx$? **1**

(A) 0 (B) 4 (C) 8 (D) 12

3. The diagram shows the graph of $y = f'(x)$. **1**



Which of the following represents the x coordinate of a maximum turning on the graph of $f(x)$?

(A) $x = p$ (B) $x = q$ (C) $x = r$ (D) $x = s$

4. How many solutions are there for the following equation within $0 \le x \le 2\pi$? **1**

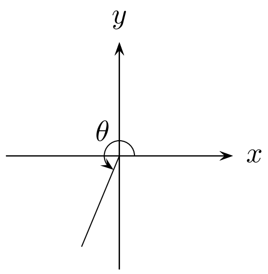
$$\sin x \left(\tan^2 x - 1\right) = 0$$

(A) 4 (B) 5 (C) 6 (D) 7

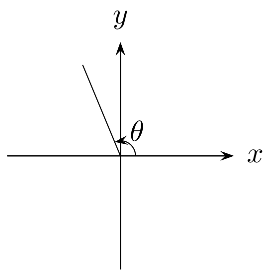
5. For the angle θ which is negative, $\tan\theta = \dfrac{12}{5}$ and $\cos\theta = -\dfrac{5}{13}$. **1**

Which diagram best shows the angle θ?

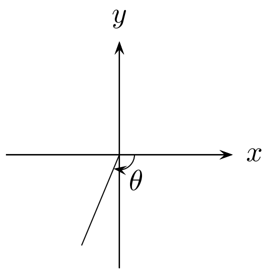
(A)



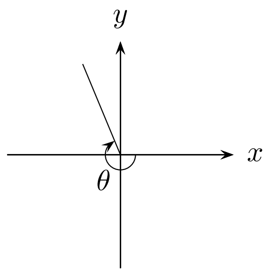
(C)



(B)

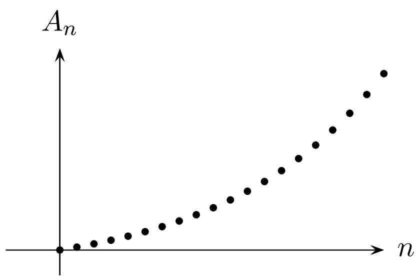


(D)

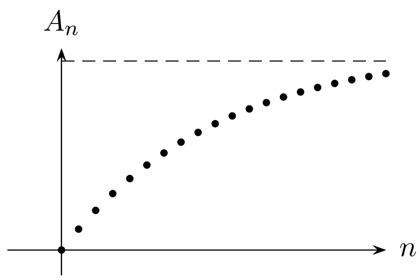


6. Which of the following graphs describes the total amount A_n in a bank account where it attracts a fixed interest rate and also receives regular fixed amount deposits? **1**

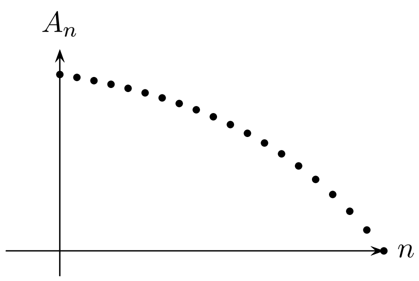
(A)



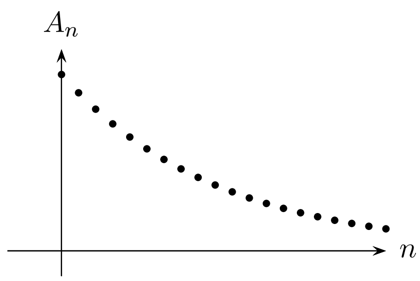
(C)



(B)

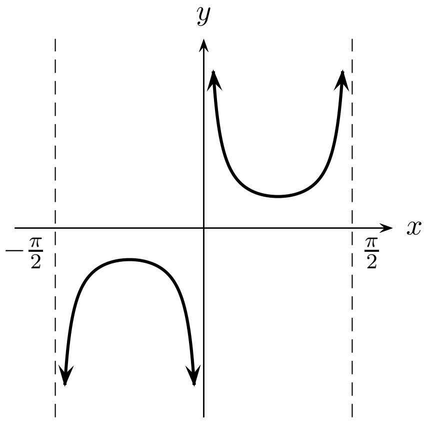


(D)

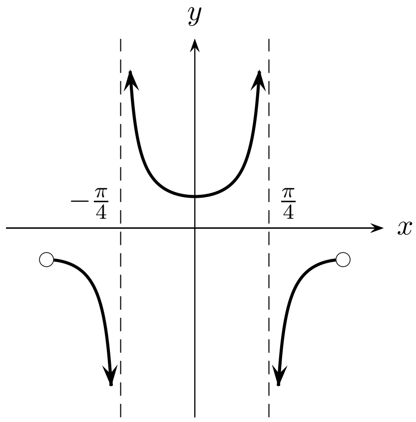


7. Which of the following is the graph of $y = \sec 2x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$? **1**

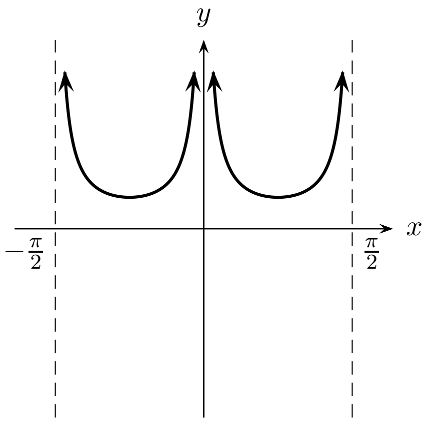
(A)



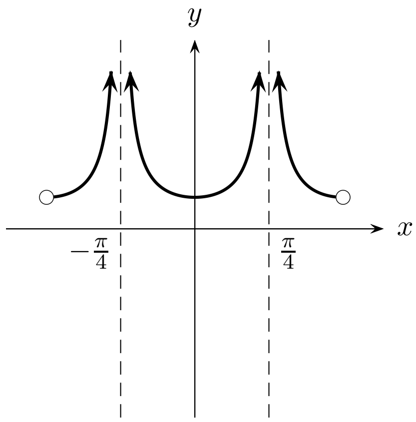
(C)



(B)



(D)



8. What is the focal length of the parabola described by the equation $y = x^2 - 4x + 1$? **1**

(A) $\frac{1}{4}$ (B) 1 (C) 2 (D) 4

9. Which of the following best describes the function $y = \dfrac{e^x + e^{-x}}{2}$? **1**

(A) Odd

(B) Even

(C) Always increasing

(D) Always decreasing

10. Which of the following equations would produce the same graph as $y = \sin\left(x - \frac{\pi}{3}\right)$? **1**

(A) $y = \cos\left(x - \frac{5\pi}{6}\right)$

(B) $y = -\cos\left(x - \frac{5\pi}{6}\right)$

(C) $y = \sin\left(\frac{2\pi}{3} - x\right)$

(D) $y = -\sin\left(\frac{2\pi}{3} - x\right)$

Examination continues overleaf...

