North Sydney Girls High School

2020 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 30)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

STUDENT NUMBER

Question	1 – 10	11 – 16	17 – 20	21 – 24	25 – 29	30 – 33	34 – 35	Total
Total	/10	/16	/16	/15	/16	/14	/13	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 What is the value of $e^2 \sin(1.2)$ to 3 significant figures?

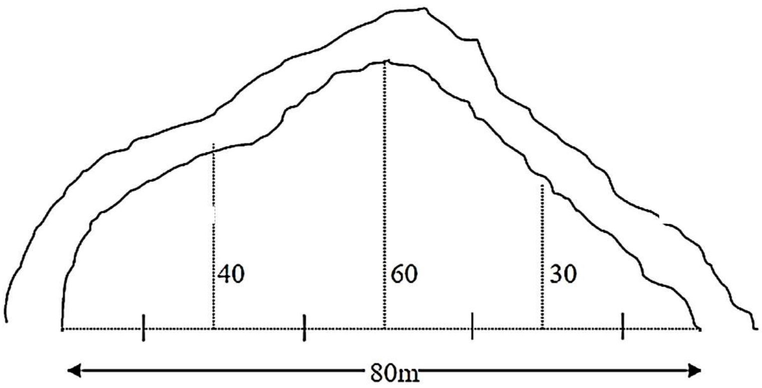
A. 0.154

B. 0.155

C. 6.886

D. 6.89

2 A paddock is bounded by a fence and a river as illustrated below:



Four applications of trapezoidal rule were used to determine the area of the paddock. What is the approximate area of the paddock?

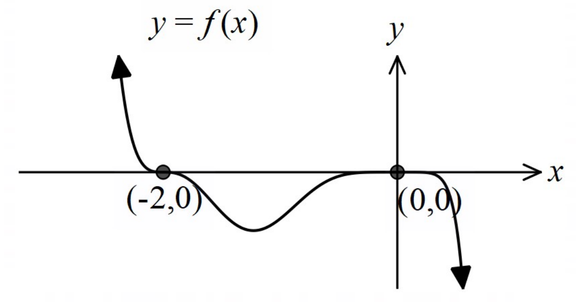
A. 1300 m^2

B. 1900 m^2

C. 2600 m^2

D. 5200 m^2

3 The following diagram shows the graph of $y = f(x)$.



Which of the following is the solution to $f(x) \geq 0$?

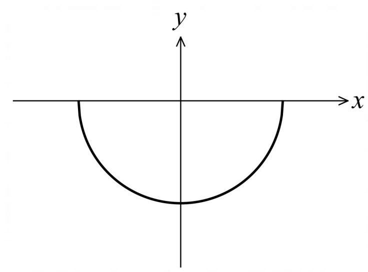
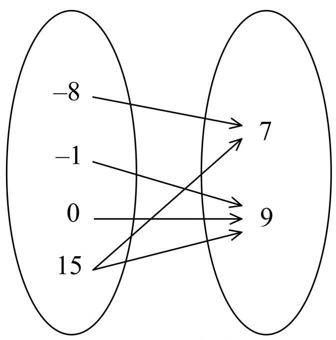
A. $(\infty, -2)$

B. $(-2, \infty) \cup (0,0)$

C. $(-2, \infty) \cup [0,0]$

D. $(-\infty, -2] \cup [0,0]$

4 Which of the following represents a many-to-one relation?

I	**II**	**III**
	$\{(-2,5),\ (4,5),\ (5,-2),\ (5,1)\}$	

A. I only

B. I and II only

C. I and III only

D. I , II and III

5 If 4 is added to each score in a set, which one of the following statements will be true?

A. The mean and standard deviation will remain the same.

B. The mean will increase by 4 and the standard deviation will remain the same.

C. The mean will increase by 4 and the standard deviation will increase by 2.

D. The mean will increase by 4 and the standard deviation will increase by 4.

6 James plays a game involving the tossing of two coins. One turn at this game costs $1. The possible outcomes are listed below along with their payoffs:

- 2 Heads pays $5
- 1 Head and 1 Tail pays $2
- 2 Tails pays nothing

What is the financial expectation for this game?

A. $1.25

B. −$1.00

C. $2.00

D. $2.25

7 The maximum speed of a train going up a hill is inversely proportional to the square root of its weight. A train weighing 3600 tonnes can go up a hill at 30 km/h. What is the maximum speed at which a train weighing 2500 tonnes could go up the same hill?

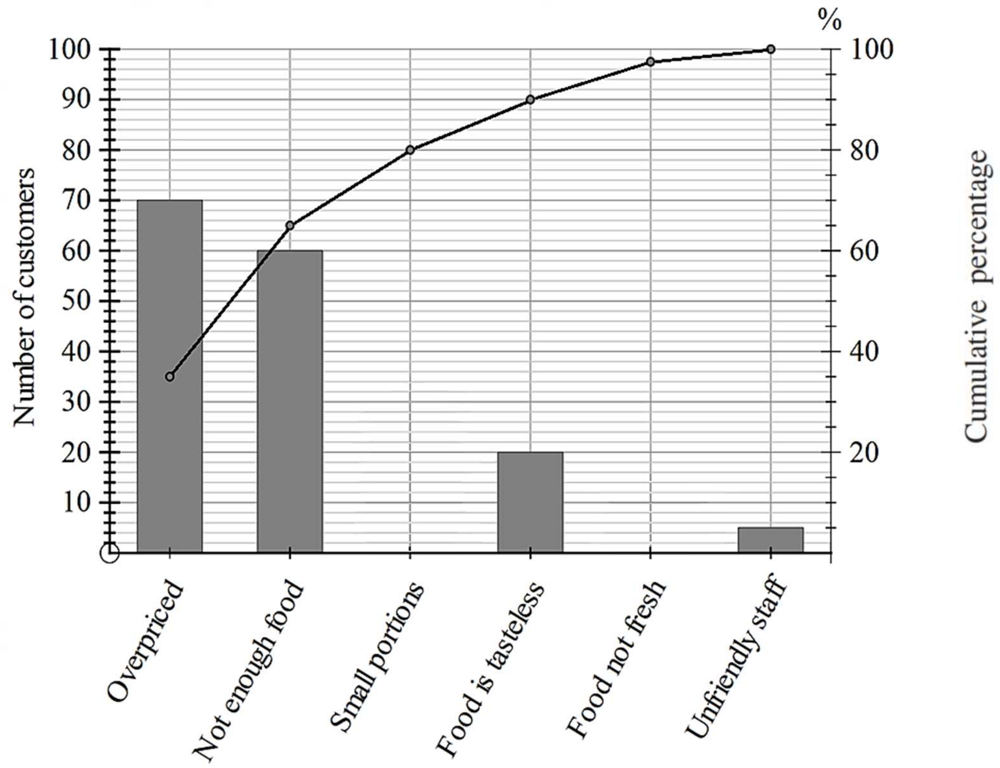
A. 43.2 km/h

B. 36 km/h

C. 25 km/h

D. 20.8 km/h

8 The following Pareto chart shows 200 customer complaints from a restaurant survey. The columns for 'small portions' and 'food not fresh' are missing.



