100 Years

NORTH SYDNEY BOYS HIGH SCHOOL

1912 ~ 2012

MATHEMATICS

2012 HSC Course Assessment Task 3 (Trial Examination)

June 21, 2012

General instructions

- Working time – 3 hours.
  (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer sheet provided (numbered as page 9)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12M2A – Mr Berry

◯ 12M3C – Ms Ziaziaris

◯ 12M3D – Mr Lowe

◯ 12M3E – Mr Lam

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	Total
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{100}$

Section I: Objective response

Mark your answers on the multiple choice sheet provided. **Marks**

1. What is the value of $|-8| - |10|$? **1**

(A) 2 (B) 1 (C) -1 (D) -2

2. What is the sum of the exterior angles of a polygon? **1**

(A) 90° (B) 180° (C) 360° (D) none of these

3. Which conditions make the quadratic $y = ax^2 + bx + c$ positive definite? **1**

(A) $a < 0$, $\Delta < 0$ (B) $a < 0$, $\Delta > 0$ (C) $a > 0$, $\Delta < 0$ (D) $a > 0$, $\Delta > 0$

4. Which of the following is *not* a condition for congruent triangles? **1**

(A) SSS (B) AAA (C) SAS (D) AAS

5. What is 1.9926 to two significant figures? **1**

(A) 2.0 (B) 1.9 (C) 2.09 (D) 2.01

6. Which of the following is the locus of a point that is equidistant from a fixed point and a fixed line? **1**

(A) a parabola (B) a hyperbola (C) a circle (D) an ellipse

7. Evaluate $\displaystyle\lim_{x\to\infty} \frac{3x^5 - 2x^2 + 7x - 3}{6x^5 - 3x + 7}$. **1**

(A) ∞ (B) 0 (C) $\frac{1}{2}$ (D) 2

8. Which of the following conditions for $\dfrac{dP}{dt}$ and $\dfrac{d^2P}{dt^2}$ describe the slowing growth of a variable P? **1**

(A) $\dfrac{dP}{dt} > 0$ and $\dfrac{d^2P}{dt^2} > 0$. (C) $\dfrac{dP}{dt} > 0$ and $\dfrac{d^2P}{dt^2} < 0$.

(B) $\dfrac{dP}{dt} < 0$ and $\dfrac{d^2P}{dt^2} < 0$. (D) $\dfrac{dP}{dt} < 0$ and $\dfrac{d^2P}{dt^2} > 0$.

9. If $a > b$, which of the following is always true? **1**

(A) $a^2 > b^2$ (B) $\frac{1}{a} > \frac{1}{b}$ (C) $-a > -b$ (D) $2^a > 2^b$

10. What is the exact value of b if the area beneath the curve $y = \dfrac{2}{x}$ between $x = 1$ and $x = b$ $(b > 1)$ is equal to 3 units^2? **1**

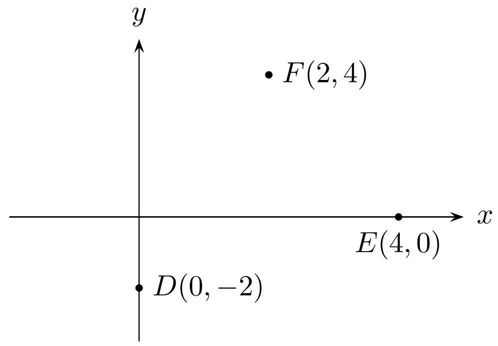
(A) $e^{\frac{3}{2}}$ (B) e^2 (C) $e^{\frac{5}{2}}$ (D) e^3

Section II: Short answer

Question 11 (15 Marks) Commence a NEW page. **Marks**

(a) Solve the equation $x^2 + \frac{9}{x^2} = 10$. **3**

(b) $D(0, -2)$, $E(4, 0)$ & $F(2, 4)$ are three points on the number plane.



i. Calculate the length of the interval DF. **1**

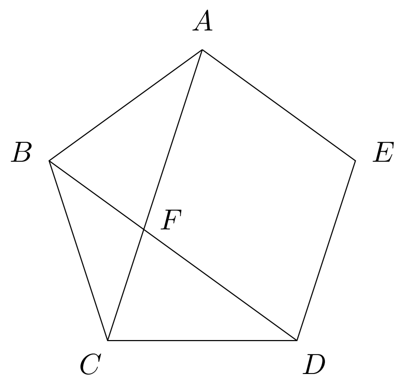
ii. Calculate the gradient of DF. **1**

iii. Write the equation of the line DF in general form. **1**

iv. Calculate the perpendicular distance from E to the line DF. **2**

v. Calculate the area of $\triangle DEF$. **2**

(c) $ABCDE$ is a regular pentagon. The diagonals AC and BD intersect at F.