Section II

90 marks
Attempt Questions 11 to 16
Allow approximately 2 hours and 50 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Commence a NEW booklet. **Marks**

(a) If $\left(2\sqrt{2}-\sqrt{10}\right)^2 \equiv a - b\sqrt{5}$, find the values of a and b. **2**

(b) What is the domain of the function $f(x) = \dfrac{1}{\sqrt{1-4x^2}}$? **2**

(c) Solve $|x+3| \geq 2$, and graph the solution on a number line. **2**

(d) Differentiate with respect to x:

i. $y = \left(4x^3+10\right)^6$ **1**

ii. $y = \dfrac{\log_e(x+1)}{\sin x}$. **1**

iii. $y = e^{x^2}\cos x$ **2**

(e) Find:

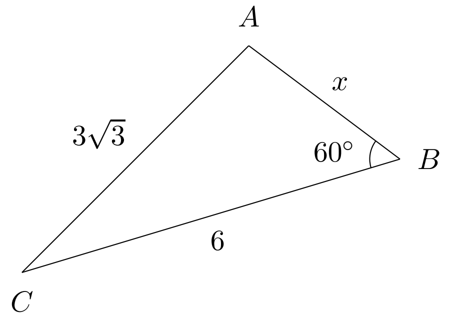
i. $\displaystyle\int \left(\frac{1}{x^2} + \frac{1}{2x+1}\right)\,dx$ **2**

ii. $\displaystyle\int (3x+4)^7\,dx$. **1**

(f) Evaluate: $\displaystyle\int_0^{\frac{\pi}{18}} \sec^2 3x\,dx$ **2**

Question 12 (15 Marks) Commence a NEW booklet. **Marks**

(a) $\triangle ABC$ has side lengths x, 6 and $3\sqrt{3}$ centimetres. $\angle ABC = 60°$.



i. Use the cosine rule to find the *exact* value of x. **3**

ii. Hence, find the *exact* area of $\triangle ABC$. **2**

(b) i. On the same set of axes, accurately draw the graphs of **2**

$$y = \sin x \text{ and } y = \frac{3}{2}x$$

for $0 \leq x \leq \pi$.

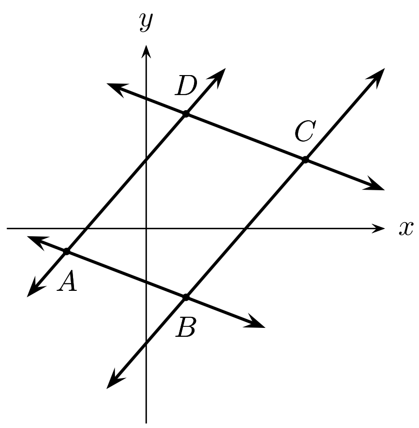
ii. Find the gradient of the tangent to $y = \sin x$ at the origin. **1**

iii. For what values of m does the equation **2**

$$\sin x = mx$$

have a solution within the domain $0 < x < \pi$? Justify your answer.

(c) In the diagram, the coordinates of A and B are $(-2, -1)$ and $(1, -3)$ respectively. The line AD has equation $y = 2x + 3$ and the line CD has equation $2x + 3y = 17$. $BC \parallel AD$.



Copy or trace the diagram into your writing booklet, clearly labelling all given information.

i. Find the equation of the line BC. **1**

ii. Find in exact form, the perpendicular distance from B to the line AD. **1**

iii. Show that the coordinates of D, the point of intersection of the lines AD and CD are $(1, 5)$. **2**

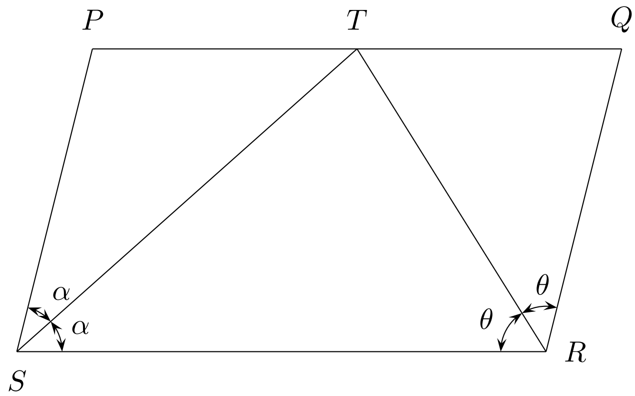
iv. Hence or otherwise, find the area of the parallelogram $ABCD$. **1**

Question 13 (15 Marks) Commence a NEW booklet. **Marks**

(a) Find the values of p such that $px^2 - 3x + p > 0$ for all values of x. **3**

(b) The diagram shows the parallelogram $PQRS$. The point T lies on PQ. It is given that $\angle PST = \angle RST = \alpha$ and $\angle QRT = \angle SRT = \theta$. **3**

Copy or trace the diagram into your writing booklet and prove that $PQ = 2PS$.



(c) The first three terms of an arithmetic series are $(x + 5)$, $(3x + 9)$ and $(5x + 13)$.

i. State the common difference in terms of x. **1**

ii. Find the value of x if the sum of the first ten terms is equal to the 31st term. **3**

(d) The p-th term of a certain geometric progression is c times the first term. **2**

Show that the common ratio r is given by

$$\log r = \frac{\log c}{p - 1}$$

(Hint: the base of the logarithm is not important)

(e) i. Show that $\dfrac{1}{2x - 3} - \dfrac{1}{2x + 3} = \dfrac{6}{4x^2 - 9}$. **1**

ii. Hence find $\displaystyle\int \frac{dx}{4x^2 - 9}$, expressing your answer in simplest form. **2**

Question 14 (15 Marks) Commence a NEW booklet. **Marks**

(a) Sketch the graph of the parabola $x = \frac{1}{2}y^2$, showing clearly the coordinates of the focus and the equation of the directrix. **2**

(b) Solve the equation: $2\log_e x = \log_e (5 + 4x)$. **2**

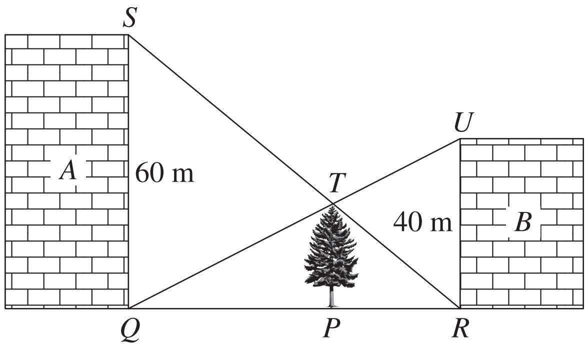
(c) Tom plays BUGP and has a probability of 0.7 of not winning a game, and a probability of 0.3 of winning.

i. Find the probability of Tom winning at least once if he plays three games. **2**

ii. What is the least number of consecutive games Tom must play in order to be 90% or more certain, that he will win at least one game? **2**

(d) A tree is situated at P between two buildings A and B. The tree and the buildings are on horizontal ground and are vertical.