How many customers complained about the restaurant having 'small portions'?

A. 15 people

B. 30 people

C. 40 people

D. 80 people

9 What is the derivative of $e^{3\ln x}$?

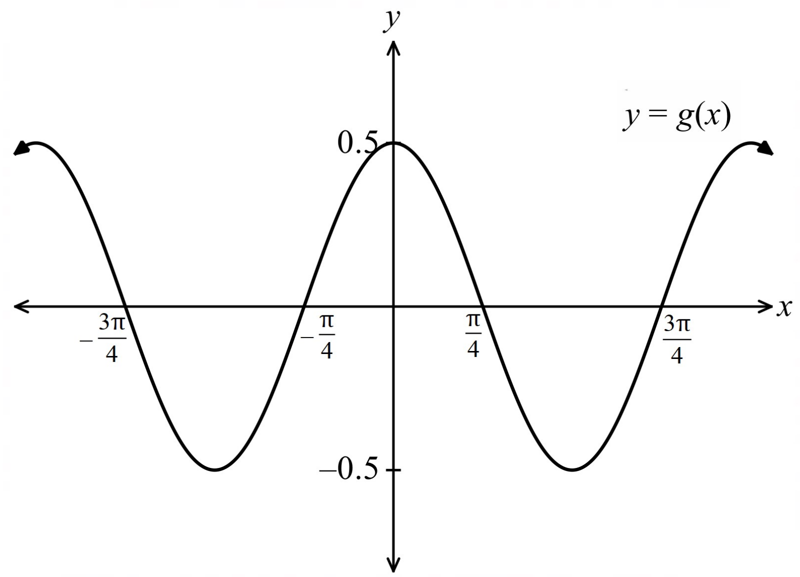
A. $3x^2$

B. $3e^{3\ln x}$

C. $(3\ln x)e^{3\ln x-1}$

D. $(3\ln x)e^{\ln x} \times \dfrac{1}{x}$

10 The graph of $f(x) = \sin x$ is transformed to the graph of $y = g(x)$ as shown below.



Which of the following describes the CORRECT order of transformations that have been applied to $f(x)$?

	Step 1	**Step 2**	**Step 3**
A.	A vertical dilation by a factor of 2.	A horizontal translation to the right by $\frac{\pi}{2}$.	A horizontal dilation by a factor of 2.
B.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal translation to the left by $\frac{\pi}{2}$.	A horizontal dilation by a factor of $\frac{1}{2}$.
C.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal dilation by a factor of 2.	A horizontal translation to the right by $\frac{\pi}{4}$.
D.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal dilation by a factor of $\frac{1}{2}$.	A horizontal translation to the left by $\frac{\pi}{2}$.

End of Section I

Student number

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11 – 35
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Differentiate $y = \dfrac{2x}{3-5x}$. **2**

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Question 12 (2 marks)

Given that $f'(x) = 3x^2 - 7$ and $f(3) = 5$, find $f(x)$. **2**

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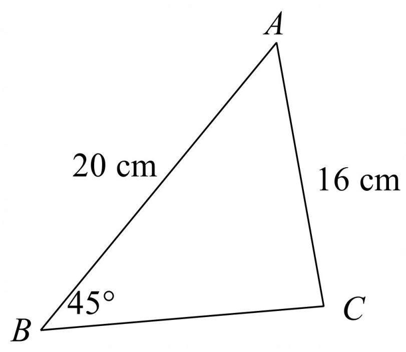
Question 13 (3 marks)

In triangle ABC, $AB = 20$ cm, $AC = 16$ cm and $\angle B = 45^\circ$. Find $\angle C$, correct to the nearest degree. **3**



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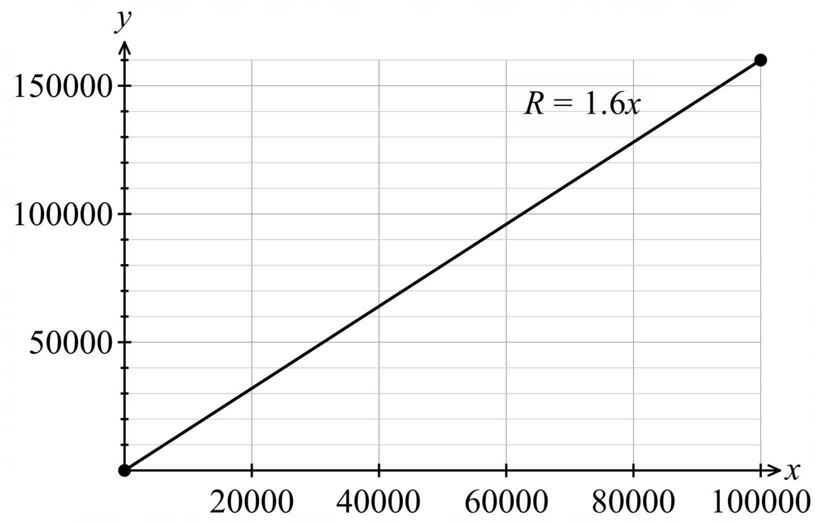
Question 14 (3 marks)

The NSGHS brick company manufactures bricks for the building industry. The production capacity for the year is 100 000 standard bricks. The cost of production is fixed at \$60 000 plus \$0.60 per brick. The selling price of each brick is \$1.60. The equation of the revenue (R) is shown on the graph below. The y-axis displays the cost/revenue in dollars and the x-axis represents the amount of bricks.



(a) Find the equation for the cost (C) of x bricks and plot this graph on the grid above. **2**

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(b) How many bricks must be sold to break even? **1**

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Question 15 (2 marks)

Find the gradient of the tangent to the curve $y = 3^{2x+1} \ln x$ at the point $(1, 0)$. **2**

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Question 16 (4 marks)

Find:

(a) $\int 4e^{-x}(e^{-3x} + 1)\, dx$ **2**

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(b) $\int x(x^2 + 4)^5\, dx$ **2**

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Question 17 (5 marks)

(a) Show that the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as $4\sin^2\theta - 15\sin\theta - 4 = 0$. **2**

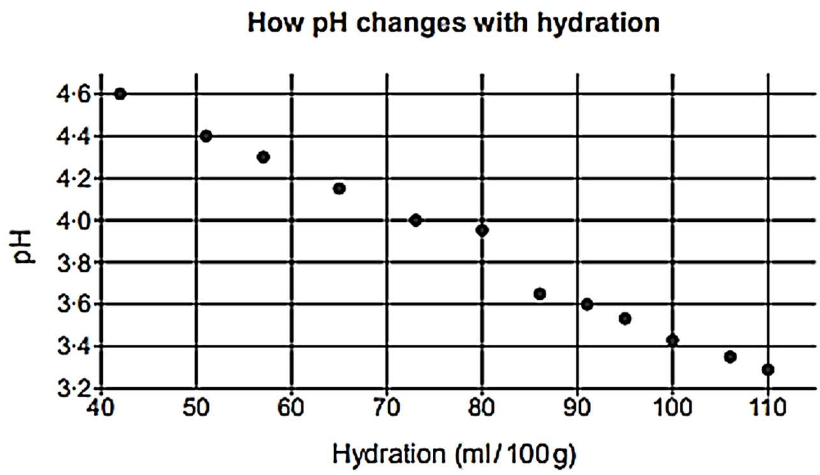
(b) Hence solve the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0 \le \theta \le 2\pi$. **3**

Question 18 (4 marks)

A baker researches how the pH of sourdough (y) changes with the hydration (x). Hydration is measured in mL of water per 100 g of flour (mL/100g).

