NORTH SYDNEY GIRLS HIGH SCHOOL

2014
TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11-16.

Total Marks – 100

Section 1 – Q 1-10 worth 10 marks
Section 2 – Q 11-16 each worth 15 marks

At the end of the examination, place your solution booklets in order and put inside this question paper. Submit one bundle. The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number:____________________ **Teacher:**____________________

Student Name:____________________

QUESTION	MARKS
1-10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
TOTAL	/100

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SECTION 1

Ten questions worth 10 marks
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet provided.

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1 What is the solution of $x^2 - 5x > 6$?

(A) $x < -2$ or $x > 3$
(B) $2 < x < 3$
(C) $x < -1$ or $x > 6$
(D) $x < -6$ or $x > 1$

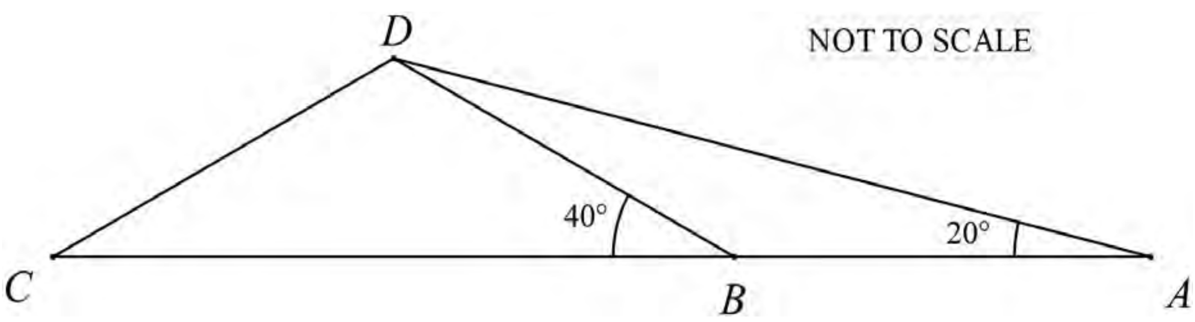
2 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.

At a certain time, $\dot{x} = -3 \text{ ms}^{-1}$ and $\ddot{x} = 2 \text{ ms}^{-2}$.

Which statement describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed.
(B) The particle is moving to the left with increasing speed.
(C) The particle is moving to the right with decreasing speed.
(D) The particle is moving to the left with decreasing speed.

3 In the diagram below ABC is a straight line. If $BD = CD$ and AB=10 cm, what is the value of BC, correct to the nearest cm?



(A) 8 cm
(B) 13 cm
(C) 14 cm
(D) 15 cm

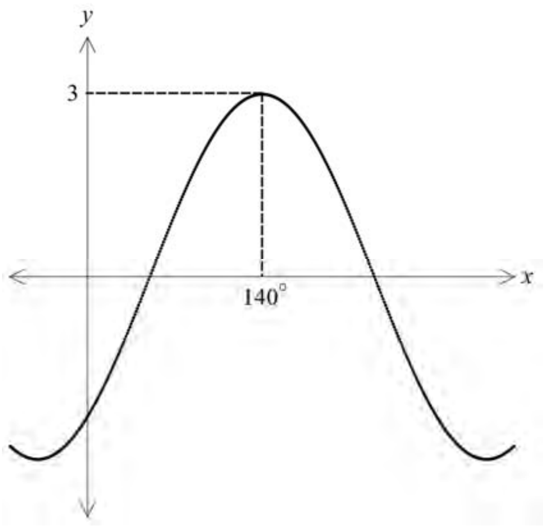
4 If $x^2 + p(x+5) + q \equiv (x-2)(x+5)$, where p and q are constants, then what is the value of q?

(A) -25
(B) -10
(C) 3
(D) 5

5 What are the coordinates of the vertex of the parabola $x^2 - 4x - 12 = 8y$?

(A) $(-2, 2)$
(B) $(2, -2)$
(C) $(0, 2)$
(D) $(2, 0)$

6 Let a be a constant and $-90° < b < 90°$. If the figure below shows the graph of $y = a\cos(x° + b)$, then what are the values of a and b?



(A) $a = -3$ and $b = -40°$
(B) $a = -3$ and $b = 40°$
(C) $a = 3$ and $b = -40°$
(D) $a = 3$ and $b = 40°$

