NORTH SYDNEY GIRLS HIGH SCHOOL

2008

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this page
- All necessary working should be shown in every question

Total Marks – 120

Attempt Questions 1–10

All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top. Submit one bundle. The bundle will be separated before marking commences so that anonymity will be maintained

Student Number: ______________________ **Teacher:** ______________________________

Student Name: _____________________________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
8	/12
9	/12
10	/12
TOTAL	/120
	%

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Total marks – 120
Attempt Questions 1 – 10
All questions are of equal value.

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate correct to 3 significant figures: $e^3 + \log_e 200$. **2**

(b) Find the values of x for which $|2x-7| \leq 5$. **2**

(c) The cost of a laptop computer is \$1 287. This includes 10% G.S.T. calculated on the original price. Calculate the original price of the computer. **1**

(d) Find integers a and b such that $\left(1-2\sqrt{3}\right)^2 = a - b\sqrt{3}$. **2**

(e) Solve $\frac{x}{4} - \frac{x-1}{3} = 5$. **2**

(f) Differentiate $\log_e x - \frac{1}{x}$ with respect to x. **2**

(g) Evaluate $\lim\limits_{x\to\infty} \frac{3x^2-1}{2x^2+1}$. **1**

Marks

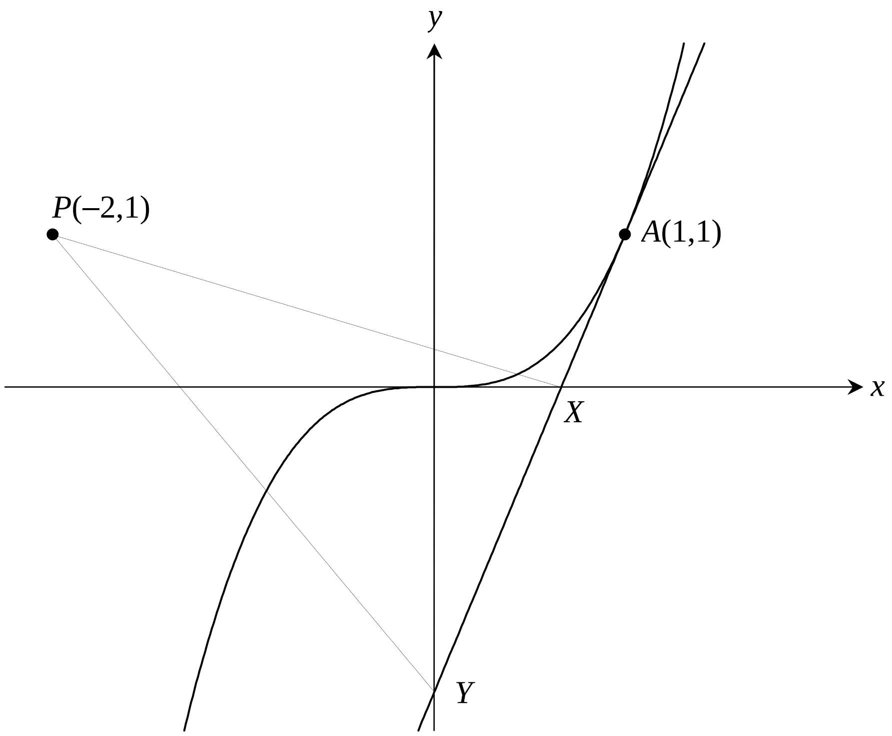
Question 2 (12 marks) Use a SEPARATE writing booklet

(a) Consider the parabola $x = y^2 - 6y + 5$. Find:

(i) the coordinates of the vertex; **2**

(ii) the coordinates of the focus. **2**

(b) The diagram shows the graph $y = x^3$. A is the point (1, 1) and the tangent at A cuts the x and y axes at X and Y respectively.



