Question 1 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\frac{\sqrt{25-e^3}}{\pi^2}$ to 3 significant figures: **2**

(b) Write down the exact value of tan (315°) **1**

(c) Find x when: $\frac{3}{5}+\frac{2x-1}{3} = 1$ **2**

(d) Write $1.2\overline{87}$ as a fraction **2**

(e) Rationalise the denominator of: $\frac{8}{3-\sqrt{7}}$ **2**

(f) Solve $|3x + 2| < 11$ and sketch your solution on a number line **3**

Question 2 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate:

(i) $\dfrac{2}{\sqrt{x^3}}$ **1**

(ii) $\cos^2(3x)$ **1**

(iii) $\dfrac{x^2}{e^x+1}$ **2**

(b) Find the equation of the normal to $y = x^3 - 7x^2 + 4x + 11$ when $x = 2$ **4**

(c) A plane flew 250 km on a bearing of 070°T and then 100 km due east. Find:

(i) Its distance from the starting point **2**

(ii) Its bearing from the starting point to the nearest degree **2**

Question 3 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) A (2,1), B (-5, -6) , C (-6,-1) and D (1,6) form a parallelogram

i) Plot points A, B, C and D on a number plane **1**

ii) Find the gradient of AB **1**

iii) Show that the equation of AB is: $x - y - 1 = 0$ **1**

iv) Find the exact length of AB **1**

v) Find the coordinates where the diagonals of $ABCD$ intersect.
Label it as point E on your diagram **2**

vi) Find the exact area of triangle ABE **3**

b) Find the values of A, B and C for the identity:

$$A(x\text{-}1)^2 + B(x\text{-}1) + C \equiv 3x^2 - x + 3$$ **3**

Question 4 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Find:

(i) $\int \frac{x^3+1}{x^2}dx$ **2**

(ii) $\int \frac{x^2}{x^3+1}dx$ **2**

(b) Evaluate $\int_2^5 e^{3x}dx$. Round your answer to 2 decimal places. **2**

(c) For the function $f(x) = x^3$:

(i) Evaluate $\int_{-1}^{1} f(x)dx$ **1**

(ii) Find the area between $y = f(x)$, $x = 1$, $x = -1$ and the x-axis **2**

(iii) Is your answer for (i) the same as (ii). Give a reason. **1**

(d) Sketch the graph of $y = 2 + \cos 2x$ for $0 \leq x \leq 2\pi$ **2**

Question 5 (12 Marks) Use a SEPARATE writing booklet

(a) Find the value(s) of k in $x^2 + (k + 6)x - 2k = 0$ such that:

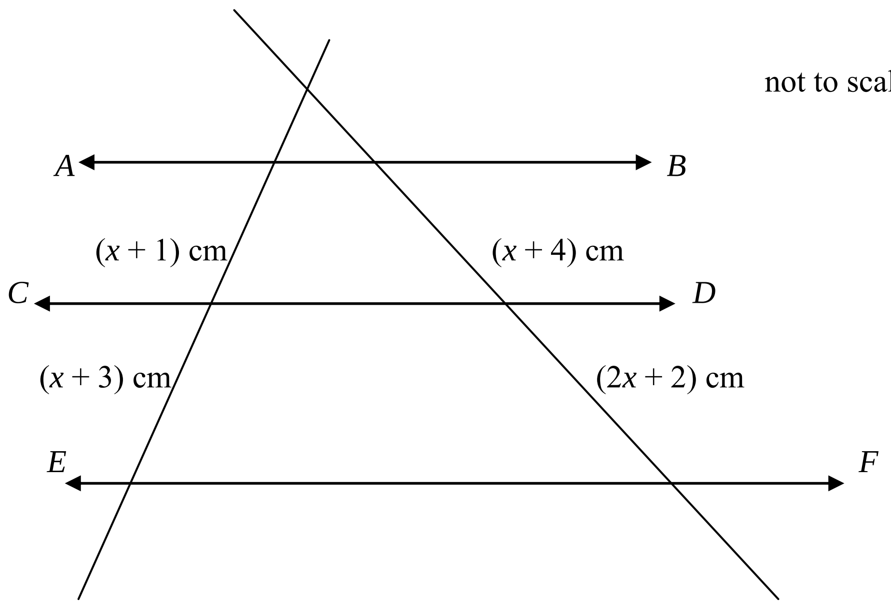
(i) 3 is a root of the quadratic **1**

(ii) The roots are equal in magnitude but opposite in sign **1**

(iii) The roots are reciprocals of one another **1**

(iv) The roots are real **3**

(b) not to scale



AB, CD and EF are parallel lines. Find the exact length(s) of x, giving reasons. **3**

(c) Find all possible values of θ when $3\tan^2\theta - 1 = 0$ and $0 \le \theta \le 2\pi$. **3**

Question 6 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) A curve is given by the function $f(x) = x^3 + 2x^2 - 4x - 8$

(i) Find the y – intercept **1**

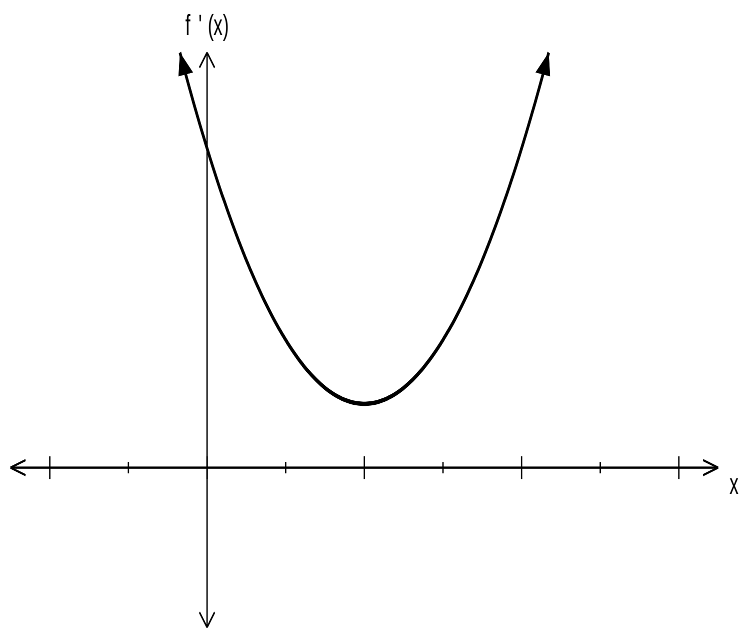
(ii) Factorise in pairs to find the x – intercept(s) **2**