The results of his research are shown in the table and diagram below:

Hydration	42	51	57	65	73	80	86	91	95	100	116	110
pH	4.60	4.40	4.10	4.18	4.00	3.98	3.65	3.60	3.55	3.42	3.37	3.10



(a) Describe the relationship between pH and hydration. **1**

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(b) Find the equation of the least-squares regression line using your calculator. Use this equation to estimate the pH of the sourdough when the hydration is 20 mL/100 g. Also comment on the reliability of this estimate. **3**

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Question 19 (4 marks)

The continuous random variable X has cumulative distribution function F given by

$$F(x)=\begin{cases} 0 & \text{for } x<1, \\ k(x^2-x) & \text{for } 1\le x\le 2, \\ 1 & \text{for } x>2. \end{cases}$$

where k is a constant.

(a) Show that $k=\dfrac{1}{2}$, and hence find the 90th percentile value of X, giving your answer correct to three significant figures. **3**

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(b) Find an expression for $f(x)$ which represents the probability density function of X. **1**

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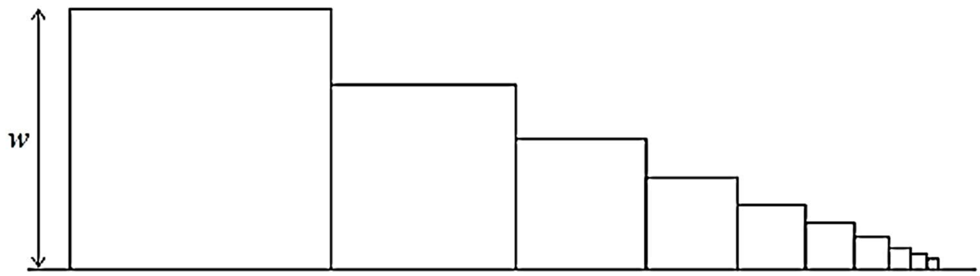
Question 20 (3 marks)

Helen is creating a mosaic pattern by placing square tiles next to each other in a straight line. Each subsequent tile is half the area of the previous one.



(a) Find, in terms of w, the length of the sides of the second largest tile. **1**

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(b) Assume the tiles are in contact with the adjacent tiles, but do not overlap. Show that, no matter how many tiles are in the pattern, the total length of the line of tiles will be less than $3.5w$. **2**

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Question 21 (4 marks)

The heights of female students at NSGHS are normally distributed with a mean of 168 cm and a standard deviation of 4.5 cm.

(a) Find the probability that the height of a randomly selected female student attending NSGHS is between 159 cm and 163.5 cm. **1**

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It is also known that the heights of male students at NSBHS are normally distributed with a mean of 170 cm and standard deviation of 6.5 cm.

(b) Wendy is a student at NSGHS and she is 173.5 cm tall. Caleb is a student at NSBHS and he is 176 cm tall. Who is the taller student relative to their peers in their own school? Justify your answer. **2**

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(c) What height would Caleb need to be in order to be comparable to Wendy's height in relation to her school peers? **1**

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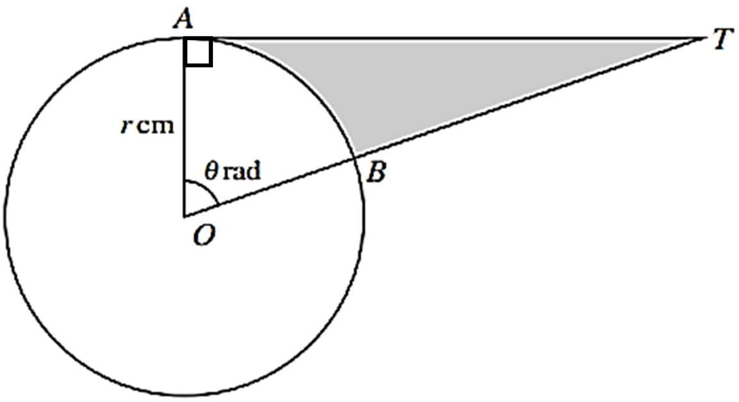
Question 22 (5 marks)

The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle.

OBT is a straight line and $\angle AOB = \theta$ radians.



(a) Express the area of the shaded region in terms of r and θ. **2**

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(b) Given that $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. Give your answer to 2 decimal places. **3**

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Question 23 (3 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \left(\tan^2 x + \sin 2x\right) dx$. **3**

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Question 24 (3 marks)

The discrete random variable *X* has the following probability distribution: **3**

x	1	2	3	4	5
$P(X=x)$	0.3	0.2	0.1	a	b

where *a* and *b* are constants.

Given that $E(X) = 2.85$, find the values of *a* and *b*.

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Questions 11–24 are worth 47 marks in total

Question 25 (2 marks)

The derivative of a function is defined as $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. **2**

Differentiate $f(x)=5x-x^2$ from first principles.

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Question 26 (2 marks)

Consider the function $f(x)=x^3-kx^2+3x+3$. For what values of k is $f(x)$ an increasing function? **2**

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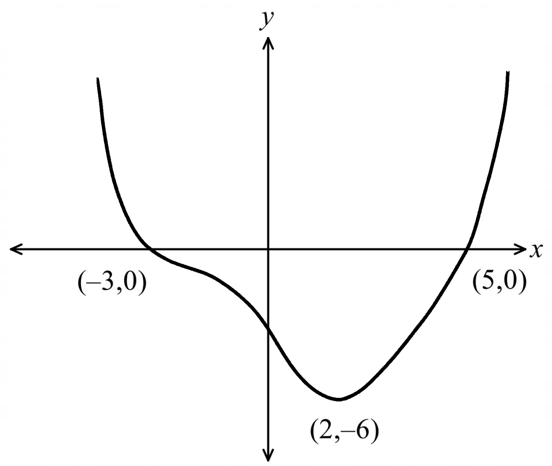
Question 27 (3 marks)

The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-3,0)$ and $(5,0)$, and has a minimum turning point at $(2,-6)$.



(a) The graph of $y = 4f(x+a)$ passes through the origin. Write down the possible values of a. **2**

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(b) The curve $y = f(x)$ is now transformed so that the y-coordinate of the stationary point on the graph of $y = bf(x+2)$ is 4. Write down the value of b. **1**

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Question 28 (5 marks)

Consider the curve $f(x) = 2x^3 - x^2 - 4x + 6$. Find the stationary points of $f(x)$ and determine their nature. Then sketch the curve, clearly labelling the stationary points. You do not need to find the x-intercepts of $f(x)$. **5**

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Question 29 (4 marks)

A teacher in college asks her mathematics students what other subjects they are studying. She finds that, of her 24 students, 12 study physics, 8 study geography and 4 study both.

