Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2010 HSC ASSESSMENT TASK 3 (TRIAL HSC)

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Fletcher
- O Mr Ireland
- O Mr Lowe
- O Mr Rezcallah
- O ....

Student Number:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	/100

Question 1 (12 marks)

i) Simplify $2\sqrt{11} \times 5\sqrt{11}$ **(2)**

ii) Solve correct to 2 decimal places $x^2 - 3x - 7 = 0$ **(3)**

iii) If $f(x) = \begin{cases} x - 2 & for\ x < 0 \\ x^2 & for\ x \geq 0 \end{cases}$

Evaluate f (-1) + f (0) + f (2) **(2)**

iv) Give the exact value of

a) $Sin\ \frac{7\pi}{6}$ **(1)**

b) $log_2 1$ **(1)**

v) A rectangle has sides of $\frac{1}{\sqrt{2}+1}\ cm\ and\ \frac{1}{\sqrt{2}-1} cm.$ Find **(3)**

a) its area.

b) its perimeter in simplest surd form.

Question 2 (12 marks) **Start a new page.**

i) Differentiate with respect to x **(8)**

a) $\frac{4x^2-3x+2}{x}$

b) e^{3x^2-5}

c) x^3e^{-x}

d) $\ln(\sin x)$

ii) Explain, using calculus, why the curve $y = (3x - 2)^4$, has no points of inflexion. **(2)**

iii) Solve for x, $|3 - 2x| \leq 5$ **(2)**

Question 3 (12 marks) **Start a new page.**

i) What are the period and amplitude of the curve of $y = 3\cos 2x$?

Sketch the curve for $0 \le x \le 2\pi$ **(4)**

ii) Write down a quadratic equation which is negative within

the interval $-1 < x < 2$ and non negative for all other values of x. **(2)**

iii) Find a primitive of **(6)**

a) $\dfrac{2x}{x^2+1}$

b) $\dfrac{2}{x} + 5e^x$

c) $\dfrac{e^{2x}}{e^{2x}+4}$

Question 4 (12 marks) **Start a new page.**

i) The volume $V(cm^3)$ of water remaining in a leaking bucket

after t seconds is $V = 2000 - 40t + 0.2t^2$ **(3)**

a) How much water was there initially in the bucket ?

b) How long does it take to empty the bucket ?

c) How fast is the volume of water in the bucket decreasing when t=30 ?

ii) Find the value of **(2)**

$$8 - 4 + 2 - 1 + 0.5 - 0.25 + \ldots\ldots\ldots$$

iii) Use Simpson's rule with five function values to estimate the area

bounded by the curve $y = \sqrt{x+2}$, the x axis and the lines $x = 1$ and $x = -1$,

correct to 2 decimal places **(3)**

iv) Simplify $\dfrac{sin^2 30^0}{1 - sin^2 30^0}$ **(2)**

v) Find the coordinates of the point on the curve $y = 2x^2 + 4x - 11$

where the tangent is parallel to the line $y = 8x + 9$ **(2)**

Question 5 (12 marks) Start a new page.

i) Find the equation of the normal to the curve $y = x\sin x$ where $x = \dfrac{\pi}{2}$ **(3)**

ii) For the quadratic equation $x^2 - (2+k)x + 3k = 0$ **(2)**

Find the value of k

a) If the sum of the roots is 5.

b) If the product of the roots is 12.

iii) Given that $sin\theta = \frac{3}{4}$ *and* θ *is acute*, find the value of **(3)**

a) $tan\theta$

b) $sec\theta$

c) $cosec^2\theta - cot^2\theta$

iv) For the parabola $y = 5 + 2x - x^2$, find: **(4)**

a) The equation of the axis of symmetry

b) The co-ordinates of the vertex

c) The focal length

d) Sketch the parabola

Question 6 (12 marks) Start a new page.

i) What is the equation of the line through (-1,3) and perpendicular to the line $5x - 2y - 3 = 0$? **(2)**

ii) For $y = 2x^3 - 3x^2 - 12x + 2$ **(6)**

a) Find all stationary points and determine their nature.

b) Find point(s) of inflexion

c) Sketch the curve

iii) Find the volume generated when the area between the curve $y = x^3$ and the line $y = x$, where $x \geq 0$, is rotated around the x axis. **(4)**

Question 7 (12 marks) Start a new page.

i) Inflation is still increasing, but various policies to reduce inflation over the last few years have had an effect. Draw a graph to represent this information with, **I**, inflation against , **t** , time. **(2)**

ii) Show that $\frac{\cos\alpha}{1+sin\alpha} + \frac{1+sin\alpha}{cos\alpha} = \frac{2}{cos\alpha}$ **(2)**

iii) Draw the graph of $y = e^{-x}$. By drawing another graph on the same set of axes, show that $f(x) = e^{-x} - x + 1$ has exactly one root. **(2)**

iv) If A is the point (1,3) and B is the point (5,6) and P is the point (x,y) **(4)**

a) Find the lengths PA and PB in terms of x and y.

b) Find the locus of points whose distance from A is double its distance from B.

v) The p^{th} term of a certain geometric progression is c times the first term.

Show that the common ratio, r, is given by $\log r = \frac{\log c}{p-1}$ **(2)**

Question 8 (12 marks) Start a new page.

i) The series 7 + 13 + 31+ .... has as its n^{th} term $3^n + 4$ **(3)**

Write down a) the seventh term

b) The sum of the first n terms

ii) Prove that the line $4x - y - 8 = 0$ is a tangent to the curve $x^2 = 2y$ and find the point of contact . **(3)**

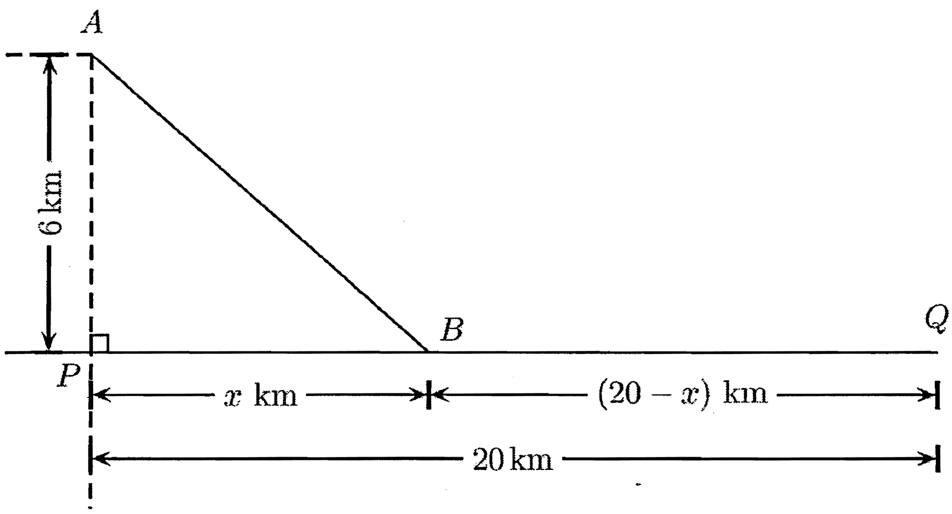
iii) Solve the following where $0 \leq x \leq 2\pi$ **(6)**

a) $(2\cos x + 1)(\cos x - 1) = 0$

b) $3\, tan^2 x = 1$

c) $2\sin\left(x - \frac{\pi}{3}\right) = -1$

Question 9 (12 marks) Start a new page.



