Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:
(Please tick or highlight)
- O Mr Rezcallah
- O Mr Ireland
- O Mr Lowe
- O Mr Trenwith
- O ....

Student Number:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	/100

Question 1 (12 marks) **Marks**

(a) Evaluate $\dfrac{3.24^2 - 2.1^2}{\sqrt{36+2.1}}$ correct to 3 significant figures. 2

(b) Rationalise the denominator of $\dfrac{5}{3-\sqrt{7}}$ 2

(c) Solve $\frac{1}{3}(x-2)=\frac{1}{12}(1-3x)+4$. 2

(d) If $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$ 2

(e) Solve $|15-4x| \leq 3$ 2

(f) Find the sum of the first 15 terms of the series

$1+3+3^2+3^3+3^4+\ldots\ldots.$ 2

Question 2 (12 marks) Start a NEW page. **Marks**

(a) Differentiate with respect to x:

(i) $(2x^2+1)^8$ 2

(ii) $x^2 \ln x$ 2

(iii) $\dfrac{\sin x}{e^x}$ 2

(b) Find: $\int (\cos 2x + e^{5x})\, dx$ 2

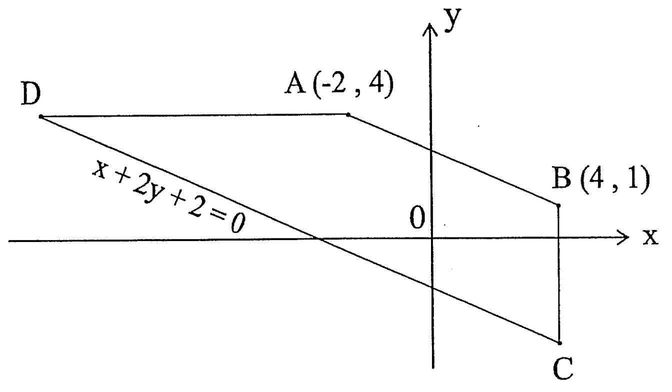
(c) Evaluate $\int_0^1 \dfrac{3}{x+1}\, dx$ 2

(d) Solve $2\sin\theta + 1 = 0$ for $0 \le \theta \le 2\pi$ 2

Question 3 (12 marks) Start a NEW page. **Marks**

(a) In the quadrilateral ABCD the coordinates of the points A and B are $(-2, 4)$ and $(4, 1)$ respectively.

The equation of the line DC is $x + 2y + 2 = 0$.



NOT TO SCALE

(i) Find the gradients of AB and DC. Hence, explain why the quadrilateral ABCD is a trapezium. 3

(ii) Find the length of AB in exact form. 2

(iii) The line BC is parallel to the y axis, find the coordinates of point C. 1

(iv) Find the perpendicular distance from A to the line DC. 2

(v) If the length of DC is $7\sqrt{5}$ units, find the area of the trapezium ABCD. 2

(b) An infinite geometric series has a limiting sum of 3. If the first term of this series is equal to the common ratio, find the first term of this series. 2

Question 4 (12 marks) Start a NEW page. **Marks**

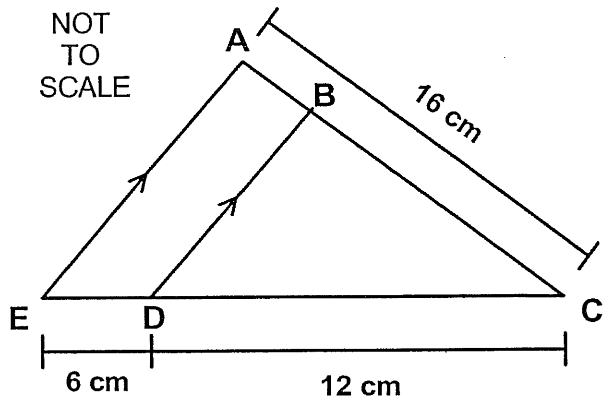
(a) Given that $\log_a m = 4.5$ and $\log_a n = 1.5$, find the value of:

(i) $\log_a \left(\frac{n}{m}\right)$ 1

(ii) $\log_a (mn)^2$ 2

(b) Find the equation of the normal to the curve $y = \log_e x - 1$ at the point $(e, 0)$. 2

(c)



The diagram above shows a triangle ACE. AE is parallel to BD, AC = 16cm, CD = 12 cm and DE = 6 cm.

(i) Prove that ΔACE is similar to ΔBCD. 2

(ii) Hence, or otherwise, find the length of AB. 2

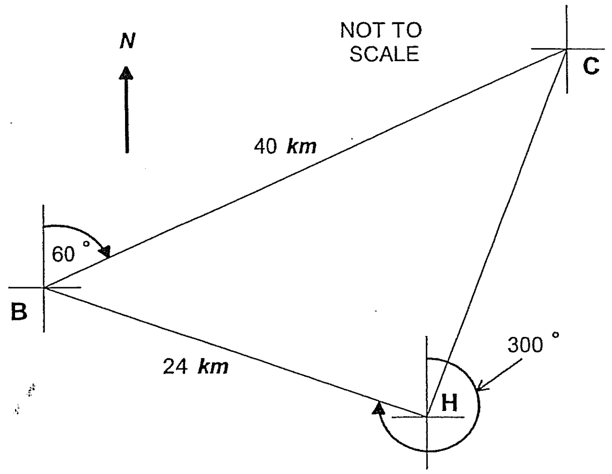
(d) Consider the parabola $y^2 = 8x + 16$.