(a) A student is chosen at random from the class. Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent. **2**

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(b) It is known that for the whole college: **2**

- The probability of a student studying mathematics is $\frac{1}{5}$
- The probability of a student studying biology is $\frac{1}{6}$
- The probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

Calculate the probability that in the whole college a student studies mathematics or biology or both.

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Question 30 (3 marks)

Consider the functions $f(x) = x + 1$ and $g(x) = \sqrt{1 - x^2}$.

(a) Find the value of $g(f(-0.5))$. **1**

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(b) Find the domain of $g(f(x))$. **2**

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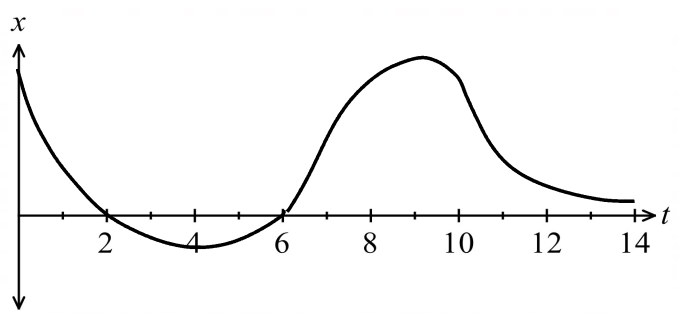
Question 31 (3 marks)

A particle P moves along a straight line. The graph of the particle's displacement $x(t)$ from the origin, is shown in the diagram below.



(a) At what time(s) is the particle at rest? **1**

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(b) What are the two times the acceleration of the particle is equal to 0? **2**

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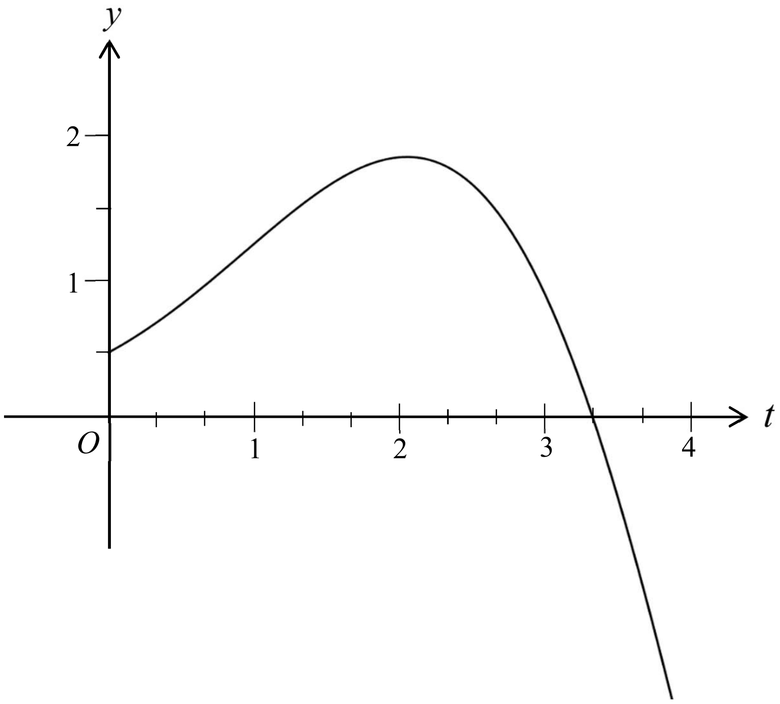
Question 32 (5 marks)

During a recent flood, the level of water in a river was measured at regular time intervals starting from midnight. The height *h* metres, by which the water level exceeded normal levels was recorded. The rate at which *h* increased at time *t*, is given by $R(t)$.

The graph of $y = R(t)$ is shown below.



(a) The equation of $R(t)$ is given by $R(t) = 0.5(1 + e^{0.5t}\sin t)$. Calculate the missing values in the table below to 1 decimal place. **1**

t	0	$\frac{5}{6}$	$\frac{5}{3}$	$\frac{5}{2}$	$\frac{10}{3}$
$R(t)$		1.1		1.5	

Question 32 continues on page 25

Question 32 (continued)

(b) The water level was 0.25 m higher than the normal levels at midnight, and the town needs to be evacuated if the water rises to 4 m above the normal levels. Use the trapezoidal rule with five function values to determine if the town needed to be evacuated. Show all working and provide reasons. **3**

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(c) If the water level returned to normal at time p, find the value of the definite integral **1**

$$\int_0^p R(t)\,dt\,.$$

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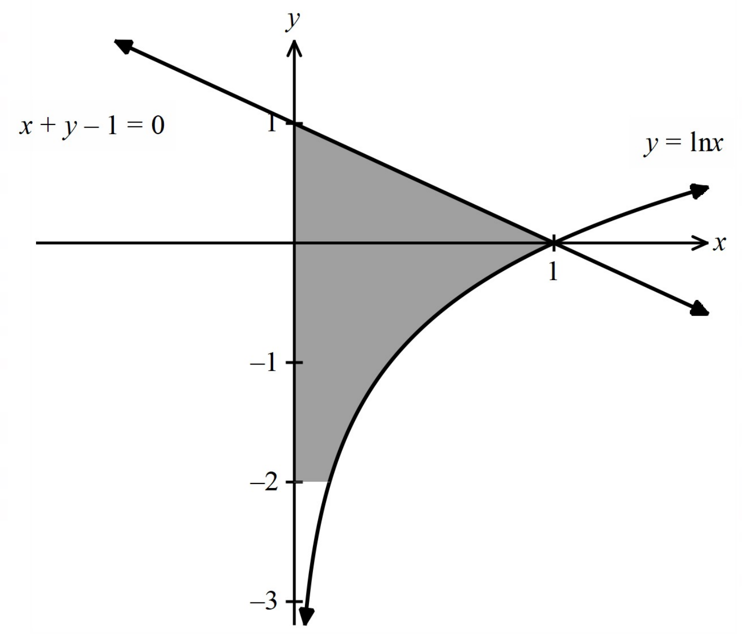
Question 33 (3 marks)

The curves $y = \ln x$ and $x + y - 1 = 0$ are shown in the diagram below. Find the exact shaded area bounded by the curves, the y-axis, and the line $y = -2$. **3**



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Question 34 (5 marks)

A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using $m = m_0 e^{-kt}$ where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

It takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

(a) The scientist drinks two strong cups of coffee at 8 am. By first finding the value of k, use the model to estimate the mass of caffeine in the scientist's body at midday to the nearest gram. **3**

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Question 34 continues on page 28

Question 34 (continued)

(b) The scientist wants the mass of caffeine in her body to stay below 480 mg. Use the model to find the earliest time that she could drink another cup of strong coffee. Give your answer to the nearest minute. **2**

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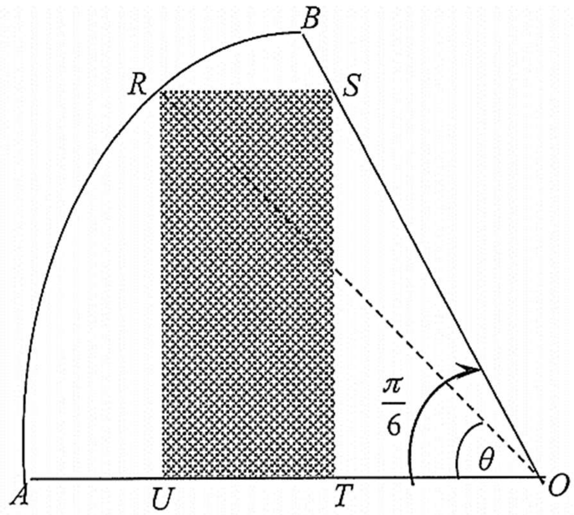
Question 35 (8 marks)

Consider the diagram below.