i) **(6)**

A courier on a bike is in open country at the point A and is 6 km from the nearest point P on a straight sealed road. He wishes to get as quickly as possible to a point Q on the sealed road 20 km from P. If the maximum speed across open country is 40 km/hr and along the sealed road is 50 km / hr,

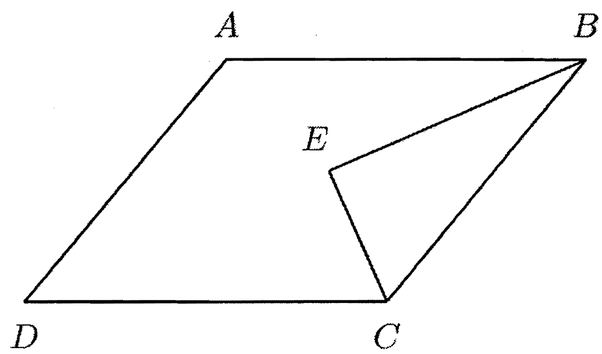
a) Write an expression for the times taken to traverse AB and BQ, if B is a point on the sealed road between P and Q.

b) Write an expression for the total time, T, for the journey

c) Find the distance from P that he should find the road.

ii) Find the volume of revolution if the curve $y = \sec x$ from $x = 0$ *to* $x = \frac{\pi}{4}$ is rotated about the x axis. **(3)**

iii) In the attached figure ABCD, not drawn to scale, is a parallelogram. EB bisects $\angle ABC$, $\angle ADC = 64^0$ and EC is perpendicular to DC.



a) Draw a diagram showing all the information given.

b) Calculate the size of $\angle BEC$, giving reasons.

(3)

Question 10 (12 marks) Start a new page.

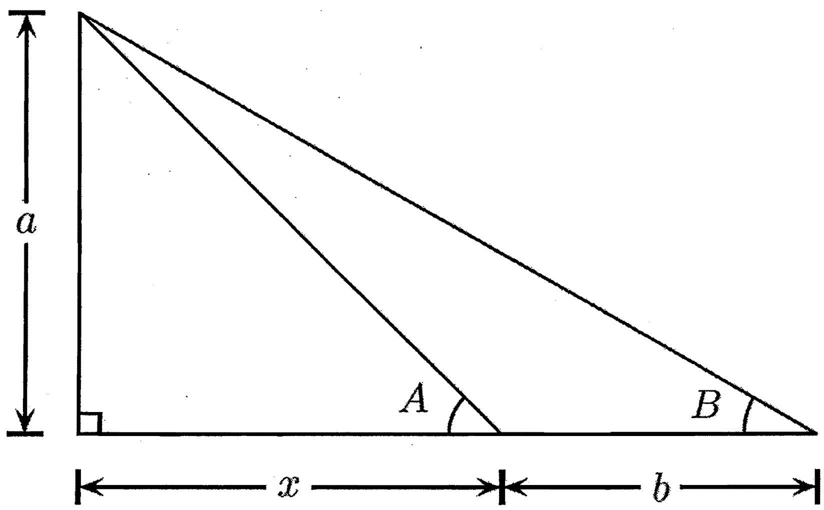
i) Evaluate $\tan\frac{\pi}{3} + \operatorname{cosec}\frac{\pi}{4}$ **(1)**

ii) **(3)**

a) Show that $\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{6}{4x^2-9}$

b) Hence find $\int \frac{dx}{4x^2-9}$

iii) **(8)**



It is given that $\tan A - \tan B = 1$

(a) Prove that $x^2 + bx - ab = 0$

(b) If $a = 3$ and $b = 6$, find x in simplest surd form.

(c) Prove that $\tan A = \frac{\sqrt{3}+1}{2}$ and find $\tan B$ in similar form.

(d) Find A and B to the nearest degree.

NORTH SYDNEY BOYS HIGH SCHOOL

2010 HSC ASSESSMENT TASK 4

Mathematics

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
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- O Mr Berry
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- O ....

Student Number:______________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/7	/13	/12	/12	/6	/50	/100

Question 1 (Start on a new page)

a) The surface area of a balloon is given by the formula:

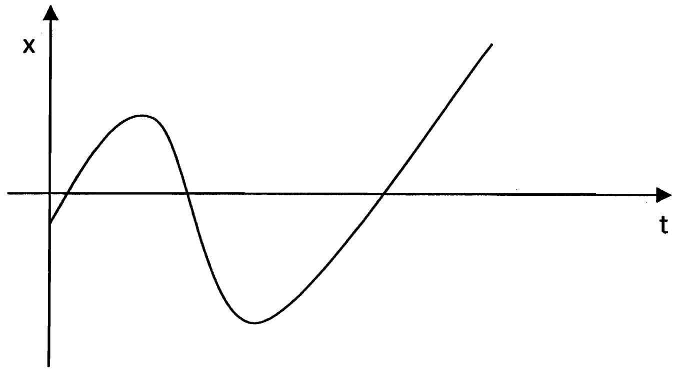
$$SA = 2t^2 + 5t + 7$$

where SA is the surface area in cm^2 and t is the time in seconds:

i) Find the initial surface area of the balloon **1**

ii) How fast is the surface area increasing after 5 seconds? **2**

b) The displacement of a particle is shown in the graph below. Copy the diagram into your answer booklet.



On two different sets of axes draw the graphs for the velocity and acceleration showing all relevant features (i.e. x-intercepts and stationary points) **4**

Question 2 (Start on a new page)

a) A number is selected at random from the numbers 1-100 (inclusive). What is the probability that the number is even or divisible by 3? **2**

b) 5 black balls, 4 white balls and 3 red balls are placed inside a bag. Two balls are drawn without replacement.

i) Draw a probability tree diagram to represent the situation. **1**

ii) What is the probability of:

a) Both the balls being black **1**

b) Neither of the balls being white **1**

c) At least one of the balls being red **1**

c) 150 Year 12 students were asked what they had for breakfast that day.
80 had toast, 60 had cereal and 50 skipped breakfast altogether.

i) Draw a Venn diagram to represent the situation. **2**

ii) If a student is chosen at random, what is the probability that they had cereal but not toast that day? **2**

d) Warren has 4 keys that look the same. Only one of them can unlock the store room door in his office. Each day when he arrives at work he unlocks this door.

i) On a certain day he tries to unlock the door. What is the probability that he will need to try all 4 keys before unlocking the door ? **2**

ii) What is the probability that on 3 consecutive days, Warren will need to try all 4 keys before unlocking the door ? **1**