Copy or trace this diagram into your writing booklet. By giving full reasons for your answer,

i. Prove that $\angle ABC = 108°$. **2**

ii. Find the size of $\angle BAC$. **1**

(d) Find the exact value of $3 \tan 210° + 2 \sin 300°$. **2**

Question 12 (15 Marks) Commence a NEW page. **Marks**

(a) Find the values of p, $p > 0$ for which the roots of the equation $x^2 - px + p = 0$ are

i. opposite in sign. **1**

ii. real **2**

(b) i. Sketch the parabola with equation **2**

$$(y-2)^2 = 2(x+2)$$

Show the vertex of the parabola on your sketch.

ii. Find the coordinates of the focus and the equation of the directrix of the parabola. **2**

(c) The sum to n terms of a sequence of numbers is given by $S_n = 102n - 2n^2$.

i. Find an expression for T_n, the n-th term of the sequence. **2**

ii. What type of a sequence is this? **1**

(d) Differentiate the following expressions:

i. $\dfrac{2}{x^3}$ **1**

ii. $3\cos 4x$ **2**

iii. $\log_e(2x)$ **2**

Question 13 (15 Marks) Commence a NEW page. **Marks**

(a) For the curve $y = x^3(4-x)$

i. Find the stationary point(s) and determine their nature. **3**

ii. Find the point(s) of inflexion. **2**

iii. Draw a neat sketch of the curve showing the intercepts with the coordinate axes, any stationary points and any point(s) of inflexion. **3**

(b) Find $f(x)$ if $f'(x) = 2x + \dfrac{1}{x^2}$ and the curve passes through the point $(1, 2)$. **3**

(c) Find the primitive of

i. $\sqrt[3]{x}$ **2**

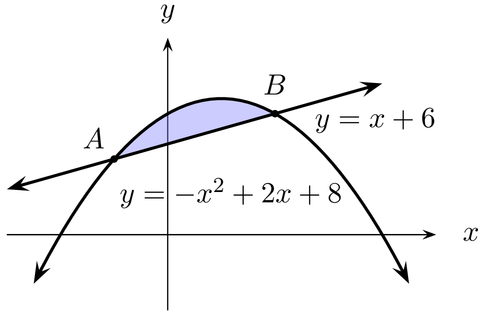
ii. $3\sec^2\dfrac{x}{3}$ **2**

Question 14 (15 Marks) Commence a NEW page. **Marks**

(a) Use Simpson's Rule with five function values to evaluate $\int_1^3 f(x)\,dx$ given the following table: **2**

x	1	1.5	2	2.5	3
$f(x)$	0	3	5	2	1

(b) The diagram below shows the graphs of $y = -x^2 + 2x + 8$ and $y = x + 6$.



i. Show that the x coordinate of A and B are $x = -1$ and $x = 2$ respectively. **2**

ii. Hence or otherwise, find the shaded area bounded by the curves and the straight line. **3**

(c) i. State the period and amplitude of $y = 3\sin 2x$. **2**

ii. Draw a neat sketch of $y = 3\sin 2x$, where $0 \le x \le 2\pi$. **3**

iii. Hence or otherwise, state the number of solutions to the equation **1**

$$3\sin 2x = \frac{2}{3}$$

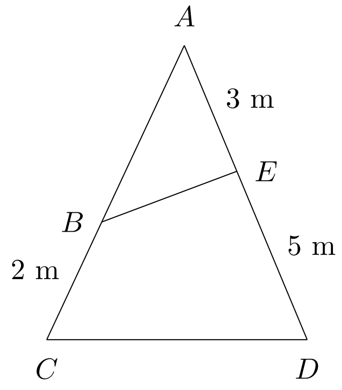
within the domain $0 \le x \le 2\pi$.