OAB is a sector of a circle with centre at *O* and radius *r* such that $\angle AOB = \frac{\pi}{6}$.

RSTU is a rectangle drawn inside the sector and $\angle ROA = \theta$ as shown in the diagram where $0 < \theta < \frac{\pi}{6}$.

Question 35 continues on page 29

Question 35 (continued)

(a) Show that $UT = r\cos\theta - \sqrt{3}r\sin\theta$. **3**

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Question 35 continues on page 30

Question 35 (continued)

(b) Show that the area of the rectangle can be expressed as $A = r^2\left(\sin\theta\cos\theta - \sqrt{3}\sin^2\theta\right)$. Hence find the value of θ which will maximise the area of the rectangle. **5**

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End of paper

North Sydney Girls High School

2020	**HSC TRIAL EXAMINATION**

Mathematics Advanced

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Total marks: 100

Section I – 10 marks (pages 2 – 6)

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Question	1 – 10	11 – 16	17 – 20	21 – 24	25 – 29	30 – 33	34 – 35	Total
Total	/10	/16	/16	/15	/16	/14	/13	/100

Section I

10 marks
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Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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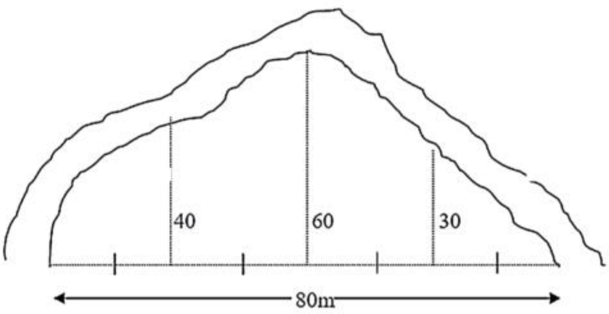

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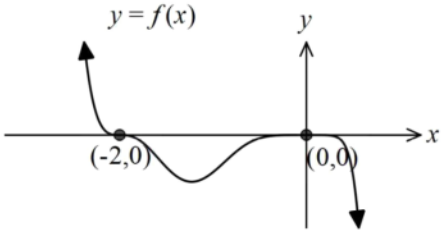
3 The following diagram shows the graph of $y = f(x)$.



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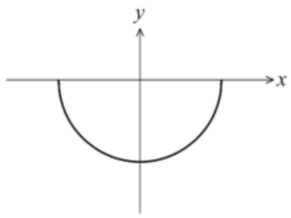
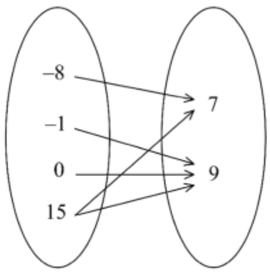
B. $(-2, \infty) \cup (0, 0)$

C. $(-2, \infty) \cup [0, 0]$

D. $(-\infty, -2] \cup [0, 0]$

4 Which of the following represents a many-to-one relation?

I	II	III
	$\{(-2,5), (4,5), (5,-2), (5,1)\}$	

A. I only

B. I and II only

C. I and III only

D. I , II and III

5 If 4 is added to each score in a set, which one of the following statements will be true?

A. The mean and standard deviation will remain the same.

B. The mean will increase by 4 and the standard deviation will remain the same.

C. The mean will increase by 4 and the standard deviation will increase by 2.

D. The mean will increase by 4 and the standard deviation will increase by 4.

6 James plays a game involving the tossing of two coins. One turn at this game costs $1. The possible outcomes are listed below along with their payoffs:

- 2 Heads pays $5
- 1 Head and 1 Tail pays $2
- 2 Tails pays nothing

What is the financial expectation for this game?

A. $1.25

B. –$1.00

C. $2.00

D. $2.25

7 The maximum speed of a train going up a hill is inversely proportional to the square root of its weight. A train weighing 3600 tonnes can go up a hill at 30 km/h. What is the maximum speed at which a train weighing 2500 tonnes could go up the same hill?

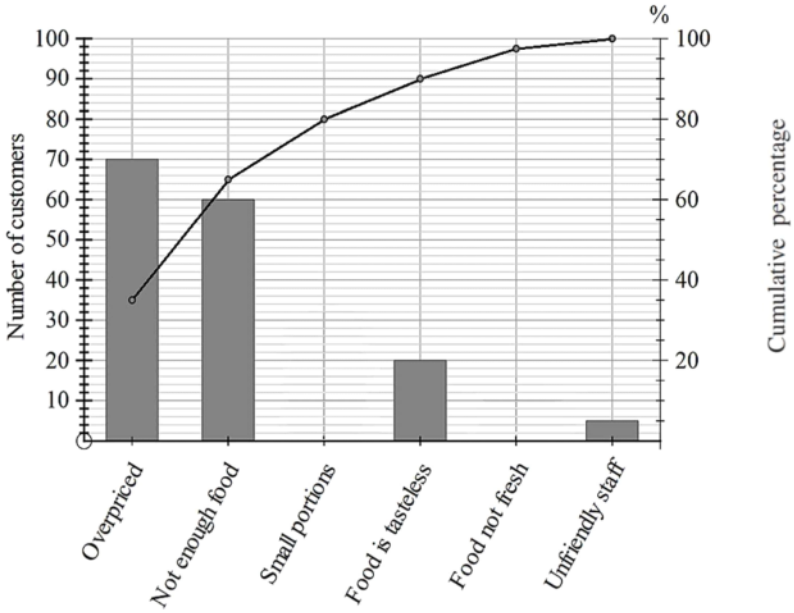
A. 43.2 km/h

B. 36 km/h

C. 25 km/h

D. 20.8 km/h

8 The following Pareto chart shows 200 customer complaints from a restaurant survey. The columns for 'small portions' and 'food not fresh' are missing.



How many customers complained about the restaurant having 'small portions'?