(iii) Find the stationary point(s) and determine their nature **4**

(iv) Find the point(s) of inflexion **2**

(v) Draw a neat sketch of the curve showing all the above features **1**

(b) The gradient of a function $f(x)$ is shown in the graph below such that $y = f'(x)$



Sketch a possible graph for $y = f(x)$. **2**

Question 7 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) George plays a game where he rolls 2 dice. The first die has 3 red faces, 2 blue faces and 1 green face. The other die has 2 red faces, 2 blue faces, and 2 green faces

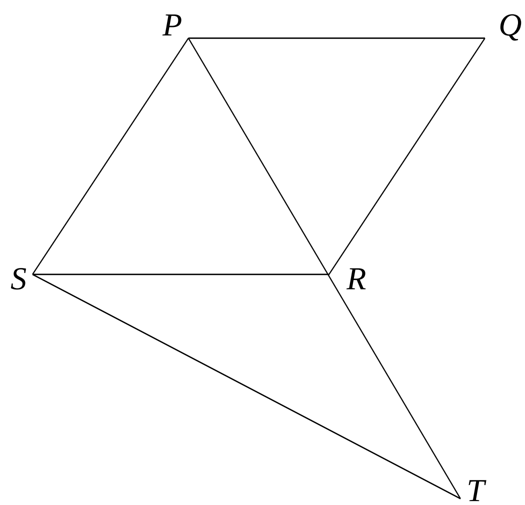
(i) Find the probability that both dice show red **1**

(ii) Find the probability that one shows red and one shows blue **1**

(iii) Find the probability that both dice do not show red, nor do both show green **1**

(b) Evaluate $\int_0^1 \pi^x dx$ using Simpson's rule with 5 function values.
Answer to two decimal places **3**

(c) $PQRS$ is a rhombus. PR is produced to T such that $SR = TR$



(i) Show that $\angle SPQ = 4\angle STR$ **3**

(ii) Show that R is the midpoint of PT, given than $\angle PST = 90^0$ **3**

Question 8 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$ **2**

(b) For the series $8 + 12x + 18x^2 + 27x^3 + \ldots$

(i) For what values of x will this series have a limiting sum **2**

(ii) Find the limiting sum if $x = \frac{1}{4}$ **2**

(c) A parabola has a focus of (3,2) and a directrix $x = 5$

(i) Find the vertex **1**

(ii) State the equation of the parabola **1**

(iii) Show that the points of intersection of the parabola and the line $2x + y - 6 = 0$ are (3,0) and (0,6) **1**

(iv) Find the area between the parabola and the line in the first quadrant **3**

Question 9 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\sum_{n=0}^{4} \cos^2\left(\frac{n\pi}{3}\right)$ **2**

(b) For the function y = 2 tan x:

(i) Sketch y = 2 tan x for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ **2**

(ii) State the range of the curve **1**

(iii) On your graph, shade the regions bounded by the function and the x-axis. **1**

(iv) Show that $\frac{d}{dx}\ln(\cos x) = -\tan x$ **1**

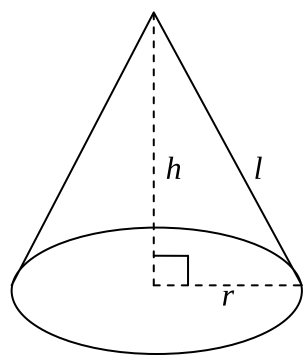
(v) Hence find the exact area shaded in part (iii) **2**

(vi) Using the identity $1 + \tan^2 x = \sec^2 x$:
Find the volume of the solid generated when the area bound by the curve and the x-axis is rotated about the x-axis. **3**

Question 10 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Find the sum of the first 50 terms in the series $\ln 3 + \ln 9 + \ln 27 + \ln 81 + \ldots$ **3**

(b)



A cone has radius r, height h and slant height l.

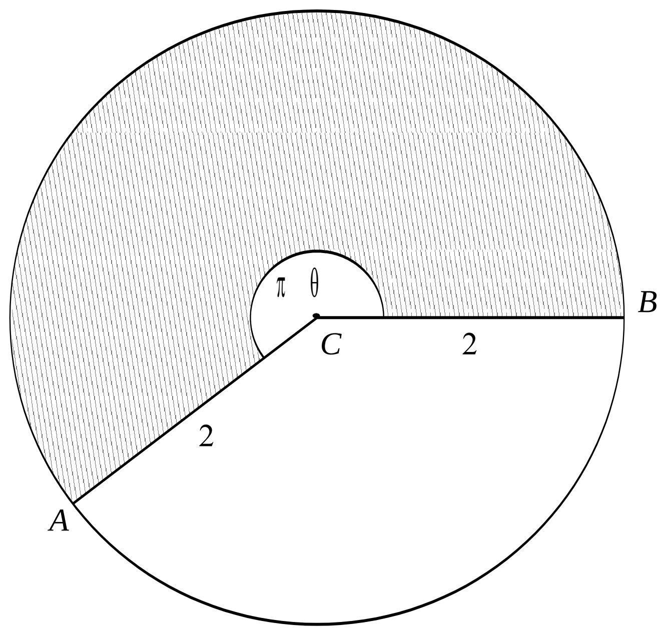
The volume of the cone is given by $V = \frac{\pi}{3} r^2 h$

Show that the volume of the cone can be expressed as $V = \frac{\pi}{3}\sqrt{l^2 r^4 - r^6}$ **2**

Question 10 continues on next page

Question 10 (continued)

(c)



The angle at the centre C of a circle of radius 2cm is $\pi\theta$ radians, $0 < \theta < 2$, as shown on the diagram.

(i) Write down the length of the arc of the shaded sector **1**

(ii) The sector is cut from the circle along the radii CA and CB and and folded to make a cone.

