North Sydney Girls High School.
2u Trial. 1989.

QUESTION 1:

(a) Find correct to three decimal places the value of:

$$\frac{5.6 \times 4.9^3}{\sqrt{7.3} + 4.1}$$

(b) Solve the equation:

$$x - \frac{x+1}{3} = 2 + \frac{x}{5}$$

(c) Last year Council rates increased by 7½%.
The new rate for a property is \$1735.
What was the old rate for this property?
Give your answer correct to the nearest dollar.

(d) The point P $(0, -4\frac{1}{2})$ is the mid-point of A $(g, -2)$ and B $(-5, h)$.
Find the values of g and h.

(e) A carton contains a dozen eggs, 3 or which have a double yolk.
If 3 eggs are required to make a cake, find the probability that at least one of the eggs used is a double yolk.

QUESTION 2:

(a) Determine: $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

(b) Differentiate the following with respect to x:

(i) $\dfrac{1}{\sqrt{x}}$

(ii) $x^2 e^{-x}$

(iii) $(\cos x + \sin x)^3$

(iv) $\log \left(\dfrac{x + 2}{x - 2}\right)$

(c) If α and β are the roots of the equation

$2x^2 - 4x + 1 = 0$ find:

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\alpha^2 + \alpha\beta + \beta^2$

(d) Find the area of the sector of a circle in which the arc length is 15 cm and the radius is 7 cm.

QUESTION 3:

(a) Find the primitive function of each of the following:

(i) $12x^2 - 4$

(ii) $e^{7x} + 14$

(iii) $(3x + 5)^6$

(b) Evaluate: $\displaystyle\int_1^{13} \frac{dx}{2x + 6}$

(c) Find all values of k for which the quadratic equation $kx^2 - 8x \div k = 0$ has real roots.

(d) The gradient function of a curve is given by

$$\frac{dy}{dx} = 3 - 4x$$

Find the equation of the curve if it passes through the point $(6, -5)$.

QUESTION 4

(a) P is the point $(-2, 7)$

d is the line $2x + 5y + 1 = 0$

k is the line through P, perpendicular to d.

(i) Find the equation of k.

(ii) If d meets the y-axis at D, and k meets the y-axis at K, find the area of the triangle PKD.

(b) Solve: $m^4 - 6m^2 - 40 = 0$

(c) Find the equation of the circle which is concentric with the circle $x^2 + y^2 + 8x + 2y + 8 = 0$ and which passes through the point $(1, 7)$.

(d) Find the values of x for which

$$|2x - 1| \leq 5$$

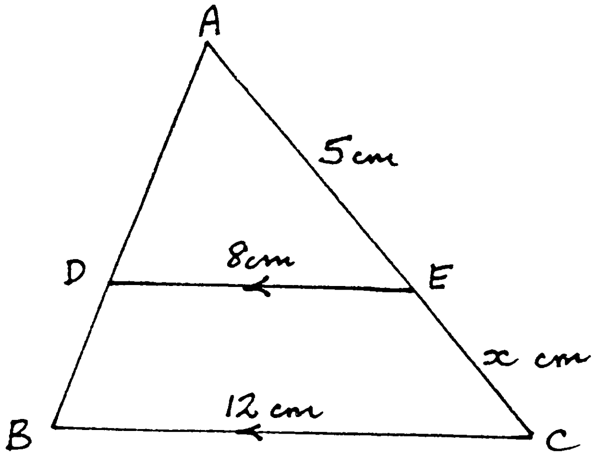
QUESTION 5

(a) The fifth term of an arithmetic series is 14, and the sum of the first 10 terms is 165.

Find the first term of this series.

(b) Find the size of each internal angle of a regular pentagon. Hence or otherwise find the size of each of the external angles.

(c)



Find x, giving reasons in full.

(Diagram not drawn to scale)

(d) PQRS is a quadrilateral. Its diagonals are perpendicular, meeting at T, which is the midpoint of QS, but not of PR.

(i) Draw a diagram showing this information.

(ii) Prove that $P\hat{Q}R = P\hat{S}R$

QUESTION 6

(a) If $\cos\theta° = -\frac{5}{11}$ and $180° < \theta < 360°$

give the exact value of $\operatorname{cosec}\theta°$

(b) From a lighthouse, L, a ship, S, bears 053° T and is at a distance of 8 nautical miles. From L a boat B bears 293° T and is a distance of 6 nautical miles.

(i) Draw a diagram marking on it the information supplied.

(ii) Find the distance of Ship S from boat B.
Give your answer as a surd.

(iii) Find the bearing of ship S from boat B.
Give your answer to the nearest degree.