7 For $0 \le \theta \le \frac{\pi}{2}$, what is the least value of $\frac{30}{3\sin^2\theta + 2\cos^2\theta}$?

(A) 5
(B) 6
(C) 10
(D) 15

8 If $b > 1$ and $a = \log_{12} b$, which expression is equal to $\frac{1}{a}$?

(A) $-\log_{12} b$

(B) $\log_b 12$

(C) $\log_{12}\frac{1}{b}$

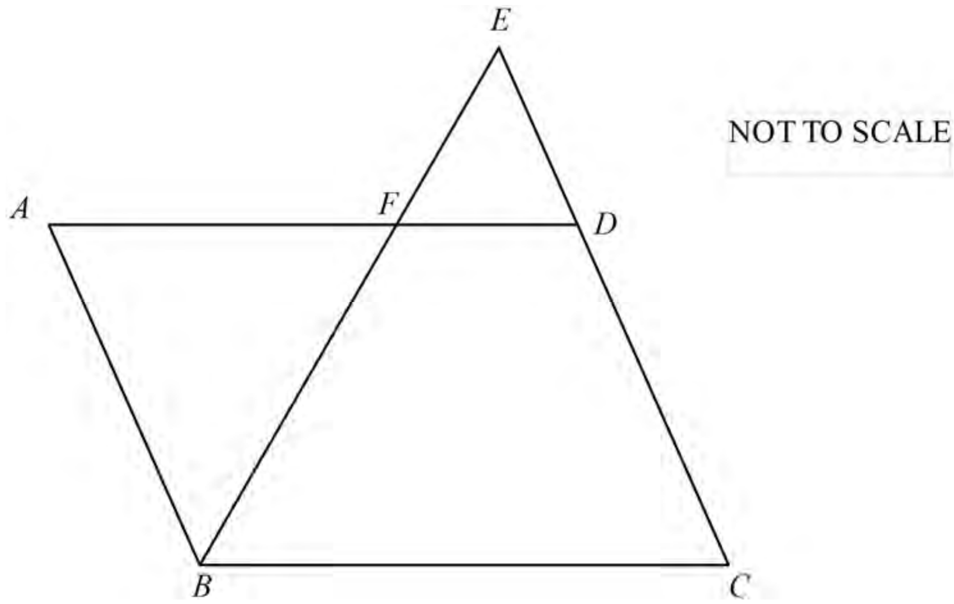
(D) $\frac{1}{\log_b 12}$

9 The quadratic equation $x^2 - kx + 3 = 0$ has roots α and β. What is the value of $\alpha^2\beta + \beta^2\alpha$?

(A) $k - 3$
(B) $k + 3$
(C) $-3k$
(D) $3k$

10 In the diagram below *ABCD* is a parallelogram. *F* is a point lying on *AD*. *BF* produced and *CD* produced meet at *E*. If *CD:DE*=2:1, then what is the ratio of *AF:BC?*

(A) 1:2
(B) 2:3
(C) 3:4
(D) 8:9



End of Section 1

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SECTION 2

90 marks
Attempt Questions 11–16.
All questions are of equal value (15 marks each).
Allow about 2 hours and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Evaluate $\dfrac{2.1^2 \times 4.5^2}{\sqrt{2.1^2 + 4.5^2}}$, correct to 3 significant figures. **1**

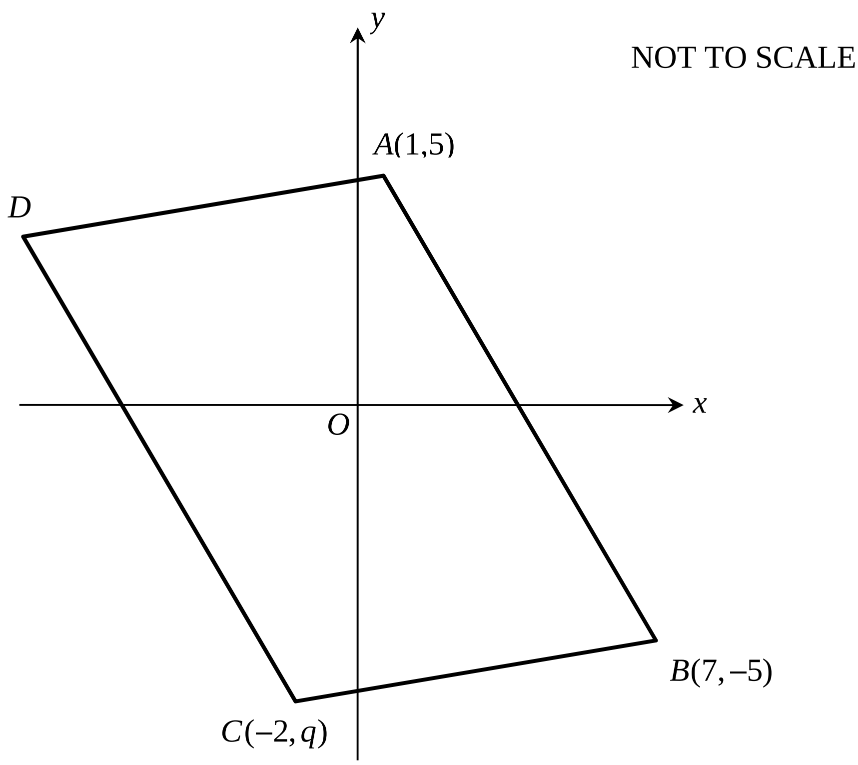
(b) Fully factorise $128x - 16x^4$. **2**

(c) Express $6\sqrt{5} - \dfrac{1}{\sqrt{5}-2}$ in the form $a + b\sqrt{5}$ where a and b are integers. **2**

(d) If the straight line $y = x$ and the circle $x^2 + y^2 + 6x + ky - k = 0$ do not intersect each other, what are the possible values of k? **3**

Question 11 continues on Page 9

(e) In the diagram below, A, B and C have coordinates $(1,5)$, $(7,-5)$ and $(-2,q)$ respectively. C is in the third quadrant and $ABCD$ is a parallelogram.



