NORTH SYDNEY GIRLS HIGH SCHOOL

2017 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet has been provided
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 - 5

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6 - 14

90 marks

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Student Name:_______________________________

Student Number:_______________________________

Class: _______________________________

QUESTION	MARKS
1 – 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
TOTAL	/100

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1- 10.

---

1 How many significant figures are there in the number 14.320?

(A) 2

(B) 3

(C) 4

(D) 5

2 What is the shortest distance between the two parallel lines $y = 3x + 2$ and $y = 3x + 10$?

(A) 8

(B) $\dfrac{8}{\sqrt{10}}$

(C) $\dfrac{8}{\sqrt{26}}$

(D) $\dfrac{18}{\sqrt{10}}$

3 Which of the following is closest to the angle the line $3x + 2y - 7 = 0$ makes with the positive direction of the x-axis?

(A) 56°

(B) 124°

(C) 34°

(D) 146°

4 Which expression is equal to $\int \frac{1}{2x+1}\,dx$?

(A) $\log_e|2x+1|+c$

(B) $\frac{1}{2}\log_e|2x+1|+c$

(C) $2\log_e|2x+1|+c$

(D) $\log_e|2x|+c$

5 What is the value of $\sum_{k=5}^{10}(3k+2)$?

(A) 147

(B) 137

(C) 187

(D) 167

6 A car windscreen wiper traces out the area *ABCD* where *AB* and *CD* are arcs of circles with centre *O*. Angle *AOB* measures $\frac{5\pi}{9}$ and $OC = 9$ cm.

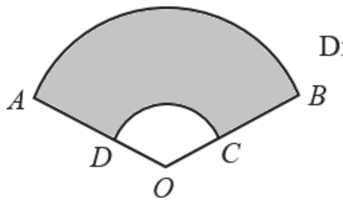


Diagram not drawn to scale

If the shaded region is 40π, what is the length of *BC*?

(A) 4.75 cm (2 decimal places)

(B) 22.75 cm (2 decimal places)

(C) 72 cm

(D) 6 cm

7 The table below shows the values of a function $f(x)$.

x	0	2	4	6	8
$f(x)$	10	42	26	10	42

What is an estimate for $\int_0^8 f(x)\,dx$ using Simpson's rule with these five function values?

(A) 416

(B) 208

(C) 312

(D) 832

8 The graph of $y = \tan\left(2x - \frac{\pi}{6}\right)$ is translated a units to the right. The equation of the new graph is $y = \tan\left(2x - \frac{\pi}{3}\right)$. What is the value of a?

(A) $-\frac{\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{12}$

9 Water is flowing in and out of a rock pool. The volume of water in the pool at time t hours is V litres. The rate of change of the volume is given by

$$\frac{dV}{dt} = 80\sin\left(0.5t\right).$$

After what time does the volume of water first start to decrease?

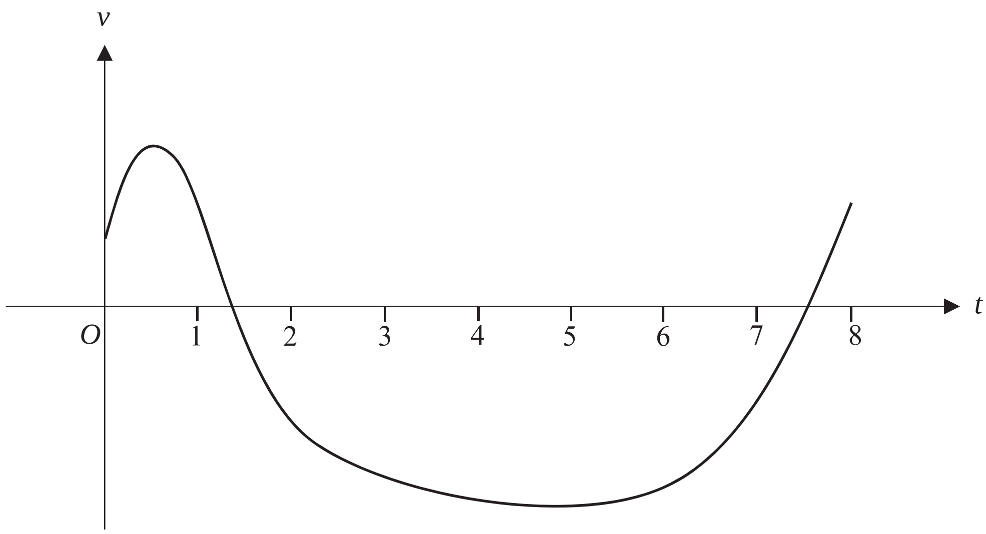
(A) $\frac{\pi}{2}$ hours

(B) π hours

(C) $\frac{3\pi}{2}$ hours

(D) 2π hours

10 A particle is moving along the x-axis. The graph shows its velocity v metres per second at time t seconds.



Initially, the particle is at the origin.

Which of the following describes the position of the particle after 8 seconds?

(A) The particle is to the left of the origin.

(B) The particle is to the right of the origin.

(C) The particle is to the at the origin.

(D) There is not enough information to determine its position.