(i) Find the coordinates of the vertex. 2

(ii) Find the equation of the directrix. 1

Question 5 (12 marks) Start a NEW page. **Marks**

(a)



Mona left home (**H**) and travelled for 24 km to **B** on a bearing of $300^o T$.
She then travelled for 40 km to **C** on a bearing of $60^o T$.

Copy the diagram in your solution booklet.

(i) Show that $\angle CBH = 60^0$. 1

(ii) Use the Cosine Rule to show that the length of CH=34.87 km. 2

(iii) Find the bearing of **H** from **C**. Leave your answer to the nearest minute. 2

(b) David was training for his school marathon. On the first day, he ran 1250m, on the second day he ran 1340m, and on each of the following days the distances he ran continued to increase by the same amount.

(i) What distance did he run on the 10^{th} day? 2

(ii) What is the total distance he ran in the first 10 days? 1

(iii) On which day did the distance he ran first exceed 2.5km? 2

(c) Find the values of *k* for which the quadratic equation $2x^2 - kx + 5 = 0$ has real roots. 2

Question 6 (12 marks) Start a NEW page. **Marks**

(a) Consider the curve $y = x^3 - 6x^2 + 5$

(i) Determine the coordinates of any stationary points and determine their nature. 4

(ii) Find the coordinates of the point of inflexion. 2

(iii) Sketch the curve $y = x^3 - 6x^2 + 5$ 2

(iv) For what values of x is the curve $y = x^3 - 6x^2 + 5$ concave down? 1

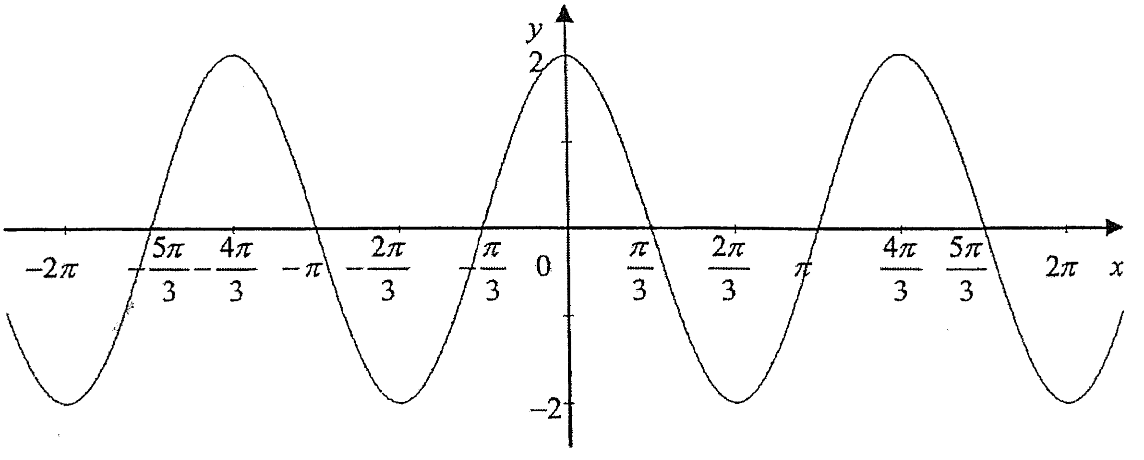
(b) The displacement x metres of a particle moving in a straight line at time t seconds is given by $x = 2t - 4\ log_e(2t + 1)$

(i) Find the initial velocity of the particle. 2

(ii) Show that the acceleration of the particle is always positive. 1

Question 7 (12 marks) Start a NEW page. **Marks**

(a)



The graph above can be represented by an equation in the form $y = a \cos nx$. Find the values of a and n. 2

(b) A circle has radius 12 cm. Find the area of a sector of this circle that subtends an angle at the centre of $\frac{4\pi}{3}$. 2

(c) Is the following series an arithmetic or geometric progression?