(i) Show that the equation of AB is $5x+3y-20=0$. **2**

(ii) Write down an expression, in terms of q, for the perpendicular distance from C to the line AB. **1**

(iii) Find the length of the interval AB. **1**

(iv) Given that the area of $ABCD$ is 100 square units, find the value of q. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $\sqrt{1+\log_e x}$ **1**

(ii) $\dfrac{\tan x}{1-2x}$ **2**

(b) Find:

(i) $\displaystyle\int \sin\tfrac{x}{3}\,dx$ **1**

(ii) $\displaystyle\int \frac{e^{2x}}{e^{2x}+4}\,dx$ **2**

(c) Evaluate $\displaystyle\int_1^2 \left[2x+7(3x-4)^6\right]\,dx$ **2**

(d) Write down the equation of the parabola with vertex $(-1,3)$ and focus $(1,3)$. **2**

(e) The temperature A (in ˚C) inside a house at t hours after 5 a.m. is given by $A = 20-3\cos\dfrac{\pi t}{12}$ for $0 \le t \le 24$. The temperature B (in ˚C) outside the house at the same time t is given by $B=21-5\cos\dfrac{\pi t}{12}$ for $0\le t\le 24$.

(i) Find the temperature inside the house at 9 a.m. **1**

(ii) Write down an expression for $T = B - A$, the difference between the outside and inside room temperatures. **1**

(iii) By sketching a graph or otherwise, find when the outside temperature is higher than the inside temperature. **3**

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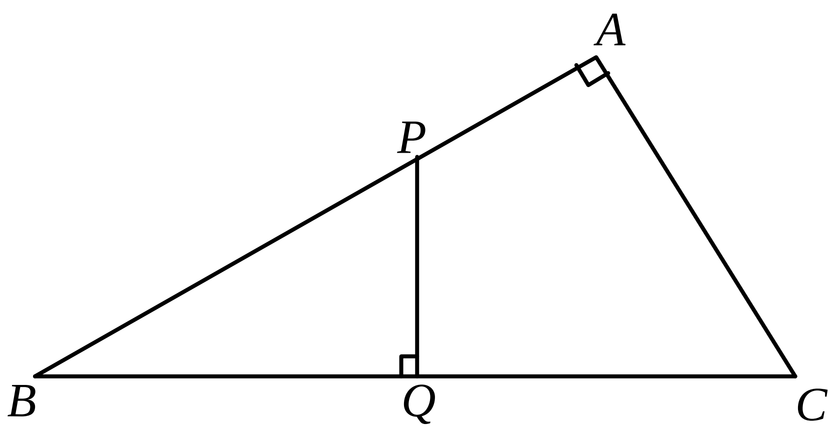
Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) For the last phase of preparations before the Olympic Games, swimmers started a new training schedule. On the first day they had to complete 26 laps of the pool. Each succeeding day they increased their training by 6 laps, until their daily schedule reached 200 laps. They then continued swimming 200 laps daily for another 14 days to fully complete their training schedule. (Note : Length of the pool = 50 m.)

(i) On which day did they first swim 200 laps? **2**

(ii) Find the total distance (in km) completed by the swimmers following this training schedule. **3**

(b)



In the diagram, ABC is a right angled triangle with $AB = 8$ cm and $AC = 6$ cm. If the triangle is folded along the line PQ, vertex B coincides with vertex C.

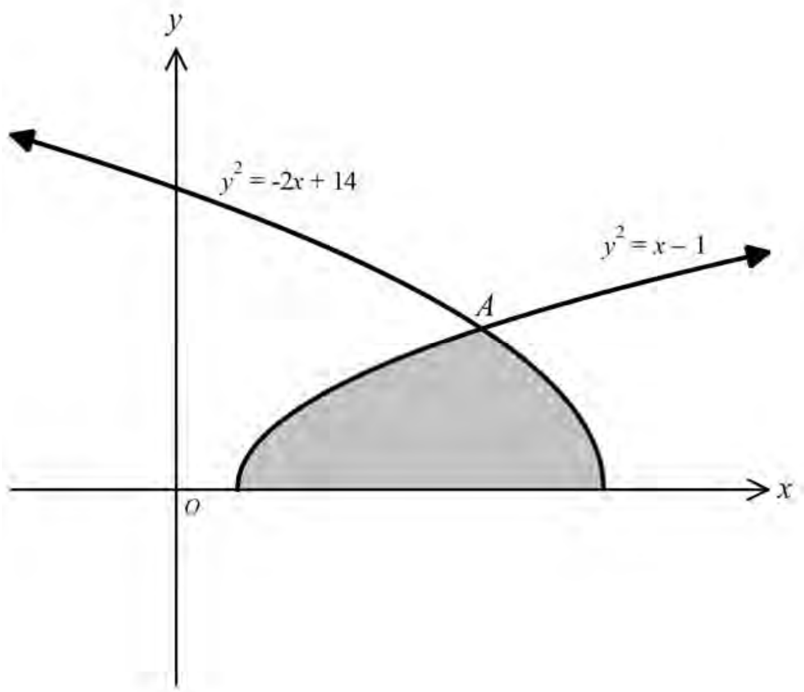
(i) Explain why $\angle PCQ = \angle PBQ$ **1**

(ii) Show that triangles ABC and CPQ are similar. **2**

(iii) Find the length of PQ. **3**

Question 13 continues on Page 13

(c) The shaded region in the diagram below, is bounded by the two curves $y^2 = x - 1$ and $y^2 = -2x + 14$ and the $x-$axis. The curves intersect at A in the first quadrant.



(i) Find the coordinates of A. **1**

(ii) If the region is revolved around the x-axis, find the exact volume of the solid generated. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle moves in a straight line so that its velocity, v metres per second, at time t is given by $v = 3 - \frac{2}{1+t}$.

The particle is initially 1 metre to the right of the origin.