Find the radius of the cone. **1**

(iii) Show that the volume of the cone is given by $V = \frac{\pi}{3}\sqrt{4\theta^4 - \theta^6}$ **1**

(iv) Find the value of θ to 2 decimal places, for which the volume of the cone is maximised. **4**

End of Examination

Question 1

e) $\dfrac{8}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \dfrac{8(3+\sqrt{7})}{3^2 - (\sqrt{7})^2}$

$= \dfrac{24 + 8\sqrt{7}}{2}$

$= 12 + 4\sqrt{7}$

f) $|3x + 2| < 11$

case 1: $3x + 2 < 11$

$3x < 9$

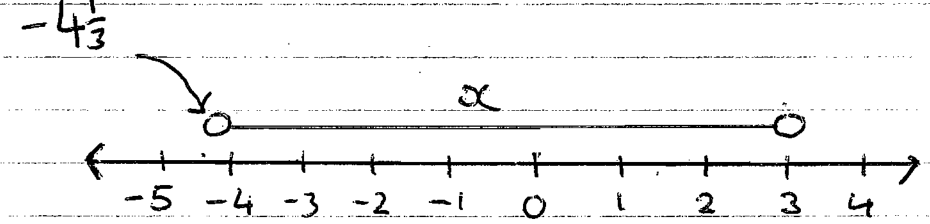
$x < 3$

case 2 $-(3x+2) < 11$

$-3x - 2 < 11$

$-3x < 13$

$x > -\dfrac{13}{3}$ or $-4\frac{1}{3}$



Question 1

✓ answer

a) 0.225 (3 s.f.) ✓ rounding

b) -1 ✓

c) $\frac{3}{5} + \frac{2x-1}{3} = 1$

$\frac{9}{15} + \frac{5(2x-1)}{15} = 1$ ✓

$\therefore 9 + 10x - 5 = 15$

$10x + 4 = 15$

$10x = 11$

$x = \frac{11}{10}$ ✓

d) let $x = 1.28787\ldots$

$100x = 128.7878\ldots$

$\therefore 99x = 128.7878\ldots - 1.2878\ldots$ ✓

$99x = 127.5$

$\therefore x = \frac{127.5}{99}$

$\therefore x = \frac{85}{66}$ ✓

Question 2

a) i) $\frac{d}{dx} \frac{2}{\sqrt{x^3}}$

$= \frac{d}{dx} 2x^{-3/2}$

$= -3x^{-5/2}$ ✓

ii) $\frac{d}{dx} \cos^2(3x)$

$= -6\sin 3x \cos 3x$ ✓

iii) let $y = \frac{x^2}{e^x + 1}$

$u = x^2$ and $u' = 2x$

$v = e^x + 1$ and $v' = e^x$ ✓

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{2x(e^x + 1) - (e^x)(x^2)}{(e^x + 1)^2}$

$= \frac{2xe^x + 2x - x^2 e^x}{(e^x + 1)^2}$ ✓

$= \frac{x(2e^x + 2 - xe^x)}{(e^x + 1)^2}$

Question 2

b) $y = x^3 - 7x^2 + 4x + 11$

when $x = 2$

$y = 2^3 - 7(2)^2 + 4(2) + 11$

$y = -1$ ✓

$y' = 3x^2 - 14x + 4$ ✓

$\therefore m_{tgt} = 3(2)^2 - 14(2) + 4$

$= -12$

$\therefore m_{norm} = 1/12$ ✓

$y - y_1 = m(x - x_1)$

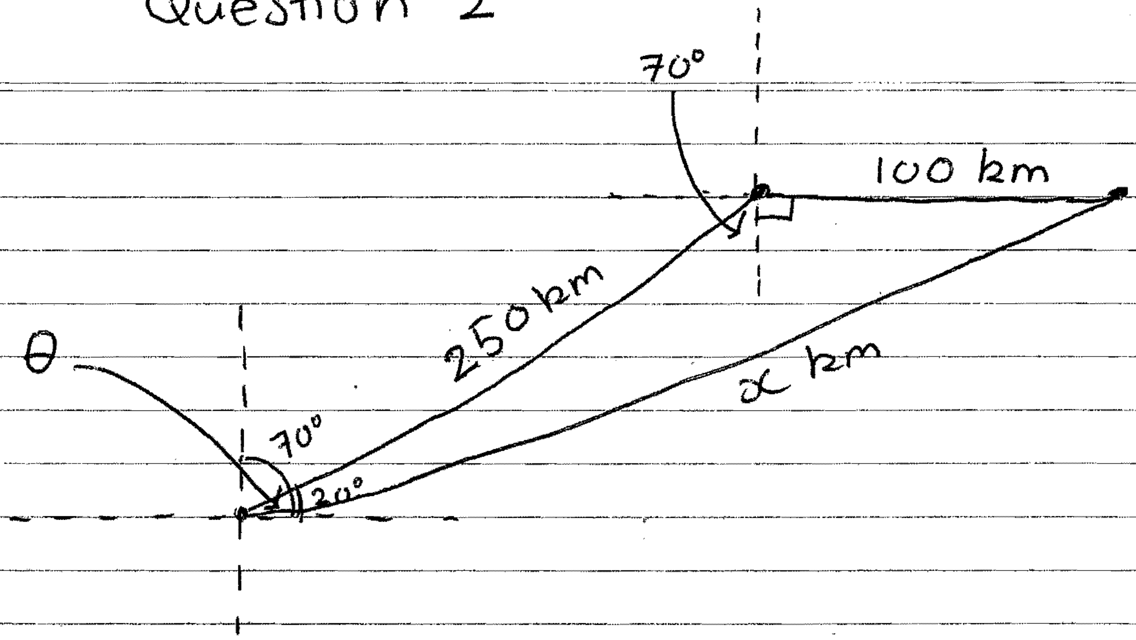
$y + 1 = 1/12\,(x - 2)$

$12y + 12 = x - 2$

$\therefore x - 12y - 14 = 0$ ✓

Question 2

c)



i) let x be the distance

$x^2 = 250^2 + 100^2 - 2 \times 250 \times 100 \times \cos 160°$

$\therefore x = 346$ km (nearest km)

ii) let θ be as shown.