$\ln(x) + \ln(x^2) + \ln(x^3) + \ln(x^4) + \ldots\ldots$

Justify your answer. 2

(d) Solve the equation $9^x - 10(3^x) + 9 = 0$ 3

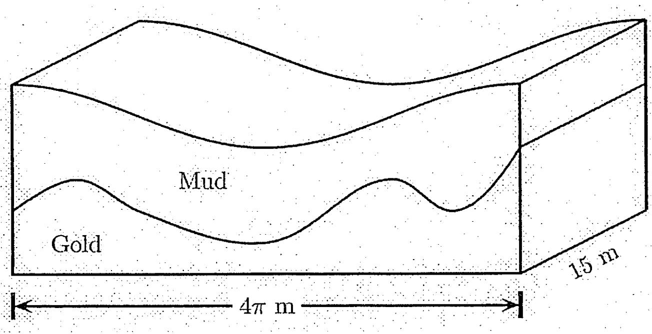
(e) (i) Differentiate $\sin(x^2)$ 1

(ii) Use this result to find the exact area bounded by $y = x\cos(x^2)$, the x-axis and the lines $x = 0$ and $x = 1$. 2

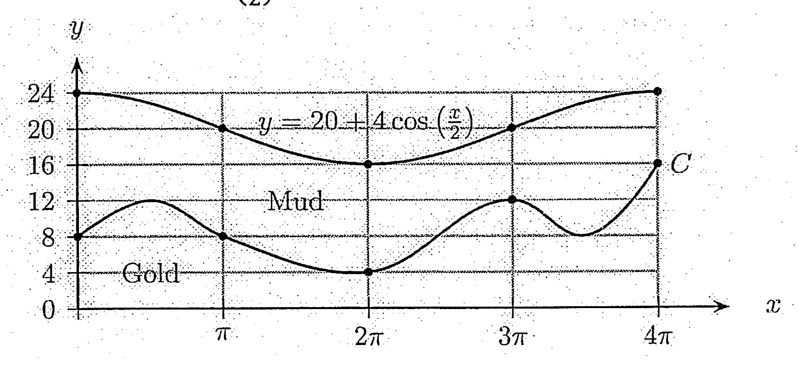
Question 8 (12 marks) Start a NEW page. **Marks**

(a) The volume V cm^3 of a balloon is increasing such that its volume at any time t seconds is given by $V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$. Find the rate at which the volume is increasing when $t = 2$ seconds. 2

(b) The diagram below shows an amount of gold, which is in the shape of a prism underneath a large amount of mud. The width of the prism is 4π metres and its length is 15 metres.



The graph below shows the cross-section of the prism. The top of the mud is given by the function $y = 20 + 4\cos\left(\frac{x}{2}\right)$ and the top of the gold is shown by the curve C.



(i) Find, by integration, the total area of the cross-section, i.e. the area of both the mud and gold. 2

(ii) Using Simpson's Rule with the five function values shown on the graph, Find an estimate for the area of the cross-section of the gold. 3

(iii) Find the volume of the mud. 1

(c) A coin is tossed four times. Find the probability that:

(i) the first three tosses are heads 2

(ii) there are at least three heads in the four tosses. 2

Question 9 (12 marks) Start a NEW page. **Marks**

(a) A population of 100 birds was introduced at the start of 2004, on an enclosed reserve where no natural predators exist. Two years later the population had grown to 312.
The equation that is believed to best model this growth process is given by $N(t) = N_o e^{kt}$, where $N(t)$ represents the number of birds present at time t, and t is the number of years since the introduction of the birds on the reserve.

(i) What does N_o represent? State its value. 2

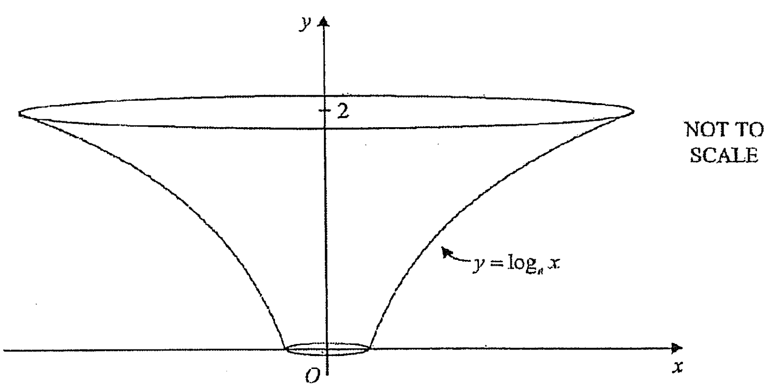
(ii) Show that $k = 0.5689$ (to 4 decimal places). 2

(iii) Find the time it takes for the population to double. 2

(iv) Find the number of birds that will be on the reserve at the end of 2009. 1

(b) If α, β are the roots of $3x^2 - 4x - 7 = 0$, find the value of $\alpha^2 + \beta^2$. 2

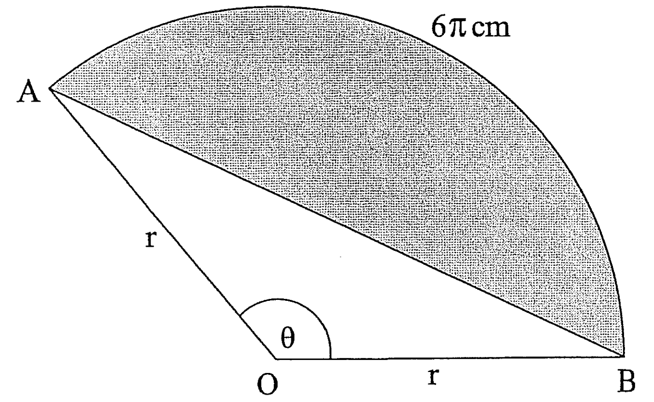
(c)