A. 15 people

B. 30 people

C. 40 people

D. 80 people

9 What is the derivative of $e^{3\ln x}$?

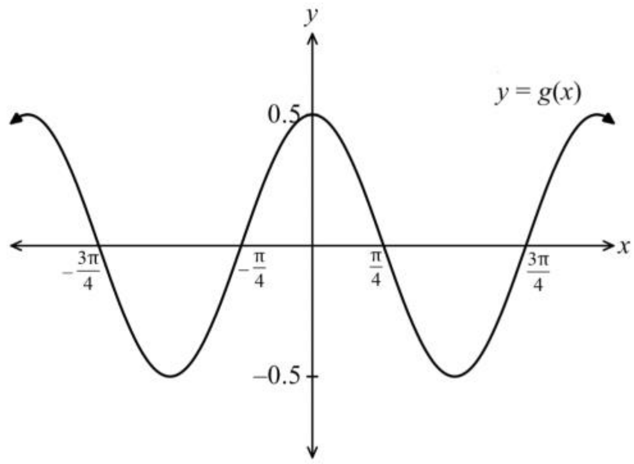
A. $3x^2$

B. $3e^{3\ln x}$

C. $(3\ln x)e^{3\ln x-1}$

D. $(3\ln x)e^{\ln x}\times\frac{1}{x}$

10 The graph of $f(x) = \sin x$ is transformed to the graph of $y = g(x)$ as shown below.



Which of the following describes the CORRECT order of transformations that have been applied to $f(x)$?

	Step 1	Step 2	Step 3
A.	A vertical dilation by a factor of 2.	A horizontal translation to the right by $\frac{\pi}{2}$.	A horizontal dilation by a factor of 2.
B.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal translation to the left by $\frac{\pi}{2}$.	A horizontal dilation by a factor of $\frac{1}{2}$.
C.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal dilation by a factor of 2.	A horizontal translation to the right by $\frac{\pi}{4}$.
D.	A vertical dilation by a factor of $\frac{1}{2}$.	A horizontal dilation by a factor of $\frac{1}{2}$.	A horizontal translation to the left by $\frac{\pi}{2}$.

End of Section I

2020 | HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Student number

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11 – 35
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Differentiate $y = \dfrac{2x}{3-5x}$. **2**

$$\frac{dy}{dx} = \frac{2(3-5x) - 2x(-5)}{(3-5x)^2}$$

$$= \frac{6 - 10x + 10x}{(3-5x)^2}$$

$$= \frac{6}{(3-5x)^2}$$

Very well done. A few students reversed the order in the numerator. If in doubt, look at the reference sheet.

Question 12 (2 marks)

Given that $f'(x) = 3x^2 - 7$ and $f(3) = 5$, find $f(x)$. **2**

$$f(x) = \frac{3x^3}{3} - 7x + C$$

$$= x^3 - 7x + C$$

$$5 = 3^3 - 7 \times 3 + C$$

$$5 = 27 - 21 + C$$

$$5 = 6 + C$$

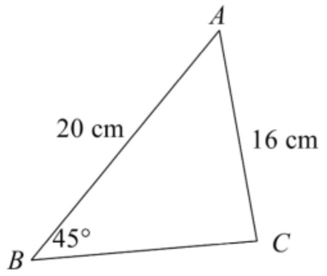
$$C = -1$$

$$\therefore f(x) = x^3 - 7x - 1$$

Very well done. Only silly errors caused marks to be lost.

Question 13 (3 marks)

In triangle ABC, $AB = 20$ cm, $AC = 16$ cm and $\angle B = 45^\circ$. Find $\angle C$, correct to the nearest degree. **3**



NOT TO SCALE

$\frac{\sin C}{20} = \frac{\sin 45}{16}$

$\sin C = \frac{\sqrt{2}}{32} \times 20$

$= \frac{5\sqrt{2}}{8}$

$\therefore \angle C = \sin^{-1} \frac{5\sqrt{2}}{8}$

$= 62^\circ 7'$ or $180^\circ - 62^\circ 7' = 117^\circ 53'$

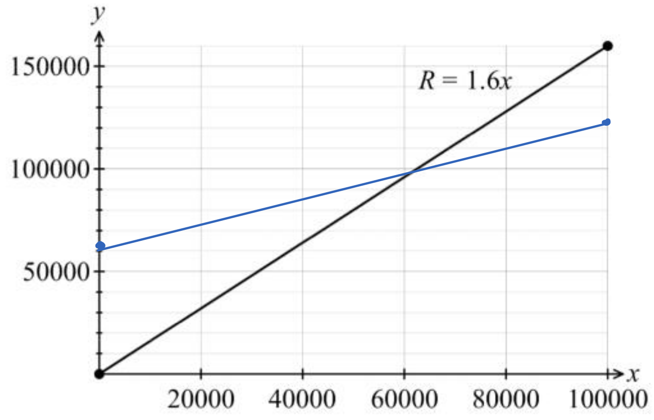
$= 62^\circ$ or 118° (nearest degree)

The majority of students did not find the second angle i.e. $180^\circ - 62^\circ = 118^\circ$.

Please turn over

Question 14 (3 marks)

The NSGHS brick company manufactures bricks for the building industry. The production capacity for the year is 100 000 standard bricks. The cost of production is fixed at $60 000 plus $0.60 per brick. The selling price of each brick is $1.60. The equation of the revenue (R) is shown on the graph below. The y-axis displays the cost/revenue in dollars and the x-axis represents the amount of bricks.



(a) Find the equation for the cost (C) of x bricks and plot this graph on the grid above. **2**

$C = 60000 + 0 \cdot 6x$

Very well done.

(b) How many bricks must be sold to break even? **1**

$60000 + 0 \cdot 6x = 1 \cdot 6x$

$x = 60000$

$\therefore$ 60000 bricks must be sold to break even

Very well done.

Question 15 (2 marks)

Find the gradient of the tangent to the curve $y = 3^{2x+1}\ln x$ at the point $(1,0)$. **2**

$\frac{dy}{dx} = (2\ln 3)(3^{2x+1})\ln x + 3^{2x+1} \times \frac{1}{x}$

When $x = 1$,

$\frac{dy}{dx} = (2\ln 3)(3^3)\ln 1 + 3^3 \times \frac{1}{1}$

$= 27$

There were quite a few errors in this differentiation. Students need to practise $\int a^{f(x)}\,dx$ type of questions making sure to use the reference sheet. Some students changed 3^{2x+1} into $e^{(2x+1)\ln 3}$ and mostly had success.

Question 16 (4 marks)

Find:

(a) $\int 4e^{-x}(e^{-3x}+1)\,dx$ **2**

$= \int 4e^{-4x} + 4e^{-x}\,dx$

$= -e^{-4x} - 4e^{-x} + C$

Students needed to expand the brackets before integrating. As long as they did this they had a good chance of success.

(b) $\int x(x^2+4)^5\,dx$ **2**

$= \frac{1}{2}\int 2x\,(x^2+4)^5\,dx$

$= \frac{(x^2+4)^6}{12} + c$

Students needed to recognise that $\int x(x^2+4)^5\,dx = \frac{1}{2}\int 2x(x^2+4)^5\,dx$.

Those that did generally scored 2 marks. Some students attempted a sort of "product rule integration".