(c) Tennis balls are often supplied in a cylindrical container which holds 3 balls in a neat fit (the centre of the balls are collinear.) What percentage of the volume of the container is occupied by the balls?

QUESTION 7

(a) If $S_n = n(n - 2)$, find an expression for the n th term of the series.

(b) Which term of 2, 6, 18, ... is 486?

(c) A person invests \$500 at the beginning of each year in a superannuation fund.
Compound interest is paid at 10% per annum on the investment.
The first \$500 is to be invested at the beginning of 1989 and the last is to be invested at the beginning of 2018.
Calculate, to the nearest dollar:

(i) the amount to which the 1989 investment will have grown by the beginning of 2019.

(ii) the amount to which the total investment will have grown by the beginning of 2019.

(d) Solve $2x^2 + 4x - 7 = 0$ in exact form.

QUESTION 8

(a) For the parabola $x^2 - 8y = 0$

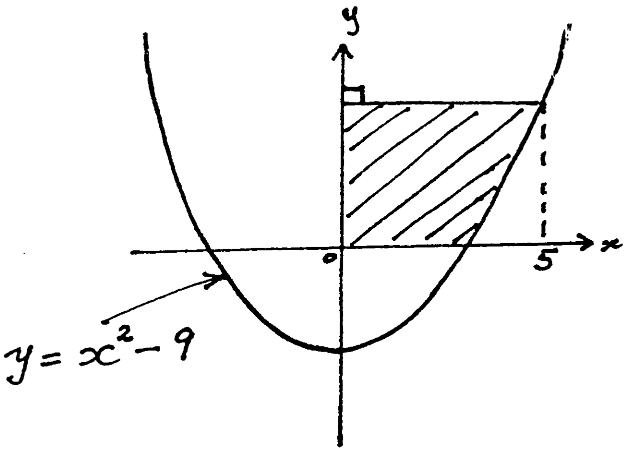
(i) find the coordinates of the Vertex

(ii) find the coordinates of the Focus

(iii) find the equation of the Directrix

(iv) find the equation of the tangent to the parabola at the point where $x = 4$

(b) If the shaded region is rotated around the y axis, find the volume of the solid of revolution generated.



(c) Differentiate e^{x^2}. Hence or otherwise find $\int_0^1 x\, e^{x^2}\, dx$.

QUESTION 9

(a) Sketch the curve $y = e^{-x} + 2$. Evaluate the area bounded by the curve, the x axis and the ordinates $x = 0$ and $x = 3$.

(b) For the curve $y = x^3 + 3x^2 - 9x$

(i) Find the stationary points and determine their nature.

(ii) Find any points of inflection.

(iii) Using the above information and any other relevant information, sketch the curve $y = x^3 + 3x^2 - 9x$.

(iv) Find the set of values for which the curve is monotonically increasing.

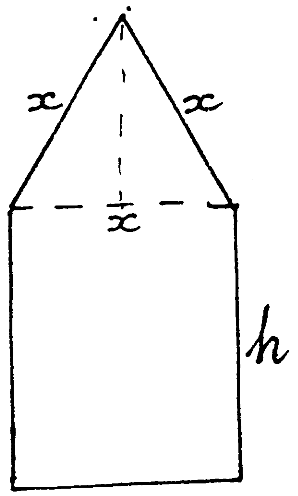
QUESTION 10

(a) The following table gives values of $f(x) = x \log x$

x	1	2	3	4	5
f(x)	0	1.39	3.30	5.55	8.05

Use Simpson's Rule with five function values to find an approximation for the value of $\int_1^5 x \log x \, dx$

(b)



The figure represents a large window made up of a rectangle and an equilateral triangle.
The sides of the equilateral triangle are x m and the dimensions of the rectangle are x m and h m as shown.

(i) If the beading around the outside of the window is 16 m long show that $h = 8 - \frac{3}{2}x$.

(ii) Show that an expression for the Area of the window is given by

$$A = 8x + \frac{x^2}{4}(\sqrt{3} - 6)$$

(iii) Hence calculate the dimensions of the window which allow the maximum light to pass through (correct to 2 decimal places).