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Write $\dfrac{1}{4-\sqrt{7}}$ in the form $a+b\sqrt{c}$, where a, b, and c are rational. **2**

(b) Simplify $\dfrac{a^3+b^3}{a^2-b^2}$. **2**

(c) Solve $|4-2x|<3$. **2**

(d) Differentiate xe^{5x}. **2**

(e) Solve $\cos 2x=\dfrac{1}{2}$ for $0\le x\le 2\pi$. **2**

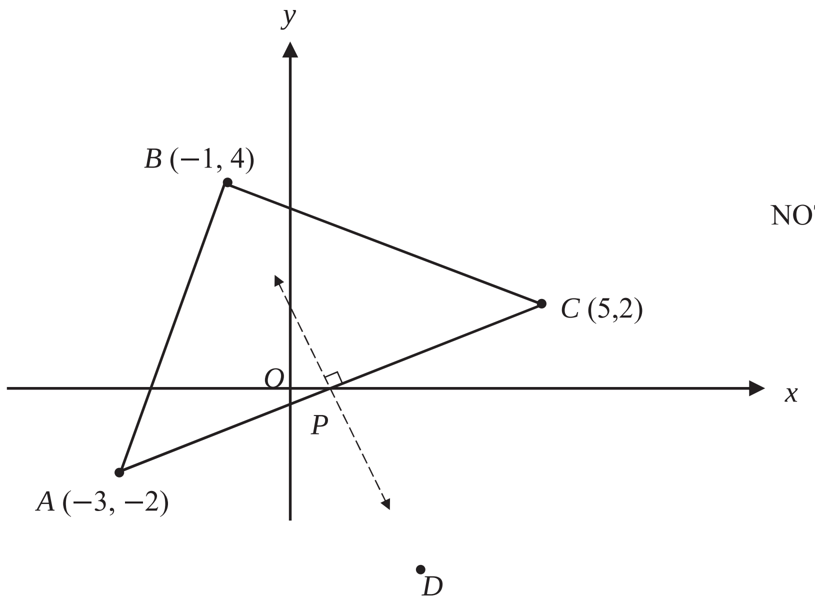
(f) Find $\displaystyle\int \frac{1}{\sqrt{4x-1}}\,dx$. **2**

(g) On a number plane, shade the region where the points (x, y) satisfy both of the inequalities $x^2+(y-2)^2\le 4$ and $x+y>2$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows points $A(-3,-2)$, $B(-1,4)$ and $C(5,2)$.

Point P is the midpoint of AC.



NOT TO SCALE

(i) Show that the equation of the line perpendicular to AC and passing through the point P is $2x + y - 2 = 0$. **3**

(ii) Show that B lies on the line $2x + y - 2 = 0$. **1**

(iii) If P is also the midpoint of BD, find the coordinates of D. **1**

(iv) What type of quadrilateral is $ABCD$? Explain your answer. **1**

(b) Consider the parabola $y^2 + 4y + 8x - 12 = 0$.

(i) By completing the square, or otherwise, find the focal length of the parabola. **2**

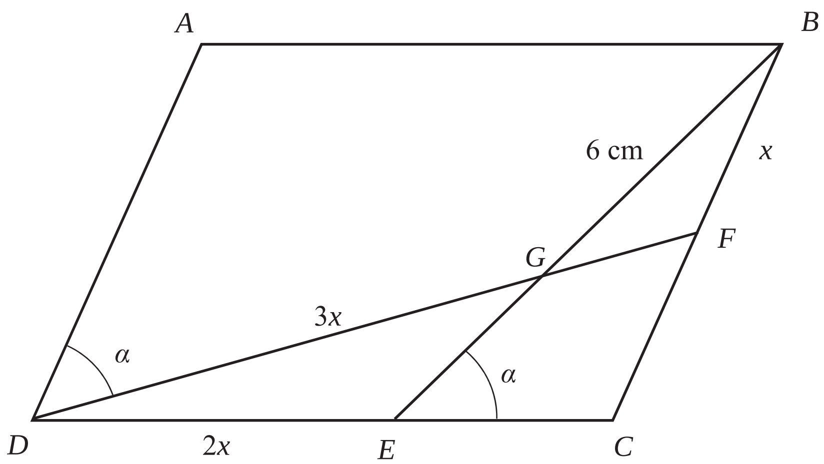
(ii) Find the coordinates of the focus. **1**

Question 12 continues on page 8

(c) *ABCD* is a parallelogram.

F and *E* are two points on *BC* and *DC* respsectively such that $BF = x$, $DE = 2x$ and $\angle ADF = \angle BEC = \alpha$.

FD meets *BE* at *G* such that $DG = 3x$ and $BG = 6\,\text{cm}$.



(i) Prove that $\angle GDE = \angle GBF$. **2**

(ii) Prove that ΔGDE is similar to ΔGBF. **2**

(iii) Hence find the value of *x*. **2**

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet.

(a) Consider the function $y = x^3 - 9x^2 + 24x$.

(i) Find all stationary points and determine their nature. **4**

(ii) Find the point of inflexion. **2**

(iii) Sketch the curve showing all important features. **2**

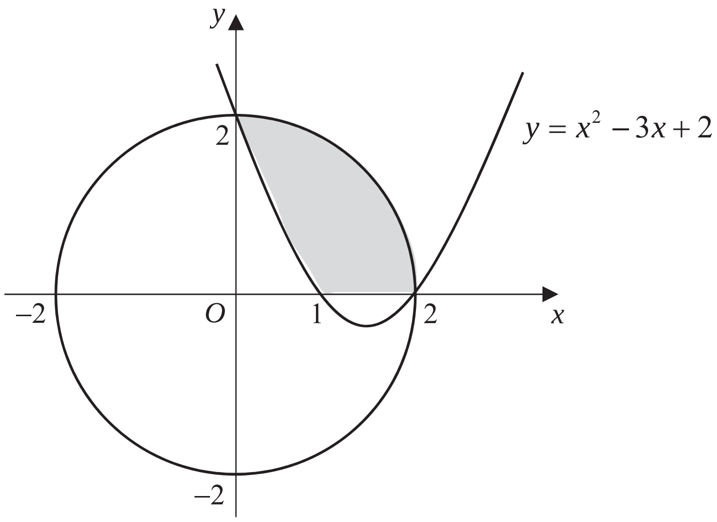
(b) A new design of shoes is introduced to the market and 15 000 shoes are sold in the first month. Each month thereafter, the sales are 80% of the sales in the previous month.