$\frac{\sin\theta}{100} = \frac{\sin \cancel{70}\ 160}{x}$

$\therefore \sin\theta = \frac{100 \sin \cancel{70}\ 160}{x}$

$\therefore \theta = \sin^{-1}\left(\frac{100 \sin \cancel{70}\ 160}{x}\right)$

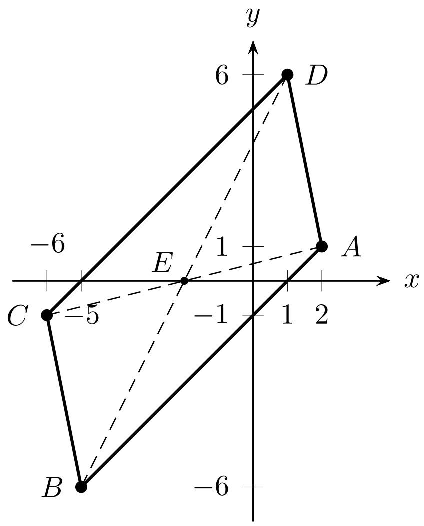
$\theta = 6°$ (nearest degree

$\therefore$ bearing $= 70° + 6°$

$= 076°T$ (nearest degree)

Question 3 (Lam)

(a) i. (1 mark)



ii. (1 mark)

$$m = \frac{1-(-6)}{2-(-5)} = \frac{7}{7} = 1$$

iii. (1 mark)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 1(x-2) \\ y - 1 &= x - 2 \\ x - y - 1 &= 0 \end{aligned}$$

iv. (1 mark)

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(2-(-5))^2 + (1-(-6))^2} \\ &= \sqrt{98} = 7\sqrt{2} \end{aligned}$$

v. (2 marks)
Diagonals intersect at midpoints (property of a parallelogram). Find midpoint E of AB:

$$E = \left(\frac{2+(-6)}{2}, \frac{1+(-1)}{2}\right) = (-2, 0)$$

vi. (3 marks)

✓ [1] for correct substitution into perpendicular dist formula.

✓ [1] for perpendicular height

✓ [1] for area of $\triangle ABE$

$$A_{\triangle ABE} = \frac{1}{2}bh$$

Perpendicular height from $E(2,0)$ to $x - y - 1 = 0$

$$\begin{aligned} h &= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} \\ &= \frac{|1(2) + (-1)(0) + (-1)|}{\sqrt{1^2 + (-1)^2}} \\ &= \frac{3}{\sqrt{2}} \\ A &= \frac{1}{2} \times 7\cancel{\sqrt{2}} \times \frac{3}{\cancel{\sqrt{2}}} = \frac{21}{2} \end{aligned}$$

(b) (3 marks)

✓ [1] each for A, B and C

$$A(x-1)^2 + B(x-1) + C \equiv 3x^2 - x + 3$$

By inspection,

$$A = 3$$

Letting $x = 1$,

$$\begin{aligned} 0 + 0 + C &= 3 - 1 + 3 \\ \therefore C &= 5 \end{aligned}$$

Letting $x = 2$,

$$\begin{aligned} A + B + C &= 3(2)^2 - 2 + 3 \\ 3 + B + 5 &= 13 \\ \therefore B &= 5 \end{aligned}$$

Question 4

a) i) $\int \frac{x^3+1}{x^2}\,dx$

$= \int (x + x^{-2})\,dx$ ✓

$= \frac{1}{2}x^2 + (-1)x^{-1} + c$

$= \frac{x^2}{2} - \frac{1}{x} + c$ ✓

ii) $\int \frac{x^2}{x^3+1}\,dx$

$= \frac{1}{3}\int \frac{3x^2}{x^3+1}\,dx$ ✓

$= \frac{1}{3}\log_e(x^3+1) + c$ ✓

b) $\int_2^5 e^{3x}\,dx$

$= \left[\frac{1}{3}e^{3x}\right]_2^5$ ✓

$= \frac{1}{3}\left[e^{15} - e^{6}\right]$

$= 1\ 089\ 538$ (nearest whole number) ✓

Question 4

c) i) $\int_{-1}^{1} x^3 \, dx$

$= \left[\frac{1}{4}x^4\right]_{-1}^{1}$

$= \frac{1}{4}(1)^4 - \frac{1}{4}(-1)^4$

$= \frac{1}{4} - \frac{1}{4}$

$= 0$

ii) $A = 2\int_0^1 x^3 \, dx$ (since it's an odd function)

$= 2\left[\frac{1}{4}x^4\right]_0^1$

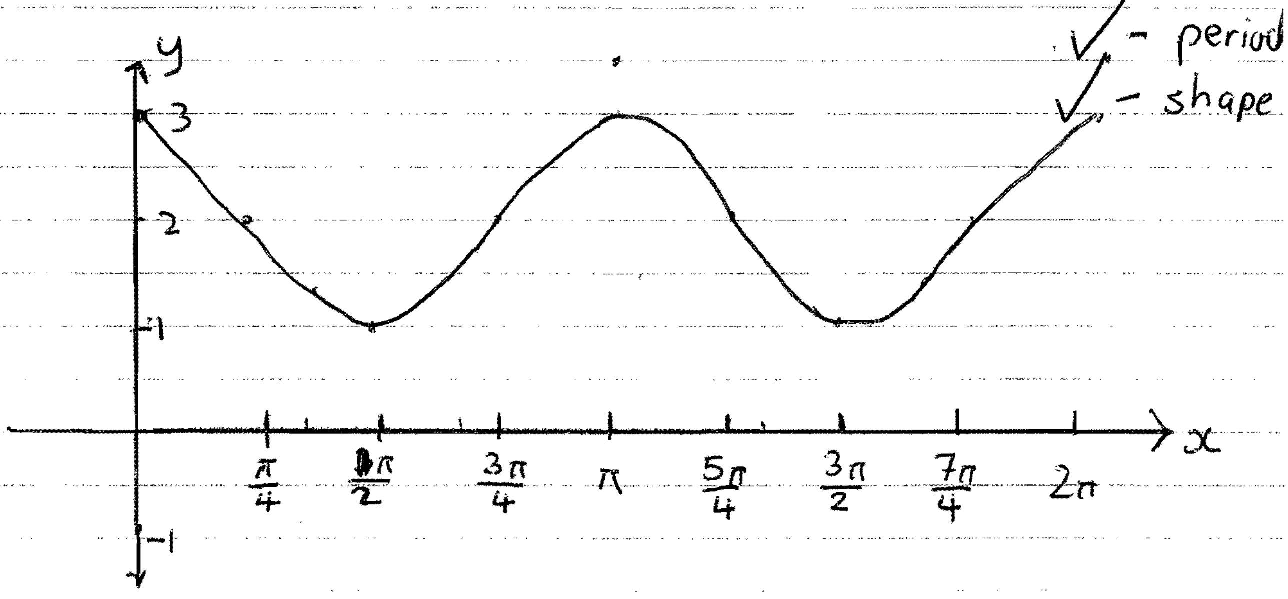
$= 2\left(\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4\right)$