One day Shelley observes that the roof edge of building A, from the top of the tree and the base of building B form a straight line. She also observes that the base of building A, the top of the tree and the roof edge of building B form a straight line.



Let QR be the horizontal distance between the two buildings, PT being height of the tree, SQ the height of building A and UR the height of building B. All distances are measured in metres.

Copy or trace the diagram into your writing booklet.

i. Show that $\triangle TQP$ and $\triangle UQR$ are similar. **2**

ii. Let $PT = h$, $QP = a$ and $PR = b$. **1**

Show that $h = \dfrac{40a}{a+b}$.

iii. Find another expression for h in terms of a and b. **2**

iv. Find h, the height of the tree. **2**

Question 15 (15 Marks) Commence a NEW booklet. **Marks**

(a) A function is given by $f(x) = 12x - 3x^2 - 2x^3$.

i. Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

ii. Hence sketch the graph of $y = f(x)$, showing the stationary points and the y intercept. **2**

iii. Hence or otherwise, for what values of x is the function decreasing? **1**

iv. For what values of k will $12x - 3x^2 - 2x^3 - k = 0$ have two real and distinct solutions? **2**

(b) i. Find the exact value of $\displaystyle\int_0^{\frac{2\pi}{3}} \sin x \, dx$. **1**

ii. Using Simpson's Rule with five function values, find an approximation to **2**

$$\int_0^{\frac{2\pi}{3}} \sin x \, dx$$

leaving your answer in terms of π and $\sqrt{3}$.

iii. Using parts (i) and (ii), show that **1**

$$\pi \approx \frac{18}{\sqrt{3} + 4}$$

(c) Consider the series $\cos x + \cos^3 x + \cos^5 x + \cdots$, where $0 < x < \dfrac{\pi}{2}$.

i. Show that a limiting sum exists. **1**

ii. Show that the limiting sum is $\cot x \operatorname{cosec} x$. **2**

Question 16 (15 Marks) Commence a NEW booklet. **Marks**

(a) The projected figures for the number of vehicles V (in thousands) travelling in a newly built, tolled road tunnel that opens on 1 April 2019 is given by

$$V = 4(t-5)e^{-\frac{t}{10}} + 70$$

where t is measured in days.

i. How many vehicles are projected to use the tunnel on 1 April 2019, as the tunnel opens? **1**

ii. According to these projections, after how many days of opening would the number of vehicles using the tunnel reach its peak? **3**

iii. As time passes, how many vehicles are projected to pass through the tunnel on a daily basis? **1**

iv. Draw a sketch of V as a function of t, showing the maximum number of vehicles that are projected to use the tunnel. **2**

(b) Mr Lam borrows \$900 000 at 3% per annum for his residence. The repayments of \$$M$ are made on a fortnightly basis, after interest has been applied. The loan is to be repaid over 30 years. Let A_n be the total amount owing.

i. Show that the amount owing after the second repayment is **1**

$$A_2 = 900\,000F^2 - M\,(1+F)$$

given $F = \dfrac{2\,603}{2\,600}$.

ii. Show that **2**

$$A_n = 900\,000F^n - \frac{2\,600M}{3}\,(F^n - 1)$$

iii. Find M, the minimum fortnightly repayment amount. **2**

iv. Find the time in years that Mr Lam will be able to fully repay the loan if he were to be able to afford repayments of \$2 200 per fortnight. **3**

(Moral of the story: make extra repayments if you can!)

End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

NESA STUDENT #: ....

Class (please ✔)

◯ 12MAT.3 – Mr Tan

◯ 12MAT.4 – Mrs Gan

◯ 12MAT.5 – Mr Lam

◯ 12MAT.6 – Ms Park

◯ 12MAT.7 – Mrs Dupuche

Directions for multiple choice answers

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

  Ⓐ Ⓑ ● Ⓓ

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

  Ⓐ Ⓑ ~~●~~ ●

- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:

  Ⓐ Ⓑ ~~●~~ ~~●~~ correct

1 – Ⓐ Ⓑ Ⓒ Ⓓ

2 – Ⓐ Ⓑ Ⓒ Ⓓ

3 – Ⓐ Ⓑ Ⓒ Ⓓ

4 – Ⓐ Ⓑ Ⓒ Ⓓ

5 – Ⓐ Ⓑ Ⓒ Ⓓ

6 – Ⓐ Ⓑ Ⓒ Ⓓ

7 – Ⓐ Ⓑ Ⓒ Ⓓ

8 – Ⓐ Ⓑ Ⓒ Ⓓ

9 – Ⓐ Ⓑ Ⓒ Ⓓ

10 – Ⓐ Ⓑ Ⓒ Ⓓ

2018 Mathematics HSC Course Assessment Task 4 STUDENT SELF REFLECTION

1. In hindsight, did I do the best I can? Why or why not?

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2. Which topics did I need more help with, and what parts specifically?

- Q1, 8, 13(a), 14(a) - Quadratic Polynomial/Locus

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- Q4, 5, 7, 10, 11(f), 12(a)(b), 15(b)(c) - Trigonometry/Trigonometric functions

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- Q6, 9, 13(d), 14(a), 16(a) - Exponential/Logarithmic Functions (with a mix of series, probability and calculus!)

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- Q2, 3, 11, 12(c), 13(e), 15 - Pre-calculus, differential/integral Calculus plus applications

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- Q16(a) - Applications of calculus to the physical world

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3. What other parts from the feedback session can I take away to refine my solutions for future reference?