(i) Show that the equation of the tangent at A is given by $y = 3x - 2$. **2**

(ii) Write down the coordinates of X and Y. **2**

(iii) Find the perpendicular distance from the point $P(-2, 1)$ to the tangent at A. **2**

(iv) Find the area of ΔPXY. **2**

Marks

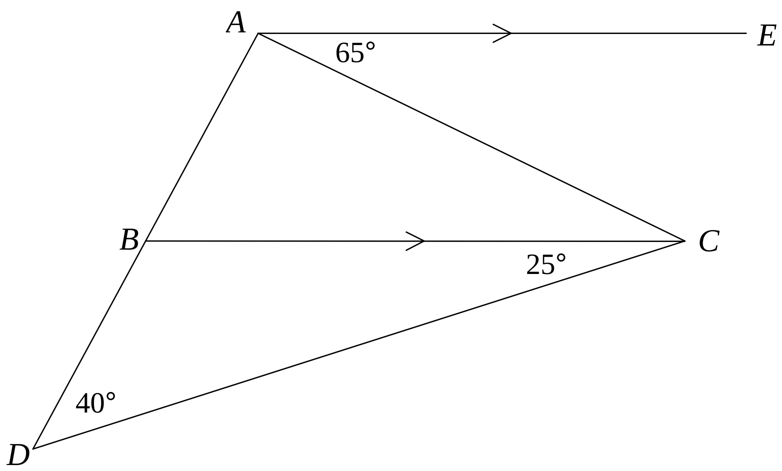
Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

(i) $\log_e \sqrt{2x+3}$ **2**

(ii) $\dfrac{x^2}{e^x}$ **2**

(b) In the diagram below, AE is parallel to BC.
$\angle BDC = 40°$, $\angle BCD = 25°$, $\angle EAC = 65°$.



NOT TO SCALE

(i) Copy the diagram into your booklet.

(ii) Prove that ΔABC is isosceles. **3**

(c) Find:

(i) $\int e^{2x}dx$ **1**

(ii) $\int \dfrac{dx}{(2x-3)^2}$ **2**

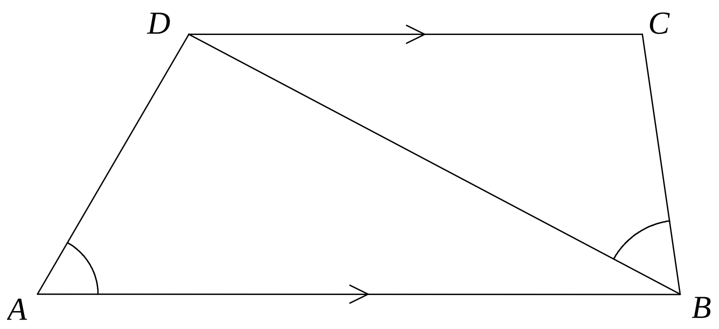
(iii) $\int_0^{\frac{\pi}{4}} \left(\sec^2 x + x\right) dx$ **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\log_3(2x-1)=4$ **2**

(b) $ABCD$ is a trapezium such that $AB \parallel CD$ and $\angle DAB = \angle DBC$.



Copy the diagram into your booklet and prove $BD=\sqrt{AB\times CD}$ **3**

(c) (i) Draw a neat sketch of $y=3\cos 2x$ for $-\pi \le x \le \pi$ **2**

(ii) On the same set of axes sketch $y=x$. **1**

(iii) How many solutions has the equation $3\cos 2x = x$? **1**

(d) The quadratic equation $x^2-x+3=0$ has roots α and β. Find

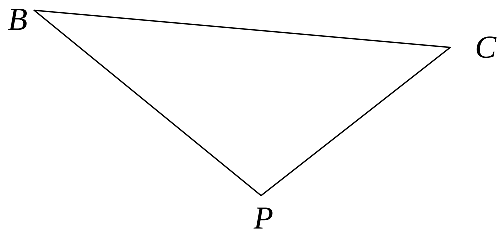
(i) $\dfrac{1}{\beta}+\dfrac{1}{\alpha}$ **1**

(ii) $\alpha^2+\beta^2$ **2**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) A fishing boat located at P observes two islands B and C where B is in a westerly direction from C.
The bearings of B and C from P are 295° and 058° respectively.
P is 7 km from B and 5 km from C.