Question 3 (Start on a new page)

a) The velocity of a particle given by $V = 8t - 16\ ms^{-1}$. When $t = 0$ the particle is at the origin.

i) What is the acceleration of the particle? **1**

ii) How fast is the particle moving at 2 seconds? **1**

iii) How far does the particle travel in the first 5 seconds? **3**

b) The population N of a certain type of ant is growing according to $N = 400e^{kt}$, where t is the time in weeks after the ants were first counted. At the end of 4 weeks the population has doubled.

i) How many ants were first counted? **1**

ii) Calculate the value of the constant k. **2**

iii) At what rate will the population be increasing after 5 weeks? **2**

iv) Calculate the number of weeks required for this population of ants to reach 12800. **2**

Examination continues over page

Question 4 (Start on a new page)

a) A particle is moving along the x axis. Its displacement x metres from the origin after t seconds is given by

$$x = 1 - 2\cos 4t \quad \text{for} \quad 0 \le t \le \frac{\pi}{2}.$$

i) Sketch the graph of the displacement of the particle. **2**

ii) Find when and where the particle first comes to rest after passing through the origin. **2**

iii) Find the maximum speed of this particle. **2**

b) A particle starts to move from the origin along the x axis. Its velocity in ms^{-1} is given by:

$$v = 3 - \frac{1}{2t+1}$$

i) What is the initial velocity of the particle? **1**

ii) Explain why the particle is never at rest. **2**

iii) Find the time taken for the acceleration to reach $0.08ms^{-2}$. **3**

Question 5 (Start on a new page)

a) A biologist wishes to estimate the number of butterflies living in a region.
She captures 120 butterflies, tags their wings, and releases them back into the area.

The next day she captures 150 butterflies and notes that 20 of them are tagged.
Estimate the approximate butterfly population. **2**

b) Ada, Hypatia, Sofia and Shafi go to a restaurant for dinner. The menu at a restaurant has 3 entrees, 5 main courses and 4 desserts.

i) How many different possible meals could be ordered? **1**

ii) What is the probability that

a) Ada and Shafi order the exact same meal? **1**

b) At least two of the women order identical meals? **2**

End of Examination

Q1

2u - Task 3 - 2010.

(i) $2\sqrt{11} \times 5\sqrt{11} = 10 \times 11$

$= 110$ ✓✓

(ii) $x^2 - 3x - 7 = 0 \quad \therefore x = \dfrac{3 \pm \sqrt{9 - 4(1)(-7)}}{2}$

$= \dfrac{3 \pm \sqrt{37}}{2}$ ✓

$\doteqdot 4.54138, \ -1.54138$

$= 4.54, \ -1.54 \quad (2.dp)$ ✓✓

(iii) $f(-1) + f(0) + f(2)$

$= (-1-2) + 0^2 + 2^2$ ✓

$= -3 + 4$

$= 1$ ✓

(iv) (a) $\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$

$= -\frac{1}{2}$ ✓

(b) $\log_2 1 = 0$ ✓

(v) (a) $A = \dfrac{1}{\sqrt{2}+1} \cdot \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{1}{2 - 1}$

$= 1 \ cm^2$ ✓

(b) $P = 2 \cdot \dfrac{1}{\sqrt{2}+1} + 2 \cdot \dfrac{1}{\sqrt{2}-1}$

$= \dfrac{2}{\sqrt{2}+1} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} + \dfrac{2}{\sqrt{2}-1} \cdot \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$= 2\sqrt{2} - 2 + 2\sqrt{2} + 2 \quad = 4\sqrt{2}.$ ✓✓

/12

(2) i) a) $\frac{d}{dx}\left(4x - 5 + 2x^{-1}\right)$

$= 4 - 2x^{-2}$

$= 4 - \frac{2}{x^2}$ 2

b) $\frac{d}{dx}\left(e^{3x^2-5}\right) = 6x e^{3x^2-5}$ 2

c) $\frac{d}{dx}\left(x^3 e^{-x}\right) = 3x^2 e^{-x} - e^{-x} x^3$

$= x^2 e^{-x}(3 - x)$ 2

d) $\frac{1}{\sin x} \times \cos x = \cot x$ 2

[$\cot x$ is worth 1 mark]

ii) $y = (3x-2)^4$

$y' = 12(3x-2)^3$

$y'' = 108(3x-2)^2$ 1

As y'' is a square it will never change sign through the derivative $\therefore$ no inflexion 2

iii) $-5 \le 3 - 2x \le 5$

$-8 \le -2x \le 2$

$4 \geq x \geq -1$ 2

i.e. $-1 \le x \le 4$

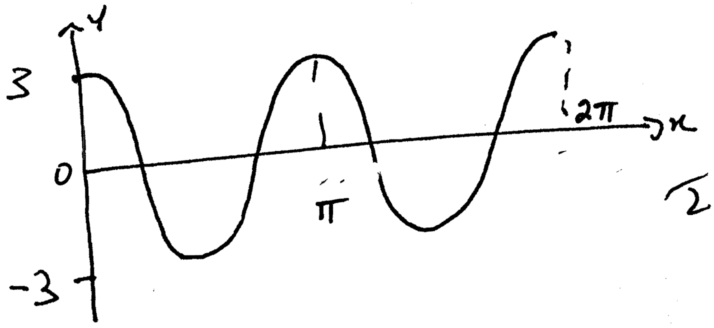
Accepted: $x \geq -1$ and $x \leq 4$.

2(ii) The second mark is for a valid explanation. i.e.
$y'' \geq 0$ for all real x $\therefore$ no change in concavity.
OR even testing one point before + one point after.

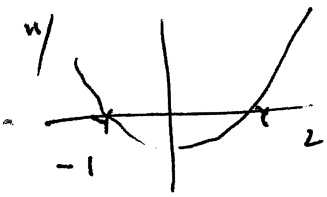
ii) ...

$p = \frac{2\pi}{2} = \pi.$ 1

$a = 3$ 1



2

ii) 

$y = k(x+1)(x-2)$ 2

iii) a) $\ln(x^2+1)+c$ 2

b) $2\ln x + 5e^x + c$ 2

c) $\frac{1}{2}\ln(e^{2x}+4)+c$ 2

4) i) $V = 2000 - 40t + 0.2t^2$

a) $t=0$ $\underline{V = 2000\,\text{cm}^3}$ 1

b) either solve quadratic to get $t=100$ or $\frac{dV}{dt}=0$ to get $t=100$ 1

c) $\frac{dV}{dt} = .4t - 40$ 1

when $t = 30$

$\frac{dV}{dt} = -28$

Decreasing at rate of $28\ \text{cm}^3/\text{s}$

ii) $S_\infty = \frac{a}{1-r}$ G.P $r = -\frac{1}{2}$, $a = 8$ 1

$\doteq \frac{8}{3/2} = \underline{\frac{16}{3}}$ 1

iii)

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	1	$\sqrt{\frac{3}{2}}$	$\sqrt{2}$	$\sqrt{\frac{5}{2}}$	$\sqrt{3}$