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Sample Band 6 Responses

Section I

1. (D) **2.** (C) **3.** (A) **4.** (D) **5.** (B)
6. (A) **7.** (C) **8.** (A) **9.** (B) **10.** (A)

Section II

Question 11 (Gan)

(a) (2 marks)

✓ [1] for each correct value of a and b.

$$\begin{aligned}\left(2\sqrt{2}-\sqrt{10}\right)^2 &= \left(\sqrt{2}\right)^2\left(2-\sqrt{5}\right)^2\\ &= 2\left(4-4\sqrt{5}+5\right)\\ &= 2\left(9-4\sqrt{5}\right)\\ &= 18-8\sqrt{5}\\ \therefore a = 18 \qquad & b = 8\end{aligned}$$

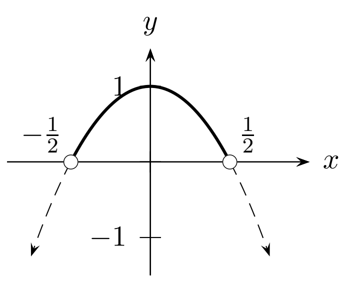
(b) (2 marks)

✓ [1] for restricting $(1-2x)(1+2x)>0$.
✓ [1] for final answer.

$$f(x)=\frac{1}{\sqrt{1-4x^2}}$$

Natural domain dictates that $1-4x^2$ is strictly positive:

$$\begin{gathered}1-4x^2>0\\(1-2x)(1+2x)>0\end{gathered}$$

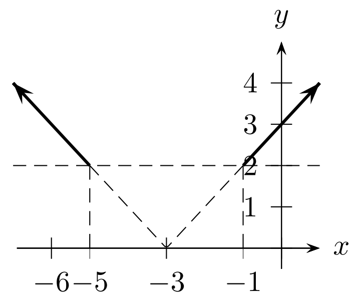


$$\therefore -\frac{1}{2}<x<\frac{1}{2}$$

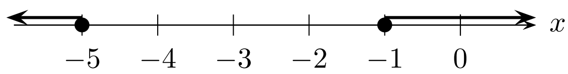
(c) (2 marks)

✓ [1] for correct solution.
✓ [1] for correct sketch on to the number line.

$$|x+3|\ge 2$$



$$\therefore x\ge -1 \text{ or } x\le -5$$



(d) i. (1 mark)

$$\begin{aligned}y &= \left(4x^3+10\right)^6\\ \frac{dy}{dx} &= 6\left(4x^3+10\right)^5\times 12x^2\\ &= 72x^2\left(4x^2+10\right)^5\end{aligned}$$

ii. (1 mark)

$$y=\frac{\ln(x+1)}{\sin x}$$

$$\left|\begin{array}{ll} u=\ln(x+1) & v=\sin x\\ u'=\dfrac{1}{x+1} & v'=\cos x\end{array}\right.$$

$$\frac{dy}{dx}=\frac{\frac{\sin x}{x+1}-\cos x\ln(x+1)}{\sin^2 x}$$

iii. (2 marks)

✓ [1] for all of u, v, u' and v' correct.

✓ [1] for correct application of the product rule.

$$y = e^{x^2}\cos x$$

$$\left|\begin{array}{ll} u = e^{x^2} & v = \cos x \\ u' = 2xe^{x^2} & v' = -\sin x \end{array}\right.$$

$$\begin{aligned}\frac{dy}{dx} &= uv' + vu' \\ &= -e^{x^2}\sin x + 2xe^{x^2}\cos x \\ &= e^{x^2}(2x\cos x - \sin x)\end{aligned}$$

(e) i. (2 marks)

✓ [1] for each correctly integrated term. Loss of one mark here if constant is not present.

$$\int\left(x^{-2} + \frac{1}{2x+1}\right)\,dx$$

$$= -x^{-1} + \frac{1}{2}\ln(2x+1) + C$$

ii. (1 mark)

$$\begin{aligned}\int (3x+4)^7\ dx &= \frac{1}{8}(3x+4)^8 \times \frac{1}{3} + C \\ &= \frac{1}{24}(3x+4)^8 + C\end{aligned}$$

(f) (2 marks)

✓ [1] for correct primitive.

✓ [1] for correct evaluation of limits.

$$\begin{aligned}\int_0^{\frac{\pi}{18}} \sec^3 3x\,dx &= \frac{1}{3}\Big[\tan 3x\Big]_0^{\frac{\pi}{18}} \\ &= \frac{1}{2}\left[\tan\frac{\pi}{6} - \tan 0\right] \\ &= \frac{1}{2} \times \frac{1}{\sqrt{3}} \\ &= \frac{1}{2\sqrt{3}}\end{aligned}$$

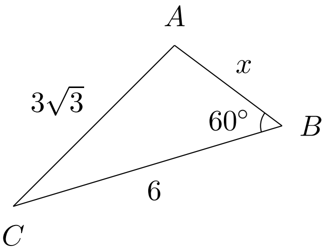
Question 12 (Gan)

(a) i. (3 marks)

✓ [1] for correct application of cosine rule with the values provided.

✓ [1] for correct simplification to a quadratic.

✓ [1] for final answer.



$$\begin{aligned}AC^2 &= x^2 + 6^2 - 2(x)(6)\cos 60^\circ \\ \left(3\sqrt{3}\right)^2 &= x^2 + 36 - 12x \times \frac{1}{2} \\ 27 &= x^2 + 36 - 6x \\ x^2 - 6x + 9 &= 0 \\ (x-3)^2 &= 0 \\ \therefore x &= 3\end{aligned}$$

ii. (2 marks)

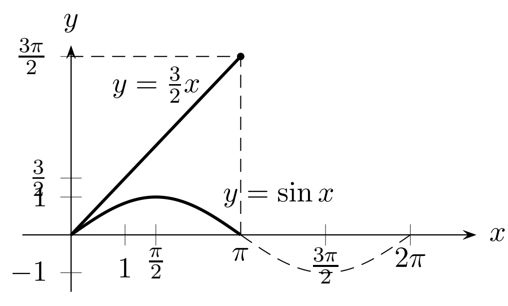
✓ [1] for correct application of area via sine rule

✓ [1] for final answer.

$$\begin{aligned}A &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2} \times 3 \times 6 \times \sin 60^\circ \\ &= 9 \times \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{2}\end{aligned}$$

(b) i. (2 marks)

✓ [1] for each correct graph.



ii. (1 mark)

$$
\begin{aligned}
y &= \sin x\\
\frac{dy}{dx} &= \cos x\Big|_{x=0}\\
&= \cos 0 = 1
\end{aligned}
$$

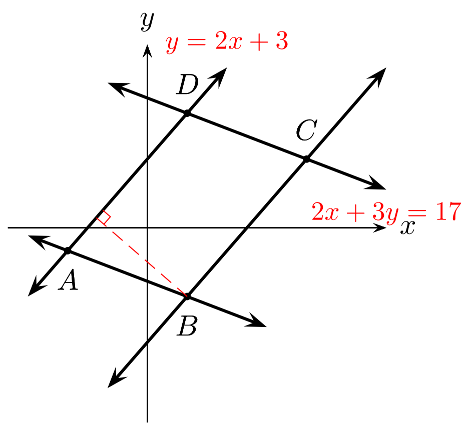
iii. (2 marks)

✓ [1] for correct justification

✓ [1] for correct range of values of m derived from justification.