(d) Find the angle subtended at the centre of the circle of sector with radius 4 cm and area 20 cm^2. Give your answer correct to the nearest degree. **2**

Question 15 (15 Marks) Commence a NEW page. **Marks**

(a) Solve $8^x = 16^{x+1} \times 4^{-x}$. **2**

(b) In this diagram, $\angle BCD + \angle BED = 180°$.



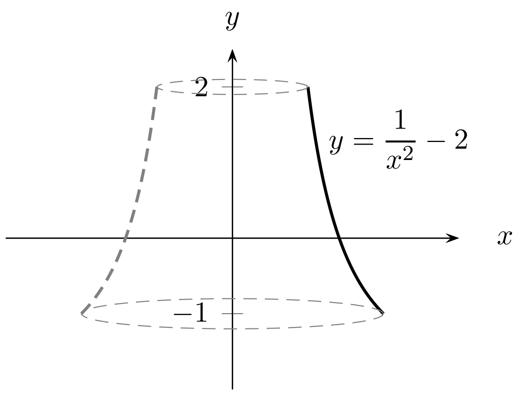
i. Prove that $\triangle ABE$ is similar to $\triangle ADC$. **3**

ii. Given that $AE = 3$ m, $ED = 5$ m and $BC = 2$ m, calculate the length of AB. **3**

(c) i. Evaluate $\dfrac{d}{dx}\left(\log_e(\sin x)\right)$. **1**

ii. Hence or otherwise, find $\displaystyle\int \cot x\, dx$. **2**

(d) A liquor bottle is obtained by rotating about the y axis the part of the curve $y = \dfrac{1}{x^2} - 2$ between $y = -1$ and $y = 2$. **4**

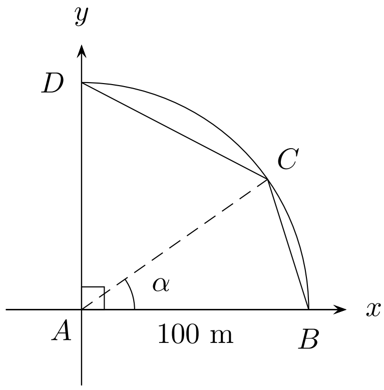


Find the exact volume of the bottle.

Examination continues overleaf...

Question 16 (15 Marks) Commence a NEW page. **Marks**

(a) $ABCD$ is a quadrilateral inscribed in a quarter of a circle centred at A with radius 100 m. The points B and D lie on the x and y axes and the point C moves on the circle such that $\angle CAB = \alpha$ as shown in the diagram below.



i. Solve the equation $\sin(x + 15°) = \cos 24°$. **1**

ii. Show that the area of the quadrilateral $ABCD$ can be expressed as **3**

$$A = 5\,000(\sin\alpha + \cos\alpha)$$

iii. Show that the maximum area of this quadrilateral is $5\,000\sqrt{2}$ m^2. **4**

(b) i. Sketch the curve $y = 4e^{-2x}$. **2**

ii. Consider the series $2e^{x} + 8e^{-x} + 32e^{-3x} + \cdots$.

α) Show that this series is geometric. **1**

β) Find the values of x for which this series has a limiting sum. **2**

γ) Find the limiting sum of this series in terms of x. **2**

End of paper.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE: $\ln x = \log_e x, \; x > 0$

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

STUDENT NUMBER: ....

Class (please ✔)

◯ 12M2A – Mr Berry

◯ 12M3C – Ms Ziaziaris

◯ 12M3D – Mr Lowe

◯ 12M3E – Mr Lam

1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ
6 – Ⓐ Ⓑ Ⓒ Ⓓ
7 – Ⓐ Ⓑ Ⓒ Ⓓ
8 – Ⓐ Ⓑ Ⓒ Ⓓ
9 – Ⓐ Ⓑ Ⓒ Ⓓ
10 – Ⓐ Ⓑ Ⓒ Ⓓ

Suggested Solutions

Section I

(Lowe) **1.** (D) **2.** (C) **3.** (D) **4.** (B) **5.** (A) **6.** (A) **7.** (C) **8.** (C) **9.** (D) **10.** (A)

Question 11 (Lowe)

(a) (3 marks)

✓ [1] for quartic.