1

$A \doteq \frac{1}{6}\left[\sqrt{2}+1+4\sqrt{\frac{3}{2}}\right] + \frac{1}{6}\left[\sqrt{2}+\sqrt{3}+4\sqrt{\frac{5}{2}}\right]$

$\doteq 2.80$ (2dp) 1

(only 2.80 for last mark).

iv) $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ 2

v) $y' = 4x+4$

$\parallel$ when $4x+4=8$ $\longrightarrow$ worth 1

$4x = 4$

$x = 1$

1

i.e. at $(1, -8)$

(5) i) $y' = x\cos x + \sin x$

when $x = \pi/2$ $\quad y' = 1 \quad y = \frac{\pi}{2}$

∴ Normal is

$$y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = -x + \pi/2$$

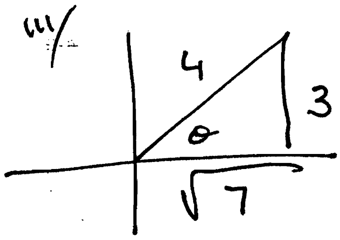
$$x + y = \pi$$

ii) a) $2 + k = 5$

$k = 3$

b) $3k = 12$

$k = 4$

iii) 

a) $\tan\theta = \frac{3}{\sqrt{7}}$ 1.13

b) $\sec\theta = \frac{4}{\sqrt{7}}$ 1.51

c) $\csc^2\theta - \cot^2\theta = 1$

iv) $y = -x^2 + 2x + 5$

a) $x = -\frac{b}{2a} = \frac{-2}{-2}$

$x = 1$

b) vertex is $(1, 6)$

c) $y - 5 = -x^2 + 2x$

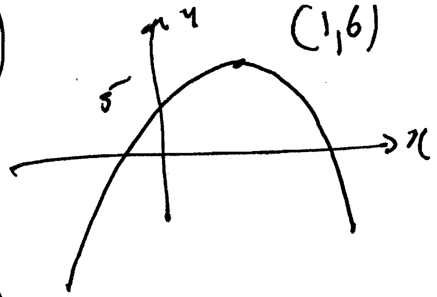
$-y + 5 + 1 = x^2 - 2x + 1$

$-y + 6 = (x-1)^2$

$(x-1)^2 = -1(y-6)$

$4a = 1$

$a = 1/4$

d) 

(6) i) $2x + 5y + k = 0$ (grad $= -2/5$)

through $(-1, 3)$

$-2 + 15 + k = 0$

$k = -13$

∴ $2x + 5y - 13 = 0$

ii) $y = 2x^3 - 3x^2 - 12x + 2$

$y' = 6x^2 - 6x - 12$

$y'' = 12x - 6$

a) stat when $y' = 0$

i.e. $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1$ or $x = 2$

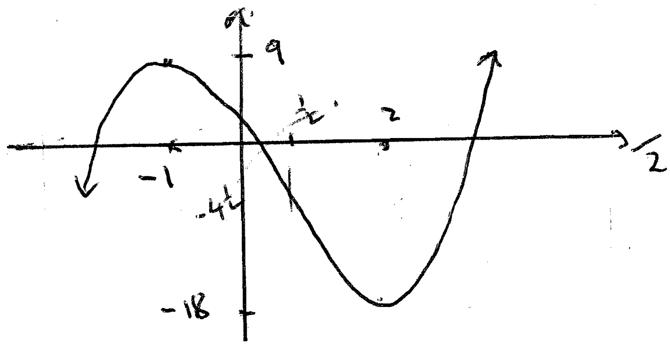
$y'' < 0$ $\quad$ $y'' > 0$

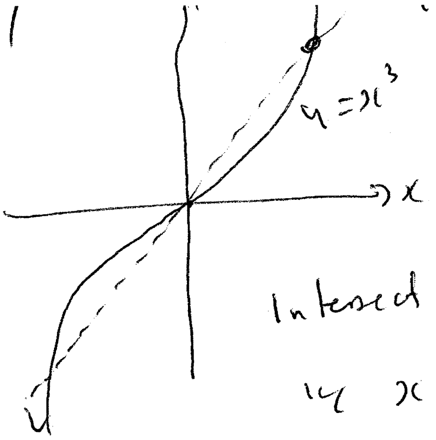
∴ max at $(-1, 9)$ $\quad$ min at $(2, -18)$

b) inflexion when $y'' = 0$ & sign changes

i.e. $x = 1/2$

test $f''(\frac{1}{2} - \epsilon)$ −ve, $f''(\frac{1}{2} + \epsilon)$ +ve } sign change ∴ inflexion at $(\frac{1}{2}, -4\frac{1}{2})$

c) 



Intersect when $x^3 = x$

i.e. $x = 0, x = 1$

$$V = \pi \int_0^1 y^2 \, dx$$

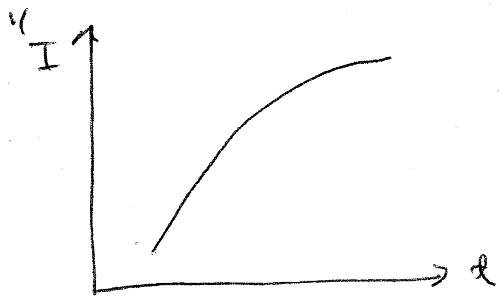
$$= \pi \int_0^1 x \, dx - \pi \int_0^1 x^3 \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_0^1 - \pi \left[\frac{x^4}{4} \right]_0^1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4} \text{ u}^3$$

7) i)



1 - increasing
1 - concavity

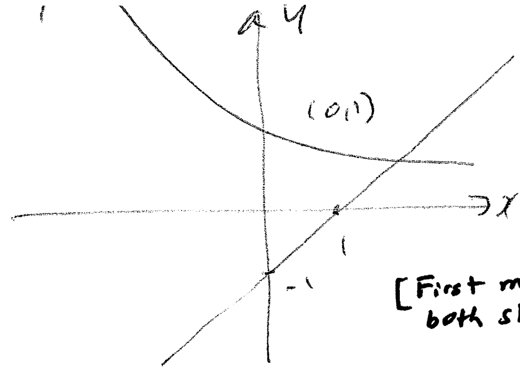
ii) $$\text{LHS} = \frac{\cos^2\alpha + 1 + 2\sin\alpha + \sin^2\alpha}{\cos\alpha(1+\sin\alpha)}$$

$$= \frac{2 + 2\sin\alpha}{\cos\alpha(1+\sin\alpha)}$$

$$= \frac{2(1+\sin\alpha)}{\cos\alpha(1+\sin\alpha)}$$

$$= \frac{2}{\cos\alpha}$$

$= \text{RHS}$



[First mark for both shapes] ✓

$e^{-x} - x + 1 = 0$. Because 1 point of intersection, 1 root. ✓

other graph is $y = x - 1$

iv) a) $PA = \sqrt{(x-1)^2 + (y-3)^2}$

$PB = \sqrt{(x-5)^2 + (y-6)^2}$

b) $PA = 2PB$

$PA^2 = 4PB^2$

$$(x-1)^2 + (y-3)^2 = 4\left[(x-5)^2 + (y-6)^2\right]$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 = 4(x^2 - 10x + 25 + y^2 - 12y + 36)$$