- The line $y = mx$ intersects the graph $y = \sin x$ once.
- Hence $0 < m < 1$.

(c) i. (1 mark)



$$y = 2x + b$$

When $x = 1$, $y = -3$:

$$
\begin{aligned}
-3 &= 2 + b\\
b &= -5\\
\therefore BC : y &= 2x - 5
\end{aligned}
$$

ii. (1 mark)

- Perpendicular distance from B to AD:
  - $B(1, -3)$
  - $AD : 2x - y + 3 = 0$

$$
\begin{aligned}
d_\perp &= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|2(1) + (-1)(-3) + 3|}{\sqrt{2^2 + (-1)^2|}}\\
&= \frac{|2 + 3 + 3|}{\sqrt{5}} = \frac{8}{\sqrt{5}}
\end{aligned}
$$

iii. (2 marks)

✓ [1] for each correct coordinate.

$$
\begin{cases}
2x + 3y = 17 & (1)\\
2x - y = -3 & (2)
\end{cases}
$$

Subtract (2) from (1):

$$
\begin{aligned}
4y &= 20\\
y &= 5
\end{aligned}
$$

Substitute back to (2):

$$
\begin{aligned}
2x - 5 &= -3\\
2x &= 2\\
x &= 1\\
\therefore D&(1, 5)
\end{aligned}
$$

iv. (1 mark)

$$
\begin{aligned}
AD &= \sqrt{(-2 - 1)^2 + (-1 - 5)^2}\\
&= \sqrt{9 + 36}\\
&= \sqrt{45} = 3\sqrt{5}
\end{aligned}
$$

Applying area of parallelogram,

$$
\begin{aligned}
A &= bh\\
&= AD \times d_\perp\\
&= 3\sqrt{5} \times \frac{8}{\sqrt{5}}\\
&= 24
\end{aligned}
$$

Question 13 (Tan)

(a) (3 marks)

- ✓ [1] for expression in p for $\Delta < 0$
- ✓ [1] for excluding one particular value of p due to it being $p < 0$.
- ✓ [1] for final answer.

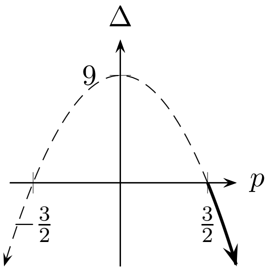
$$p^2 - 3x + p > 0$$

Need $p^2 - 3x + p$ to be positive definite, i.e. $\Delta < 0$ and $p > 0$ simultaneously.

$$\begin{aligned}\Delta &= 3^2 - 4(p)(p)\\ &= 9 - 4p^2\end{aligned}$$

Now when $\Delta < 0$,

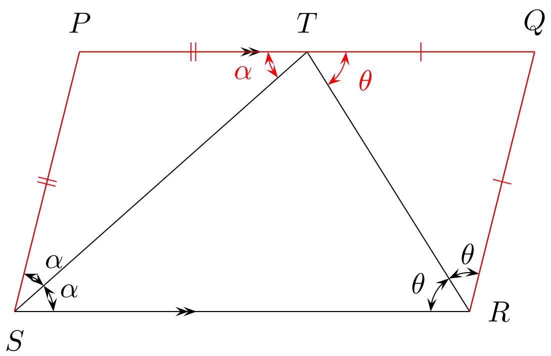
$$\begin{aligned}9 - 4p^2 &< 0\\ (3-2p)(3+2p) &< 0\end{aligned}$$



$\therefore p > \frac{2}{3}$ only as $p > 0$ as well.

(b) (3 marks)

- ✓ [1] for each correct statement listed below.



- $\angle QTR = \angle TRS = \theta$ (alternate $\angle$, $PQ \parallel SR$) $\therefore QT = QR$ (equal sides of isosceles $\triangle QTR$ since $\angle QTR = \angle QRT$).
- Similarly, $\triangle PTS$ is isosceles with $PT = PS$ and base angle equal to α.
- Since $QR = PS$ (opposite sides of parallelogram), then $PT = TQ$.

$$\begin{aligned}\frac{PQ}{QS} &= \frac{2}{1}\\ PQ &= 2PS\end{aligned}$$

(c) i. (1 mark)

$$T_1 = (x+5) \qquad T_2 = (3x+9)$$
$$T_3 = (5x+13)$$
$$\begin{aligned}d &= T_2 - T_1\\ &= (3x+9) - (x+5)\\ &= 2x+4\end{aligned}$$

ii. (3 marks)

- ✓ [1] for correct expression for S_{10} in terms of x, in simplest form
- ✓ [1] for correct expression for T_{31} in terms of x, in simplest form
- ✓ [1] for final value of x

$$\begin{aligned}S_{10} &= \frac{n}{2}(2a + d(n-1))\\ &= \frac{10}{2}(2(x+5) + (2x+4)(9))\\ &= 5(2x+10+18x+36)\\ &= 5(20x+46)\\ &= 10(10x+23)\\ T_{31} &= a + d(n-1)\\ &= (x+5) + (2x+4)(30)\\ &= x+5+60x+120\\ &= 61x+125\end{aligned}$$

As $S_{10} = T_{31}$,

$$\begin{aligned}10(10x+23) &= 61x+125\\ 100x+230 &= 61x+125\\ 39x &= -105\\ x &= -\frac{105}{39}\end{aligned}$$

(d) (2 marks)

✓ [1] for $ar^{p-1} = ac$, and showing the cancellation.