(i) Find an expression for the position x, of the particle at time t. **2**

(ii) Explain why the velocity of the particle is never 3 metres per second. **1**

(iii) Find the acceleration of the particle when $t = 2$ seconds. **2**

(b) Sunlight transmitted into water loses intensity as it penetrates to greater depths. The intensity, I, at a depth s metres below the surface is given by

$$I = I_0 e^{-ks}$$

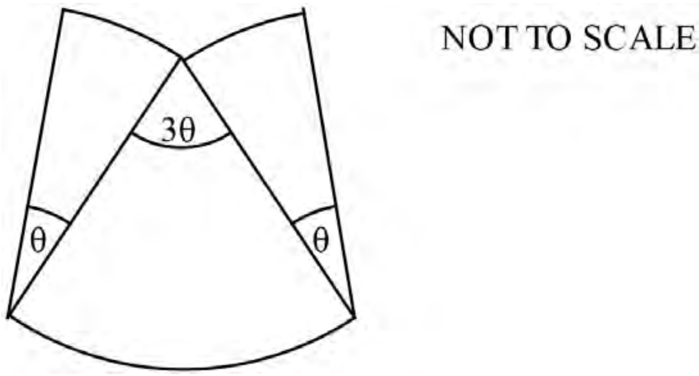
where I_0 and k are constants.

Given that at a depth of 75 m, the intensity is 40% of its intensity at the surface, find the depth at which the intensity of sunlight would be decreased by 90%. **4**

Question 14 continues on Page 15

Question 14 (continued)

(c) The diagram below shows a design to be used on a new brand of jam. The design consists of three circular sectors each of radius r cm. The angle of two of the sectors is θ radians and the angle of the third sector is 3θ radians as shown.



Given that the area of the design is 25cm^2,

(i) Show that $\theta = \dfrac{10}{r^2}$. **1**

(ii) Find the external perimeter of the design, P, in terms of r. **2**

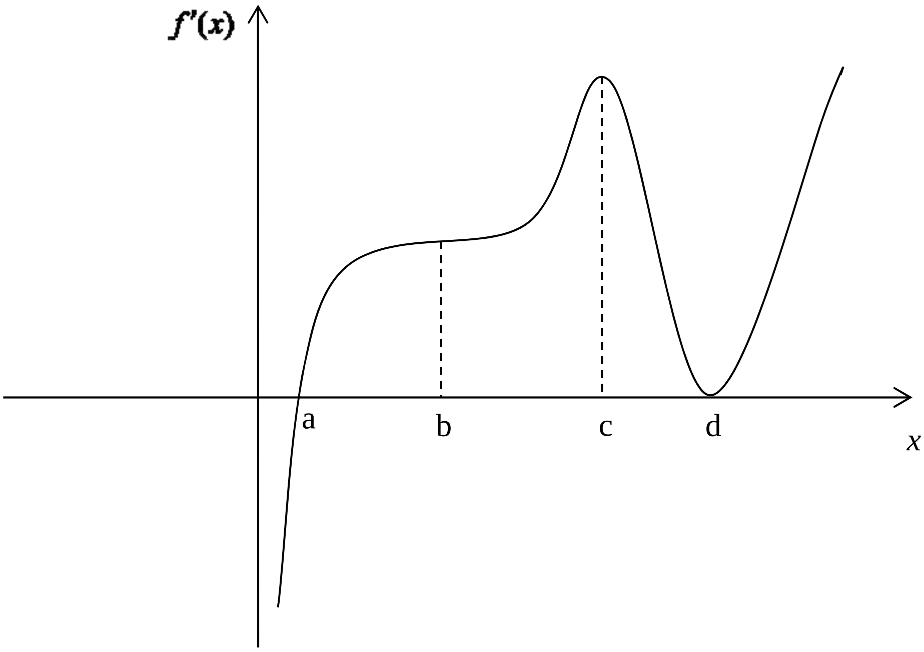
(iii) Given that r can vary, find the value of r for which P is a minimum. **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim\limits_{x \to \infty} \dfrac{3x^2 - 1}{2x^2 + 1}$. **1**

(b) The following is a sketch of $f'(x)$.



In your writing booklet sketch the graph of $f''(x)$, showing the position of *a, b, c* and *d* on your diagram. **2**

Question 15 continues on Page 17

Question 15 (continued)

(c) Susan has planned a holiday which she decides to take in 3 years' time. She has estimated that the holiday will cost about \$8 000 and plans to save a fixed amount each month. She invests her savings at the beginning of each month in an account which pays interest at 6% pa compounded monthly.

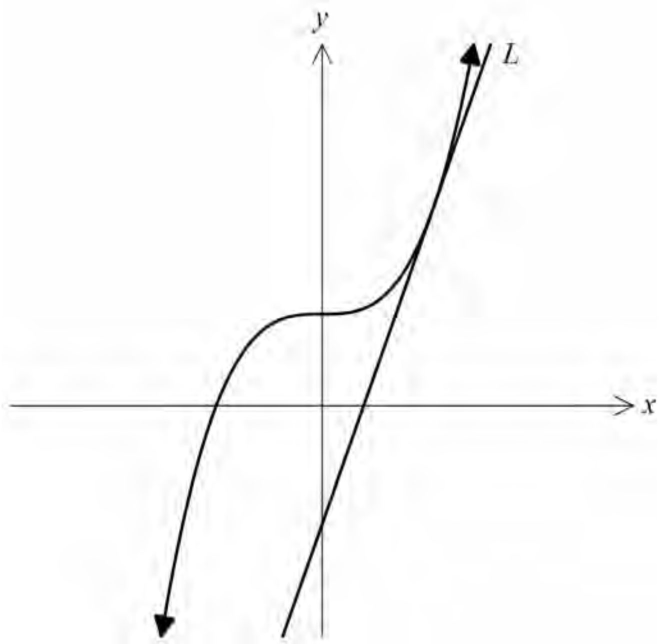
(i) Let the amount she saves each month be $\$A$ and let V_n be the value of her investment after n months. Show that the value of her investment at the end of 3 months is given by

$$V_3 = A\left(1.005 + 1.005^2 + 1.005^3\right).$$ **1**

(ii) Find correct to the nearest dollar, the least amount of money that Susan would need to save each month to reach her target. **3**