$= 2 \times 1/4$

$= 1/2 \; u^2$

iii) Because half of the area is below the x-axis it will cancel out when evaluating the definite integral.

d)



Question 5

a) i) $3^2 + (k+6)3 - 2k = 0$

$9 + 3k + 18 - 2k = 0$

$27 + k = 0$

$k = -27$ ✓

ii) let the roots be α and $-\alpha$

$\therefore \alpha + -\alpha = \frac{-b}{a}$

$\therefore 0 = \frac{-(k+6)}{1}$

$\therefore k = -6$ ✓

iii) let the roots be α and $\frac{1}{\alpha}$

$\therefore \alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$\therefore 1 = \frac{-2k}{1}$

$\therefore -2k = 1$

$k = -\frac{1}{2}$ ✓

Question 5

iv) $b^2 - 4ac \geqslant 0$

$(k+6)^2 - 4(1)(-2k) \geqslant 0$

$k^2 + 12k + 36 + 8k \geqslant 0$

$k^2 + 20k + 36 \geqslant 0$

$(k+18)(k+2) \geqslant 0$

$\therefore k \leqslant -18 \quad \text{or} \quad k \geqslant -2$

b) $\frac{x+1}{x+3} = \frac{x+4}{2x+2}$ (ratio of intercepts)

$(x+1)(2x+2) = (x+4)(x+3)$

$2x^2 + 2x + 2x + 2 = x^2 + 3x + 4x + 12$

$2x^2 + 4x + 2 = x^2 + 7x + 12$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\therefore x = 5$ cm (since x cannot be negative)

c) $3\tan^2\theta - 1 = 0$

$3\tan^2\theta = 1$

$\tan^2\theta = 1/3$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore \theta = \pi/6, \ 5\pi/6, \ \frac{7\pi}{6}, \ \frac{11\pi}{6}$