$$3x^2 - 38x + 3y^2 - 42y + 234 = 0$$

v) $T_p = ar^{p-1}$

$\therefore ca = ar^{p-1}$ ✓

$c = r^{p-1}$

$\log c = \log r^{p-1}$

$\log r \, (p-1) = \log c$

$$\log r = \frac{\log c}{p-1}$$

(8) i) $T_n = 3^n + 4$

a) $T_7 = 3^7 + 4$

b) $S_n = $ Sum of $3^1 + 3^2 + \dots + 3^n + 4n$

$= \dfrac{3(3^n - 1)}{3 - 1} + 4n$

$= \dfrac{3(3^n - 1)}{2} + 4n$

ii) $y = 4x - 8 \cap x^2 = 2y$

when $x^2 = 8x - 16$

i.e. $x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0$

$\therefore x = 4$

$y = 8$

$(4, 8) \therefore$ 1 point of intersection

$\therefore$ line is a tangent

iii) a) $\cos x = -\frac{1}{2}$ or $\cos x = 1$

S | A
T | C

Base $x = \frac{\pi}{3}$

$x = 0, 2\pi$

$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$

b) $\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

S | A
T | C

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c) $\sin(x - \pi/3) = -\frac{1}{2}$

Base $\angle$ $\frac{\pi}{6}$

S | A
T | C

$x - \frac{\pi}{3} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ 1

i.e. $x = \frac{7\pi}{6} + \frac{\pi}{3}$ or $\frac{11\pi}{6} + \frac{\pi}{3}$

$= \frac{9\pi}{6}$ $\quad \frac{13\pi}{6}$

$= \frac{3\pi}{2}$ $\quad$ i.e. $\frac{\pi}{6}$ 1

not in Domain

9(a)(i) $T = \frac{D}{S}$

$T_{AB} = \frac{\sqrt{x^2+36}}{40}$ ✓

Accepted in minutes.

$T_{BQ} = \frac{20-x}{50}$ ✓ (2)

(b) $T = \frac{\sqrt{x^2+36}}{40} + \frac{20-x}{50}$ ✓ (1)

(c) $\frac{dT}{dx} = \frac{\frac{1}{2}(x^2+36)^{-\frac{1}{2}}\,2x}{40} - \frac{1}{50}$

$= \frac{x}{40\sqrt{x^2+36}} - \frac{1}{50}$ ✓

Let $\frac{x}{40\sqrt{x^2+36}} - \frac{1}{50} = 0$

$\frac{x}{40\sqrt{x^2+36}} = \frac{1}{50}$

$5x = 4\sqrt{x^2+36}$

$25x^2 = 16(x^2+36)$

$9x^2 = 16 \cdot 36$

$x^2 = 16 \cdot 4$

$x = 8 \quad (x>0)$ ✓

$f'(8-\varepsilon) < 0$

$f'(8) = 0$ (3)

$f'(8+\varepsilon) > 0$

$\therefore$ ~~Max~~ at $x = 8$ ✓

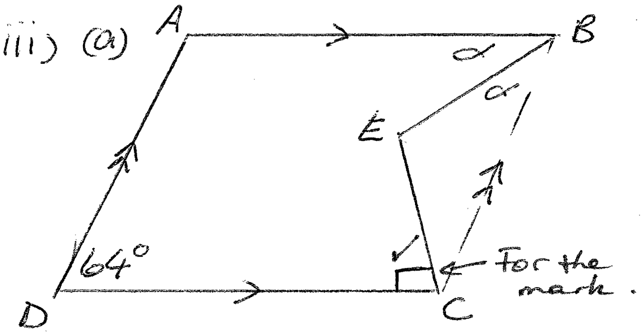
Min

(ii) (d) $V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$ ✓ (3)

$= \pi u^3$ ✓

(iii) (a)



(b) $\angle ECB = 180 - (90+64)$ (coint. ∠'s, parallel lines)

$= 26°$

$\angle ABC = 64°$ (opp. ∠'s of parm)

$\angle EBC = 32°$ (data)

$\therefore \angle BEC = 180 - (32+26)$ (∠ sum of △)

$= 122°$ ✓ ✓

If right angle not between EC and DC, question much easier (3)

$\frac{2}{3}$ km/min

$\sqrt{36+x^2}$

p 5 C10 Williams

Q10

(i) $\tan\frac{\pi}{3} + \operatorname{cosec}\frac{\pi}{4} = \sqrt{3} + \sqrt{2}$ ✓

(ii)
(a) $$\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{2x+3 - 2x+3}{(2x-3)(2x+3)}$$

$$= \frac{6}{4x^2-9}$$ ✓

(b) $$\int \frac{dx}{4x^2-9} = \frac{1}{6}\int \frac{6}{4x^2-9}\,dx$$

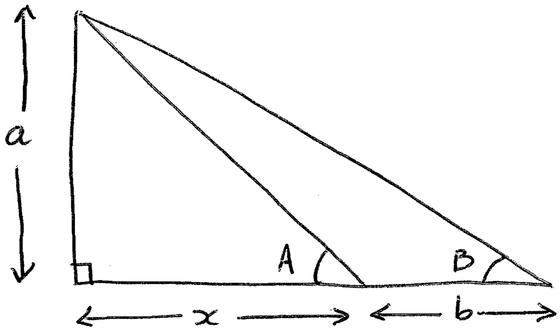
$$= \frac{1}{6}\int\left(\frac{1}{2x-3} - \frac{1}{2x+3}\right)dx$$ ✓

$$= \frac{1}{12}\ln(2x-3) - \frac{1}{12}\ln(2x+3) + c$$

$$= \frac{1}{12}\ln\left(\frac{2x-3}{2x+3}\right) + c$$ ✓

Q10 (cont'd)

(iii)



Given:
$\tan A - \tan B = 1$

(a) $\tan A - \tan B = 1$

$\therefore \quad \dfrac{a}{x} - \dfrac{a}{b+x} = 1$

$\therefore \quad \dfrac{ab + \cancel{ax} - \cancel{ax}}{x(x+b)} = 1$

$\therefore \quad ab = x(x+b)$

$\therefore \quad x^2 + bx - ab = 0.$

(b) $x^2 + 6x - 18 = 0$

$x = \dfrac{-6 \pm \sqrt{36 - 4(1)(-18)}}{2}$

$= \dfrac{-6 \pm \sqrt{108}}{2} = \dfrac{-6 \pm 6\sqrt{3}}{2}$

$= -3 \pm 3\sqrt{3}$

[But $x > 0$ $\quad \therefore \quad x = -3 + 3\sqrt{3}$]

(c) $\tan A = \dfrac{a}{x} = \dfrac{3}{-3+3\sqrt{3}} = \dfrac{1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}+1}{2}$

$\tan B = \dfrac{3}{6 + (-3+3\sqrt{3})} = \dfrac{3}{3+3\sqrt{3}} = \dfrac{1}{1+\sqrt{3}}$