(iii) If, after 2 years of her saving plan, the interest rate rose to 9% pa, how much extra spending money would Susan have if she maintained the amount she was saving as calculated in part (ii) above? **3**

(d) The line L is the tangent to the curve $y = x^3 + 7$ at $x = 2$.



(i) Show that the equation of the tangent L is $y = 12x - 9$. **2**

(ii) Find the area bounded by the y-axis, the tangent L and the curve $y = x^3 + 7$. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) Let $f(t)$ be a function defined for all $t \geq 0$. The gradient function is given by

$$f'(t) = e^{2bt} + ae^{bt} + 8.$$

where a and b are negative constants and $f(0) = 0$, $f'(0) = 3$ and $f'(1) = 4.73$.

(i) Find the value of a. **1**

(ii) Show that the value of $b = -0.5$ correct to 1 decimal place. **3**

(iii) Hence find $f(12)$ correct to 4 decimal places. **3**

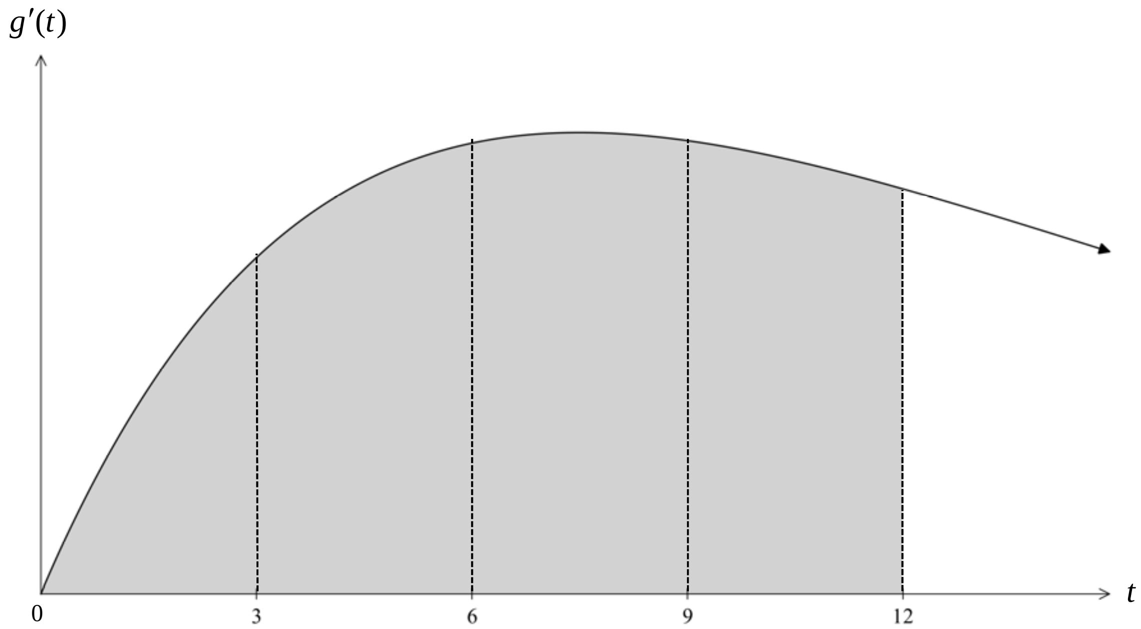
Question 16 continues on Page 19

Question 16 (continued)

(b) Let $g(t)$ be another function defined for all $t \geq 0$. The gradient function is given by

$$g'(t) = \frac{33}{10} t e^{-kt}.$$

where k is a positive constant. The diagram below shows a sketch of $g'(t)$ against t. It is given that $g'(t)$ attains the greatest value at $t = 7.5$ and $g(0) = 0$.



(i) Show that $k = \frac{2}{15}$. **2**

(ii) Use the trapezoidal rule with four sub-intervals to estimate the shaded area in the diagram above. **3**

(iii) Explain why your answer to b(ii) is an estimate for $g(12)$. **1**

(c) From part (a) and part (b), Jenny claims that $g(12) > f(12)$. Do you agree? Explain your answer. **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Mathematics Trial HSC Examination 2014 – Solutions

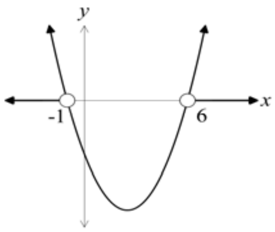
Section 1

1 $x^2 - 5x - 6 > 0$

$(x-6)(x+1) > 0$

$x < -1,\ \ x > 6$

Answer : C



2 $\dot{x} = -3 \text{ ms}^{-1}$

$\therefore$ particle is moving to the left

$\ddot{x} = 2 \text{ ms}^{-2}$

$\therefore$ acceleration is +ve and since particle is moving to the left it is slowing down.