A mould for a vase is formed by rotating that part of the curve $y = \log_e x$ between $y = 0$ and $y = 2$ about the y axis.
Find the volume of the mould. Leave your answer in simplest exact form. 3

Question 10 (12 marks) Start a NEW page. **Marks**

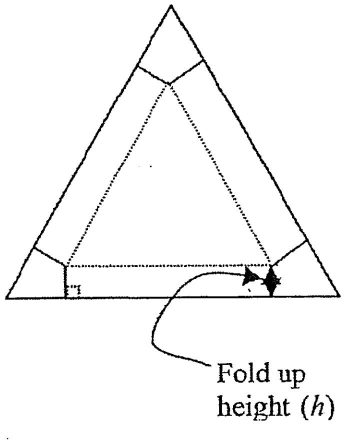
(a) The diagram shows a sector with angle θ at the centre and radius r cm. The arc length is 6π cm.

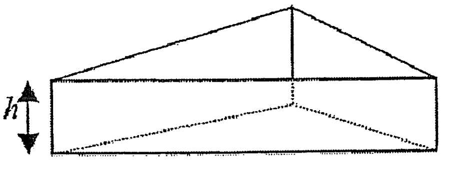


Calculate the area of the shaded minor segment when $\theta = \frac{3\pi}{4}$. 3

(b) The Logs are a baseball team and the Sectors are a soccer team. A total of 18 people are members of one or the other or both teams. The soccer team has 14 members and the baseball team has 10 members. A player is selected at random from the baseball team.
What is the probability that the player is **not** in the soccer team? 2

(c) A piece of paper in the shape of an equilateral triangle with edge length 20cm is to be used to make an open-ended box. Quadrilateral shapes are cut out of the comers and the sides folded up in the manner shown.



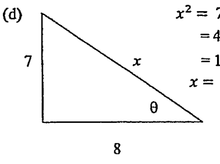


(i) Show that the side of the equilateral triangle base is $20 - 2h\sqrt{3}$ 2

(ii) Prove that the volume of the box is $V = h\sqrt{3}(10 - h\sqrt{3})^2$. 2

(iii) Find the height of the box that will produce the maximum volume. 3

END OF PAPER

Solutions	Marks/Comments
Question 1 (Marked by Mr Lowe)	
(a) $\frac{3\cdot24^2-2\cdot1^2}{\sqrt{36+2\cdot1}}=0\cdot986242288$ $=0\cdot986$	(a) ✓✓ for correct rounded answer
(b) $\frac{5}{3-\sqrt7}=\frac{5}{3-\sqrt7}\times\frac{3+\sqrt7}{3+\sqrt7}=\frac{5(3+\sqrt7)}{2}$	(b) ✓for correct method ✓for correct answer
(c) $\frac{1}{3}(x-2)=\frac{1}{12}(1-3x)+4$ $4(x-2)=1-3x+48$ $4x-8=49-3x$ $7x=57$ $x=8\frac{1}{7}$	(c) ✓for multiplying every term by 12 (if that is not done no marks) ✓for correct answer
(d)  $x^2=7^2+8^2$ $=49+64$ $=113$ $x=\sqrt{113}$ Since $\tan\theta=\frac{7}{8}$ and $\cos\theta<0$ 3rd Quadrant $\therefore \operatorname{cosec}\theta<0$ $\therefore \operatorname{cosec}\theta=-\frac{\sqrt{113}}{7}$	(d) ✓for correct x ✓for correct answer
(e) $\lvert 15-4x\rvert\le3$ $-3\le 15-4x\le3$ $-18\le -4x\le -12$ $3\le x\le4.5$ $x\ge3$ and $x\le4\frac{1}{2}$	(e) 2 Marks for correct answer (If 2 correct inequalities written, then the word AND should be used)
(f) G.S of a=1 and r=3 $S_{15}=\frac{1(3^{15}-1)}{(3-1)}$ $S_{15}=7174453$	✓for correct formula ✓for correct answer

Solutions	Marks/Comments
Question 2 (Marked by Mr Lowe)	
(a) (i) $\frac{d}{dx}(2x^2+1)^8$ Let $u=2x^2+1$, then $\frac{d}{dx}(2x^2+1)^8=\frac{d}{du}u^8\times\frac{d}{dx}(2x^2+1)$ $=8(2x^2+1)^7\times4x$ $=32x(2x^2+1)^7$	(i) ✓for $8(2x^2+1)^7$ or correct start ✓for correct answer
(ii) $\frac{d}{dx}x^2\ln x=x^2\frac{1}{x}+\ln x\times2x=x(1+2\ln x)$	(ii) ✓for product rule ✓ for simplification.
(iii) $\frac{d}{dx}\left[\frac{\sin x}{e^x}\right]=\frac{(e^x)(\cos x)-(\sin x)(e^x)}{(e^x)^2}$ $=\frac{e^x[\cos x-\sin x]}{(e^x)^2}$ $=\frac{[\cos x-\sin x]}{e^x}$	(iii) ✓for quotient rule or correct product rule if changed ✓for correct answer
(b) $\frac{\sin 2x}{2}+\frac{e^{5x}}{5}+C$	(b)✓ ✓ 1 mark for each part including +C
(c) $\int_0^1\frac{3}{x+1}dx=[3\ln(x+1)]_0^1=3\ln2-3\ln1=3\ln2$	(c) ✓for correct integration ✓for correct answer
(d) $2\sin\theta+1=0$ $2\sin\theta=-1$ $\sin\theta=-\frac{1}{2}$ $\theta=\frac{7\pi}{6},\frac{11\pi}{6}$	(d) ✓ Correct rearrangement of trigonometric equation and one correct solution. OR ✓ ✓ Two correct solutions in radians. (Award ✓ for degrees)