Question 17 (5 marks)

(a) Show that the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as $4\sin^2\theta - 15\sin\theta - 4 = 0$. **2**

$\frac{4\cos\theta}{\tan\theta} + 15 = 0$

$4\cos\theta \times \frac{\cos\theta}{\sin\theta} + 15 = 0$

$4\cos^2\theta + 15\sin\theta = 0$

$4(1-\sin^2\theta) + 15\sin\theta = 0$

$4 - 4\sin^2\theta + 15\sin\theta = 0$

$4\sin^2\theta - 15\sin\theta - 4 = 0$

Very well done.

(b) Hence solve the equation $\frac{4\cos\theta}{\tan\theta}+15=0$ for $0\le\theta\le 2\pi$. **3**

$\frac{4\cos\theta}{\tan\theta}+15=0$

$\therefore 4\sin^2\theta-15\sin\theta-4=0$

$4\sin^2\theta-16\sin\theta+\sin\theta-4=0$

$4\sin\theta(\sin\theta-4)+(\sin\theta-4)=0$

$(\sin\theta-4)(4\sin\theta+1)=0$

$\sin\theta-4=0$, $4\sin\theta+1=0$

$\sin\theta=4$ $4\sin\theta=-1$

no solution $\sin\theta=-\frac{1}{4}$

$\theta=3.394\ldots, 6.031\ldots$

$=3.34, 6.03$ (2dp)

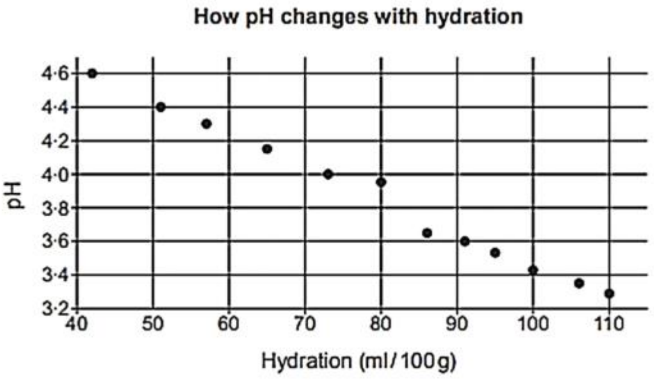
Quite well done. Some students answered in degrees.

Question 18 (4 marks)

A baker researches how the pH of sourdough (y) changes with the hydration (x). Hydration is measured in mL of water per 100 g of flour (mL/100g).

The results of his research are shown in the table and diagram below:

Hydration	42	51	57	65	73	80	86	91	95	100	116	110
pH	4.60	4.40	4.10	4.18	4.00	3.98	3.65	3.60	3.55	3.42	3.37	3.10



(a) Describe the relationship between pH and hydration. **1**

There is a strong negative linear correlation between pH and hydration.

Looked for 2 of the three descriptors "linear", "negative", "strong".

(b) Find the equation of the least-squares regression line using your calculator. Use this equation to estimate the pH of the sourdough when the hydration is 20 mL/100 g. Also comment on the reliability of this estimate. **3**

$y = 5.328 - 0.0186x$

$x = 20, \quad y = 5.328 - 0.0186 \times 20$

$= 4.956$

This estimate is not reliable as we are extrapolating outside of the range of data given.

Calculations generally well done. The comment was only paid if it made mention of the hydration level being outside the range of data given i.e. extrapolation.

Question 19 (4 marks)

The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & \text{for } x < 1, \\ k(x^2 - x) & \text{for } 1 \le x \le 2, \\ 1 & \text{for } x > 2. \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{2}$, and hence find the 90th percentile value of X, giving your answer correct to three significant figures. **3**

$F(2) = 1$

$\therefore\ 1 = k(2^2 - 2)$

$1 = k \times 2$

$k = \frac{1}{2}$

$0.9 = \frac{1}{2}(x^2 - x)$

$1.8 = x^2 - x$

$0 = x^2 - x - 1.8$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1.8)}}{2}$

$= \dfrac{1 \pm \sqrt{8.2}}{2}$

$= 1.932,\ -0.932$

Since $1 \le x \le 2$ $\quad x = 1.932$ (3dp)

(b) Find an expression for $f(x)$ which represents the probability density function of X. **1**

$f(x) = F'(x) = \frac{1}{2}(2x - 1)$

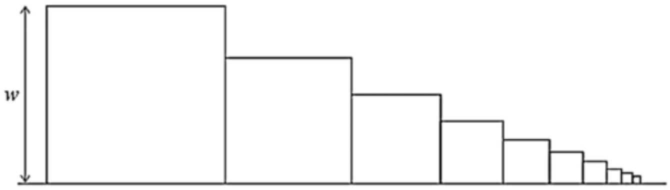
$= x - \frac{1}{2}$

$\therefore f(x) = \begin{cases} x - \frac{1}{2} & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

No constructive comments. The question tended to be done either very well or very poorly.

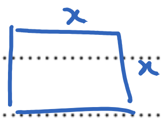
Question 20 (3 marks)

Helen is creating a mosaic pattern by placing square tiles next to each other in a straight line. Each subsequent tile is half the area of the previous one.



(a) Find, in terms of w, the length of the sides of the second largest tile. **1**

$A_1 = w^2$



$A_2 = \frac{1}{2}w^2$

$x^2 = \frac{1}{2}w^2$

$x = \frac{1}{\sqrt{2}}w \quad (x>0)$

Well done.

(b) Assume the tiles are in contact with the adjacent tiles, but do not overlap. Show that, no matter how many tiles are in the pattern, the total length of the line of tiles will be less than $3.5w$. **2**

$A_3 = \frac{1}{4}w^2$ ∴ side length of square is $\frac{1}{2}w$

$A_4 = \frac{1}{8}w^2$ ∴ side length of square is $\frac{1}{2\sqrt{2}}w$

∴ side lengths of the squares form a geometric sequence with common ratio $r = \frac{1}{\sqrt{2}} = 0.707\ldots$

Since $|r| < 1$ a limiting sum exists

$S_\infty = \frac{a}{1-r}$

$= \frac{w}{1-\frac{1}{\sqrt{2}}}$

$= \frac{1}{1-\frac{1}{\sqrt{2}}} w$

$= 3.414w$ (3dp)

∴ no matter how many tiles are in the pattern the total length will not exceed $3.5w$

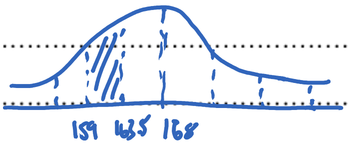
Generally well done. Some students tried to sum areas rather than sum length of sides.

Question 21 (4 marks)

The heights of female students at NSGHS are normally distributed with a mean of 168 cm and a standard deviation of 4.5 cm.

(a) Find the probability that the height of a randomly selected female student attending NSGHS is between 159 cm and 163.5 cm. **1**

$\frac{1}{2}(95\% - 68\%) = 13.5\%$



It is also known that the heights of male students at NSBHS are normally distributed with a mean of 170 cm and standard deviation of 6.5 cm.