Answer : D

3 $BC^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos 100°$

$BC = 15$ (nearest cm)

Answer : D

4 $x^2 + px + 5p + q \equiv x^2 + 3x - 10$

$p = 3,\ \ 5p + q = -10$

$q = -25$

Answer : A

5 $x^2 - 4x = 8y + 12$

$x^2 - 4x + (-2)^2 = 8y + 12 + (-2)^2$

$(x-2)^2 = 8y + 16$

$(x-2)^2 = 8(y+2)$

vertex $(2, -2)$

Answer : B

6 $a = -3,\ b = 40°$

Answer : B

7 $\dfrac{30}{2\sin^2\theta + 2\cos^2\theta + \sin^2\theta}$

$= \dfrac{30}{2 + \sin^2\theta}$

$= \dfrac{30}{3}$

$= 10$

Answer : C

8 $\dfrac{1}{a} = \dfrac{1}{\log_{12} b}$

$= \dfrac{1}{\left(\dfrac{\log_b b}{\log_b 12}\right)}$

$= \dfrac{\log_b 12}{\log_b b}$

$= \log_b 12$

Answer : B

9 $x^2 - kx + 3 = 0$

$\alpha + \beta = k$

$\alpha\beta = 3$

$\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$

$= 3k$

Answer : D

10 **B**

Section 2

Question 11

(a) 18.0 (3 s.f.)

(b) $16x(8-x^3)$

$16x(2-x)(4+2x+x^2)$

(c) $6\sqrt{5}-\frac{1}{\sqrt{5}-2}=6\sqrt{5}-\frac{\sqrt{5}+2}{5-4}$

$=6\sqrt{5}-\sqrt{5}-2$

$=-2+5\sqrt{5}$

(d) sub. $y=x$ into $x^2+y^2+6x+ky-k=0$

$x^2+x^2+6x+kx-k=0$

$2x^2+(6+k)x-k=0$

No intersection points $\Rightarrow \Delta<0$

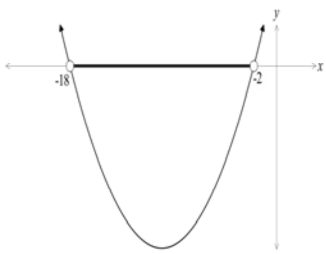
$\Delta=(6+k)^2-4(2)(-k)$

$=k^2+20k+36$

$\Delta<0$

$(k+18)(k+2)<0$

$-18<k<-2$



(e) (i) $m_{AB}=\frac{-5-5}{7-1}=\frac{-5}{3}$

Equation of AB:

$y-5=\frac{-5}{3}(x-1)$

$3y-15=-5x+5$

$5x+3y-20=0$

(ii) perp. distance = $\frac{|5(-2)+3(q)-20|}{\sqrt{25+9}}$

$=\frac{|3q-30|}{\sqrt{34}}$

(iii) $AB=\sqrt{(1-7)^2+(5+5)^2}$

$=\sqrt{36+100}$

$=2\sqrt{34}$ units

(e) (iv) Area = 100 units^2

Area $=2\sqrt{34}\times\frac{|3q-30|}{\sqrt{34}}$

$100=2|3q-30|$

$3q-30=50,\quad 3q-30=-50$

$3q=80,\quad 3q=-20$

$q=\frac{80}{3},\quad q=-\frac{20}{3}$

Since C is in 3rd quadrant, q is negative

$\therefore q=-\frac{20}{3}$

Question 12

(a) (i) $\frac{d}{dx}(1+\log_e x)^{\frac{1}{2}}=\frac{1}{2}(1+\log_e x)^{-\frac{1}{2}}\times\frac{1}{x}$

$=\frac{1}{2x\sqrt{1+\log_e x}}$

(ii) $\frac{d}{dx}\frac{\tan x}{1-2x}=\frac{\sec^2 x(1-2x)+2\tan x}{(1-2x)^2}$

(b) (i) $\int \sin\frac{x}{3}\,dx=\frac{-\cos\frac{x}{3}}{\frac{1}{3}}$

$=-3\cos\frac{x}{3}+C$

(ii) $\int \frac{e^{2x}}{e^{2x}+4}\,dx=\frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+4}\,dx$

$=\frac{1}{2}\log_e(e^{2x}+4)+C$

(c) $\int_1^2 \left[2x+7(3x-4)^6\right]dx$

$=\left[x^2+\cancel{7}\times\frac{(3x-4)^7}{\cancel{7}\times 3}\right]_1^2$

$=\left(\left(4+\frac{2^7}{3}\right)-\left(1-\frac{1}{3}\right)\right)$

$=46$

(d) Focal length $a = 2$

Equation is $(y-3)^2 = 8(x+1)$

(e) (i) $A = 20 - 3\cos\left(\dfrac{\pi t}{12}\right)$

At 5 a.m. $t = 0$ and at 9 a.m. $t = 4$.