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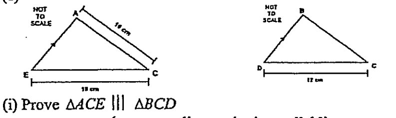
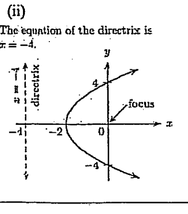
Solutions	Marks/Comments
Question 3 (Marked by Mr Rezcallah)	
(a) (i) $m_{AB}=\frac{1-4}{4+2}=-\frac{3}{6}=-\frac{1}{2}$ $m_{DC}=-\frac{A}{B}=-\frac{1}{2}=m_{AB}$ ∴ one pair of opposite sides are parallel, and so $ABCD$ is a trapezium.	(i) ✓for correct m_{DC} ✓for correct m_{AB} ✓for correct reason
(ii) $d_{AB}=\sqrt{(4+2)^2+(1-4)^2}$ $=\sqrt{36+9}=\sqrt{45}=3\sqrt5$ units	(ii) ✓for correct substituted formula ✓for correct exact answer
(iii) At C, $x=4$. $x+2y+2\rvert_{x=4}=0$ $4+2y+2=0$ $2y=-6$ $y=-3$ $\therefore C(4,-3)$	(iii) ✓for correct answer
(iv) $d_\perp=\frac{\lvert1(-2)+2(4)+2\rvert}{\sqrt{1^2+2^2}}$ $=\frac{\lvert8\rvert}{\sqrt5}=\frac{8\sqrt5}{5}$ units N.B: Students who find the intersection of the Perp. should get x=-18/5, y=4/5 to get the first mark, then the second mark is for the correct answer.	(iv) ✓for correct substituted formula ✓for correct answer (wrong formula no marks)
(v) $A=\frac{1}{2}\times\frac{8}{\sqrt5}\times(3\sqrt5+7\sqrt5)$ $=10\sqrt5\times\frac{4}{\sqrt5}=40$ units²	(v) ✓for correct area ✓for correct answer
(b) $S=\frac{a}{1-r}$ When $a=r$ $3=\frac{a}{1-a}$ $3-3a=a$ $4a=3$ $a=\frac{3}{4}$	(b) ✓for correct method ✓for correct answer

Solutions	Marks/Comments
Question 4 (Marked by Mr Ireland)	
(a) (i) $\log_a\left(\frac{n}{m}\right)=\log_a n-\log_a m=1.5-4.5=-3.$ (ii) $\log_a(mn)^2=2\log_a m+2\log_a n$ $=2(4.5)+2(1.5)=12$	(a) (i)✓for -3 (ii) ✓for correct method ✓ for 12
(b) $y=\log_e x-1$ $\frac{dy}{dx}=\frac{1}{x}$ at $x=e$, $m_t=\frac{1}{e}$ $m_n=-e$ (Gradient of normal) equation of normal at point $(e,0)$ $y-0=-e(x-e)$ $y=e^2-ex$	(b) ✓ Correct differentiation and gradient of tangent in terms of e ✓ Correct gradient of normal and equation of normal that doesn't include x^2
(c) 	(c)
(i) Prove $\Delta ACE \parallel\!\mid \Delta BCD$ $\angle EAC=\angle DBC$ (corresponding angles in parallel lines are equal) $\angle AEC=\angle BDC$ (corresponding angles in parallel lines are equal) $\angle C$ is common $\therefore \Delta ACE \parallel\!\mid \Delta BCD$ (equiangular or AAA)	(i) ✓ correct terminology mentioning parallel lines and corresponding angles ✓ uses correct test to prove similarity
(ii) $\frac{BC}{16}=\frac{12}{18}$ (corresponding sides in similar triangles are in ratio) $BC=10\frac{2}{3}$ $\therefore AB=5\frac{1}{3}$	(ii) ✓ Uses ratios to find the length of BC ✓ Correct answer
(d) (i) $y^2=8(x+2)$. The coordinates of the vertex V are $(-2,0)$ The focal length is $4a=8$, i.e. $a=2$.	(d) (i) ✓ for V(-2,0) ✓ for a=2
(ii) The equation of the directrix is $x=-4$. 	(ii) ✓ for x=-4