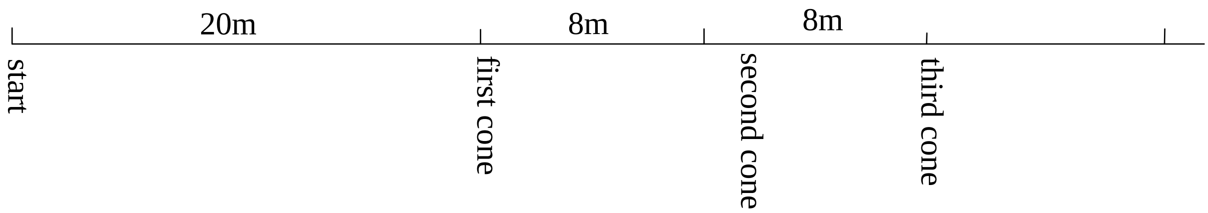


(i) Find the size of $\angle BPC$. **1**

(ii) Find the distance from B to C. (Answer correct to 1 decimal place). **2**

(iii) The boat is only permitted to operate within the triangular area PBC. Find this area correct to the nearest square kilometre. **1**

(b) As part of her training program Jana places a series of cones on the ground.
The first cone is 20 m from her starting point and each successive cone is 8 m from the previous one.



She runs from her starting point to the first cone, then back to the start.
She then runs to the second cone back to the start and repeats the process until she has visited all the cones.

(i) How many cones did she put out if the last cone is 116 m from the start? **2**

(ii) How far does she run to complete her training? **3**

(c) Solve $\sin\theta = -0\cdot 7$ for $0 \leq \theta \leq 2\pi$ giving your answers in radians correct to 2 decimal places. **3**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) The gradient function for a curve is given by $f'(x) = 3(x+2)(x-1)$ and the curve passes through the point $(0, 7)$.

(i) Find the equation of the curve $y = f(x)$. **2**

(ii) Find the coordinates of all stationary points of the curve $y = f(x)$ and determine their nature. **3**

(iii) Find the values of x for which the curve $y = f(x)$ is concave down. **1**

(iv) Sketch the curve $y = f(x)$. **2**

(b) A total of 15 tickets are sold in a raffle which has three prizes. There are 5 green, 5 blue and 5 red tickets.
At the drawing of the raffle, winning tickets are **not** replaced before the next draw.

(i) What is the probability that each of the winning tickets is red? **1**

(ii) What is the probability that at least one of the winning tickets is not red? **1**

(iii) What is the probability that there is one winning ticket of each colour? **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\sum_{n=2}^{5}(3n-2)$ **1**

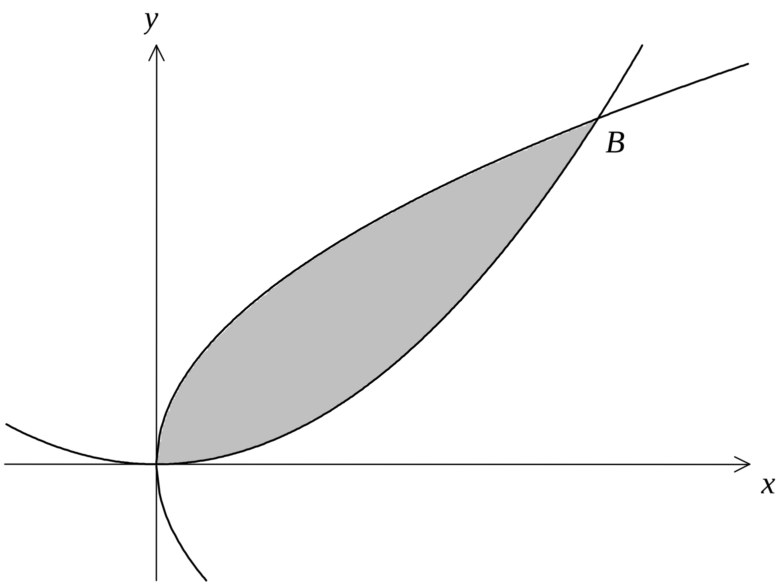
(b) Consider the function $y=\log_e(x+2)$ for $x>-2$.

(i) Sketch the function showing its essential features. **2**

(ii) Use the trapezoidal rule using two trapezia to find an approximation for $\int_0^4 \log_e(x+2)dx$ expressing your answer in the form $\log_e K$. **3**

(iii) Is this approximation more or less than the actual value? Justify your answer. **1**

(c) The curves $y=x^2$ and $y^2=8x$ intersect at the origin and the point B.