Question 6

a) i) $y = 0^3 + 2(0)^2 - 4(0) - 8$

$= -8$ ✓

ii) $y = x^2(x+2) - 4(x+2)$

$= (x+2)(x^2-4)$

$= (x+2)(x+2)(x-2)$

$= (x+2)^2(x-2)$

$\therefore x = -2$ ✓ and $x = 2$ ✓

iii) $y' = 3x^2 + 4x - 4$ ✓

$= (3x-2)(x+2)$

$\therefore x = -2$ or $x = 2/3$. ✓

$y'' = 6x + 4$

<u>when $x = -2$</u>

$y = (-2)^3 + 2(-2)^2 - 4(-2) - 8$

$= 0$

$y'' = 6(-2) + 4$

$= -8$

< 0 ✓

$\therefore (-2, 0)$ is a maxima

Question 6

<u>when $x = 2/3$</u>

$y = (2/3)^3 + 2(2/3)^2 - 4(2/3) - 8$

$= -9\frac{13}{27}$

$y'' = 6(2/3) + 4$

$= 8$

> 0

$\therefore (2/3, -9\frac{13}{27})$ is a minima. ✓

iv) $y'' = 0$

$\therefore 6x + 4 = 0$

$6x = -4$

$x = -2/3$. ✓

$y = (-2/3)^3 + 2(-2/3)^2 - 4(-2/3) - 8$

$= -4\frac{20}{27}$

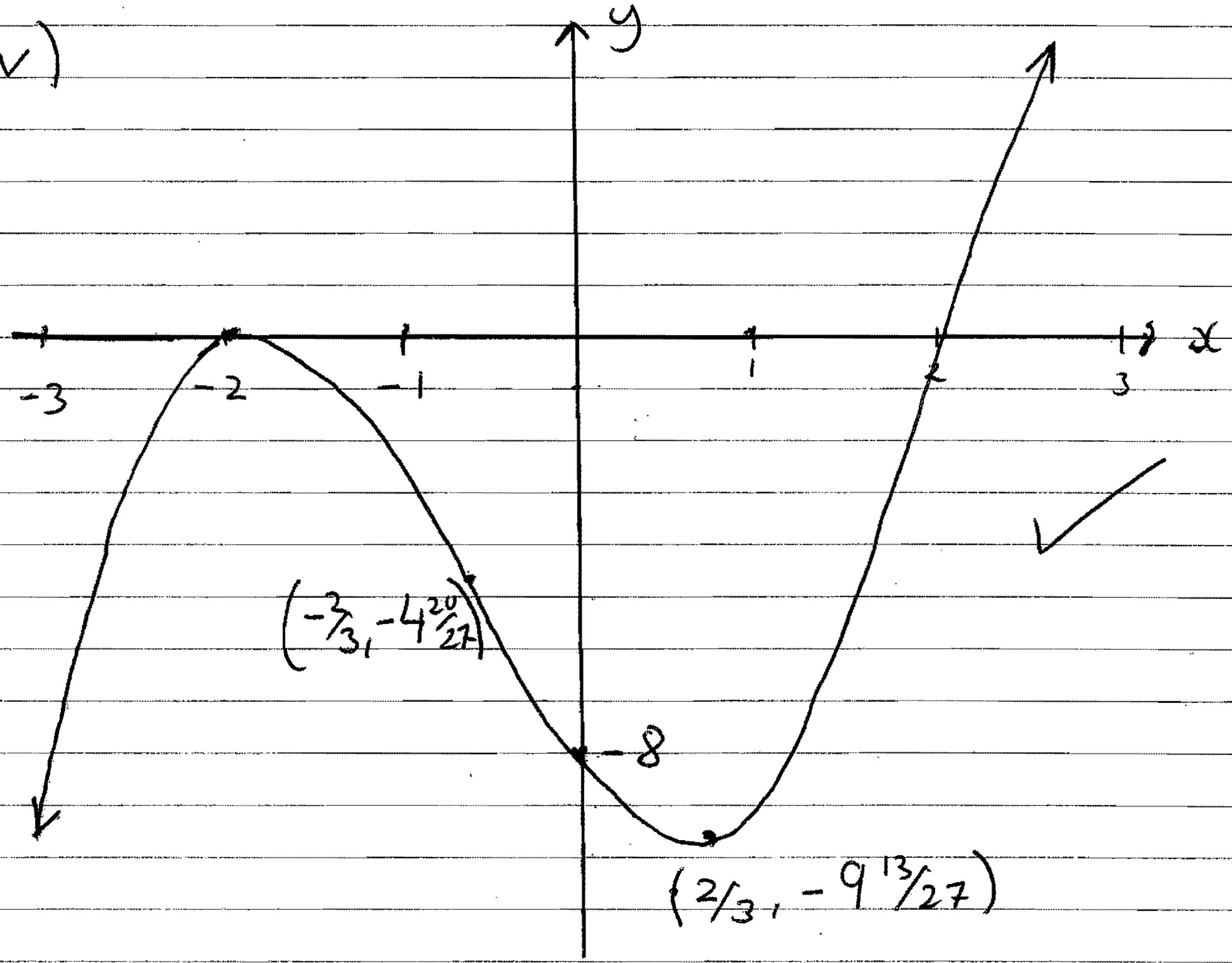
x	-1	$-2/3$	0
y''	$-$	0	$+$

$\therefore$ concavity change ✓

$\therefore (-2/3, -4\frac{20}{27})$ is a point of inflexion

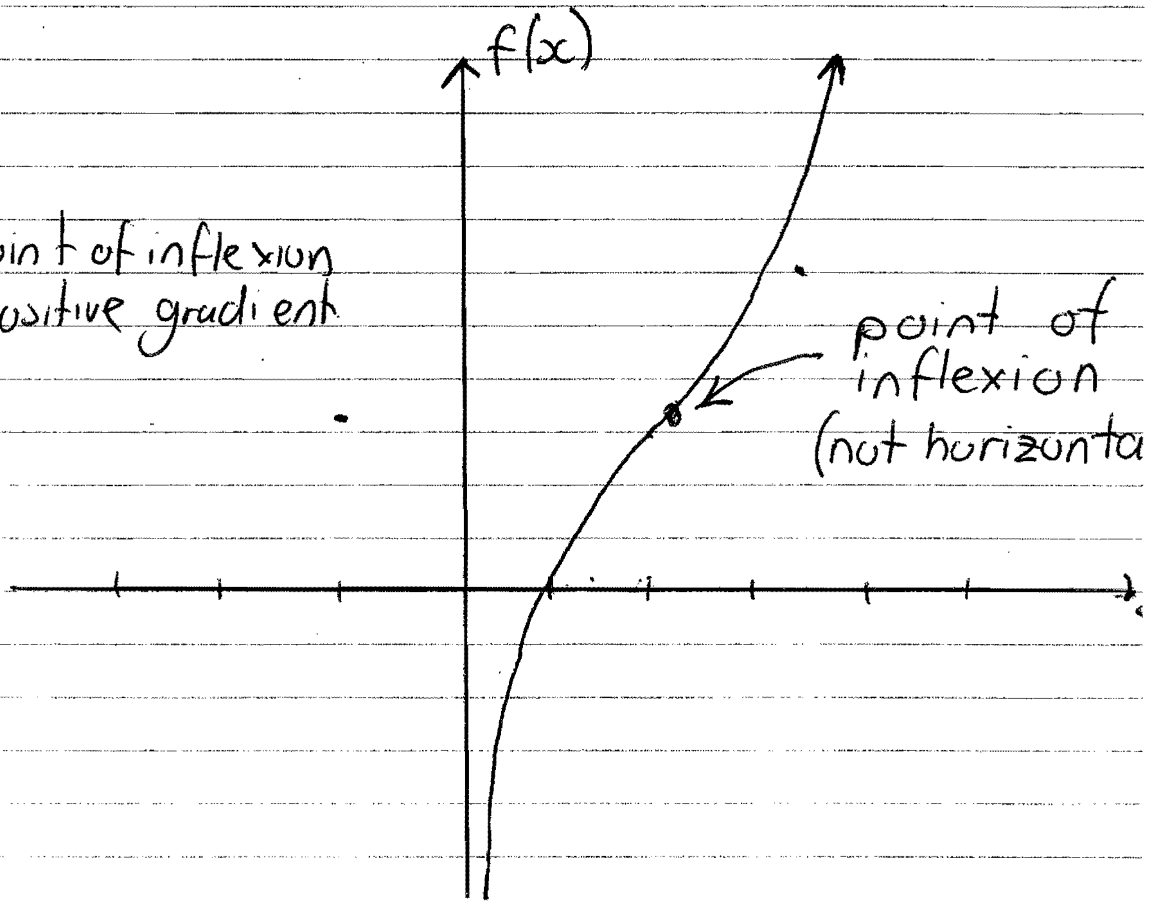
Question 6

v)



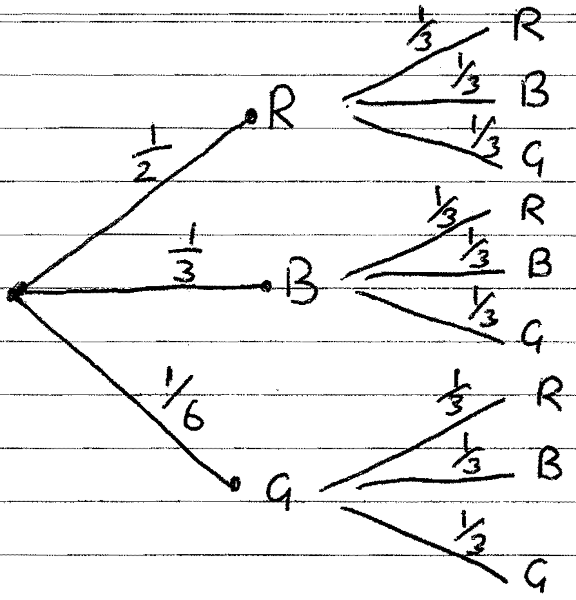
b)

✓ - point of inflexion
✓ - positive gradient



Question 7

a)



i) $P(RR) = \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$ ✓

ii) $P(RB) + P(BR) = \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right)$

$= \frac{5}{18}$ ✓

iii) $1 - (P(RR) + P(GG)) = 1 - \left(\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right)\right)$