✓ [1] for final solutions.

$$\underset{\times x^2}{x^2} + \underset{\times x^2}{\frac{9}{x^2}} = \underset{\times x^2}{10}$$
$$x^4 + 9 = 10x^2$$
$$x^4 - 10x^2 + 9 = 0$$
$$\left(x^2-9\right)\left(x^2-1\right) = 0$$
$$\therefore x = \pm 1, \pm 3$$

(b) i. (1 mark)

$$\begin{aligned} DF &= \sqrt{(2-0)^2 + (4-(-2))^2} \\ &= \sqrt{2^2+6^2} = \sqrt{40} \\ &= 2\sqrt{10} \end{aligned}$$

ii. (1 mark)

$$m_{DF} = \frac{6}{2} = 3$$

iii. (1 mark)

$$\begin{aligned} \frac{y-4}{x-2} &= 3 \\ y-4 &= 3x-6 \\ 3x-y-2 &= 0 \end{aligned}$$

iv. (2 marks)

✓ [1] for correctly recalling perpendicular dist formula.

✓ [1] for final answer.

$$\begin{aligned} d_\perp &= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \\ &= \frac{|3(4)+(-1)(0)-2|}{\sqrt{3^2+1^2}} \\ &= \frac{10}{\sqrt{10}} = \sqrt{10} \end{aligned}$$

v. (2 marks)

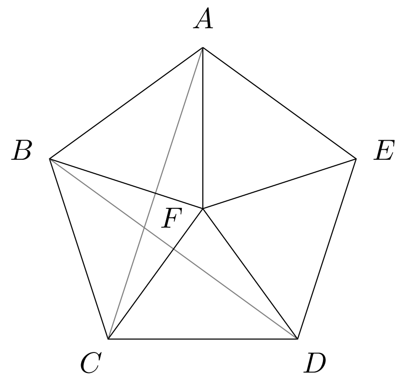
✓ [1] for using parts (iii) & (iv)

✓ [1] for final answer.

$$\begin{aligned} A &= \frac{1}{2} \times DF \times d_\perp \\ &= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} \\ &= 10 \text{ units}^2 \end{aligned}$$

(c) i. (2 marks)

- Divide pentagon into five equilateral triangles.



- Apex angle of one of the triangles $\frac{360°}{5} = 72°$.
- Angle sum of the two base angles is thus

$$180° - 72° = 108°$$

ii. (1 mark)

- $\triangle BAC$ is isosceles.
- $\therefore \angle BAC = \frac{180° - 108°}{2} = 36°$.

(d) (2 marks)

$$\begin{aligned} &3\tan 210° + 2\sin 300° \\ &= 3 \times \left(\frac{1}{\sqrt{3}}\right) + 2 \times \left(-\frac{\sqrt{3}}{2}\right) \\ &= \frac{3}{\sqrt{3}} - \sqrt{3} = 0 \end{aligned}$$

Question 12 (Lowe)

(a) i. (1 mark)

$$x^2 - px + p = 0$$
$$\alpha = -\beta$$
$$\therefore \alpha + \beta = 0 = -\frac{b}{a} = p$$
$$\therefore p = 0$$

But as $p > 0$, therefore there are no real solutions.

ii. (2 marks)

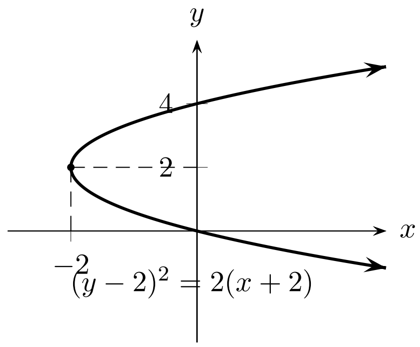
✓ [1] for $p \le 0$ or $p \ge 4$.

✓ [1] justify why $p \ge 4$ only.

$$\Delta \ge 0$$
$$\therefore b^2 - 4ac = p^2 - 4p \ge 0$$
$$p(p-4) \ge 0$$
$$\therefore p \le 0 \text{ or } p \ge 4$$

But as $p > 0$, hence $p \ge 4$ only.

(b) i. (2 marks)



ii. (2 marks)

$$4a = 2$$
$$\therefore a = \frac{1}{2}$$
$$S\left(-\frac{3}{2}, 2\right)$$

Directrix is $x = -\frac{5}{2}$.