1) (a) $\underline{\underline{19 \cdot 768}}$ (1)

(b) $x - \frac{(x+1)}{3} = 2 + \frac{x}{5}$

$\therefore 15x - 5(x+1) = 30 + 3x$ ← 1 mark

$\therefore 15x - 5x - 5 = 30 + 3x$

$10x - 5 = 30 + 3x$

$\therefore 7x = 35$

$\therefore \underline{\underline{x = 5}}$ (2)

(c) Old rate 100%

New rate $107\frac{1}{2}\%$ is \$1735

$\therefore$ 1% is $\$\frac{1735}{107 \cdot 5}$

$\therefore$ 100% is $\$\frac{1735}{107 \cdot 5} \times 100$

$= \underline{\underline{\$1614}}$ (2)

(to nearest dollar)

(d) $0 = \frac{g-5}{2}$ $\qquad -4\frac{1}{2} = \frac{-2+h}{2}$

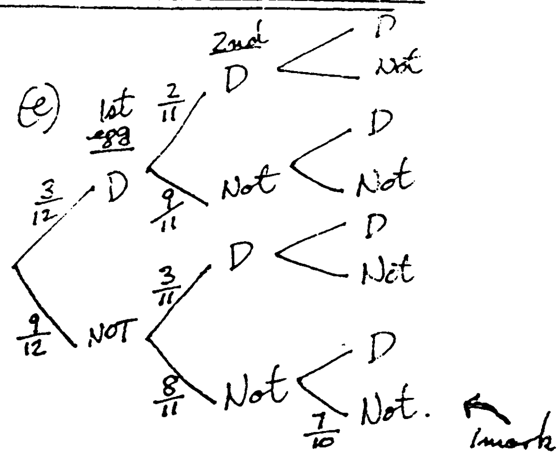
$\therefore 0 = g - 5$ $\qquad \therefore -\frac{9}{2} = \frac{-2+h}{2}$

$\therefore \underline{\underline{g = 5}}$ $\qquad \therefore -9 = -2 + h$

1 each $\qquad \therefore \underline{\underline{h = -7}}$

(2)

(e)



P(at least 1 D) 1 mark

$= 1 - P(\text{No double-yolks})$

$= 1 - P(\text{Not} \times \text{Not} \times \text{Not})$

$= 1 - \frac{9}{12} \times \frac{8}{11} \times \frac{7}{10}$

$= 1 - \frac{21}{55}$

$= \underline{\underline{\frac{34}{55}}}$ (3)

$\boxed{10}$

2) (a) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

$= \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

$= \lim_{x \to 3} (x+3)$

$= \underline{\underline{6}}.$ (1)

(b) (i) $y = x^{-\frac{1}{2}}$

$\therefore \frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{-1}{2x^{3/2}}$ (1)

(ii) $y = x^2 e^{-x}$

$\frac{dy}{dx} = (x^2).(e^{-x}.-1) + (e^{-x}).(2x)$

$= xe^{-x}(2-x)$ (1)

(iii) $y = (\cos x + \sin x)^3$

$\therefore \frac{dy}{dx} = 3(\cos x + \sin x)^2(-\sin x + \cos x)$ (1)

$= 3(\cos x + \sin x)^2(\cos x - \sin x)$

(iv) $y = \log\left(\frac{x+2}{x-2}\right)$

$y = \log(x+2) - \log(x-2)$

$\therefore \frac{dy}{dx} = \frac{1}{x+2} - \frac{1}{x-2}$

$= \frac{(x-2)-(x+2)}{x^2-4}$

$= \frac{x-2-x-2}{x^2-4}$

$= \frac{-4}{x^2-4}$ (1)

(c) $2x^2 - 4x + 1 = 0$

(i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{-(-4)}{2}$

$= 2$ (1)

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{1}{2}$ (1)

(iii) $\alpha^2 + \alpha\beta + \beta^2$

$= (\alpha+\beta)^2 - \alpha\beta$

$= (2)^2 - (\frac{1}{2})$

$= 4 - \frac{1}{2}$ (1)

$= 3\frac{1}{2}$

(d) $l = r\theta$

$\therefore \theta = \frac{l}{r}$

$= \frac{15}{7}$ ← 1 mark.

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 49 \times \frac{15}{7}$

$= \frac{7 \times 15}{2}$

$= \frac{105}{2}$

$= 52\frac{1}{2}$ cm^2 (2)

$\boxed{10}$

3) (a)

(i) $\int 12x^2 - 4 \, dx$

$= \frac{12x^3}{3} - 4x + c$

$= \underline{4x^3 - 4x + c}$ (1)

(ii) $\int e^{7x} + 14 \, dx$

$= \underline{\frac{e^{7x}}{7} + 14x + c}$ (1)

(iii) $\int (3x+5)^6 \, dx$

$= \frac{(3x+5)^7}{7.3} + c$

$= \underline{\frac{(3x+5)^7}{21} + c}$ (1)

(b) $\int_1^{13} \frac{dx}{2x+6}$

$= \frac{1}{2}\left[\log(2x+6)\right]_1^{13}$ ← 1 mark.