✓ [1] for final proven result

$$\begin{aligned} T_n &= ar^{n-1} \\ T_p &= ar^{p-1} = ac \\ \therefore \cancel{a}r^{p-1} &= \cancel{a}c \\ r^{p-1} &= c \\ (p-1)\log r &= \log c \\ \log r &= \frac{\log c}{p-1} \end{aligned}$$

(e) i. (1 mark)

$$\begin{aligned} &\frac{1}{2x-3} - \frac{1}{2x+3} \\ &= \frac{2x+3-(2x-3)}{4x^2-9} \\ &= \frac{6}{4x^2-9} \end{aligned}$$

ii. (2 marks)

✓ [1] for using previous part to obtain $\frac{1}{6}\int\left(\frac{1}{2x-3} - \frac{1}{2x+3}\right)dx$

✓ [1] for final answer in simplest form.

$$\begin{aligned} &\int \frac{dx}{4x^2-9} \\ &= \frac{1}{6}\int \frac{6\,dx}{4x^2-9} \\ &= \frac{1}{6}\int\left(\frac{1}{2x-3} - \frac{1}{2x+3}\right)\,dx \\ &= \frac{1}{6}\left(\frac{1}{2}\ln(2x-3) - \frac{1}{2}\ln(2x+3)\right) + C \\ &= \frac{1}{12}\ln\left(\frac{2x-3}{2x+3}\right) + C \end{aligned}$$

Question 14 (Lam)

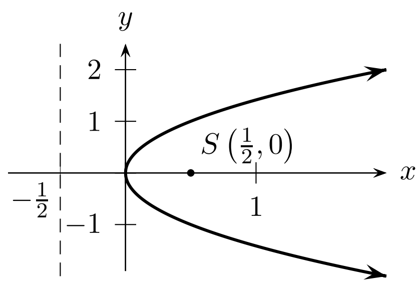
(a) (2 marks)

✓ [1] for correct shape.

✓ [1] for correct focus, vertex and directrix.

$$\begin{aligned} x &= \frac{1}{2}y^2 \\ y^2 = 2x &= 4\left(\frac{1}{2}\right)x \end{aligned}$$

- Vertex: $V(0,0$, Focus: $S\left(\frac{1}{2},0\right)$
- Directrix: $x = -\frac{1}{2}$



(b) (2 marks)

✓ [1] for using logarithm law to convert $2\log_e x$ to $\log_e x^2$

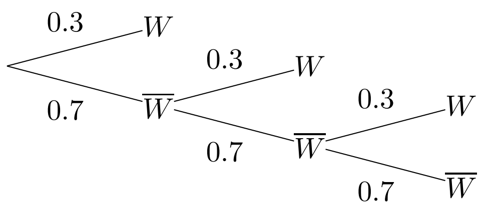
✓ [1] for final answer, excluding the negative solution in x.

$$\begin{aligned} 2\log_e x &= \log_e(5+4x) \\ \log_e x^2 &= \log_e(5+4x) \\ x^2 &= 5+4x \\ x^2-4x-5 &= 0 \\ (x-5)(x+1) &= 0 \\ \therefore x &= 5 \text{ only as } x > 0 \end{aligned}$$

(c) i. (2 marks)

✓ [1] for using the complementary situation.

✓ [1] for final answer.



$$P(\text{win at least once}) = 1 - P(\overline{W}\,\overline{W}\,\overline{W}) = 1 - 0.7^3 = 0.657$$

ii. (2 marks)

✓ [1] for generalising to n games

✓ [1] for final answer

- $P(\text{win at least once in three games}) = 1 - 0.7^3$
- $P(\text{win at least once in } n \text{ games}) = 1 - 0.7^n > 0.9$

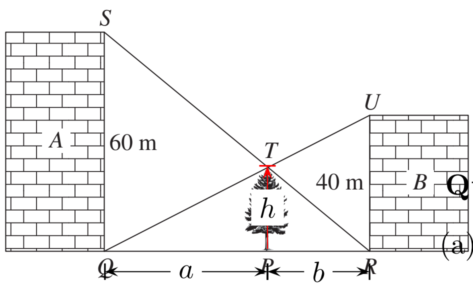
$$\begin{aligned} 1 - 0.7^n &> 0.9 \\ 0.7^n &< 0.1 \\ n \log \frac{7}{10} &< \log \frac{1}{10} \\ \therefore n &> \frac{\log 10}{\log \frac{10}{7}} \approx 6.455\cdots \end{aligned}$$

Hence he needs to play at least 7 games to be 90% or more certain to win at least one game.

(d) i. (2 marks)

✓ [1] for correct reasons leading to similarity condition.

✓ [1] for correct similarity statement.



In $\triangle TQP$ and $\triangle UQR$:

- $\angle TQR$ is common.
- $\angle TPQ = \angle URQ = 90°$ (given)

$\therefore \triangle TQP \,|||\, \triangle UQR$ (equiangular).

ii. (1 mark)

As the ratio of corresponding sides of $\triangle TQP$ and $\triangle UQR$ are equal,

$$\frac{h}{a} = \frac{40}{a+b}$$
$$h = \frac{40a}{a+b}$$

iii. (2 marks)

✓ [1] for proving $\triangle TPR \,|||\, \triangle SQR$

✓ [1] for obtaining another value of h based on the similarity ratio expression for $\triangle TPR$ and $\triangle SQR$

In $\triangle TPR$ and $\triangle SQR$:

- $\angle TRP$ is common
- $\angle SQP = \angle TPR = 90°$ (given)

Hence $\triangle TPR|||\triangle SQR$ (equiangular).

Similarity ratio:

$$\frac{h}{b} = \frac{60}{a+b}$$
$$\therefore h = \frac{60b}{a+b}$$

iv. (2 marks)

✓ [1] for obtaining $a = \frac{3}{2}b$

✓ [1] for final answer.

$$h = \frac{40a}{a+b} = \frac{60b}{a+b}$$
$$\therefore 40a = 60b$$
$$a = \frac{3}{2}b$$

Substitute into $h = \frac{60b}{a+b}$:

$$h = \frac{60b}{\frac{3}{2}b + b} = \frac{60\cancel{b}}{\frac{5}{2}\cancel{b}} = 60 \times \frac{2}{5} = 24\,\text{m}$$

Question 15 (Dupuche)

(a) i. (3 marks)

✓ [1] for correct x values of stationary points.

✓ [1] for correct y values of stationary points.

✓ [1] for correct testing and classifying the stationary points.

$$\begin{aligned} f(x) &= 12x - 3x^2 - 2x^3 \\ &= -x\left(2x^2 + 3x - 12\right) \\ f'(x) &= 12 - 6x - 6x^2 \\ &= -6\left(x^2 + x - 2\right) \\ &= -6(x+2)(x-1) \end{aligned}$$

Stationary points occur when $f'(x) = 0$:

$$(x+2)(x-1) = 0$$
$$\therefore x = -2 \text{ or } 1$$

Finding the second derivative:

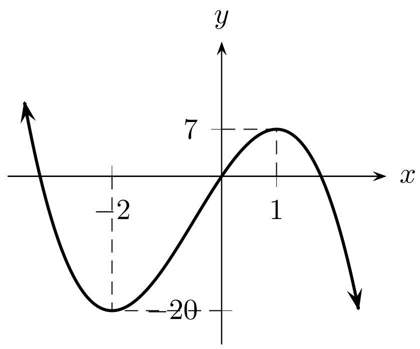
$$f''(x) = -6 - 12x$$

- When $x = 1$:
  $$f(1) = 12 - 3 - 2 = 7$$
  $$f''(1) = -6 - 12 < 0$$
  Hence $(1, 7)$ is a local max.
- When $x = -2$:
  $$\begin{aligned} f(-2) &= 12(-2) - 3(-2)^2 - 2(-2)^3 \\ &= -20 \\ f''(-2) &= -6 - 12(-2) \\ &= 20 > 0 \end{aligned}$$
  Hence $(-2, -20)$ is a local min.

ii. (2 marks)

✓ [1] for correct shape.