(c) i. (2 marks)

$$S_n = 102n - 2n^2$$
$$\begin{aligned} T_n &= S_n - S_{n-1} \\ &= 102n - 2n^2 \\ &\quad - \left(102(n-1) - 2(n-1)^2\right) \\ &= \cancel{102n} - 2n^2 \\ &\quad - \left(\cancel{102n} - 102 - 2(n^2 - 2n + 1)\right) \\ &= \cancel{-2n^2} + 102 + \cancel{2n^2} - 4n + 2 \\ &= 104 - 4n \end{aligned}$$

ii. (1 mark)

$$T_1 = 104 - 4(1) = 100$$
$$T_2 = 104 - 4(2) = 96$$
$$T_3 = 104 - 4(3) = 92$$
$$T_3 - T_2 = T_2 - T_1$$

Arithmetic sequence.

(d) i. (1 mark)

$$\frac{d}{dx}\left(2x^{-3}\right) = -6x^{-4}$$

ii. (2 marks)

$$\frac{d}{dx}(3\cos 4x) = -12\sin 4x$$

iii. (2 marks)

$$\frac{d}{dx}(\log_e 2x) = \frac{d}{dx}(\log_e 2 + \log_e x) = \frac{1}{x}$$

Question 13 (Berry)

(a) i. (3 marks)

$$y = 4x^3 - x^4 = x^3(4-x)$$
$$\frac{dy}{dx} = 12x^2 - 4x^3$$

Stationary pts occur when $\frac{dy}{dx} = 0$:

$$4x^2(3-x) = 0$$
$$\therefore x = 0, 3$$

x		0		3	
$\frac{dy}{dx}$	+	0	+	0	−
y	↗	0	↗	27	↘

Hence $(0, 0)$ is a horizontal point of inflexion and $(3, 27)$ is a local maximum.

ii. (2 marks)
Points of inflexion occur when $\frac{d^2y}{dx^2} = 0$:

$$\frac{d^2y}{dx^2} = 24x - 12x^2 = 12x(2-x)$$
$$\therefore x = 0, 2$$

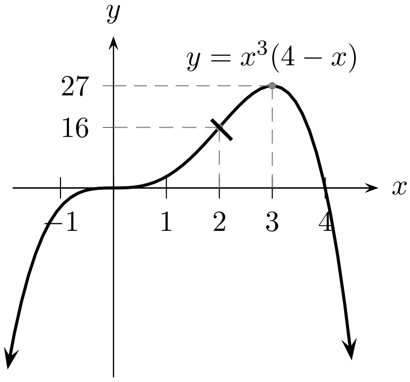
x		0		2	
$\frac{d^2y}{dx^2}$	−	0	+	0	−
	⌢		⌣		⌢

When $x = 2$,

$$y = x^3(4-x)\Big|_{x=2}$$
$$= 2^3(4-2) = 16$$

Hence points of inflexion occur at $(0, 0)$ and $(2, 16)$ as concavity changes at these two pts.

iii. (3 marks)
✓ [−1] for each omission from requirements of the question, provided graph is correct.



(b) (3 marks)
✓ [1] for correct integral.
✓ [1] for correct value of C.
✓ [1] for final answer.

$$f(x) = \int \left(2x + x^{-2}\right)\, dx = x^2 - x^{-1} + C$$
$$f(1) = 1 - 1^{-1} + C = 2$$
$$\therefore C = 2$$
$$\therefore f(x) = x^2 - \frac{1}{x} + 2$$

(c) i. (2 marks)
✓ [−1] if missing arbitrary constant.

$$\int x^{\frac{1}{3}}\, dx = \frac{3}{4}x^{\frac{4}{3}} + C$$

ii. (2 marks)
✓ [−1] if missing arbitrary constant.

$$\int 3\sec^2 \frac{x}{3}\, dx = 9\tan\frac{x}{3} + C$$

Question 14 (Berry)

(a) (2 marks)

$$\begin{aligned}\int_1^3 f(x)\,dx &= \frac{h}{3}\left(y_0 + 4\sum y_{\text{odd}} + 2\sum y_{\text{even}} + y_\ell\right)\\ &= \frac{\frac{1}{2}}{3}\left(0 + 1 + 4(3+2) + 2(5)\right)\\ &= \frac{31}{6}\end{aligned}$$