$= \frac{7}{9}$ ✓

b) sub-interval width $= 0.25$ ✓

$A \doteqdot \frac{h}{3}(f + 4m + l)$

$A \doteqdot \frac{1}{12}\left(\pi^0 + 4\pi^{0.25} + \pi^{0.5}\right)$ ✓

$+ \frac{1}{12}\left(\pi^{0.5} + 4\pi^{0.75} + \pi^1\right)$

$\doteqdot 1.87$ (2 d.p.) ✓

Question 7

c) i) let $\angle STR = x$

$\therefore \angle TSR = x$ (since $SR = TR$ ΔRST is isoceles and base angles are equal)

$\therefore \angle PRS = 2x°$ (external angle o ΔRST)

$\therefore \angle SPR = 2x$ (since $PQRS$ is a rhombus $PS = SR$ and ΔPRS is isoceles and base angles are equal)

$\therefore \angle SPQ = 4x$ (diagonals bisect the angles of a rhombus)

$\therefore \angle SPQ = 4\angle STR$ as required

ii) $x + 2x + 90° = 180°$ (angle sum of ΔPST)

$\therefore x = 30°$

$\angle PSR = 90° - 30°$ (since $TSR = x$ from i))
$= 60°$

$\therefore \angle PSR = \angle SPR = \angle PRS$ (from i))

$\therefore \Delta PRS$ is equilateral

$\therefore PR = SR$ (sides of equilateral triangle)

$\therefore PR = TR$ (both $= SR$)

$\therefore R$ is the midpoint of PT

Question 8 (Lam)

(a) (2 marks)

✓ [1] correctly factorises numerator.

✓ [1] final answer.

$$\begin{aligned}\lim_{x\to 4}\frac{x^2-16}{x-4} &= \lim_{x\to 4}\frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}}\\ &= 8\end{aligned}$$

(b) i. (2 marks)

✓ [1] for observing $r = \frac{3x}{2}$.

✓ [1] for final answer.

$$\begin{aligned}8 + 12x + 18x^2 + 27x^3 + \cdots\\ r = \frac{T_2}{T_1} = \frac{12x}{8} = \frac{3x}{2}\end{aligned}$$

A limiting sum exists when $|r| < 1$,

$$\begin{aligned}\left|\frac{3x}{2}\right| &< 1\\ \underset{\times 2}{-1} < \underset{\times 2}{\frac{3x}{2}} &< \underset{\times 2}{1}\\ \underset{\div 3}{-2} < \underset{\div 3}{3x} &< \underset{\div 3}{2}\\ \therefore -\frac{2}{3} < x &< \frac{2}{3}\end{aligned}$$

ii. (2 marks)

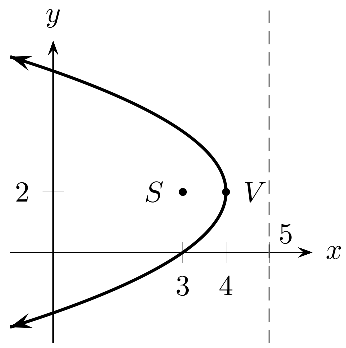
✓ [1] recall limiting sum formula.

✓ [1] for final answer.

$$\begin{aligned}x &= \frac{1}{4}\\ \therefore r &= \frac{3\times\frac{1}{4}}{2} = \frac{3}{8}\\ S &= \frac{a}{1-r} = \frac{8}{1-\frac{3}{8}}\\ &= \frac{8}{\frac{5}{8}} = \frac{64}{5}\end{aligned}$$

(c) i. (1 mark)

$$\begin{aligned}S(3,2)\text{ directrix } x = 5\\ \therefore V(4,2)\end{aligned}$$



ii. (1 mark)

$$(y-2)^2 = -4(x-4)$$

iii. (1 mark)

$$\begin{cases}(y-2)^2 = -4(x-4)\\ 2x+y-6 = 0\end{cases}$$

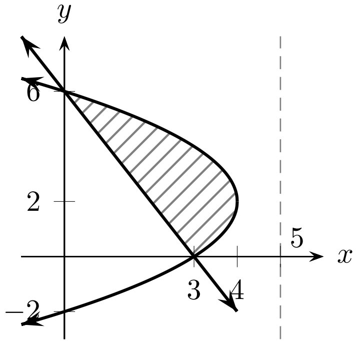
Straight line: $y = -2x + 6$. Substitute into equation of parabola to find pts of intersection:

$$\begin{aligned}(-2x+6-2)^2 &= -4(x-4)\\ (-2x+4)^2 &= -4(x-4)\\ \cancel{4}(x-2)^2 &= \cancel{-4}(x-4)\\ x^2 - \underset{+x}{4x} + 4 &= \underset{+x}{-x} + 4\\ x^2 - 3x &= 0\\ \therefore x &= 0, 3\\ \therefore y &= 6, 0\end{aligned}$$

Alternatively substitute points into line and parabola and verify.

iv. (3 marks)

- ✓ [1] for equation of parabola in terms of y.
- ✓ [1] for equation of line in terms of y.
- ✓ [1] for final answer.
- ✓ [0] for any attempt to integrate w.r.t. x.



$$\begin{aligned}(y-2)^2 &= -4(x-4)\\ \left(y^2-4y+4\right) &= -4(x-4)\\ \therefore x &= -\frac{1}{4}\left(y^2-4y+4\right)+4\\ &= -\frac{1}{4}y^2+y+3\end{aligned}$$

Change subject of the line to x,

$$\begin{aligned}2x+y-6 &= 0\\ 2x &= -y+6\\ \therefore x &= -\frac{1}{2}y+3\end{aligned}$$

Shaded area is between the parabola and the y axis, subtracting the area between the line and the y axis.

$$\begin{aligned}A &= \int_0^6 \left(-\frac{1}{4}y^2+y+3\right)\,dy\\ &\quad -\int_0^6\left(-\frac{1}{2}y+3\right)\,dy\\ &= \int_0^6\left(-\frac{1}{4}y^2+\frac{3}{2}y\right)\,dy\\ &= \left[-\frac{1}{12}y^3+\frac{3}{4}y^2\right]_0^6\\ &= -\frac{1}{12}(6^3)+\frac{3}{4}(6^2)\\ &= 9\end{aligned}$$