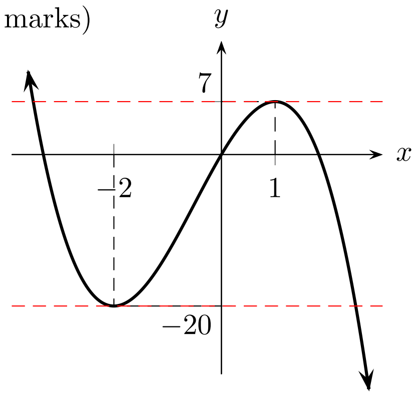
✓ [1] for correctly applied data from previous part.



iii. (1 mark)

$$x > 1 \text{ or } x < -2$$

iv. (2 marks)



From the diagram, $y = 12x - 3x^2 - 2x^3$ intersects the line $y = k$ at two distinct (unique) locations only when $k = -20$ or $k = 7$.

(b) i. (1 mark)

$$\begin{aligned} \int_0^{\frac{2\pi}{3}} \sin x\, dx &= -\left[\cos x\right]_0^{\frac{2\pi}{3}} \\ &= -\left[\cos\frac{2\pi}{3} - \cos 0\right] \\ &= -\left[-\frac{1}{2} - 1\right] \\ &= \frac{3}{2} \end{aligned}$$

ii. (2 marks)

✓ [1] for correct values in the table.

✓ [1] for correct application of Simpson's Rule.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$

Applying Simpson's Rule,

$$\begin{aligned} A &\approx \frac{\frac{\pi}{6}}{3}\left(0 + 4\left(\frac{1}{2} + 1\right) + 2\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\right) \\ &= \frac{\pi}{18}\left(6 + \frac{3\sqrt{3}}{2}\right) \end{aligned}$$

iii. (1 mark)

$$\underset{\times 2}{\frac{3}{2}} \approx \underset{\times 2}{\frac{\pi}{18}}\left(6 + \frac{3\sqrt{3}}{2}\right)$$

$$\underset{\times 9}{\cancel{3}} \approx \underset{\times 9}{\frac{\pi}{9}} \times \cancel{3}\left(2 + \frac{1}{2}\sqrt{3}\right)$$

$$\underset{\times 2}{9} \approx \pi\left(2 + \underset{\times 2}{\frac{1}{2}}\sqrt{2}\right)$$

$$18 \approx \pi\left(4 + \sqrt{3}\right)$$

$$\therefore \pi \approx \frac{18}{4 + \sqrt{3}}$$

(c) i. (1 mark)

$$\cos x + \cos^3 x + \cos^5 x + \cdots \qquad \left(0 < x < \frac{\pi}{2}\right)$$
$$r = \cos^2 x$$

As $-1 < \cos x < 1$ due to the restriction placed on x,

$$\therefore 0 < \cos^2 x < 1$$
$$\therefore 0 < r < 1$$

Hence a limiting sum exists.

ii. (2 marks)

✓ [1] for transforming into $\frac{\cos x}{\sin^2 x}$.

✓ [1] for final result.

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{\cos x}{1-\cos^2 x} = \frac{\cos x}{\sin^2 x} \\ &= \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \\ &= \cot x \operatorname{cosec} x \end{aligned}$$

Question 16 (Park)

(a) i. (1 mark)

$$V = 4(t-5)e^{-\frac{t}{10}} + 70$$

When $t = 0$,

$$\begin{aligned} V &= 4(0-5)e^0 + 70 \\ &= -20 + 70 \\ &= 50 \end{aligned}$$

Hence the were 50 000 vehicles on the first day.

ii. (3 marks)

✓ [1] for correct $\dfrac{dV}{dt}$.

✓ [1] for correct value of t.

✓ [1] for checking type of stationary point.

$$\left|\begin{array}{ll} u = t-5 & v = e^{-\frac{t}{10}} \\ u' = 1 & v' = -\dfrac{1}{2}e^{-\frac{t}{10}} \end{array}\right.$$

$$\begin{aligned} \frac{dV}{dt} &= uv' + vu' \\ &= 4\left(\frac{-(t-5)}{10}e^{-\frac{t}{10}} + e^{-\frac{t}{10}}\right) \\ &= 4e^{-\frac{t}{10}}\left(1 - \frac{t-5}{10}\right) \end{aligned}$$

Peak number of vehicles (maximum) will occur when $\frac{dV}{dt} = 0$. As $e^{-\frac{t}{10}} \neq 0$ (exponential always positive),

$$\begin{aligned} \therefore 1 - \frac{t-5}{10} &= 0 \\ 1 &= \frac{t-5}{10} \\ t &= 15 \end{aligned}$$

t	14	15	16
$\frac{dV}{dt}$	+ 0.098	0	− −0.081
V	↗	$78.93\cdots$	↘

iii. (1 mark)

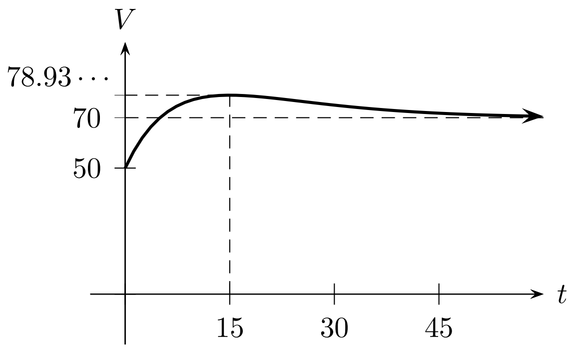
$$t \to \infty, e^{-\frac{t}{10}} \to 0$$
$$\therefore V \to 70$$

(70 000 vehicles per day)

iv. (2 marks)

✓ [1] for correctly applied data from parts (i) and (ii)

✓ [1] for correctly applied data from part (iii)



(b) i. (1 mark)

$$\begin{aligned} P &= 90\,000 \\ r &= 0.03\,\text{p.a.} \\ &= \frac{0.03}{26} \text{ per fortnight} \\ &= \frac{3}{2\,600} \text{ per fortnight} \end{aligned}$$