(b) i. (2 marks)

$$\begin{cases} y = -x^2 + 2x + 8\\ y = x + 6\end{cases}$$

Solve by equating,

$$\begin{aligned}-x^2 + 2x + 8 &= x + 6\\ x^2 - x + 2 &= 0\\ (x-2)(x+1) &= 0\\ \therefore x &= -1, 2\end{aligned}$$

ii. (3 marks)

$$\begin{aligned}A &= \left|\int_{-1}^{2}\left(x^2 - x + 2\right)\,dx\right|\\ &= \left|\left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{-1}^{2}\right|\\ &= \left|-\frac{1}{3}\left(2^3 - (-1)^3\right)\right.\\ &\qquad + \frac{1}{2}\left(2^2 - (-1)^2\right)\\ &\qquad \left. + 2(2 - (-1))\right|\\ &= \left|-3 + \frac{3}{2} + 6\right| = \frac{9}{2}\end{aligned}$$

(c) i. (2 marks)

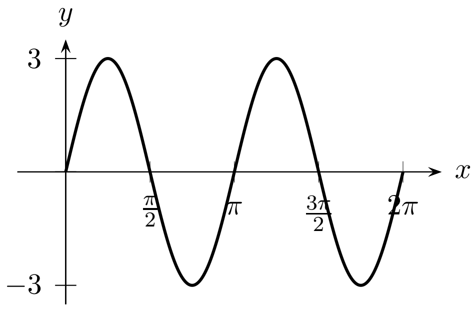
$$T = \frac{2\pi}{2} = \pi \qquad a = 3$$

ii. (3 marks)

✓ [1] for shape.

✓ [1] for correct period.

✓ [1] for amplitude.



iii. (1 mark)
4 solutions.

(d) (2 marks)

✓ [1] for answer in radians.

✓ [1] for answer in degrees.

$$\begin{aligned}A &= \frac{1}{2}r^2\theta\\ 20 &= \frac{1}{2}\times 4^2 \times \theta\\ \theta = \frac{5}{4} &= \frac{5}{4}\times\frac{180^\circ}{\pi} \approx 71^\circ\end{aligned}$$

Question 15 (Ziaziaris)

(a) (2 marks)

✓ [1] for resolving into powers of 2.

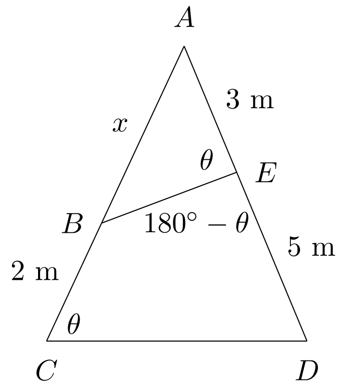
✓ [1] for final answer.

$$\begin{aligned} 8^x &= 16^{x+1} \times 4^{-x} \\ \left(2^3\right)^x &= \left(2^4\right)^{x+1} \times \left(2^2\right)^{-x} \\ 2^{3x} &= 2^{4x+4} \times 2^{-2x} \\ 2^{3x} &= 2^{2x+4} \\ 3x &= 2x+4 \\ x &= 4 \end{aligned}$$

(b) i. (3 marks)

✓ [1] for each correct reason.

In $\triangle ABE$ and $\triangle ADC$



- $\angle CAD$ (common)
- Let $\angle BCD = \theta$. From the information,

$$\angle BED = 180^\circ - \theta$$

  Hence $\angle AEB = \theta$ (supplementary), and $\angle ACD = \angle AEB$.
- $\therefore \angle ABE = \angle ADC$ (remaining $\angle$)

Hence $\triangle ABE ||| \triangle ACD$ (equiangular)

ii. (3 marks)

✓ [1] for ratio of lengths.

✓ [1] for setting up quadratic.

✓ [1] for final answer.

Let $AB = x$. As the ratio of the side lengths of corresponding sides are equal,

$$\begin{aligned} \frac{AB}{AD} &= \frac{AE}{AC} \\ \frac{x}{8} &= \frac{3}{x+2} \\ x(x+2) &= 24 \\ x^2+2x-24 &= 0 \\ (x+6)(x-4) &= 0 \\ \therefore x &= 4, -6 \end{aligned}$$

As $x > 0$ (length), $\therefore x = 4$ only.