$= \frac{1}{2}\{\log 45 - \log 3\}$

$= \frac{1}{2} \log \frac{45}{3}$

$= \frac{1}{2} \underline{\log} 15$ (or $\log\sqrt{15}$.) (2)

(c) For real roots $\Delta \geq 0$ ← 1 mark

$\Delta = b^2 - 4ac$

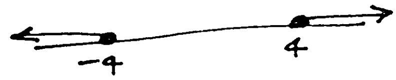
$= (-8)^2 - 4(k)(k)$

$= 64 - 4k^2$

$\therefore 4k^2 - 64 \overset{\leq}{\geq} 0$ 1 mark

$\therefore 4(k^2 - 16) \overset{\leq}{\geq} 0$

$\therefore (k-4)(k+4) \geq 0$



$\therefore \underline{k \leq -4 \; , \; k \geq 4}$ (3)

(d) $\frac{dy}{dx} = 3 - 4x$

$\therefore y = 3x - \frac{4x^2}{2} + c$

$\therefore y = 3x - 2x^2 + c$ ← 1 mark.

Thru' $(6, -5)$

$\therefore -5 = 18 - 72 + c$.

$\therefore -5 = -54 + c$

$49 = c$

$\therefore y = \underline{-2x^2 + 3x + 49}$ (2)

10

4 (a) (i) d is $2x+5y+1=0$

$\therefore 5y = -2x-1$

$\therefore y = -\frac{2}{5}x - \frac{1}{5}$

$\therefore$ slope of k is $\frac{5}{2}$ ← 1 mark

Thru' $(-2,7)$ eqn of k is

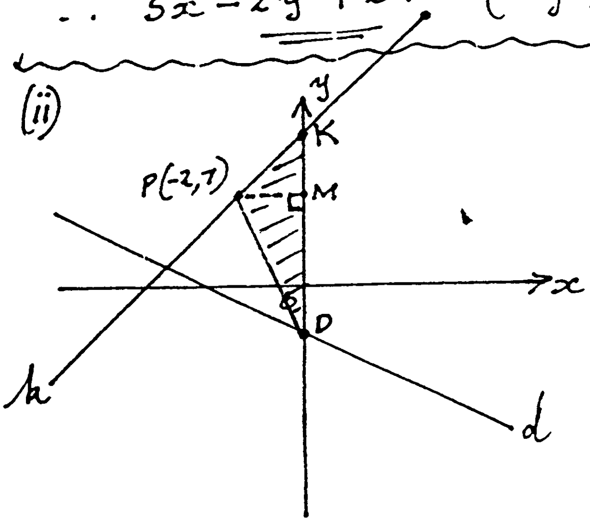
$(y-7) = \frac{5}{2}(x+2)$

$\therefore 2(y-7) = 5(x+2)$

$2y - 14 = 5x+10$ (2)

$\therefore 5x - 2y + 24 = 0$ (or $y = \frac{5}{2}x + 12$)

(ii)



D is point $(0, -\frac{1}{5})$

K is point $(0, 12)$

$\therefore$ length $KD = 12 + \frac{1}{5}$

$= 12\frac{1}{5}$

From sketch PM is 2

} 1 mark

Area $\Delta PKD = \frac{1}{2}$ base × height

$= \frac{1}{2} \times KD \times PM$

$= \frac{1}{2} \times 12\frac{1}{5} \times 2$

$= \frac{1}{2} \times \frac{61}{5} \times 2$ (2)

$= 12\frac{1}{5}$ units^2

(b) $m^4 - 6m^2 - 40 = 0$

Let $x = m^2$

$\therefore x^2 - 6x - 40 = 0$

$(x+4)(x-10) = 0$ ← 1 mark

$\therefore x = -4, 10$

$\therefore m^2 = -4$ or $m^2 = 10$

NO SOLN. $\therefore m = \pm\sqrt{10}$ (2)

(c) $x^2 + y^2 + 8x + 2y + 8 = 0$

$\therefore x^2 + 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$

$\therefore (x+4)^2 + (y+1)^2 = 9$ ← 1 mark.

New circle also has centre $(-4,-1)$. $\therefore$ New circle is

$(x+4)^2 + (y+1)^2 = r^2$

Thru' $(1,7)$

$\therefore (1+4)^2 + (7+1)^2 = r^2$

$\therefore 25 + 64 = r^2$

$89 = r^2$

$\therefore$ Circle is

$(x+4)^2 + (y+1)^2 = 89$. (2)

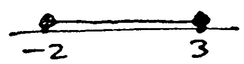
(d) 1 mark

$2x - 1 \leq 5$ or $-(2x-1) \leq 5$

$\therefore 2x \leq 6$ $\quad -2x + 1 \leq 5$

$x \leq 3$ $\quad -2x \leq 4$

$x \geq -2$



$\therefore -2 \leq x \leq 3$ (2)