(i) In which month will monthly sales first drop below 1000 per month? **2**

(ii) How many shoes are sold in total in the first year? **1**

(iii) How many shoes are eventually sold altogether? **1**

(c) The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x-axis. **3**

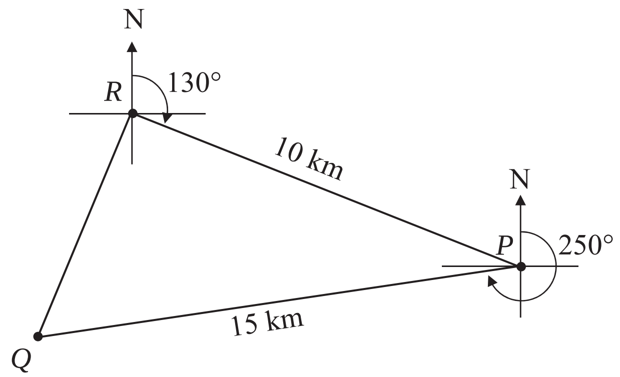


Find the exact area of the shaded region.

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) The diagram shows three bus stops P, Q and R. **2**

P is 10 km away from R on a bearing of $130°$, while Q is 15 km from P on a bearing of $250°$, as shown in the diagram below.



Find the distance of R from Q correct to one decimal place.

(b) Rhonda takes out a loan for \$48 000 on terms where no repayments are made for a year and then equal monthly repayments are made for a further 3 years.

Interest is compounded at 1.5% per month on the current monthly balance for the full duration of the loan (including the time when no repayments are made). The repayment (\$$N$) is deducted each month before the interest is calculated.

(i) Show that the amount still owing 15 months after the loan is taken out is given by: **2**

$$48000(1.015)^{15} - N\left(1.015 + 1.015^2 + 1.015^3\right).$$

(ii) Calculate the value of the monthly repayment \$$N$, to the nearest dollar. **3**

Question 14 continues on page 11

(c) A tank filled with water has a hole on its side. The volume of water in the tank is changing at a rate of $R = t^2 - 9t - 10$ L/min for $0 \le t \le T$, where t is the time in minutes after the water begins to flow. Initially, the tank held 400L of water.

(i) What is the largest value of T for which the expression for R is physically reasonable? **2**

Explain your answer.

(ii) Calculate the exact amount of water left in the tank once the water stops leaking. **2**

(d) Coal is extracted from a mine at a rate that is proportional to the amount of coal remaining in the mine. The amount of coal (in tonnes) A, remaining after t years is given by

$$A = R_0 e^{-kt}$$

where k is a constant and R_0 is the initial amount of coal.

After 25 years, 50% of the initial amount of coal remains.

(i) Find the value of k, correct to 4 decimal places. **2**

(ii) How many more years will elapse before only 20% of the original remains? **2**

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet.

(a) A particle moves along the x-axis so that its displacement, x metres, after t seconds is given by $x = \dfrac{(5t-3)^2}{e^t}$.

(i) Show that the velocity is given by $v = \dfrac{(5t-3)(13-5t)}{e^t}$. **2**

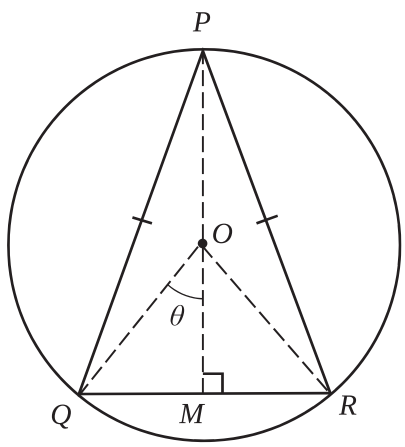
(ii) When does the particle first come to rest? **2**

(iii) Show that acceleration is zero when $t = \dfrac{13}{5} - \sqrt{2}$ and $t = \dfrac{13}{5} + \sqrt{2}$. **2**

(iv) Find the particle's maximum speed, correct to 2 decimal places. **2**

(v) What happens to the particle eventually? **1**

(b) Isosceles triangle PQR is inscribed in a circle of radius 1 unit, with centre O.
$\angle QOM = \theta$, where θ is acute.
PO is extended to meet QR at M such that $\angle OMR = 90°$.



(i) Prove that $QM = \sin\theta$ and $OM = \cos\theta$. **2**

(ii) Show that the area A, of ΔPQR is given by $A = \sin\theta(\cos\theta + 1)$. **1**

(iii) Hence find the maximum area of ΔPQR as an exact value. **3**

Question 16 (15 Marks) Use a SEPARATE writing booklet.

(a) Find $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$. **3**

(b) A particle moves in a straight line. Its velocity v m/s at time t seconds is given by $v = \sin\left(2t - \frac{\pi}{6}\right)$. The particle is at rest 4m to the right of the origin after $\frac{\pi}{12}$ seconds.