$= \dfrac{\sqrt{3}-1}{2}$

(d) $A \doteqdot 53.79^\circ, \quad B \doteqdot 20.104^\circ$

ie $\quad A = 54^\circ$ (nearest degree)

$B = 20^\circ$

/12

Question 1

a) i) $SA = 2t^2 + 5t + 7$

at $t = 0$

$SA = 2(0)^2 + 5(0) + 7$

$= 7\text{cm}^2$ ✓

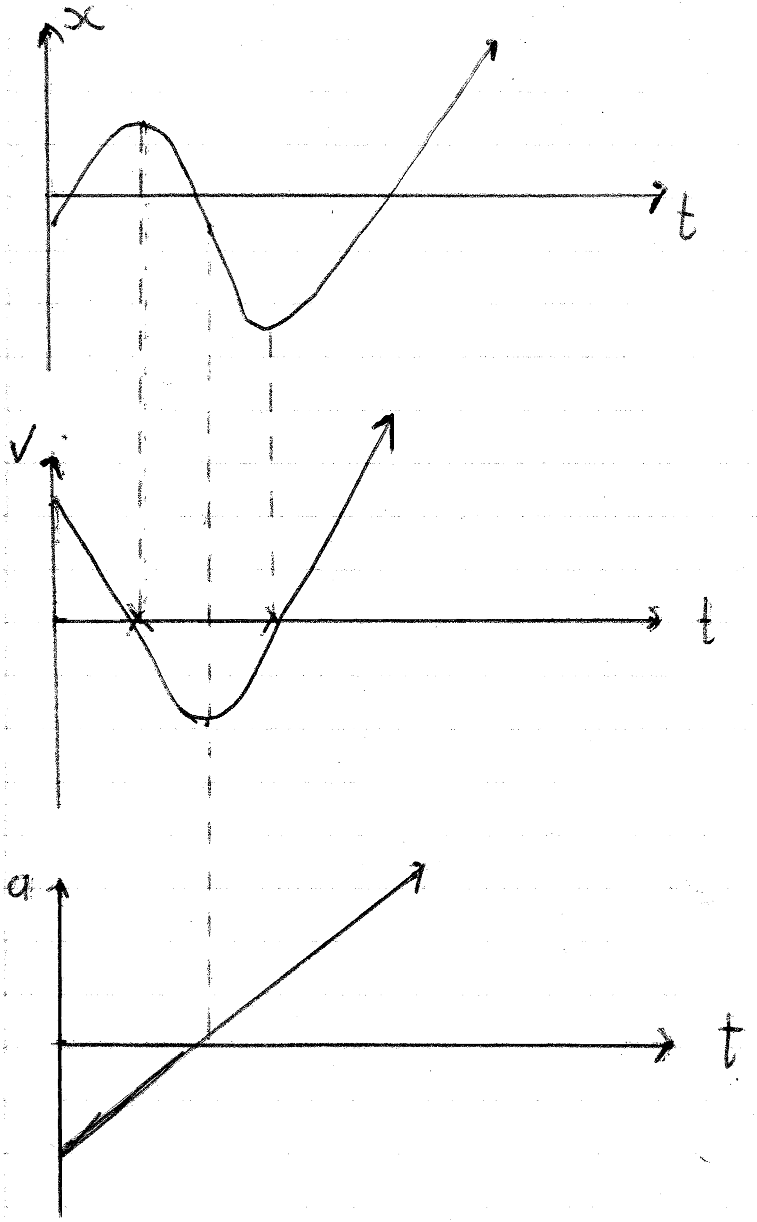
ii) $\frac{dSA}{dt} = 4t + 5$ ✓

at $t = 5$

$\frac{dSA}{dt} = 4(5) + 5$

$= 25 \text{ cm}^3/\text{s}.$ ✓

b)



✓ - correct shape
✓ - x-intercepts & S. P.

✓ - correct shape
✓ - x-intercept

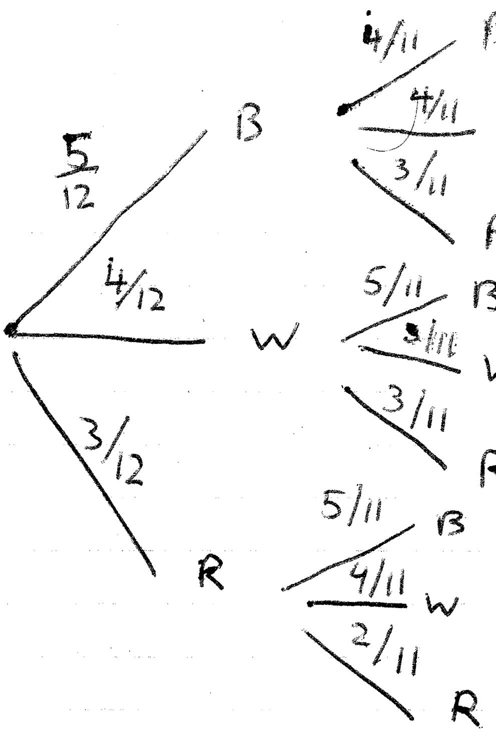
Question 2

a) $P(2 \cup 3) = P(2) + P(3) - P(2 \cap 3)$ ✓

$= \frac{50}{100} + \frac{33}{100} - \frac{16}{100}$

$= \frac{67}{100}$ ✓

b) i)



BB $\left(\frac{20}{132}\right)$

BW $\left(\frac{20}{132}\right)$

BR $(15/132)$ ✓ (tree only)

WB $(20/132)$

WW $(12/132)$

WR $(12/132)$

RB $(15/132)$

RW $(12/132)$

RR $(6/132)$

ii) a) $\frac{20}{132} = \frac{5}{33}$ ✓

~~b) $1 - \frac{12}{132} = \frac{10}{11}$~~

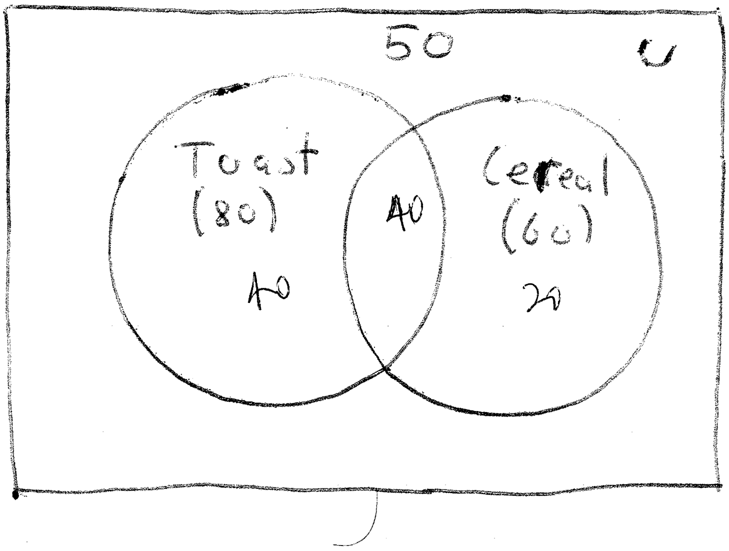
$\frac{5}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{5}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{14}{33}$ ✓

c) $\frac{15}{132} + \frac{12}{132} + \frac{15}{132} + \frac{12}{132} + \frac{6}{132}$