10

5 (a) $T_n = a + (n-1)d$

Both substitutions 1 mark

$\therefore 14 = a + 4d$ —(1)

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\therefore 165 = \frac{10}{2}\{2a + 9d\}$

$165 = 5(2a + 9d)$

$33 = 2a + 9d$ —(2)

(1)×9 $\quad 126 = 9a + 36d$ —(3)

(2)×4 $\quad 132 = 8a + 36d$ —(4)

(3)−(4) $\quad -6 = a$

$\therefore$ First term is $\underline{\underline{-6}}$ (2)

(b) Angle Sum $(S) = (2n-4)$ rt angles

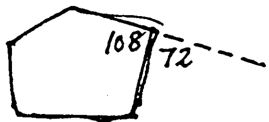
$\therefore S = (10-4) \times 90$

$S = 6 \times 90$

$S = 540$

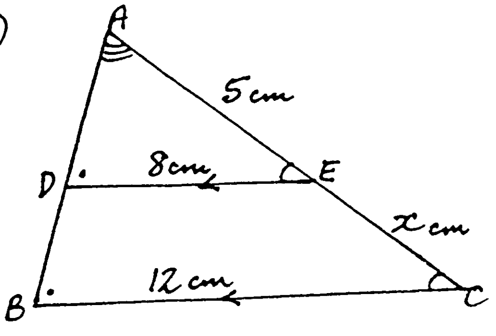
$\therefore$ Each angle is $\frac{540}{5}$

$= \underline{\underline{108^\circ}}$ ← 1 mark



$\therefore$ each external angle is $\underline{\underline{72^\circ}}$ (OR $\frac{360}{5} = 72^\circ$) (2)

(c)



$\widehat{ADE} = \widehat{ABC}$ (corr, $DE \parallel BC$)

$\widehat{AED} = \widehat{ACB}$ (corr, $DE \parallel BC$)

$\widehat{DAE}$ is common — 1 mark

$\therefore \triangle ADE \mid\mid\mid \triangle ABC$ (equiangular)

$\therefore \frac{8}{12} = \frac{5}{(x+5)}$ (corr sides in sim. $\triangle$'s)

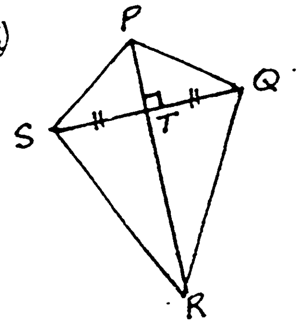
$\therefore 8(x+5) = 60$

$8x + 40 = 60$

$\therefore 8x = 20$

$\therefore x = \underline{\underline{2\frac{1}{2}}}$ (2)

(d) (i)



(2)

(ii) In $\triangle$'s PTS, PTQ

$ST = TQ$ (data)

PT is common

$\widehat{PTQ} = \widehat{PTS}$ ($=90^\circ$ data)

$\therefore \triangle PTS \equiv \triangle PTQ$ (SAS)

$\therefore \widehat{PST} = \widehat{PQT}$ (corr $\angle$'s in $\equiv \triangle$'s) ← 1 mark

Similarly $\widehat{TSR} = \widehat{TQR}$

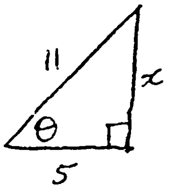
But $\widehat{PQR} = \widehat{PQT} + \widehat{TQR}$

and $\widehat{PSR} = \widehat{PST} + \widehat{TSR}$ (2)

$\therefore \underline{\underline{\widehat{PQR} = \widehat{PSR}}}$

10

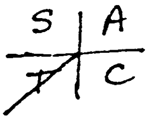
6) (a) $\cos\theta = -\frac{5}{11}$



$11^2 = x^2 + 5^2$

$121 = x^2 + 25$

$x = \sqrt{96}$ ← 1 mark

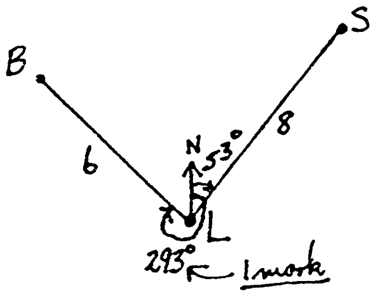


3rd quad since $180 < \theta < 360$

$\therefore \operatorname{cosec}\theta = -\frac{11}{\sqrt{96}}$ (2)

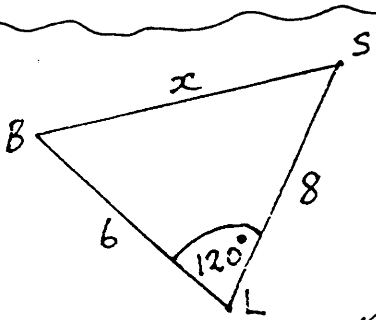
(or $-\frac{11}{4\sqrt{6}}$ or $-\frac{11\sqrt{6}}{24}$)