(i) Find the displacement after $\frac{\pi}{3}$ seconds. **2**

(ii) Find the total distance travelled in the first $\frac{\pi}{3}$ seconds. **2**

(iii) What is the maximum acceleration and when will the particle first experience this acceleration? **2**

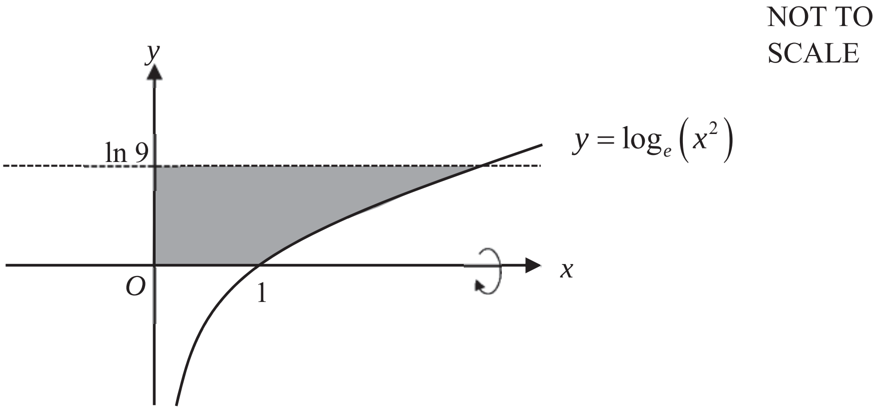
Question 16 continues on page 14

Question 16 (continued)

(c) Show that the derivative of $x\left[\left(\log_e x\right)^2 - 2\log_e x + 2\right]$ is $\left(\log_e x\right)^2$. **2**

(d) The diagram below shows the graph of $y = \log_e\left(x^2\right)$.

The region bounded by $y = \log_e\left(x^2\right)$, $y = \log_e 9$ and the x and y axes, is rotated about the x axis to form a solid.



(i) Show that the volume of the solid of revolution is given by **1**

$$V = 3\pi\left(\log_e 9\right)^2 - \pi\int_1^3 \left[\log_e\left(x^2\right)\right]^2 dx.$$

(ii) Using the result in (c), or otherwise, find the exact volume of the solid. **3**

End of paper

Question	Answer
1	D
2	B
3	B
4	B
5	A
6	D
7	B
8	D
9	D
10	A

Section II

Question 11

(a) $\frac{1}{4-\sqrt{7}} \times \frac{4+\sqrt{7}}{4+\sqrt{7}}$ **2**

$= \frac{4+\sqrt{7}}{16-7}$

$= \frac{4+\sqrt{7}}{9} = \frac{4}{9} + \frac{\sqrt{7}}{9}$

(b) $\frac{a^3+b^3}{a^2-b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a+b)(a-b)}$ **2**

$= \frac{a^2-ab+b^2}{a-b}$

(c) $-3 < 4-2x < 3$ **2**

$-7 < -2x < -1$

$\frac{1}{2} < x < \frac{7}{2}$

(d) $\frac{d}{dx}\left(xe^{5x}\right)$ **2**

$= e^{5x} + 5xe^{5x} = e^{5x}(1+5x)$

(e) $0 \le 2x \le 4\pi$ **2**

$\cos 2x = \frac{1}{2}$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

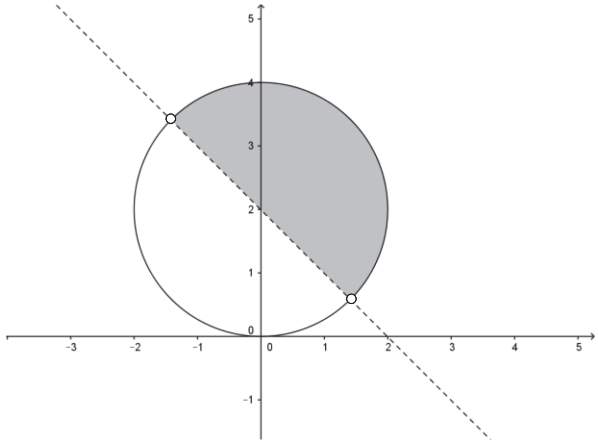
$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(f) $\int \frac{1}{\sqrt{4x-1}}\,dx = \frac{1}{4}\int 4(4x-1)^{-\frac{1}{2}}\,dx$ **2**

$= \frac{1}{4} \times \left(2(4x-1)^{\frac{1}{2}}\right) + c$

$= \frac{1}{2}\sqrt{4x-1} + c$

(g) **3**



Question 12

(a) (i) $M_{AC} = (1,0)$ **3**

$m_{AC} = \frac{2+2}{5+3} = \frac{1}{2}$

$m = -2, P(1,0)$

$y - 0 = -2(x-1)$

$y = -2x + 2$ or $2x + y - 2 = 0$

(ii) Let $x = -1$, **1**

$2(-1) + y - 2 = 0$

$\therefore y = 4$

$\therefore B(-1,4)$ lies on the line $2x + y - 2 = 0$

(iii) Let $P(1,0)$, $B(-1,4)$ and $D(a,b)$, **1**

$\frac{a-1}{2} = 1$

$\therefore a = 3$

$\frac{b+4}{2} = 0$

$\therefore b = -4$

$\therefore D(3,-4)$

(iv) This is a rhombus as diagonals *DB* and *AC* bisect each other and are perpendicular to each other. **1**

Furthermore, since diagonals are equal in length, the quadrilateral is really a square.