$= \frac{5}{11}$ ✓

OR $1 - \left(\frac{5}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{5}{11} + \frac{4}{12} \times \frac{3}{11}\right)$

c) i)



✓ - intersecting circles

✓ - universal set

ii) $|T \cup C| = 150 - 50$
$= 100$

$\therefore |T \cap C| = 40$

$\therefore |C \cap \sim T| = 60 - 40$
$= 20$ ✓

$\therefore P(C \cap \sim T) = \dfrac{20}{150}$ ✓

$= \dfrac{2}{15}$

d) i) $\dfrac{3}{4} \times \dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{6}{24}$ ✓✓

$= \dfrac{1}{4}$

ii) $\dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{64}$ ✓

Question 3.

(a) (i) $\ddot{x} = 8 \text{ m/s}^2$ ✓

(ii) @ $t = 2$ $V = 8(2) - 16 = 0$ m/s ✓

Particle is stationary.

(iii) First Method:

$x = \int (8t - 16)\, dt = 4t^2 - 16t + c$

@ $t = 0$, $x = 0$ $\therefore$ $c = 0$

$x = 4t^2 - 16.$ ✓

@ $t = 2 \longrightarrow x = -16$ m.

@ $t = 5 \longrightarrow x = 20$ m.

Distance $= (0 - -16) + (20 - (-16))$ ✓

$= 16 + 36 = 52$m ✓

Note: write x = distance = 20 only 1 mark is allocated!!
students had to use the stationary pt. to get the distance.

2ND Method:

Distance $= \left|\int_0^2 (8t - 16)\, dt\right| + \left|\int_2^5 (8t - 16)\, dt\right|$ ✓

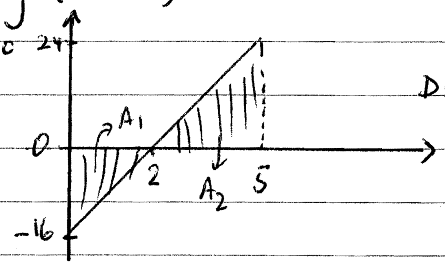
$= \left|\left[4t^2 - 16t\right]_0^2\right| + \left[4t^2 - 16t\right]_2^5$

$= |4\times4 - 16\times2 - 0| + |[4\times25 - 16\times5 - (4\times4 - 32)]|$ ✓

$= 16 + 20 + 16 = 52$m. ✓

Note: writing $\int_0^5 (8t - 16)\, dt = 20$m only gets 1 mark.

3rd method:



Distance $= A_1 + A_2$

$= \frac{1}{2} \times 2 \times 16 + \frac{24 \times 3}{2}$ ✓✓

$= 16 + 36$

$= 52$m. ✓

(b) (i) $N = 400e^0 = 400$ ants. ✓

(ii) $800 = 400e^{4k}$

$2 = e^{4k}$ ✓

$4k = \ln 2$ $\Rightarrow k = \frac{\ln 2}{4} = 0.1733\ldots$ ✓

(iii) $\frac{dN}{dt} = 400 k e^{kt}$ ✓

$= 400 \times \frac{\ln 2}{4} e^{\frac{\ln 2}{4} \times 5} = 164.86$ ants/week

$= 165$ ants / week ✓

(iv) $12800 = 400e^{kt}$

$32 = e^{\frac{\ln 2}{4} t}$ ✓

$\ln 32 = \frac{\ln 2}{4} \cdot t$ $\therefore$ $t = \frac{4 \ln 32}{\ln 2} = 4 \times 5$

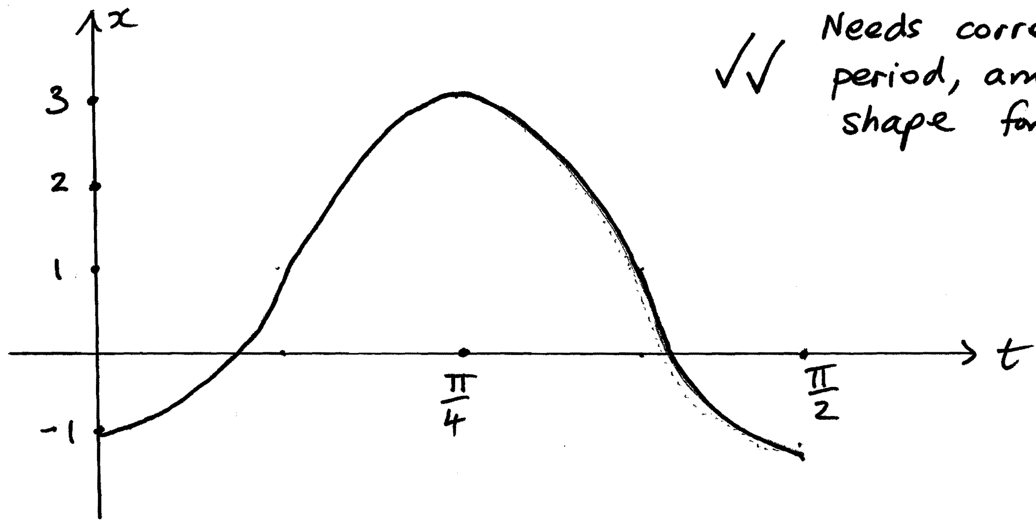
$= 20$ weeks. ✓

To get the 2ND mark students should use the exact value of k to get 20.

QUESTION 4

(a)

(i)



✓✓ Needs correct period, amplitude, shape for full marks

(ii) From the graph, particle is at rest at $t = \frac{\pi}{4}$ seconds, when $x = 3$ m.

✓ correct time
✓ correct x value

(iii) $x = 1 - 2\cos 4t$

$\therefore\ v = 8\sin 4t$ ✓ v equation

Since $|\sin 4t| \le 1$ $\therefore$ maximum speed $= 8$ m/s ✓ answer

(b) (i) at $t = 0$, $v = 3 - \frac{1}{2(0)+1} = 2$ m/s ✓

(ii) 'at rest' $\Rightarrow v = 0$ $\therefore\ 3 - \frac{1}{2t+1} = 0$

$\therefore\ 2t + 1 = \frac{1}{3}$

$\therefore\ t = -\frac{1}{3}$ ✓

but $t \ge 0$, $\therefore$ v cannot be 0

$\therefore$ particle is never at rest. ✓

* Solutions need mathematical reasoning, and words.
- Clearly demonstrating that $\frac{1}{2t+1}$ is >0 and ≤ 1, and making a conclusion, is appropriate.
- Calculating +ve acceleration is not by itself sufficient, unless linked with $v(0) = +2$.