(b)(i)



← 1 mark (2)

(ii)



1 mark

$x^2 = 6^2 + 8^2 - 2.6.8.\cos 120$

$= 36 + 64 - 96\cos(180-60)$

$= 100 + 96\cos 60$ S|A T|C

$= 100 + 96.\frac{1}{2}$

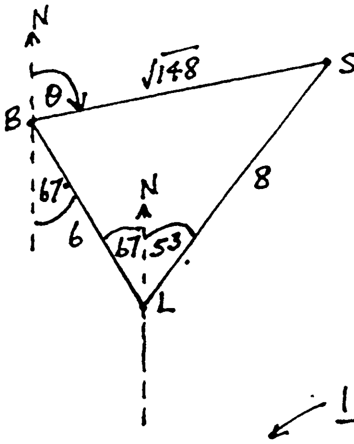
$= 100 + 48$ (2)

$= 148$

$\therefore x = \sqrt{148}$

$\therefore$ dist SB is $\underline{\sqrt{148}}$ (or $2\sqrt{37}$) n.miles.

(iii)



1 mark

$\frac{\sin \widehat{SBL}}{8} = \frac{\sin 120}{\sqrt{148}}$

$\therefore \sin \widehat{SBL} = \frac{8\sin 120}{\sqrt{148}}$

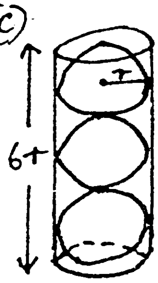
$\therefore \widehat{SBL} = 35°$

$\therefore \theta = 180 - (67+35)$

$= 78°$ (2)

$\therefore$ S is $\underline{078°T}$ from B.

(c)



V of balls $= 3 \times \frac{4}{3}\pi r^3$

$= 4\pi r^3$

V of cylinder $= \pi r^2 h$

$= \pi r^2 \times 6r$

$= 6\pi r^3$

$\therefore$ % occupied $= \frac{4\pi r^3}{6\pi r^3} \times 100\%$

1 mark (both volumes).

$= \frac{200}{3}\%$

$= \underline{66\frac{2}{3}\%}$ (2)

10

7) (a) $T_n = S_n - S_{n-1}$

$= [n(n-2)] - [(n-1)(n-3)]$ ← 1 mark

$= (n^2 - 2n) - (n^2 - 4n + 3)$

$= n^2 - 2n - n^2 + 4n - 3$

$= \underline{2n - 3}$ (2)

(b) $2, 6, 18, \ldots$ is a G.P.

$a = 2$
$r = 3$ $\quad T_n = ar^{n-1}$

$\therefore 486 = (2)(3)^{n-1}$

$243 = 3^{n-1}$ ← 1 mark → OR $3^5 = 3^{n-1}$, $\therefore 5 = n-1$, $\therefore 6 = n$

$\therefore \log 243 = \log 3^{(n-1)}$

$\therefore \log 243 = (n-1)\log 3$

$\therefore n - 1 = \dfrac{\log 243}{\log 3}$

$\therefore n - 1 = 5$ (2)

$\therefore \underline{n = 6}$.

(c) (i)

$A = P\left(1 + \frac{r}{100}\right)^n$

$= 500\left(1 + \frac{10}{100}\right)^{30}$ ← 1 mark

$= 500(1 \cdot 1)^{30}$

$= \$\,\underline{8725}$

(2)

(ii)

$A = 500(1 \cdot 1^{30} + 1 \cdot 1^{29} + \cdots + 1 \cdot 1^1)$

$= 500(\underbrace{1 \cdot 1^1 + 1 \cdot 1^2 + \cdots + 1 \cdot 1^{30}}_{\text{G.P.}})$.

1 mark

$n = 30$
$r = 1 \cdot 1$
$a = 1 \cdot 1$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_{30} = \dfrac{1 \cdot 1(1 \cdot 1^{30} - 1)}{1 \cdot 1 - 1}$

$= \dfrac{1 \cdot 1(1 \cdot 1^{30} - 1)}{0 \cdot 1}$

$\therefore A = \dfrac{500 \times 1 \cdot 1(1 \cdot 1^{30} - 1)}{0 \cdot 1}$

$= \$\,\underline{90472}$ (2)

(d) $2x^2 + 4x - 7 = 0$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-7)}}{2(2)}$