(b) (i) $y^2 + 4y + 8x - 12 = 0$ **2**

$y^2 + 4y + 4 = -8x + 12 + 4$

$(y+2)^2 = -8x + 16$

$(y+2)^2 = -8(x-2)$

$\therefore$ focal length is 2

(ii) Parabola has vertex $V(2,-2)$ **1**

and focal length 2

$\therefore S(0,-2)$

(c) (i) $AB \parallel DC$ (opposite sides of a parallelogram) **2**

$\angle ABE = \angle GEC = \alpha$ (alternate angles on parallel lines, $AB \parallel DC$)

$\angle ADC = \angle ADG + \angle GDE = \alpha + \angle GDE$

$\angle ABC = \angle ABG + \angle GBF = \alpha + \angle GBF$

$\angle ADC = \angle ABC$ (opposite vertex angles of a parallelogram)

$\alpha + \angle GDE = \alpha + \angle GBF$

$\therefore \angle GDE = \angle GBF$

(ii) In ΔGDE and ΔGBF, **2**

$\angle GDE = \angle GBF$ (from (i))

$\angle DGE = \angle BGF$ (vertically opposite angles)

$\therefore \Delta GDE \ ||| \ \Delta GBF$ (equiangular)

(iii) $\dfrac{DG}{BG} = \dfrac{DE}{BF}$ (sides of similar triangles are in proportion) **2**

$\dfrac{3x}{6} = \dfrac{2x}{x}$

$\dfrac{x}{2} = 2$

$x = 4$

Question 13

(a) (i) **4**

$y = x^3 - 9x^2 + 24x$

$y' = 3x^2 - 18x + 24$

To find stationary points,

let $y' = 0$

$3x^2 - 18x + 24 = 0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2$ or $x = 4$

$y'' = 6x - 18$

When $x = 2, y'' = -6 < 0$

$\therefore$ maximum turning point at $(2, 20)$

When $x = 4, y'' = 6 > 0$

$\therefore$ minimum turning point at $(4, 16)$

(ii) To find points of inflexion, **2**

let $y'' = 0$

$6x - 18 = 0$

$x = 3$

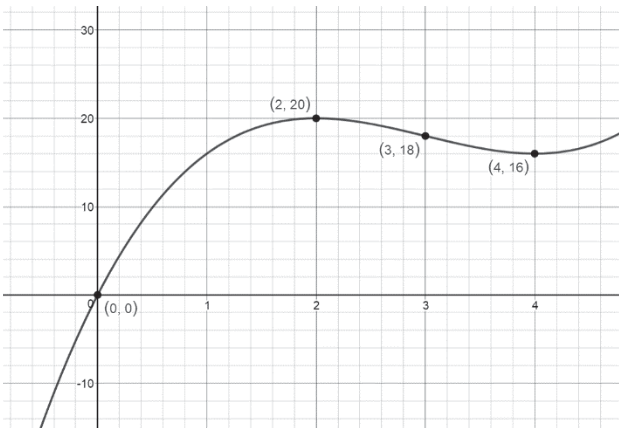
To check for change of concavity at $x = 3$,

x	2	3	4
y''	-6	0	6

$\therefore$ there is a change in concavity at $x = 3$

$\therefore (3, 18)$ is a point of inflexion.

(iii) **2**



(b) (i) $15\,000 \times (0.8)^{n-1} < 1000$ **2**

$0.8^{n-1} < \frac{1}{15}$

$\log\left(0.8^{n-1}\right) < \log\frac{1}{15}$

$(n-1)\log(0.8) < \log\frac{1}{15}$

$n-1 > \frac{\log\frac{1}{15}}{\log 0.8}$

$n > 13.135...$

$\therefore$ In the 14th month, the slaes first drop below 1000 per month.

(ii) $S_{12} = \frac{15\,000\left(1-0.8^{12}\right)}{1-0.8}$ **1**

$= 69846.039...$

$\therefore$ 69846 shoes are sold in the first year.

(iii) Since $|r| < 1$, a limiting sum exists. **1**

$S = \frac{1500}{1-0.8}$

$= 75\,000$

(c) $\int_0^1 x^2 - 3x + 2\,dx$ **3**

$= \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_0^1$

$= \frac{5}{6}\text{ u}^2$

$A_{\text{shaded region}} = \frac{1}{4} \times \pi \times 2^2 - \frac{5}{6}$

$= \pi - \frac{5}{6}\text{ u}^2$

Question 14

(a) $\angle RPQ = 60°$ **2**

$RQ^2 = RP^2 + QP^2 - 2(RP)(RQ)\cos 60°$

$RQ^2 = 10^2 + 15^2 - 2(10)(15)\cos 60°$

$\therefore RQ = 5\sqrt{7}$ or 13.2 (2 decimal places)

(b) (i) $A_{12} = 48\,000 \times 1.015^{12}$ **2**

$A_{13} = \left(48\,000 \times 1.015^{12} - N\right) \times 1.015$

$= 48\,000 \times 1.015^{13} - 1.015 \times N$

$A_{14} = \left(48\,000 \times 1.015^{13} - 1.015 \times N - N\right) \times 1.015$

$= 48\,000 \times 1.015^{14} - 1.015^2 \times N - 1.015 \times N$

$A_{15} = \left(48\,000 \times 1.015^{14} - 1.015^2 \times N - 1.015 \times N - N\right) \times 1.015$

$= 48\,000 \times 1.015^{15} - 1.015^3 \times N - 1.015^2 \times N \times 0.015 \times N$

$= 48\,000 \times 1.015^{15} - N\left(1.015 + 1.015^2 + 1.015^3\right)$

(ii) $A_{48} = 48\,000 \times 1.015^{48} - N\left(1.015 + 1.015^2 + \ldots + 1.015^{36}\right)$ **3**

Let $A_{48} = 0$

$48\,000 \times 1.015^{48} - N\left(1.015 + 1.015^2 + \ldots + 1.015^{36}\right) = 0$

$1.015 + 1.015^2 + \ldots + 1.015^{36}$ is a geometric series with

$a = 1.015$, $r = 1.015$ and $n = 36$

$$48\,000 \times 1.015^{48} - N\left(\frac{1.015\left(1.015^{36} - 1\right)}{1.015 - 1}\right) = 0$$

$$N\left(\frac{1.015\left(1.015^{36} - 1\right)}{0.015}\right) = 48\,000 \times 1.015^{48}$$

\$2044 is the value of each repayment to the nearest dollar.