(c) i. (1 mark)

$$\frac{d}{dx}\left(\log_e(\sin x)\right) = \frac{\cos x}{\sin x}$$

ii. (2 marks)

$$\begin{aligned} \int \cot x\, dx &= \int \frac{\cos x}{\sin x}\, dx \\ &= \log_e(\sin x) + C \end{aligned}$$

(d) (4 marks)

$$\begin{aligned} y &= \frac{1}{x^2} - 2 \\ y+2 &= \frac{1}{x^2} \\ x^2 &= \frac{1}{y+2} \end{aligned}$$

$$\begin{aligned} V &= \pi \int_{-1}^{2} x^2\, dy = \pi \int_{-1}^{2} \frac{dy}{y+2} \\ &= \pi \Big[\log_e(y+2)\Big]_{-1}^{2} \\ &= \pi\left(\log_e 4 - \log_e 1\right) \\ &= \pi \log_e 4 \end{aligned}$$

Question 16 (Lam)

(a) i. (1 mark)

$$\sin(x+15^\circ) = \cos 24^\circ = \sin(90^\circ - 24^\circ)$$
$$x + 15^\circ = 66^\circ$$
$$\therefore x = 51^\circ$$

ii. (3 marks)

$$A_{\triangle CAB} = \frac{1}{2} \times 100^2 \sin\alpha = 5\,000\sin\alpha$$
$$A_{\triangle CAD} = \frac{1}{2} \times 100^2 \sin\left(\frac{\pi}{2} - \alpha\right) = 5\,000\cos\alpha$$
$$\therefore A_{ABCD} = 5\,000\,(\sin\alpha + \cos\alpha)$$

iii. (4 marks)

$$A_{ABCD} = 5\,000\,(\sin\alpha + \cos\alpha)$$
$$\therefore \frac{dA}{d\alpha} = 5\,000(\cos\alpha - \sin\alpha)$$

Stationary pts occur when $\frac{dA}{d\alpha} = 0$, i.e.

$$5\,000(\cos\alpha - \sin\alpha) = 0$$
$$\underset{\div\cos\alpha}{\cos\alpha} = \underset{\div\cos\alpha}{\sin\alpha}$$
$$\tan\alpha = 1$$
$$\therefore \alpha = \frac{\pi}{4}$$

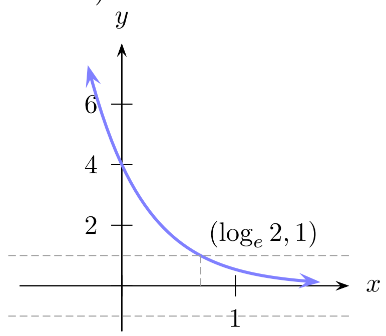
α		$\frac{\pi}{4}$	
$\frac{dA}{d\alpha}$	$+$	0	$-$
A	↗	—	↘

- $\alpha < \frac{\pi}{4}$, $\frac{dA}{d\alpha} < 0$.
- $\alpha > \frac{\pi}{4}$, $\frac{dA}{d\alpha} > 0$.

Maximum area occurs when

$$A = 5\,000\,(\sin\alpha + \cos\alpha)\Big|_{\alpha=\frac{\pi}{4}}$$
$$= 5\,000\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 5\,000\left(\frac{2}{\sqrt{2}}\right)$$
$$= 5\,000\sqrt{2}\text{ m}^2$$

(b) i. (2 marks)



ii. (α) (1 mark)
$2e^x + 8e^{-x} + 32e^{-3x}\cdots$

$$\frac{T_2}{T_1} = \frac{8e^{-x}}{2e^x} = 4e^{-2x}$$
$$\frac{T_3}{T_2} = \frac{32e^{-3x}}{8e^{-x}} = 4e^{-2x}$$
$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$\therefore 2e^x + 8e^{-x} + 32e^{-3x}\cdots$ is a geometric series with $a = 2e^x$ and $r = 4e^{-2x}$.

(β) (2 marks)
✓ [1] for $\left|4e^{-2x}\right| < 1$.
✓ [1] for justification.
A geometric series has a limiting sum when $-1 < r < 1$; i.e.

$$-1 < 4e^{-2x} < 1$$

By inspecting the graph in the previous part, $-1 < 4e^{-2x} < 1$ when $x > \log_e 2$.

$\therefore$ limiting sum exists when

$$x > \log_e 2$$

(γ) (2 marks)
✓ [1] for recalling formula
✓ [1] for final answer

$$S = \frac{a}{1-r} = \frac{2e^x}{1 - 4e^{-2x}}$$