(i) Find the coordinates of B. **2**

(ii) Find the area between the curves $y=x^2$ and $y^2=8x$. **3**

Marks

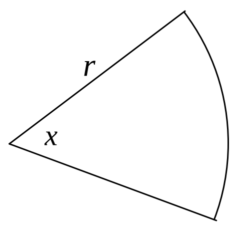
Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the series: $1+\left(\sqrt{\alpha}-2\right)+\left(\sqrt{\alpha}-2\right)^2+\left(\sqrt{\alpha}-2\right)^3+\ldots$

(i) Explain why the largest integer value of α for the series to have a limiting sum is given by $\alpha=8$. **1**

(ii) Find this sum for this value of $\alpha=8$ expressing your answer as a surd with a rational denominator. **2**

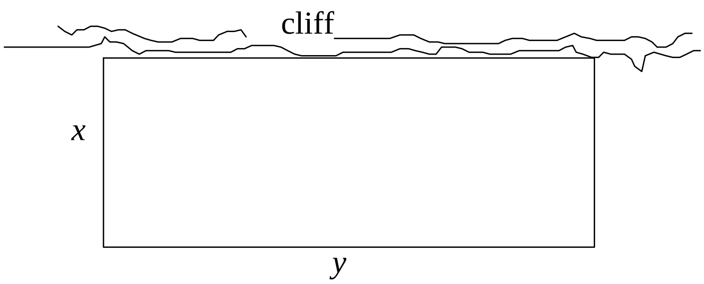
(b) A sector of a circle has a perimeter or 14 cm and an area of 12 cm^2.



(i) Use this information to form a pair of simultaneous equations. **1**

(ii) Solve these equations to find the radius and angle in radians of the sectors which satisfy this condition. **3**

(c) A farmer wishes to enclose a rectangular field of area 800 m^2. The cost of the fencing for three of the sides is $20 per metre but the fourth side is next to a cliff and this side costs $60 per metre.



(i) Show that the cost $C of fencing the field is given by **2**

$$C=40x+\frac{64000}{x}$$

(ii) Find the dimensions of the field which will allow the most economical fence to be built. **3**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

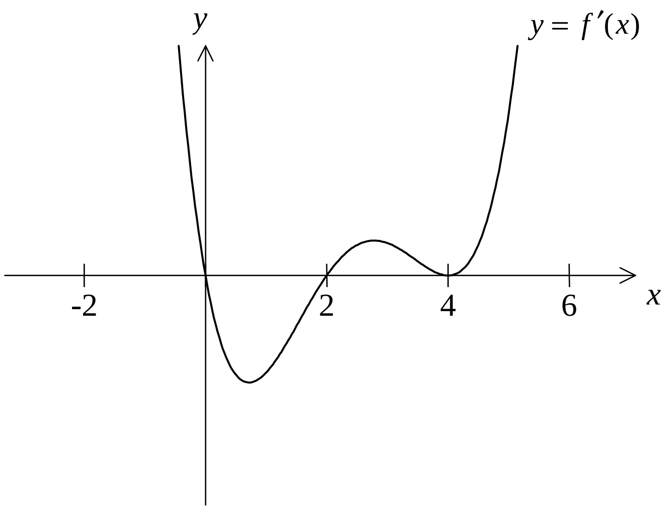
(a) Show that the function $y = \dfrac{1}{1+\tan x}$ is decreasing for all values of x. **3**

(b) Use Simpson's Rule with three function values to estimate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} x \sin x \, dx$, **3**

Giving your answer correct to 2 significant figures.

(c) The area between the curve $y = e^{2x} - 1$, the x-axis and $x = 1$ is rotated about the x-axis. Find the volume of the solid of revolution generated. (Leave your answer in terms of e and π.) **4**

(d) The diagram shows the graph of the gradient function of the curve $y = f(x)$.



For what value of x does $f(x)$ have a local minimum? Justify your answer. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet

(a) Two clowns Bart and Crusty enter a pie throwing contest in which the first to hit the other wins. Bart has a 40% chance of hitting Crusty but Crusty has an 80% chance of hitting Bart.
Both clowns have 2 pies each and throw in turn, **commencing with Bart**.

(i) Find the probability that Bart does not win with his first throw. **1**

(ii) Find the probability that there is no winner. **2**

(iii) Find the probability that Bart wins. **2**

(b) Joanne intends to save \$100 000 over the next 5 years for a deposit on a house. She deposits her entire monthly salary \$$S$ at the start of each month in an account that pays 6% p.a. compounded monthly. She withdraws \$$M$ at the end of each month for living expenses.
Let B_n represent the balance in her account at the end of each month after her living expenses have been withdrawn.