Hence when $t = 4$:

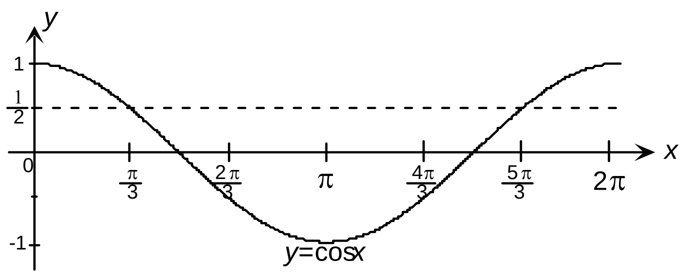
$$A = 20 - 3\cos\left(\frac{4\pi}{12}\right)$$
$$= 20 - 3\cos\left(\frac{\pi}{3}\right)$$
$$= 20 - 3\times\frac{1}{2}$$
$$= 18.5^\circ\ C$$

(ii) $T = B - A$

$$= \left[21 - 5\cos\left(\frac{\pi t}{12}\right)\right] - \left[20 - 3\cos\left(\frac{\pi t}{12}\right)\right]$$
$$= 1 - 2\cos\left(\frac{\pi t}{12}\right)$$

(iii) $1 - 2\cos\left(\dfrac{\pi t}{12}\right) > 0$

$$1 > 2\cos\left(\frac{\pi t}{12}\right)$$
$$\cos\left(\frac{\pi t}{12}\right) < \frac{1}{2}$$



$$\frac{\pi}{3} < \frac{\pi t}{12} < \frac{5\pi}{3}$$
$$4 < t < 20$$

The temperature outside is higher than the temperature inside from 9 a.m. until 1 a.m. the following day.

<u>Question 13</u>

(a) (i) $a = 26,\ d = 6$

$T_n = 26 + 6(n-1)$

$n = 30$

First swam 200 laps on 30th day

(ii) $S_{30} = \dfrac{30}{2}(26 + 200)$

$= 3390$

Total number of laps = $3390 + 200\times 14$

= 6190

Total distance = $6190\times 50 \div 1000$

$= 309.5$ km

(b) (i) $BP = PC$ (triangle is folded along line PQ)

$\angle PBC = \angle PCQ$ (angles opposite equal sides in ΔBPC)

(ii) In ΔABC and ΔQCP

$\angle CAB = \angle PQC$ (given)

$\angle ABC = \angle QCP$ (from part (i))

$\therefore \Delta ABC \,|||\, \Delta QCP$ (equiangular)

(iii) In ΔABC,

BC $= \sqrt{8^2 + 6^2}$ (Pythag. Theorem)

$= 10$

$QC = 5$

$\dfrac{PQ}{AC} = \dfrac{QC}{AB}$ (matching sides in similar triangles)

$$\frac{PQ}{6} = \frac{5}{8}$$
$$PQ = \frac{15}{4}$$

(c) (i) $-2x + 14 = x - 1$

$$-3x = -15$$
$$x = 5$$
$$y^2 = 5 - 1$$
$$y = 2 \quad (y > 0)$$
$$A(5, 2)$$

Question 13 continued

(c) (ii) $V=\pi\int_1^5 x-1\,dx \;+\pi\int_5^7 -2x+14\,dx$

$=\pi\left[\frac{x^2}{2}-x\right]_1^5+\pi\left[-x^2+14x\right]_5^7$

$=\pi\left(\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right)+$

$\pi\left(\left(-49+98\right)-\left(-25+70\right)\right)$

$=8\pi+4\pi$

$=12\pi\text{ units}^3$

Question 14

(a) (i) $v=3-\frac{2}{1+t}$

$x=3t-2\log_e(1+t)+C$

when $t=0,\ x=1$

$1=0-2\log_e 1+C$

$C=1$

$x=3t-2\log_e(1+t)+1$

(ii) Since $\frac{2}{1+t}\neq 0,$

$v=3-\frac{2}{1+t}\neq 3$

(iii) $v=3-2(1+t)^{-1}$

$a=2(1+t)^{-2}$

when $t=2,$

$a=\frac{2}{3^2}=\frac{2}{9}\text{ms}^{-2}$

(b) when $s=0,\ I=I_0$

when $s=75,\ I=0.4I_0$

$0.4I_o=I_0e^{-75k}$

$0.4=e^{-75k}$

$\log_e 0.4=-75k$

$\therefore k=-\frac{\log_e 0.4}{75}$

$0.1I_0=I_0e^{-ks}$

$0.1=e^{-ks}$

$\log_e 0.1=-ks$

$s=\log_e 0.1\times\frac{75}{\log_e 0.4}$

$s=188.47\text{ m }(2\text{ d.p.})$

(c) (i) $A=\frac{1}{2}r^2\theta$

$25=\frac{1}{2}r^2 5\theta$

$5\theta=\frac{50}{r^2}$

$\theta=\frac{10}{r^2}$

(ii) $P=2r+2r\theta+r\times 3\theta$

$P=2r+5r\theta$

$P=2r+\frac{50}{r}$

(iii) $\frac{dP}{dr}=2-\frac{50}{r^2}$

$2-\frac{50}{r^2}=0$

$r^2=25$

$r=5\ (\text{since } r>0)$

$\frac{d^2P}{dr^2}=\frac{100}{r^3}$

>0

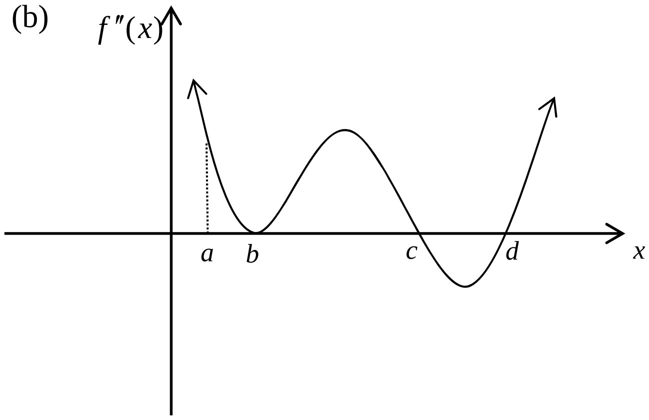
$\therefore$ P is a minimum when radius is 5 cm