Let A_n be the amount owing during any fortnight, n.

$$\begin{aligned} A_1 &= 900\,000 \times \left(1 + \frac{3}{2\,000}\right) - M \\ &= 900\,000 \times \frac{2\,603}{2\,600} - M \\ A_2 &= A_1 \times \frac{2\,603}{2\,600} - M \\ &= \left(900\,000 \times \frac{2\,603}{2\,600} - M\right) \times \frac{2\,603}{2\,600} - M \\ &= 900\,000 \times \left(\frac{2\,603}{2\,600}\right)^2 \\ &\quad - M\left(\frac{2\,603}{2\,600}\right) - M \\ &= 900\,000 \times \left(\frac{2\,603}{2\,600}\right)^2 - M\left(1 + \frac{2\,603}{2\,600}\right) \\ &= 900\,000 \times F^2 - M(1+F) \end{aligned}$$

where $F = \frac{2\,603}{2\,600}$.

ii. (2 marks)

- ✓ [1] for finding A_3 in terms of F and also showing the development of the geometric series.
- ✓ [1] for final result required.

$$\begin{aligned} A_3 &= A_2 \times F - M \\ &= 900\,000F^3 - M\left(F + F^2\right) - M \\ &= 900\,000F^2 - M\left(1 + F + F^2\right) \end{aligned}$$

Generalising,

$$A_n = 900\,000F^n - M\underbrace{\left(1 + F + F^2 + \cdots + F^{n-1}\right)}_{=S_n}$$

Summing the geometric series,

$$\begin{aligned} S_n &= \frac{1\left(F^n - 1\right)}{F - 1} \\ &= \frac{F^n - 1}{\frac{2\,603}{2\,600} - 1} \\ &= \frac{F^n - 1}{\frac{3}{2\,600}} \\ &= \frac{2\,600}{3}\left(F^n - 1\right) \end{aligned}$$

$$\therefore A_n = 900\,000F^n - \frac{2\,600}{3}M\left(F^n - 1\right)$$

iii. (2 marks)

- ✓ [1] for obtaining $\frac{2\,600}{3}M\left(F^{780} - 1\right) = 900\,000F^{780}$
- ✓ [1] for final result of \$1 750.55 per fortnight.

When the loan is repaid, $A_n = 0$, $n = 26 \times 30 = 780$.

$$\begin{aligned} 0 &= 900\,000F^{780} - \frac{2\,600}{3}M\left(F^{780} - 1\right) \\ 900\,000F^{780} &= \frac{2\,600}{3}M\left(F^{780} - 1\right) \\ M &= \frac{900\,000 \times F^{780}}{\frac{2\,600}{3}\left(F^{780} - 1\right)} \\ &= \frac{900\,000 \times \left(\frac{2\,603}{2\,600}\right)^{780}}{\frac{2\,600}{3}\left(\left(\frac{2\,603}{2\,600}\right)^{780} - 1\right)} \\ &= \$1\,750.55 \text{ per fortnight} \end{aligned}$$

iv. (3 marks)

- ✓ [1] for obtaining $0 = F^n\left(900\,000 - \frac{5\,720\,000}{3}\right) + \frac{5\,720\,000}{3}$
- ✓ [1] for explicit expression in F_n.
- ✓ [1] for final number of years.

Loan is repaid when $A_n = 0$, but this time with $M = 2\,200$.

$$\begin{aligned} 0 &= 900\,000F^n - \frac{2\,600}{3}(2\,200)\left(F^n - 1\right) \\ &= 900\,000F^n - \frac{5\,720\,000}{3}F^n + \frac{5\,720\,000}{3} \\ &= F^n\left(900\,000 - \frac{5\,720\,000}{3}\right) + \frac{5\,720\,000}{3} \\ \therefore F^n &= \frac{\frac{5\,720\,000}{3}}{\frac{5\,720\,000}{3} - 900\,000} \end{aligned}$$

Applying the logarithm,

$$\begin{aligned} n\log\left(\frac{2\,603}{2\,600}\right) &= \log\left(\frac{\frac{5\,720\,000}{3}}{\frac{5\,720\,000}{3}-900\,000}\right) \\ n &= \frac{\log\left(\frac{\frac{5\,720\,000}{3}}{\frac{5\,720\,000}{3}-900\,000}\right)}{\log\left(\frac{2\,603}{2\,600}\right)} \\ &\approx 553.8696\cdots \text{ fortnights} \end{aligned}$$

Which is approximately 21.3 years.

	ii & iii) Do not leave out steps of working. Especially for part iii which is a SHOW question worth 1 mark. c)i) Many used the wrong value of n. ii) Wrong value of n used. Those who tried to memorise and use a formula to answer the question without showing the development of the pattern and did not write the GP sum obtained the wrong answer.
Q 16	a) The Simpson's and trapezoidal rule questions should be an easy 3 marks. Some students are still losing marks through poor calculator work and incorrect application of the formula. b i) when students see the word **deduce,** they need to justify their assertions. In this case stating the fact that 'the hypotenuses is always the longest side in a triangle' was required and led to a quick and simple answer. Some students successfully used Pythagoras, which required more work. c ii) Far too many students laboured with this question. The answer in gradient point form is easily arrived in one line as both the gradient m and a point on the line $(-1,0)$ are supplied in the question. Students need to carefully read and interpret the information they have been given. As a general note on trigonometry, students need to review their year 10 trigonometry and look for simple answers first. c iii) students found this question difficult. The required insight was the fact that points P and Q are the intersection points of line PQ and the unit circle which led to a substitution of the answer from c ii) into the equation of the unit circle which led to the answer with some straightforward algebra. Students who had not answered part (ii) could not arrive at the correct answer. Many students much time unsuccessfully attempting to work backwards, often substituting the x coordinate of Q to show that it satisfied the equation, however, this approach was futile as the same cannot be done for P whose x coordinate is clearly unknown at this point. Marks are generally not awarded for information from a later part of the question being used to answer an earlier part which leads to circular arguments. c iv) Students found the question hard. Students need to read and interpret questions carefully. All this question was asking was for students to solve the quadratic equation given in part iii) and recognise that the 2 solutions were points P and Q which was again explicitly stated in part iii). For full marks students needed to use the correct result from part ii). c v) Students found this question very hard. Many students wasted ink on a pointless regurgitation and circular manipulation the double angle formula from the reference sheet. The word **deduce** in a multi-part question is usually a strong signal to students that the previous parts of the question have been leading to the answer and need to be used. Students needed the correct answer from iv) for full marks.