(i) Express B_1 in terms of S and M. **1**

(ii) Show that $B_2 = S(1\cdot005+1\cdot005^2) - M(1+1\cdot005)$ **2**

(iii) Assuming this pattern continues show that **2**

$$B_{60} = (1\cdot005^{60} - 1)(201S - 200M)$$

(iv) Calculate her monthly living expenses (M) if her monthly salary (S) is \$4 000 and she is to reach her savings goal of \$100 000 after 5 years. **2**

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SOLUTIONS — NORTH SYDNEY GIRLS HIGH MATHEMATICS 2008 (TRIAL)

Question 1

(a) $25.383\ldots$
$= 25.4$ (3sf)

(b) $|2x-7| \le 5$
$-5 \le 2x-7 \le 5$
$2 \le 2x \le 12$
$1 \le x \le 6$

(c) 110% price $= \$1287$
$\therefore 100\% = \frac{1287}{110} \times 100$
Original Price $= \$1170$

(d) $(1-2\sqrt{3})^2$
$= 1 - 4\sqrt{3} + 4\times 3$
$= 13 - 4\sqrt{3}$
$\therefore a = 13,\ b = 4$

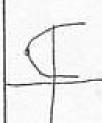
(e) $\frac{x}{4} - \frac{x-1}{3} = 5$
$3x - 4(x-1) = 60$
$3x - 4x + 4 = 60$
$-x = 56$
$x = -56$

(f) $\frac{d}{dx}\left[\ln x - \frac{1}{x}\right]$
$= \frac{1}{x} + x^{-2}$
$= \frac{1}{x} + \frac{1}{x^2}$

(g) $\lim_{x\to\infty} \frac{3x^2-1}{2x^2+1}$
$= \frac{3}{2}$

Question 2

(a) $x = y^2 - 6y + 5$
$y^2 - 6y = x - 5$
$y^2 - 6y + 9 = x + 4$
$(y-3)^2 = 4\times\frac{1}{4}(x+4)$
$\therefore$ Vertex $(-4, 3)$



$\therefore$ focus $(-3\frac{3}{4}, 3)$

(b)(i) $y = x^3$
$y' = 3x^2$
at $x=1$, $y' = m_T = 3$
$\therefore$ Equation tangent:
$y - 1 = 3(x-1)$
$y - 1 = 3x - 3$
$y = 3x - 2$

(ii) at X, $y=0$ $\therefore 3x = 2$
$x = \frac{2}{3}$
$\therefore X(\frac{2}{3}, 0)$
at Y, $x=0$ $\therefore y = -2$
$\therefore Y(0, -2)$

(iii) $P(-2,1)$ $3x - y - 2 = 0$
$d = \frac{|3\times -2 - 1 - 2|}{\sqrt{9+1}}$
$= \left|\frac{-9}{\sqrt{10}}\right|$
$= \frac{9}{\sqrt{10}}$

(iv) $XY = \sqrt{(\frac{2}{3})^2 + (-2)^2}$
$= \sqrt{4\frac{4}{9}}$
$= \frac{\sqrt{40}}{3}$
$\therefore A = \frac{1}{2}bh = \frac{1}{2} \times \frac{\sqrt{40}}{3} \times \frac{9}{\sqrt{10}}$
$= 3$ units^2

Question 3

(a)(i) $\frac{d}{dx}\left(\ln\sqrt{2x+3}\right)$
$= \frac{d}{dx}\left[\frac{1}{2}\ln(2x+3)\right]$
$= \frac{1}{2}\cdot\frac{2}{2x+3}$
$= \frac{1}{2x+3}$

(ii) $\frac{d}{dx}\left(\frac{x^2}{e^x}\right) = \frac{2xe^x - e^x\cdot x^2}{(e^x)^2}$
$= \frac{xe^x(2-x)}{e^{2x}}$
$= \frac{x(2-x)}{e^x}$