Question 4 (cont'd)

(iii)
$$V = 3 - \frac{1}{2t+1}$$
$$= 3 - (2t+1)^{-1}$$
$$\therefore a = (-1)\cdot -(2t+1)^{-2}\cdot 2$$
$$\therefore a = \frac{2}{(2t+1)^2}$$
✓ correct acceleration

Solving,
$$\frac{2}{(2t+1)^2} = 0.08$$
$$\therefore \frac{1}{(2t+1)^2} = 0.04$$
$$\therefore (2t+1)^2 = 25$$
$$(2t+1) = \pm 5$$
✓ correct working
$$\therefore 2t = -6, 4$$
$$t = -3, 2$$
But $t > 0$ $\therefore$ $t = 2$ seconds.

✓ $t = 2$ secs.

* If answers give $a = \frac{1}{(2t+1)^2}$, then answers ending in $t \doteqdot 1.268$ secs or $t = \frac{-2+5\sqrt{2}}{4}$ get max. 2 marks.

* If solutions give the integral, not derivative, ie $-\ln(2t+1)$, or similar, then no marks given [it is a terminating error].

* If solutions lose the square on the $(2t+1)^2$, then max. of 1 mark, as solution is too easy.

Question 5

a) Let x be the total number

$$\frac{20}{150} = \frac{120}{x}$$ ✓

$\therefore 20x = 18000$

$\therefore x = 900$ ✓

b) i) $3 \times 5 \times 4 = 60$ ✓

ii) a) $P(\text{same}) = 1 \times \frac{1}{60}$

$= \frac{1}{60}$ ✓

iii) $P(\text{all different}) = 1 \times \frac{59}{60} \times \frac{58}{60} \times \frac{57}{60}$

$= \frac{195054}{216000}$

$P(\geq 2 \text{ same}) = 1 - P(\text{All Different})$ ✓

$= \frac{20946}{216000}$ ✓

$= \frac{3491}{36000}$

(answers in decimal or percentage format also okay).

Question 1

a) i) $SA = 2t^2 + 5t + 7$

at $t = 0$

$SA = 2(0)^2 + 5(0) + 7$

$= 7\text{cm}^2$ ✓

ii) $\frac{dSA}{dt} = 4t + 5$ ✓

at $t = 5$

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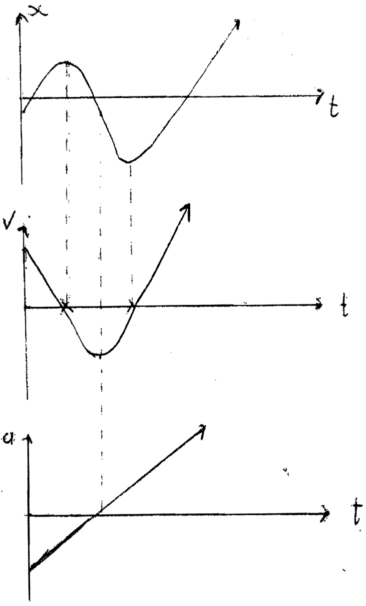
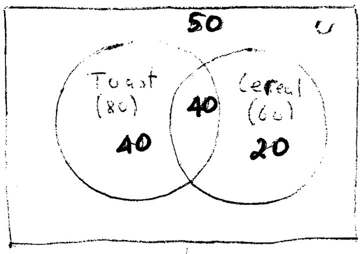


✓ - correct shape
✓ - x-intercepts & S.P.

✓ - correct shape
✓ - x-intercept.

c) i)



✓ - intersecting circles

✓ - universal set

ii) $|T \cup C| = 150 - 50$
$= 100$

$\therefore |T \cap C| = 40$

$\therefore |C \cap {}^{-}T| = 60 - 40$
$= 20$ ✓

$\therefore P(C \cap {}^{-}T) = \frac{20}{150}$ ✓

$= \frac{2}{15}$

d) i) $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{6}{24}$ ✓✓

$= \frac{1}{4}$

ii) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$ ✓

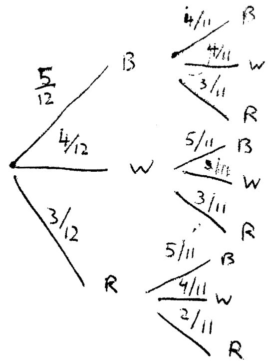
Question 2

a) $P(2 \cup 3) = P(2) + P(3) - P(2 \cap 3)$ ✓

$= \frac{50}{100} + \frac{33}{100} - \frac{16}{100}$

$= \frac{67}{100}$ ✓

b) i)



BB $(\frac{20}{132})$

BW $(\frac{20}{132})$

BR $(15/132)$ ✓

WB $(20/132)$ (tree only)

WW $(12/132)$

WR $(12/132)$

RB $(15/132)$

RW $(12/132)$

RR $(6/132)$

ii) a) $\frac{20}{132} = \frac{5}{33}$ ✓

b) $1 - \frac{12}{132} = \frac{10}{11}$ $\frac{5}{12}\times\frac{4}{11} + \frac{5}{12}\times\frac{3}{11} + \frac{3}{12}\times\frac{5}{11} + \frac{3}{12}\times\frac{2}{11} = \frac{14}{33}$ ✓

c) $\frac{15}{132} + \frac{12}{132} + \frac{15}{132} + \frac{12}{132} + \frac{6}{132}$

$= \frac{5}{11}$ ✓

OR $1 - \left(\frac{5}{12}\times\frac{4}{11} + \frac{5}{12}\times\frac{4}{11} + \frac{4}{12}\times\frac{5}{11} + \frac{4}{12}\times\frac{3}{11}\right)$

Question 3

(a) (i) $\ddot{x} = 8\text{m/s}^2$ ✓

(ii) @ $t = 2$ $V = 8(2) - 16 = 0\text{ m/s}$ ✓

Particle is stationary.

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$x = \int (8t - 16)\,dt = 4t^2 - 16t + c$

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@ $t = 2 \rightarrow x = -16\text{ m}$.

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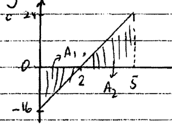

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$2 = e^{4k}$ ✓

$4k = \ln 2 \Rightarrow k = \frac{\ln 2}{4} = 0.1733\cdots$ ✓

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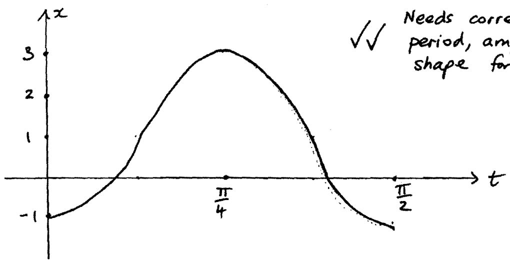

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$\therefore 2t + 1 = \frac{1}{3}$

$\therefore t = -\frac{1}{3}$ ✓

but $t \ge 0$, $\therefore$ v cannot be 0

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$\therefore a = \frac{2}{(2t+1)^2}$ ✓ correct acceleration

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$\therefore \frac{1}{(2t+1)^2} = 0.04$

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