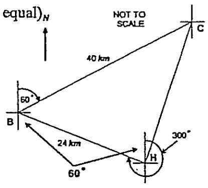
Solutions	Marks/Comments

Question 5 (Marked by Mr Trenwith)

(a) (i) From the diagram:
Use (alternate angles in parallel lines are equal)

$\angle CBH = 180^0 - (60^0 + 60^0)$

$\therefore \angle CBH = 60^0$



(ii)
$CH^2 = 24^2 + 40^2 - (2\times 24\times 40\times \cos 60^o)$

$CH = 34.87\ km$

(iii) Let $\angle BCH = \theta^0$

$\therefore \frac{\sin\theta}{24} = \frac{\sin 60}{34.87}$

$\sin\theta = \frac{24\sin 60}{34.87}$

$\therefore \theta = 36^0 35'$ *(nearest minute)*

$\therefore$ The bearing of H from $C = 270^0 - (30^0 + 36^0 35')$
$= 203^0 25' T$

(b)(i)

This can be represented by an arithmetic sequence with

$a = 1\,250 \quad d = 1\,340 - 1\,250 = 90$

$T_{10} = a + 9d$
$= 1\,250 + 9(90)$
$= 2\,060\text{ m}$

(ii)

$S_{10} = \frac{10}{2}(a + \ell) = 5(1\,250 + 2\,060)$
$= 16\,550\text{ m}$

(iii)

$T_n > 2\,500$
$\therefore 1\,250 + (n-1)\times 90 > 2\,500$
$1\,250 + 90n - 90 > 2\,500$
$90n > 1\,340$
$n > 14.8$

David will first run more than 2.5km on the 15th day.

(c)
For real roots $\Delta \ge 0$.

$\Delta = k^2 - 4\times 2\times 5$

$k^2 - 40 \ge 0$

$k^2 \ge 40$

$k \le -2\sqrt{10}$ or $k \ge 2\sqrt{10}$

Marks/Comments:

(a)
(i)✓for correct reasons

(ii)
✓✓ for correct substitution into the cosine rule

(iii)
✓ for correct value for θ

✓ for correct bearing to the nearest minute

(b)
(i)
✓for correct method

✓for 2060 m

(ii) ✓for 16 550 m

(iii)
✓for correct method

✓for 2.5km

(c)
✓for correct quad inequality with/without one answer

✓ for both answers

Solutions	Marks/Comments

Question 6 (Marked by Mr Rezcallah)

(i) $y' = 3x^2 - 12x$

For stationary points, $y' = 0$.

$3x(x-4) = 0$

$x = 0$ or 4

$(0, 5)$ and $(4, -27)$

$y'' = 6x - 12$

$f''(0) = -12 \therefore$ concave down

$(0, 5)$ is a relative maximum

$f''(4) = +12 \therefore$ concave up

$(4, -27)$ is a relative minimum

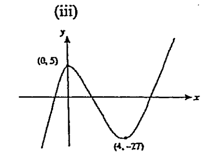
(ii)

$y'' = 6x - 12$

$6x - 12 = 0$

$x = 2$

(iii)



Test inflexion : $x < 2,\ y'' < 0$, concave down
$x > 2,\ y'' > 0$, concave up

(iv)
The curve is concave down for $x < 2$.

(b) (i)

$$\frac{dx}{dt} = v(t) = 2 - \frac{4\cdot 2}{2t+1} = 2 - \frac{8}{2t+1}$$

$$v(0) = 2 - \frac{8}{1} = -6\ \text{ms}^{-1}$$

(ii)

$$v(t) = 2 - \frac{8}{2t+1} = 2 - 8(2t+1)^{-1}$$

Using the chain rule,

$$\frac{dv}{dt} = a(t) = -8\cdot 2\cdot -1\cdot (2t+1)^{-2}$$
$$= \frac{16}{(2t+1)^2}$$

$\frac{16}{(2t+1)^2} > 0\ \forall t$

Marks/Comments:

(i)
✓for correct y'=0

✓for correct points including y values

✓ for testing each fully including concavity

✓ for nature of points

(ii)

✓for x=2, y=-11

✓ for testing inflexion

(iii) ✓✓for correct curve showing that it cuts x axis twice

(iv) ✓ for $x < 2$

(b)

(i) ✓for correct v

✓for -6 m/s (no unit no mark)

ii) ✓for correct $a(t)$ and nothing else.