(b) $\angle ACB = \angle CAE = 65^\circ$
(alternate angles, $AE \parallel BC$)
$\angle ABC = 40 + 25 = 65^\circ$
(exterior angle $\triangle DBC$)
$\therefore \triangle ABC$ isosceles
Since $\angle ABC = \angle ACB = 65^\circ$

(c)(i) $\int e^{2x}\,dx$
$= \frac{1}{2}e^{2x} + c$

(ii) $\int \frac{dx}{(2x-3)^2}$
$= \int (2x-3)^{-2}\,dx$
$= \frac{(2x-3)^{-1}}{2\cdot -1}$
$= \frac{1}{-2(2x-3)} + c$

(iii) $\int_0^{\frac{\pi}{4}} (\sec^2 x + x)\,dx$
$= \left[\tan x + \frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$
$= \tan\frac{\pi}{4} + \left(\frac{\pi}{4}\right)^2\frac{1}{2} - 0$
$= 1 + \frac{\pi^2}{32}$

Question 4

(a) $\log_3(2x-1)=4$

$2x-1=3^4$

$2x-1=81$

$2x=82$

$x=41$

(b) In Δ's ABD and BCD

$\angle DAB = \angle CBD$ (given)

$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)

$\therefore \Delta ABD \mathbin{|||} \Delta BDC$ (equiangular)

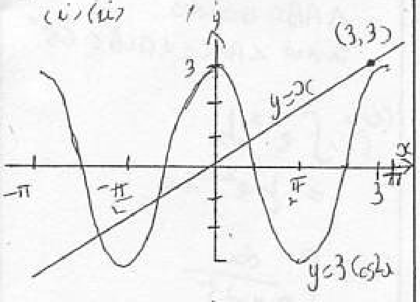
$\therefore \frac{BD}{DC}=\frac{AB}{BD}$ (ratio of sides)

$\therefore BD^2 = AB \times CD$

$\therefore BD = \sqrt{AB\times CD}$

(c)

(i)(ii)



(iii) 3 points of intersection

$\therefore$ 3 solutions

d) $x^2-x+3=0$

$\alpha+\beta=1$

$\alpha\beta=3$

(i) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{1}{3}$

(ii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=1^2-2\times3$

$=-5$

Question 5

(a) B ⟶ C, P, 65, 58

(i) $\angle BPC = 123^\circ$

(ii) $BC^2 = 7^2+5^2-2\times7\times5\cos123$

$=74-70\cos123$

$=112.12\ldots$

$BC=10.588\ldots$

$=10.6$ km (1dp)

(iii) $A=\frac{1}{2}\times7\times5\sin123$

$=14.6\ldots$

$=15$ km (nearest km)

(b) (i) $a=20$, $d=8$ $T_n=116$

$116=20+8(n-1)$

$96=8n-8$

$8n=104$

$n=13$

$\therefore$ 13 cones

(ii) $\frac{d}{2}=S_{13}=\frac{13}{2}(20+116)$

$=884$

$\therefore$ Total distance $=884\times2$

$=1768$ m

(c) $\sin\theta=-0.7$

$\theta=\pi+0.775\ldots, 2\pi-0.775$

$=3.92,\ 5.51$

Question 6

(a)(i) $f'(x)=3(x+2)(x-1)$

$=3(x^2+x-2)$

$=3x^2+3x-6$

$\therefore f(x)=x^3+\frac{3x^2}{2}-6x+C$

$(0,7)\ \therefore 7=C$

$\therefore f(x)=x^3+\frac{3x^2}{2}-6x+7$

(ii) S.P.'s $f'(x)=0$

i.e. $x=-2$, or $x=1$

$x=-2 \rightarrow y=-8+6+12+7$

i.e. $(-2, 17)$

$x=1$, $y=1+\frac{3}{2}-6+7$

i.e. $(1, 3\frac{1}{2})$

$y'=3x^2+3x-6$

$y''=6x+3$

at $x=-2$, $y''=-9<0$ ∩

$\therefore$ Maximum at $(-2,17)$

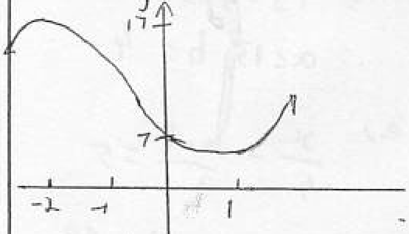
at $x=1$, $y''=9>0$ ∪

$\therefore$ minimum at $(1,3\frac{1}{2})$

(iii) Concave down $-\ y''<0$

$\therefore 6x+3<0$

$x<-\frac{1}{2}$



(b) (i) $\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}$

$=\frac{2}{91}$

(ii) P(at least 1 not red)