(c) (i) Water is leaking out from the tank and so the volume of water decreases as time goes by. **2**
Water stops flowing when $R = 0$.

$\frac{dV}{dt} \le 0$

$t^2 - 9t - 10 \le 0$

$(t - 10)(t + 1) \le 0$

$-1 \le t \le 10$

But $t \ge 0$

$\therefore 0 \le t \le 10$

Hence the largest value of T is 10

(ii) $\frac{dV}{dt} = t^2 - 9t - 10$ **2**

$\therefore V = \frac{1}{3}t^3 - \frac{9}{2}t^2 - 10t + C$

when $t = 0$, $V = 400$

$\therefore C = 400$

$\therefore V = \frac{1}{3}t^3 - \frac{9}{2}t^2 - 10t + 400$

Now, water stops leaking when $\frac{dV}{dt} = 0$

$\therefore (t - 10)(t + 1) = 0$

$t = 10$ or -1

$\therefore t = 10 \ (0 \le t \le 10)$

$\therefore V = \frac{1}{3} \times 10^3 - \frac{9}{2} \times 10^2 - 100 + 400$

$= 183\frac{1}{3}$ L

(d) (i) $A = R_0 e^{-kt}$ **2**

$\frac{1}{2}R_0 = R_0 e^{-k \times 25}$

$e^{-25k} = \frac{1}{2}$

$\log\left(e^{-25k}\right) = \log\frac{1}{2}$

$-25k = \log\frac{1}{2}$

$\therefore k = 0.0277$ (4 decimal places)

(ii) $\frac{1}{5} = e^{-0.0277t}$ **2**

$\log\frac{1}{5} = \log\left(e^{-0.0277t}\right)$

$-0.0277t = \log\frac{1}{5}$

$\therefore t = 58.1$ (1 decimal place)

Hence it will take 33 more years before only 20% of the original remains.

Question 15

(a) (i) **2**

$$x = \frac{(5t-3)^2}{e^t}$$

$$v = \frac{dx}{dt}$$

$$v = \frac{2(5t-3)\times 5\times e^t - e^t \times (5t-3)^2}{(e^t)^2}$$

$$= \frac{e^t(5t-3)(10-5t+3)}{(e^t)^2}$$

$$= \frac{(5t-3)(13-5t)}{e^t}$$

(ii) Let $v = 0$ **2**

$$\frac{(5t-3)(13-5t)}{e^t} = 0$$

$$(5t-3)(13-5t) = 0$$

$$\therefore t = \frac{3}{5} \text{ or } t = \frac{13}{5}$$

$\therefore$ the particle first comes to rest when $t = \frac{3}{5}$

(iii) **2**

$$a = \frac{dv}{dt}$$

$$v = \frac{(5t-3)(13-5t)}{e^t} = \frac{-25t^2+80t-39}{e^t}$$

$$\frac{dv}{dt} = \frac{(-50t+80)e^t - (-25t^2+80t-39)e^t}{(e^t)^2}$$

$$= \frac{e^t(-50t+80+25t^2-80t+39)}{(e^t)^2}$$

$$= \frac{25t^2-130t+119}{e^t}$$

Let $a = 0$

$$\frac{25t^2-130t+119}{e^t} = 0$$

$$25t^2-130t+119 = 0$$

$$t = \frac{130\pm\sqrt{5000}}{50}$$

$$= \frac{130\pm 50\sqrt{2}}{50}$$

$$= \frac{13}{5}\pm\sqrt{2}$$

(iv) Maximum/minimum speed occurs when $\frac{dv}{dt}=0$, that is, when $a=0$. **2**

$a=0$ when $t=\frac{13}{5}-\sqrt{2}$ and $\frac{13}{5}+\sqrt{2}$

t	1	$\frac{13}{5}-\sqrt{2}$	2	$\frac{13}{5}+\sqrt{2}$	5
$\frac{dv}{dt}$	5.15…	0	-5.54…	0	0.633…

$\therefore$ maximum occurs when $t=\frac{13}{5}-\sqrt{2}$

When $t=\frac{13}{5}-\sqrt{2}$

$v=6.327...$

$\therefore$ maximum speed is 6.3 ms^{-1}

(v) As $t\to\infty, e^t\to\infty$ then $\frac{1}{e^t}\to 0$ **1**

$\therefore x\to 0^+, v\to 0^-$ and $a\to 0^+$

Hence, the particle approaches origin from the right with decreasing speed.

(b) (i) In ΔOQM, **2**

$OQ=1$ (radii of circle)

$\sin\theta=\frac{QM}{OQ}$

$\therefore QM=\sin\theta$

$\cos\theta=\frac{OM}{OQ}$

$\therefore OM=\cos\theta$

(ii) $PM = 1 + OM$ **1**

$= 1 + \cos\theta$

$QR = 2 \times QM$

$= 2\sin\theta$

$A_{\Delta PQR} = \frac{1}{2} \times QR \times PM$

$= \frac{1}{2} \times 2\sin\theta \times (1 + \cos\theta)$

$= \sin\theta(\cos\theta + 1)$

(iv) $A = \sin\theta(\cos\theta + 1)$ **3**

$\frac{dA}{d\theta} = \cos\theta(1 + \cos\theta) - \sin\theta\sin\theta$

$= \cos\theta + \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta + \cos\theta - 1$