$= \dfrac{-4 \pm \sqrt{16 + 56}}{4}$

1 mark → $= \dfrac{-4 \pm \sqrt{72}}{4}$

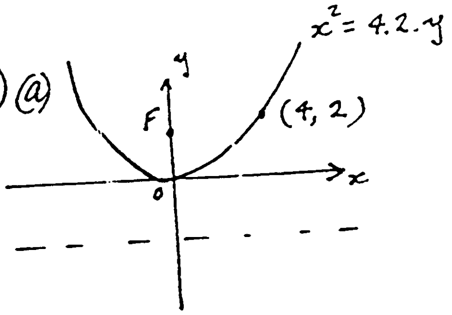
$= \dfrac{-4 \pm 6\sqrt{2}}{4}$

$= \dfrac{2(-2 \pm 3\sqrt{2})}{2}$ (2)

$= \underline{-2 \pm 3\sqrt{2}}$

$\boxed{10}$

8) (a)



(i) V $\underline{(0,0)}$ (1)

(ii) F $\underline{(0,2)}$ (1)

(iii) Dir. $\underline{y=-2}$ (1)

(iv) $y = \frac{x^2}{8}$

$\therefore \frac{dy}{dx} = \frac{2x}{8}$

$= \frac{x}{4}$

At $x=4$ $f(4) = \frac{4}{4}$

$= 1$ ← 1 mark

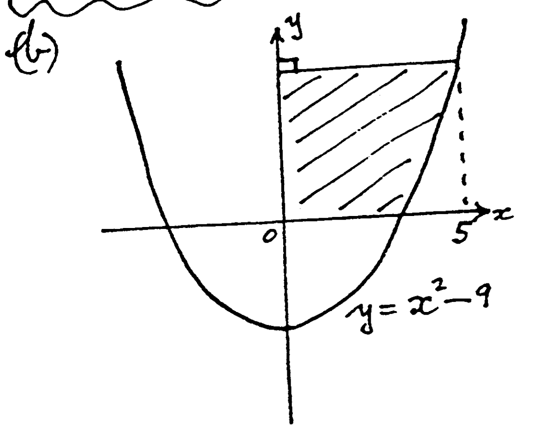
$\therefore$ equ of tang. is

$(y-2) = 1\ (x-4)$

$\therefore y-2 = x-4$

$\therefore \underline{x-y-2=0}$ (2)

(b)



$x=5 \Rightarrow y=16$

$V = \int \pi x^2\, dy$ 1 mark

$= \pi \int_0^{16} (y+9)\, dy$

$= \pi \left[\frac{y^2}{2} + 9y\right]_0^{16}$

$= \pi\{(128+144)-0\}$

$= \underline{272\pi}$ units^3 (2)

(c) $y = e^{x^2}$

$\frac{dy}{dx} = e^{x^2} \times 2x$

$= \underline{2x\,e^{x^2}}$ ← 1 mark

$\int_0^1 x\,e^{x^2}\,dx$

$= \frac{1}{2}\int_0^1 2x\,e^{x^2}\,dx$

$= \left[\frac{1}{2}\cdot e^{x^2}\right]_0^1$ ← 1 mark.

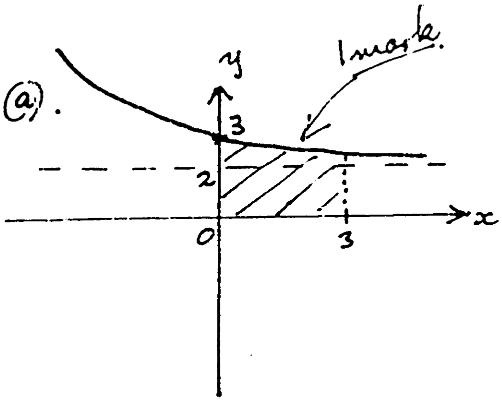
$= \frac{1}{2}\left[e^{x^2}\right]_0^1$

$= \frac{1}{2}\{e^1 - e^0\}$

$= \frac{1}{2}\underline{(e-1)}$ (3)

$\boxed{10}$

9) (a).



$$A = \int_0^3 (e^{-x}+2)\,dx$$

$$= \left[-e^{-x}+2x\right]_0^3$$

$$= (-e^{-3}+6)-(-e^{-0}+0)$$

$$= -\frac{1}{e^3}+6+1$$

$$= 7-\frac{1}{e^3} \text{ units}^2 \quad (2)$$

(b)(i) $y = x^3+3x^2-9x$

$$\frac{dy}{dx} = 3x^2+6x-9$$

For st pts $\frac{dy}{dx}=0$

$\therefore\ 3x^2+6x-9=0$

$\therefore 3(x^2+2x-3)=0$

$(x+3)(x-1)=0$

$\therefore x = 1, -3$

At $x=1$ $f''(x)=6x+6$

$= 6+6$

> 0

$\therefore$ MIN at $(1,-5)$

At $x=-3$ $f''(-3) = -18+6$

< 0 ← 1 each (2)