$=1-\frac{2}{91}$

$=\frac{89}{91}$

(iii) RBG + RGB + GRB + GBR + BRG + BGR

i.e. P(different) $=6\times\frac{5}{15}\times\frac{5}{14}\times\frac{5}{13}$

$=\frac{25}{91}$

NSGHS. 2008 MATHS TRIAL SOLUTIONS

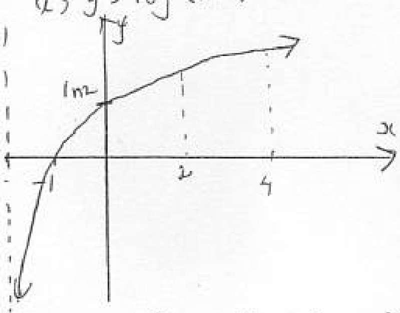
Question 7

(a) $\sum_{n=2}^{5}(3n-2)$

$= 4+7+10+13$

$= 34$

(b) (i) $y = \log(x+2)$ for $x > -2$



(ii) $A \doteq \frac{2}{2}\left[\ln 2+\ln 4\right] + \frac{2}{2}\left[\ln 4+\ln 6\right]$

$= \ln 2+\ln 4+\ln 4+\ln 6$

$= \ln(2\times 4\times 4\times 6)$

$= \ln 192$

iii) Curve concave down

∴ trapezia below graph

∴ estimate less than exact value.

(c) (i) $y = x^2$, $y^2 = 8x$

$x^4 = 8x$

$x(x^3-8)=0$

$\therefore x=0$, 8 or $x^3 = 8$

$x=2$

∴ At B $x=2$ ∴ $y=4$

∴ $B(2,4)$

(ii) $A = \int_0^2 (\sqrt{8x} - x^2)\,dx$

$= \left[\sqrt{8}\times x^{\frac{3}{2}}\times\frac{2}{3} - \frac{x^3}{3}\right]_0^2$

$= \frac{2\sqrt{8}}{3}\times 2^{\frac{3}{2}} - \frac{8}{3}$

$= \frac{2}{3}\times\sqrt{8}\times\sqrt{8} - \frac{8}{3}$

$= 8$ units²

Question 8

(a) (i) common ratio $r = \sqrt{\alpha} - 2$

For limiting sum $|\sqrt{\alpha}-2| < 1$

∴ $\alpha = 8$ is largest possible integer

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-(\sqrt{8}-2)}$

$= \frac{1}{3-\sqrt{8}}$

$= \frac{3+\sqrt{8}}{9-8}$

$= 3+\sqrt{8}$

(b) (i) $2r + rx = 14$ — ①

$\frac{1}{2}r^2x = 12$ — ②

(ii) $r(2+x) = 14$ — ③

$r^2x = 24$ ④

From ④ $x = \frac{24}{r^2}$

→③ $r\left(2+\frac{24}{r^2}\right) = 4$

$= 2r + \frac{24}{r} = 14$

$2r^2 + 24 = 14r$

$r^2 - 7r + 12 = 0$

$(r-3)(r-4) = 0$

$r = 3$ and $x = \frac{24}{9} = \frac{8}{3}$

$r = 4$ and $x = \frac{24}{16} = \frac{3}{2}$

(c) (i) $C = 20(2x+y) + 60y$

$= 40x + 80y$

but Area $= xy = 800$

$y = \frac{800}{x}$

$\therefore C = 40x + \frac{800}{x}\times 80$

$= 40x + \frac{64\,000}{x}$

(ii) $\frac{dC}{dx} = 40 - 64\,000x^{-2}$

$\frac{d^2C}{dx^2} = 128\,000x^{-3}$

$= \frac{128\,000}{x^3}$

∴ any stationary point a minimum

For SP's $40 - \frac{64000}{x^2} = 0$

$40x^2 = 64000$

$x^2 = 1600$

$x = \pm 40$

$x = 40$ only possible since $x > 0$

∴ Minimum cost when $x = 40$, $y = 20$

ie $40\text{m}\times 20\text{m}$

Question 9

(a) $y = \frac{1}{1+\tan x}$

$= (1+\tan x)^{-1}$

$\frac{dy}{dx} = -(1+\tan x)^{-2}\cdot\sec^2 x$

$= \frac{-\sec^2 x}{(1+\tan x)^2}$

$= -\left(\frac{\sec x}{1+\tan x}\right)^2$

∴ decreasing since $\frac{dy}{dx} < 0$

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} x\sin x\,dx$ $\quad h = \frac{\pi}{12}$