$= (2\cos\theta - 1)(\cos\theta + 1)$

To find maximum area, let $\frac{dA}{d\theta} = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

since θ is acute, $0° \le \theta \le 90°$

$\therefore \theta = 60°$

$\frac{d^2A}{d\theta^2} = -\sin\theta - 4\cos\theta\cos\theta$

when $\theta = 60°$,

$\frac{d^2A}{d\theta^2} = -\frac{3\sqrt{3}}{2} < 0$

$\therefore$ maximum area occurs when $\theta = 60°$

$A = \sin 60°(\cos 60° + 1)$

$= \frac{3\sqrt{3}}{4}$

Question 16

(a) **3**

$$\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

$$= \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= \left[\tan x - x \right]_0^{\frac{\pi}{4}}$$

$$= 1 - \frac{\pi}{4} - 0$$

$$= 1 - \frac{\pi}{4} = 0.2146...$$

(b) (i) **2**

$$v = \sin\left(2t - \frac{\pi}{6}\right)$$

$$x = -\frac{1}{2}\cos\left(2t - \frac{\pi}{6}\right) + c$$

when $x = 4, t = \frac{\pi}{12}$

$$\therefore c = \frac{9}{2}$$

$$\therefore x = -\frac{1}{2}\cos\left(2t - \frac{\pi}{6}\right) + \frac{9}{2}$$

Let $t = \frac{\pi}{3}$,

$$x = \frac{9}{2}$$

$\therefore$ the particle is 4.5 m to the right of the origin

(ii) **2**

$$\text{Total distance travelled} = \left| \int_0^{\frac{\pi}{12}} \sin\left(2t - \frac{\pi}{6}\right) dt \right| + \int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \sin\left(2t - \frac{\pi}{6}\right)$$

$$= \left| \frac{1}{2}\left[-\cos\left(2t - \frac{\pi}{6}\right) \right]_0^{\frac{\pi}{12}} \right| + \frac{1}{2}\left[-\cos\left(2t - \frac{\pi}{6}\right) \right]_{\frac{\pi}{12}}^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left| -1 + \frac{\sqrt{3}}{2} \right| + \frac{1}{2}[0 + 1]$$

$$= \frac{2 - \sqrt{3}}{4} + \frac{1}{2}$$

$$== \frac{4 - \sqrt{3}}{4} \text{ metres}$$

(iii) $a = \frac{dv}{dt}$ **2**

$a = 2\cos\left(2t - \frac{\pi}{6}\right)$

since the amplitude of $y = 2\cos\left(2t - \frac{\pi}{6}\right)$ is 2, the maximum value of the acceleration is 2.

To find when the particle has acceleration 2, let $a = 2$

$2 = 2\cos\left(2t - \frac{\pi}{6}\right)$

$\cos\left(2t - \frac{\pi}{6}\right) = 1$

$2t - \frac{\pi}{6} = 0, \ldots$

$2t = \frac{\pi}{6}, \ldots$

$t = \frac{\pi}{12}, \ldots$

Hence the particule first experience a maximum acceleration of 2 when $t = \frac{\pi}{12} s$

(c) $\frac{d}{dx} x\left[(\ln x)^2 - 2\ln x + 2\right]$ **2**

$= \left[(\ln x)^2 - 2\ln x + 2\right] + x\left[(2\ln x) \times \frac{1}{x} - \frac{2}{x}\right]$

$= (\ln x)^2 - 2\ln x + 2 + 2\ln x - 2$

$= (\ln x)^2$

(d) (i) For $y = \ln 9, \quad x = 3$ **1**

$V = \left(\pi \times (\ln x)^2 \times 3\right) - \left(\pi \times \int_1^3 \left(\ln x^2\right)^2 dx\right)$

$\therefore V = 3\pi \times (\ln x)^2 - \pi\int_1^3 \left(\ln x^2\right)^2 dx$

(ii) **3**

$$V = 3\pi \times (\ln 9)^2 - \pi \int_1^3 [2\ln x]^2 dx$$

$$= 3\pi \times (\ln 9)^2 - \pi \int_1^3 4(\ln x)^2 \, dx$$

$$= 3\pi \times (\ln 9)^2 - 4\pi \int_1^3 (\ln x)^2 \, dx$$

From (c), $\int (\ln x)^2 \, dx = x\left[(\ln x)^2 - 2\ln x + 2\right]$

$$= 3\pi \times (\ln 9)^2 - 4\pi \left[x(\ln x)^2 - 2\ln x + 2\right]_1^3$$

$$= 3\pi \times (\ln 9)^2 - 4\pi \left[3(\ln 3)^2 - 2\ln 3 + 2 - 2\right]$$

$$= (8\pi \ln 3)\,\text{u}^3$$

Marker's Comments

Question 11

(a) Well done except for about half the students who went on to write $a =$, $b =$, $c =$, which wasn't the question asked. (No penalty was applied and these answered were ignored).
(b) Very well done.
(c) Remember that the final solution for this question lies between two points and should be written as such. Not "or", implied or otherwise. (Half mark penalty)
(d) Very well done. Factorised form is best.
(e) Several students forgot that they needed to go around twice, so missed half of the solutions.
(f) Mostly done well. Some students still need to learn the rule. Remember the + c for some. (No penalty applied)
(g) Few students achieved the full 3 marks for this question. Most lost half a mark for not having an open circle for the intersection of the included circle and excluded line. Other errors included not stating the *x* and *y* intercepts of the circle/line, graphing the circle incorrectly and/or the line graphed incorrectly.