Solutions	Marks/Comments

Question 7 (Marked by Mr Lowe)

(a)
From graph $a = 2$

Period $= \frac{4\pi}{3}$

Period $= \frac{2\pi}{n}$

$\frac{2\pi}{n} = \frac{4\pi}{3}$

$n = \frac{3}{2}$

(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(12^2)\frac{4\pi}{3}$

$A = 96\pi\ cm^2 = 301.59\ cm^2$

(c) Test for GP:

$\frac{\ln x^2}{\ln x} = \frac{2\ln x}{\ln x} = 2$

$\frac{\ln x^3}{\ln x^2} = \frac{3\ln x}{2\ln x} = \frac{3}{2} \therefore$ not GP

Test for AP: showing $d = \ln x$

$\ln x^2 - \ln x = 2\ln x - \ln x = \ln x$

$\ln x^3 - \ln x^2 = 3\ln x - 2\ln x = \ln x$

$\therefore$ AP

(d)
$9^x - 10(3^x) + 9 = 0$

$3^{2x} - 10(3^x) + 9 = 0$

$(3^x)^2 - 10(3^x) + 9 = 0$

$v^2 - 10v + 9 = 0$ where $v = 3^x$

$(v-9)(v-1) = 0$

$v = 3^x = 9 \to x = 2$

$v = 3^x = 1 \to x = 0$

(e) (i) $2x\cos(x^2)$

(ii) $A = \int_0^1 x\cos(x^2)dx = \frac{1}{2}\int_0^1 2x\cos(x^2)dx$

$= \frac{1}{2}\left[\sin(x^2)\right]_0^1 = \frac{1}{2}\left[\sin 1 - \sin 0\right]$

$= \frac{1}{2}\left[\sin 1\right] = 0.42\ u^2$

Marks/Comments:

(a)

✓correct value of a

✓correct value of n

(b) ✓correct method

✓correct answer (exact or rounded)

(c)

✓ for AP

✓ for justification of why AP showing the common difference clearly

(d)

✓ for quadratic equation,

✓ for correctly factoring and getting one value of x
✓ for other value of x

(e) (i) ✓ for correct derivative

(ii) ✓ for correct area integral

✓ for correct answer

Solutions	Marks/Comments

Question 8 (Marked by Mr Ireland)

(a)

$V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$

$\frac{dv}{dt} = \pi t^2 - \frac{\pi t}{3}$

when $t = 2$

$\frac{dv}{dt} = 4\pi - \frac{2\pi}{3}$

$= \frac{10\pi}{3} cm^3/s$ or $10.472\ cm^3/s$

(b) (i)

$$A_{\text{mud + gold}} = \int_0^{4\pi} 20 + 4\cos\frac{x}{2}\,dx$$
$$= \left[20x + 8\sin\frac{x}{2}\right]_0^{4\pi}$$
$$= 20\times 4\pi + 8\sin(2\pi)$$
$$= 80\pi\ \text{m}^2$$

(ii)

x	0	π	2π	3π	4π
y	8	8	4	12	16

$$A_{\text{gold}} \approx \frac{h}{3}(y_1 + 4\sum y_{\text{even}} + 2\sum y_{\text{odd}} + y_\ell)$$
$$= \frac{\pi}{3}(8 + 4(8+12) + 2(4) + 16)$$
$$= \frac{112\pi}{3}$$

(iii)

$$A_{\text{gold + mud}} = 80\pi \quad A_{\text{gold}} = \frac{112\pi}{3}$$
$$\therefore A_{\text{mud}} = \left(80 - \tfrac{112}{3}\right)\pi = \frac{128\pi}{3}\ \text{m}^2$$
$$V_{\text{mud}} = \frac{128\pi}{3}\times 15 = 640\pi\ \text{m}^3$$

(c) (i)

$\mathcal{P}$(the first three coins are heads) $= \mathcal{P}(\text{HHHH}) + \mathcal{P}(\text{HHHT})$
(notice that the two events HHHH and HHHT are mutually exclusive)
$= \frac{1}{16} + \frac{1}{16}$
(since each of these two probabilities is $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$)
$= \frac{1}{8}$.

(ii)
$\mathcal{P}$(at least 3 heads) $= \mathcal{P}(\text{HHHH}) + \mathcal{P}(\text{HHHT}) + \mathcal{P}(\text{HHTH}) + \mathcal{P}(\text{HTHH}) + \mathcal{P}(\text{THH}$
$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$
$= \frac{5}{16}$.

Marks/Comments:

(a)

✓for correct rate

✓for correct answer

(b) (i)

✓for correct integral

✓ for correct answer (accept 251.33)

(ii)

✓for correct start

✓for correct rule application

✓for correct answer

(iii)

✓for correct answer (accept 2010.61)

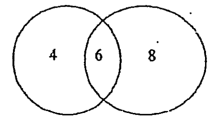
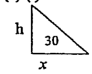
(c)(i)

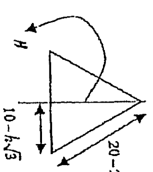
✓for correct method

✓for correct answer (award ✓✓ for correct answer only)

(ii)
✓for correct working or tree diagram
✓for correct answer (answer only 1 mark)