x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	$\frac{\pi}{12}$	$\frac{\pi}{4\sqrt{2}}$	$\frac{\sqrt{3}\pi}{6}$

$A \doteq \frac{\pi}{12}\times\frac{1}{3}\left[\frac{\pi}{12} + 4\times\frac{\pi}{4\sqrt{2}} + \frac{\sqrt{3}\pi}{6}\right]$

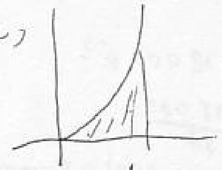
$= \frac{\pi}{36}\left[\frac{\pi}{12} + \frac{\pi}{\sqrt{2}} + \frac{\sqrt{3}\pi}{6}\right]$

$= 0.2958$

$\doteq 0.30$ square units.

Question 9 (CTD)

c)



$V = \pi \int_0^1 (e^{2x} - 1)^2 \, dx$

$= \pi \int_0^1 (e^{4x} - 2e^{2x} + 1) \, dx$

$= \pi \left[\frac{e^{4x}}{4} - e^{2x} + x \right]_0^1$

$= \pi \left[\left(\frac{e^4}{4} - e^2 + 1 \right) - \left(\frac{1}{4} - 1 \right) \right]$

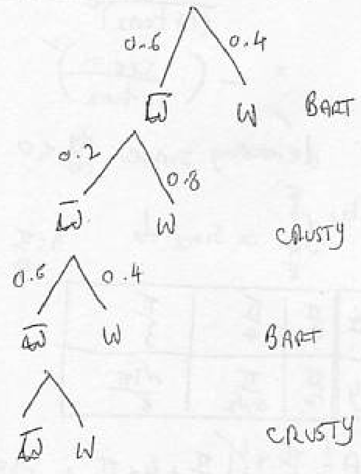
$= \pi \left[\frac{e^4}{4} - e^2 + \frac{7}{4} \right]$

d) local minimum at $x = 2$

as $\frac{dy}{dx} = 0$ and $\frac{dy}{dx}$

changes from $-$ve to $+$ve

Question 10



(a i) P(Bart does not win on first)

$= 0.6$

ii) $P(\text{no winner}) = P(\bar{W}\bar{W}\bar{W}\bar{W})$

$= 0.6 \times 0.2 \times 0.6 \times 0.2$

$= 0.0144$

iii) P(Bart wins)

$= P(\text{wins on first}) + P(\text{wins on 2nd})$

$= 0.4 + 0.6 \times 0.2 \times 0.4$

$= 0.448$

(b)

(i) $B_1 = 5 \times 1.005 - m$

(ii) $B_2 = (B_1 + 5) \times 1.005 - m$

$= [5 \times 1.005 - m + 5] \times 1.005 - m$

$= 5 \times 1.005^2 - m \times 1.005 + 5 \times 1.005 - m$

$= 5(1.005 + 1.005^2) - m(1 + 1.005)$

(iii)

$B_{60} = 5(1.005 + 1.005^2 + \cdots 1.005^{60}]$

$- m(1 + 1.005 + \cdots 1.005^{59}]$

$= 5\left[\frac{1.005(1.005^{60} - 1)}{1.005 - 1} \right]$

$- m\left[\frac{1(1.005^{60} - 1)}{1.005 - 1} \right]$

$= (201S - 200m)(1.005^{60} - 1)$

(iv) $B_{60} = 100000$, $S = 4000$

$100000 = (1.005^{60} - 1)[201 \times 4000 - 200m]$

$\left(\frac{100000}{1.005^{60} - 1} \right) + 200m = 201 \times 4000$

$200m = 201 \times 4000 - \frac{100000}{1.005^{60} - 1}$

$m = \$2586.72$ (nearest cent)