Question 12

(a) (i) Most students answered this question correctly.
(ii) Most students answered this question correctly. Some student worked LHS and RHS at the same time but they were not penalised for this.
(iii) Most students answered this question correctly.
(iv) Successful student used the previous answers and $AC = BD$ (with working) to explain *ABCD* is a square. Some students used only the previous answers to conclude that *ABCD* is a rhombus. They were not penalised for this
(b) (i) Lots of careless errors and algebraic errors. Some students answered focal length is – 2 but they were not penalised for this.
(ii) Poorly done. Some student successfully answered part (i) but put focus as $(2,-4)$ or $(2,0)$ as they considered $(y+2)^2 = -8(x-2)$ as concave up/down parabola.
(c) (i) Some students used alternate angles on parallel lines and opposite angles of parallelogram. Many students spent a lot of time proving $\Delta CDF \parallel \Delta CBF$ to show that $\angle GDE = \angle GBF$.
(ii) Most students answered this question correctly.
(iii) Most students answered this question correctly.

Question 13

(a) (i) Was done very well
(ii) Students are reminded to check for change of concavity with points of inflexion.
(iii) Some students are still not drawing a "good" sized diagram (1/3 to 1/2 page recommended). Some students haven't used a reasonable scale or ruler.
(b) Many students wrote the correct inequality but did not reverse the inequality sign when dividing by log 0.8. This often led to wrong working and the loss of 1 mark. Also, students who wrote an equation instead of an inequality are encouraged to test both A13 and A14.
(c) Many students didn't recognise one area as a quarter of a circle. Many students incorrectly found the area under the parabola from 2 to 1 rather than from 1 to 0.

Question 14

(a) Most students **proved** that angle RPQ=60 which was not necessary.
Some students got the cosine rule wrong – please check the reference sheet.

(b) (i) Mostly well done.
Some students did not take off the repayment before finding the interest and then fudged the answer or ignored the difference to the required result.

(ii) Good attempts, however a lot of students used n=36 for the total amount owing and found A_{36} when it should be A_{48}.

(c) (i) Many students misunderstood this question and found dR/dt. Answers were also given as t=10 not t=10.

(ii) Well done.

(d) (i) Well done except some misunderstanding of 4 decimal places!

(ii) Mostly well done, the majority of students did not find how many **more** years were needed.

Question 15

(a) (i) This questions was completed very well across the year.

(ii) This questions was completed very well across the year. The most common error as to not answer the question and leave the final solution as $t=\frac{3}{5}$ or $t=\frac{13}{5}$ instead of writing the conclusion $\therefore$ the particle first comes to rest when $t=\frac{3}{5}$

(iii) This was quite a challenging question but the majority of students managed to make it through.
The most common errors were algebraic in nature and occurred early in the question on the way to arriving at $a=\dfrac{25t^2-130t+119}{e^t}$.
If students were able to arrive at this equations for acceleration then the only major error here was to write the following :

$$\frac{25\left(\frac{13}{5}+\sqrt{2}\right)^2-130\left(\frac{13}{5}+\sqrt{2}\right)+119}{e^t}=0 \text{ and } \frac{25\left(\frac{13}{5}+\sqrt{2}\right)^2-130\left(\frac{13}{5}+\sqrt{2}\right)+119}{e^t}=0$$

This is not shown as it is not obvious that either of these two expressions is equal to zero.

(iv) The majority of students lost marks in this part. There are two important points that need to be addressed.

- It is only a ***possible*** max or min of v when $a=0$ and you must check that it is a turning point and not an inflexion point.
- That a maximum speed can occur at either the max velocity or min velocity since speed $=|v|$

So the correct solution required students to show that at $t=\frac{13}{5}+\sqrt{2}$ and $t=\frac{13}{5}-\sqrt{2}$
There are max and min velocities, and then test both these times in the velocity function and then take the absolute values to decide which is the larger speed.

(v) To achieve the entire mark there needed to be a reference to both the displacement approaching zero and velocity approaching zero.

(b) (i) This was completed quite well but some students did not rtefer to the triangle they were using or explain that OQ was equal to 1 and simply wrote:

$$\sin\theta = \frac{QM}{1}$$

$$\therefore \sin\theta = QM$$

This was not sufficient to show that $\sin\theta = QM$

(ii) Completed quite well

(iii) Completed quite well

Question 16

(a) A good proportion successfully integrated the given expression. For those who were unsuccessful, it was interesting see a mixture of rules for integration being used incorrectly. Students are encouraged to practise various integration techniques involving trigonometry. Other students who recognised the trigonometric identity to integrate were mostly successful in the integration part. However, a number of students did not simplify their final expression correctly.

(b) (i) Most students recognised to integrate the velocity expression to derive the expression for displacement. A number of students did not find the value of the constant or found it incorrectly. Students are advised to take particular care with their numbers.

(ii) This question was poorly done across the cohort. Many students did not recognise that the particle changes direction after $\frac{\pi}{12}$ seconds. Many students who did recognise that the particle changes direction, were not able to develop the correct integral expression. Overall, students are encouraged to practise questions on finding the total distance travelled of motion that changes direction.

(iii) This question was poorly done across the cohort.
Many students assumed that the motion is simple harmonic. Students are reminded that unless stated in the question, or proven in their response, the motion cannot be assumed to be simple harmonic. This resulted in many students stating that maximum acceleration occurs when velocity is zero. This is only true for simple harmonic motion. This is not the correct way to find the maximum acceleration for any particular in motion.
Other students stated that maximum acceleration occurs when acceleration is zero which is a puzzling statement.
Some successful students correctly found the expression for acceleration and differentiated again to maximise the expression. The more efficient students considered the amplitude of the acceleration expression, which would have given the maximum acceleration.

Many students did not answer the question, only finding the time when maximum acceleration occurs. Students are encouraged to always answer the question.

(c) Most students successfully applied the product rule or expanded the expression to differentiate.

(d) (i) This question was generally well done. However, many students referred to their expression as areas, rather than volumes.

(ii) This question was poorly done across the grade due to the fact that student needed first use logarithmic laws to simplify the expression before applying the result in (c).