Question 9

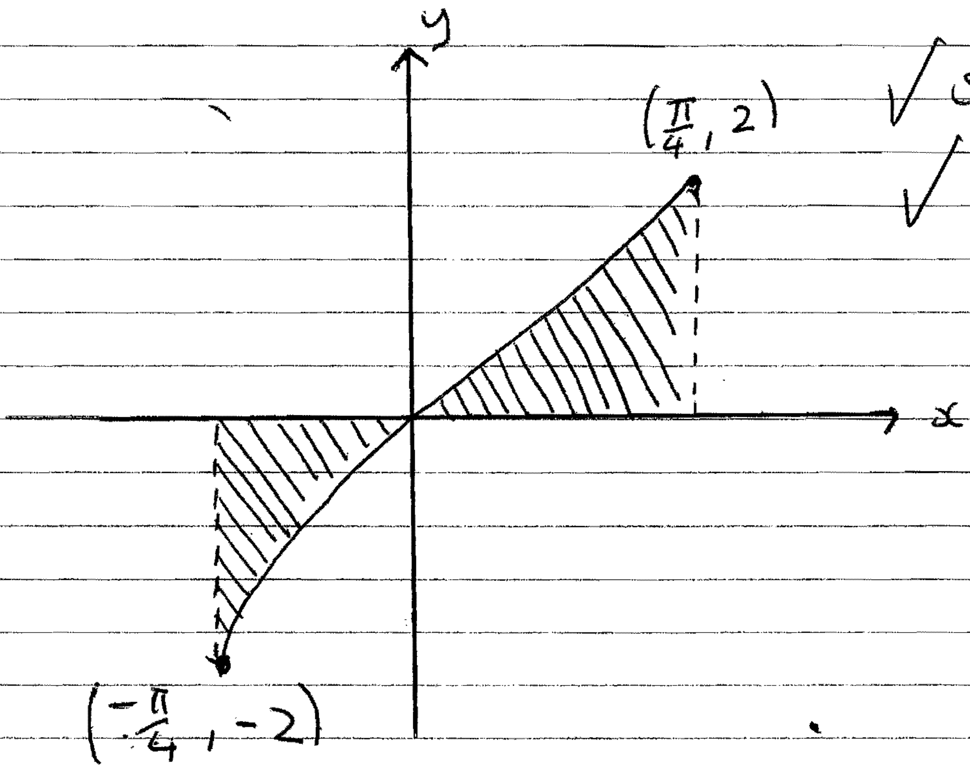
a) $(\cos 0)^2 + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{2\pi}{3}\right) + \cos^2\left(\frac{3\pi}{3}\right)$

$+ \cos^2\left(\frac{4\pi}{3}\right)$

$= 1 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + (-1)^2 + \left(-\frac{1}{2}\right)^2$ ✓

$= 2\frac{3}{4}$ ✓

b) i)



✓ shape

✓ points

ii) $-2 \leq y \leq 2$ ✓

iii) see graph ✓

Question 9

iv) $\frac{d}{dx} \ln(\cos x) = \frac{-\sin x}{\cos x}$ ✓

$= -\tan x$

v) $A = 2\int_0^{\pi/4} (2\tan x)\,dx$ (since it's an odd function)

$= 2\left[-2\ln(\cos x)\right]_0^{\pi/4}$ ✓

$= 2\left[-2(\ln 1/\sqrt{2} - \ln 1)\right]$

$= -4\ln 2^{-1/2}$

$= 2\ln 2$ units^2 ✓

vi) $V = 2\pi\int_0^{\pi/4} (2\tan x)^2\,dx$ ✓ (odd function)

$= 8\pi\int_0^{\pi/4} (\sec^2 x - 1)\,dx$

$= 8\pi\left[\tan x - x\right]_0^{\pi/4}$ ✓

$= 8\pi\left[(\tan(\pi/4) - \pi/4) - (\tan(0) - 0)\right]$

$= 8\pi(1 - \pi/4)$

$= (8\pi - 2\pi^2)$ units^3 ✓

Question 10

a) $\ln 3 + \ln 9 + \ln 27 + \ln 81$

$= \ln 3 + 2\ln 3 + 3\ln 3 + 4\ln 3 + \ldots$ ✓

this is an AP where $a = \ln 3$ and $d = \ln 3$

$S_n = \frac{n}{2}(a + l)$ ✓

$= \frac{50}{2}(\ln 3 + 50\ln 3)$

$= 25 \times 51 \ln 3$

$= 1275 \ln 3.$ ✓

b) $l^2 = r^2 + h^2$ (Pythagoras' Theorem)

$\therefore h^2 = l^2 - r^2$ ✓

$h = \sqrt{l^2 - r^2}$ (since $h > 0$)

$V = \frac{\pi}{3} r^2 h$

$= \pi/3 \; r^2 \sqrt{l^2 - r^2}$

$= \pi/3 \sqrt{r^4} \sqrt{l^2 - r^2}$ ✓

$= \pi/3 \sqrt{l^2 r^4 - r^6}$, as required.

Question 10

c) i) $l = 2\pi\theta$ ✓

ii) $C = 2\pi r$

$\therefore 2\pi\theta = 2\pi r$ (since AB is the circumference)

$\therefore r = \theta$ ✓

iii) $V = \pi/3\sqrt{l^2r^4 - r^6}$ ✓

$= \pi/3\sqrt{2^2\theta^4 - \theta^6}$

$= \pi/3\sqrt{4\theta^4 - \theta^6}$ as required.

iv) $V = \pi/3\,(4\theta^4 - \theta^6)^{1/2}$ ✓

$V' = 1/2 \times \pi/3\,(16\theta^3 - 6\theta^5)\,(4\theta^4 - \theta^6)^{-1/2}$

$= \dfrac{\pi(16\theta^3 - 6\theta^5)}{6\sqrt{4\theta^4 - \theta^6}}$

$= \dfrac{\pi\theta^3(16 - 6\theta^2)}{6\sqrt{4\theta^4 - \theta^6}}$ ✓

$\therefore$ when $v' = 0$

$16 - 6\theta^2 = 0$ (since $\theta \neq 0$)

$6\theta^2 = 16$

$\theta = \sqrt{16/6}$

$\therefore \theta = 1.63$ (2 d.p.) ✓

Question 10

θ	1	1.63	2
V'	+	0	-

✓

$\therefore$ 1.63 maximises the volume.