Solutions	Marks/Comments
Question 9 (Marked by Mr Rezcallah)	
(a) (i) N_o is the initial value $N_o = 100$	(a) (i) ✓for stating initial value ✓for 100
(ii) $N(t) = N_o e^{kt}$ $\Rightarrow 312 = 100e^{2k} \Rightarrow$ $78/25 = 3.12 = e^{2k}$ $\ln 3.12 = 2k \Rightarrow k = (\ln 3.12)/2$ $\Rightarrow k = 0.5689$	(ii) ✓for equation ✓ $k = (\ln 3.12)/2$
(iii) $200 = 100e^{0.5689165t} \Rightarrow 2 = e^{0.5689165t}$ $t = \ln 2/(0.5689165) = 1.218$ years or 14.4 months	(iii) ✓for correct doubling of population ✓for correct answer (3.22 gets no marks)
(iv) n=6 N= $100e^{(\ln 3.12)(3)} = 3037$ birds	(iv) ✓for 3037 only
(b) $\alpha + \beta = -\frac{b}{a} = -\frac{-4}{3} = \frac{4}{3}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-4}{3}\right)^2 - 2\left(\frac{-7}{3}\right) = \frac{58}{9}$	(b) ✓for correct method ✓for correct answer
(c) $x = e^y$ $V = \pi \int_0^2 e^{2y}\,dy$ $= \frac{\pi}{2}\left[e^{2y}\right]_0^2$ $= \frac{\pi}{2}\left[e^4 - e^0\right]$ $= \frac{\pi}{2}\left(e^4 - 1\right)$ units ^{3}	(c) ✓ correct volume integral ✓correct integration ✓for correct answer

Solutions	Marks/Comments
Question 10 (Marked by Mr Trenwith)	(a)
(a) Area of the minor segment: $\theta = \frac{3\pi}{4}$ $6\pi = r \times \frac{3\pi}{4}$ $r = 8\text{cm}$ $A = \frac{1}{2}r^2(\theta - \sin\theta)\Big\|_{\theta=\frac{3\pi}{4}}$ $= \frac{1}{2}8^2\left(\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$ $= 32\left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}}\right)$ cm^2	✓for correct r ✓for correct method ✓for correct answer (accept 52.77 cm^2)
(b)  $P = \frac{4}{10} = 0.4$	(b) ✓for correct Venn diagram ✓for correct answer (✓✓ for correct answer)
(c) (i)  $\tan 30 = \frac{h}{x}$ $\therefore x = \frac{h}{\tan 30} = \frac{h}{\frac{1}{\sqrt{3}}} = h\sqrt{3}$ $\therefore side = 20 - 2h\sqrt{3}$	(c) (i) ✓✓for the method of showing the side
(ii) (ii) $area = \frac{1}{2}(20 - 2h\sqrt{3})(20 - 2h\sqrt{3})\sin 60$ $= \frac{1}{2}(400 - 80h\sqrt{3} + 12h^2)\frac{\sqrt{3}}{2}$ $= (100 - 20h\sqrt{3} + 3h^2)\sqrt{3}$ $= (10 - h\sqrt{3})^2\sqrt{3}$ $Volume = h(10 - h\sqrt{3})^2\sqrt{3}$	(ii) ✓✓ for a complete derivation of the given expression

Solutions	Marks/Comments
Question 10 Cont'd	
OR  $H^2 = \left(2(10 - h\sqrt{3})\right)^2 - (10 - h\sqrt{3})^2$ $H^2 = 4(10 - h\sqrt{3})^2 - (10 - h\sqrt{3})^2 = 3(10 - h\sqrt{3})^2$ $\therefore H = \sqrt{3}(10 - h\sqrt{3})$ Surface area of equilateral triangular face: $\frac{1}{2} \times 2(10 - h\sqrt{3}) \times \sqrt{3}(10 - h\sqrt{3})$ $= \sqrt{3}(10 - h\sqrt{3})^2$ Volume: $h \times \sqrt{3}(10 - h\sqrt{3})^2 = h\sqrt{3}(10 - h\sqrt{3})^2$ General expression: $Volume = h\sqrt{3}(10 - h\sqrt{3})^2$.	(c)
(c) $V = h(100 - 20h\sqrt{3} + 3h^2)\sqrt{3} = (100h - 20h^2\sqrt{3} + 3h^3)\sqrt{3}$ $\frac{dV}{dh} = (100 - 40h\sqrt{3} + 9h^2)\sqrt{3} = 0$	✓ correct derivative
$h = \frac{40\sqrt{3} \pm \sqrt{4800 - 4(900)}}{18} = \frac{40\sqrt{3} \pm \sqrt{1200}}{18}$ $h = \frac{40\sqrt{3} \pm 20\sqrt{3}}{18}$ $h = \frac{60\sqrt{3}}{18}, \frac{20\sqrt{3}}{18} = \frac{10\sqrt{3}}{3}, \frac{10\sqrt{3}}{9}$ $\frac{d^2V}{dh^2} = (-40\sqrt{3} + 18h)\sqrt{3}$	✓correct heights
At h $= \frac{10\sqrt{3}}{9} = 1.92$ cm, $\frac{d^2V}{dh^2} = \left(-40\sqrt{3} + \frac{180\sqrt{3}}{9}\right)\sqrt{3}$ $\frac{d^2V}{dh^2} = (-40\sqrt{3} + 20\sqrt{3})\sqrt{3} = -60 < 0$ Max Volume at h $= \frac{10\sqrt{3}}{9} = 1.92$ cm	✓for correct answer after testing it