Question 15

(a) $$\lim_{x\to\infty}\frac{3x^2-1}{2x^2+1}=\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{1}{x^2}}{\frac{2x^2}{x^2}+\frac{1}{x^2}}$$

$$=\frac{3}{2}$$

(b)



(c) (i) Interest rate is $\frac{6}{12}=0.5\%$ per month

$V_1 = A(1.005)$

$V_2 = A(1.005)+A(1.005)^2$

$V_3 = A(1.005)+A(1.005)^2+A(1.005)^3$

(ii) $$V_{36}=A\ \underbrace{(1.005+1.005^2+........+1.005^{36})}_{\text{Geometric series with } a=1.005,\ r=1.005,\ n=36}$$

$$V_{36}=A\times\frac{1.005(1.005^{36}-1)}{(1.005-1)}$$

$$8000=\frac{A\times1.005(1.005^{36}-1)}{0.005}$$

$$A=\frac{8000\times0.005}{1.005(1.005^{36}-1)}$$

$A\approx\$202.36$

Susan will need to save \$203 each month to reach her target.

(iii) $A=\$203$

$$V_{24}=\frac{203\times1.005(1.005^{24}-1)}{0.005}$$

$\approx\$5188.50$

$$V_{36}=V_{24}(1.0075)^{12}+\frac{203\times1.0075(1.0075^{12}-1)}{0.0075}$$

$=\$8233.30$ (nearest cent)

She would have \$233.30 extra spending money.

(d) (i) $\frac{dy}{dx}=3x^2$

at $x=2, \frac{dy}{dx}=12$

when $x=2, y=15$

equation of tangent L:

$y-15=12(x-2)$

$\therefore y=12x-9$

(ii) $$\text{Area}=\int_0^2 x^3+7-(12x-9)\,dx$$

$$=\int_0^2 x^3-12x+16\,dx$$

$$=\left[\frac{x^4}{4}-6x^2+16x\right]_0^2$$

$$=\left(\frac{16}{4}-6(4)+32\right)-0$$

$=12\text{ units}^2$

Question 16

(a) (i) $f'(0) = e^0 + ae^0 + 8$

$3 = 1 + a + 8$

$a = -6$

(ii) $f'(t) = e^{2bt} - 6e^{bt} + 8$

$f'(1) = e^{2bt} - 6e^b + 8$

$4.73 = (e^b)^2 - 6e^b + 8$

$(e^b)^2 - 6e^b + 3.27 = 0$

$e^b = \dfrac{6 \pm \sqrt{36 - 4(1)(3.27)}}{2}$

$= 3 \pm \sqrt{9 - 3.27}$

$= 3 \pm \sqrt{5.73}$

$b = \log(3 + \sqrt{5.73}), \quad \log(3 - \sqrt{5.73})$

Since $b < 0$,

$b = \log(3 - \sqrt{5.73})$

≈ -0.5 (1dp)

(iii) $f'(t) = e^{-t} - 6e^{-\frac{t}{2}} + 8$

$f(t) = -e^{-t} + 12e^{-\frac{t}{2}} + 8t + C$

$0 = -1 + 12 + C$

$C = -11$

$f(t) = -e^{-t} + 12e^{-\frac{t}{2}} + 8t - 11$

$f(12) = -e^{-12} + 12e^{-6} + 96 - 11$

$= 85.0297$ (4dp)

(b) (i) $g''(t) = \dfrac{33}{10}(e^{-kt} + -kte^{-kt})$

when t=7.5, g″(t)=0

$0 = \dfrac{33}{10} e^{-7.5k}(1 - 7.5k)$

$1 - 7.5k = 0$

$\therefore k = \dfrac{2}{15}$

(ii)

	t	0	3	6	9	12
	$g'(t)$	0	$\frac{99}{10}e^{-\frac{6}{15}}$	$\frac{198}{10}e^{-\frac{12}{15}}$	$\frac{297}{10}e^{-\frac{18}{15}}$	$\frac{396}{10}e^{-\frac{24}{15}}$
	weight w	1	2	2	2	1
	$w \times g'(t)$	0	13.2723	17.7934	17.8909	7.9951

$h = \dfrac{12 - 0}{4} = 3$

Area $\approx \dfrac{3}{2}(0 + 13.2723 + 17.7934 + 17.8908 + 7.9951)$

≈ 85.4274 (4 dp)

(iii) Using trapezoidal rule:

Area $\approx \displaystyle\int_0^{12} g'(t)\,dt$

$= \Big[g(t) \Big]_0^{12}$

$= g(12) - g(0)$

$= g(12) - 0$

$= g(12)$

$\therefore 85.4274$ is an estimate of $g(12)$.

(c) $f(12) = 85.0297$

estimate of $g(12)$ is 85.4274

$\therefore$ estimate of $g(12) > f(12)$

Since $g'(t)$ is concave down for $0 < t < 12$, trapezoidal rule will underestimate area and hence $g(12)$.

$\therefore g(12) > 85.4274$ and $g(12) > f(12)$.

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