$\therefore$ MAX at $(-3, 27)$

(ii) Pts of infl $f''(x)=0$

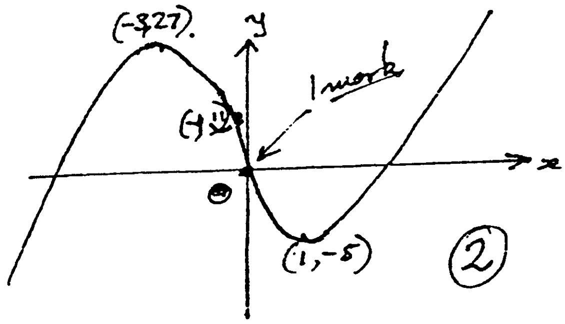
$\therefore\ 6x+6=0$

$x=-1$ ← 1 mark

$f''(-3)<0$
$f''(1)>0$ $\Big\}\ \therefore$ sign change

$\therefore$ Pt of infl at $(-1, 11)$ (2)

(iii)



(2)

(iv) for mon. increasing $\frac{dy}{dx}>0$ ← 1 mark

$\therefore\ 3x^2+6x-9>0$

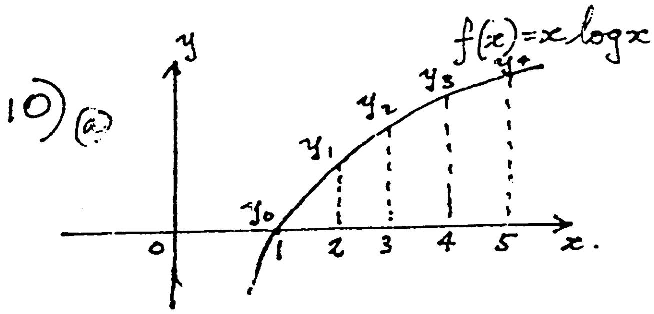
$\therefore\ x^2+2x-3>0$

$(x+3)(x-1)>0$

$-3 \qquad 1$

$\therefore\ x<-3\ ,\ x>1$ (2)

10) (a)



$$\int_1^5 x \log x \, dx$$

1 formula

$$\doteq \frac{h}{3}\{(y_0 + y_n) + 2\,\text{evens} + 4\,\text{odds}\}$$

$$= \frac{1}{3}\{(f(1)+f(5)) + 2f(3) + 4(f(2)+f(4))\}$$

1 subst.

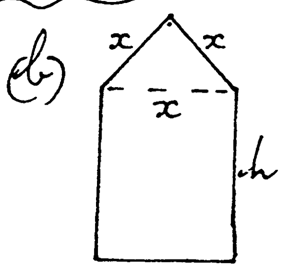
$$= \frac{1}{3}\{(0+8.05) + 2\times 3.30 + 4(1.39+5.55)\}$$

$$= \frac{1}{3}(8.05 + 6.6 + 27.76)$$

$$= \underline{14.14}$$

there are alternate methods. (3)

(b)



1 mark

(i) $x + x + h + x + h = 16$

$\therefore 2h + 3x = 16$

$\therefore 2h = 16 - 3x$

$\therefore h = 8 - \frac{3x}{2}$ (2)

(ii) $A = \frac{1}{2} x . \frac{\sqrt{3}}{2} x$ (triangle)

$+\ xh$ (rectangle)

1 mark

$$= \frac{x^2\sqrt{3}}{4} + x\left(8 - \frac{3}{2}x\right)$$

$$\therefore A = \frac{x^2\sqrt{3}}{4} + 8x - \frac{3x^2}{2} \quad (2)$$

$$= \underline{8x + \frac{x^2}{4}(\sqrt{3} - 6)}$$

(iii) $\frac{dA}{dx} = 8 + \frac{1}{2}x(\sqrt{3} - 6)$

$= 0$ when $8 + \frac{x}{2}(\sqrt{3}-6) = 0.$

$\therefore (\sqrt{3} - 6)x = -16$

1 mark $\quad x = \frac{-16}{(\sqrt{3} - 6)}$

$= \underline{3.75}$

1 mark

$\frac{d^2A}{dx^2} = \frac{\sqrt{3} - 6}{2}$

< 0

$\therefore$ MAX.

Now $h = 8 - \frac{3}{2}x$

$= 8 - \frac{3}{2} \times 3.75$

$= \underline{2.38}$

$\therefore$ Dimensions of window for max are $h = \underline{2.38\text{m}}$ (3)

$x = \underline{3.75\text{m}}$

$\boxed{10}$