(b) Wendy is a student at NSGHS and she is 173.5 cm tall. Caleb is a student at NSBHS and he is 176 cm tall. Who is the taller student relative to their peers in their own school? Justify your answer. **2**

Wendy: $z = \frac{173.5 - 168}{4.5} = \frac{11}{9} = 1.2\ldots$

Caleb: $z = \frac{176 - 170}{6.5} = \frac{12}{13} = 0.92\ldots$

Since Wendy has the higher z-score, she is more standard deviations above the mean compared to Caleb so she is the taller student relative to her peers.

(c) What height would Caleb need to be in order to be comparable to Wendy's height in relation to her school peers? **1**

Caleb would need a z-score of $\frac{11}{9}$

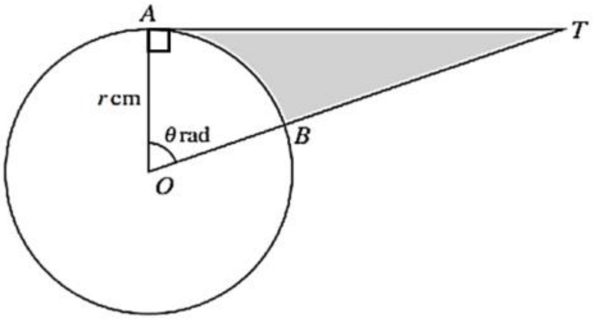
$170 + \frac{11}{9} \times 6.5 = \frac{3203}{18} = 177.94 \approx 178$ cm

Generally, most students did well.

Question 22 (5 marks)

The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle.

OBT is a straight line and $\angle AOB = \theta$ radians.



(a) Express the area of the shaded region in terms of r and θ. **2**

$\tan\theta = \frac{AT}{r}$ $\therefore AT = r\tan\theta$

Shaded area $=$ area$_{\Delta OAT}$ $-$ sector$_{OAT}$

$= \frac{1}{2} \times r \times r\tan\theta - \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2\tan\theta - \frac{1}{2}r^2\theta$

Some students did not know the formula that could be used to find arc length $l = r\theta$.

(b) Given that $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. Give your answer to 2 decimal places. **3**

Perimeter = AT + BT + arc AB

= 3 tan 1.2 + (OT − OB) + 3×1.2

= 3 tan 1.2 + ($\frac{3}{\cos 1.2}$ − 3) + 3×1.2 $\cos\theta = \frac{r}{OT}$

= 16.5955... $OT = \frac{3}{\cos 1.2}$

= 16.60 cm

Some students did not read the question carefully.

Question 23 (3 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \left(\tan^2 x + \sin 2x\right) dx$. **3**

$= \int_0^{\frac{\pi}{4}} \sec^2 x - 1 + \sin 2x \, dx$

$= \left[\tan x - x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}}$

$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4} - \frac{1}{2}\cos\frac{\pi}{2}\right) - \left(\tan 0 - 0 - \frac{1}{2}\cos 0\right)$

$= \left(1 - \frac{\pi}{4} - 0\right) - \left(0 - 0 - \frac{1}{2}\right)$

$= \frac{3}{2} - \frac{\pi}{4}$

Some students could not do $\int \tan^2 x \, dx$. Some students got the wrong arrangement from $\tan^2 x = \sec^2 x - 1$.

Question 24 (3 marks)

The discrete random variable X has the following probability distribution: **3**

x	1	2	3	4	5
$P(X = x)$	0.3	0.2	0.1	a	b

where a and b are constants.

Given that $E(X) = 2.85$, find the values of a and b.

$\Sigma p(x) = 1$

$\therefore 0.3 + 0.2 + 0.1 + a + b = 1$

$a + b = 0.4$ — ①

$E(X) = 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 + 4a + 5b$

$2.85 = 1 + 4a + 5b$

$4a + 5b = 1.85$ — ②

① ×4 $4a + 4b = 1.6$ — ③

② − ③ $b = 0.25$

$\therefore a = 0.15$

Generally, most students did well.

Questions 11–24 are worth 47 marks in total

$$f'(x) = \lim_{h \to 0} \frac{[5(x+h) - (x+h)^2] - [5x - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - x^2 - 2xh - h^2 - 5x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{5h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} 5 - 2x - h$$

$$= 5 - 2x$$

$f'(x) = 3x^2 - 2kx + 3$

For $f(x)$ to be increasing, $f'(x) > 0$

since $3x^2 - 2kx + 3$ is concave up,

$\Delta < 0$

$(-2k)^2 - 4 \times 3 \times 3 < 0$

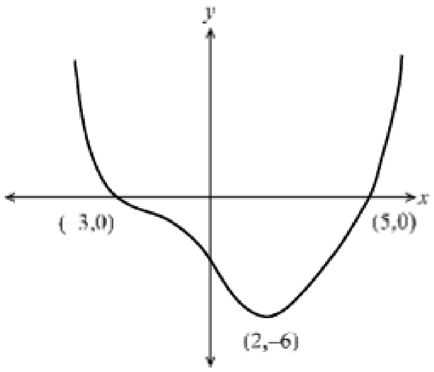
$4k^2 < 36$

$k^2 < 9$

$\therefore -3 < k < 3$

Question 27 (3 marks)

The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-3,0)$ and $(5,0)$, and has a minimum turning point at $(2,-6)$.



(a) The graph of $y = 4f(x+a)$ passes through the origin. Write down the possible values of a. **2**

$a = -3 , 5$

Done very well. The common mistake was getting the signs of a wrong.

(b) The curve $y = f(x)$ is now transformed so that the y-coordinate of the stationary point on the graph of $y = bf(x+2)$ is 4. Write down the value of b. **1**

$b \times -6 = 4$

$b = -\frac{2}{3}$

Done well.

Question 28 (5 marks)

Consider the curve $f(x)=2x^3-x^2-4x+6$. Find the stationary points of $f(x)$ and determine their nature. Then sketch the curve, clearly labelling the stationary points. You do not need to find the x-intercepts of $f(x)$. **5**

$f(x) = 2x^3 - x^2 - 4x + 6$

$f'(x) = 6x^2 - 2x - 4$

$= 2(3x^2 - x - 2)$

stationary points occur when $f'(x) = 0$

$0 = 2(3x+2)(x-1)$

$\therefore x = -\frac{2}{3}, 1$

$f''(x) = 12x - 2$

$f''(-\frac{2}{3}) = 12(-\frac{2}{3}) - 2$

$= -8 - 2$

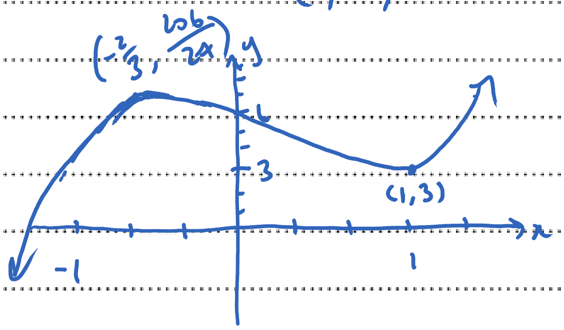
$= -10$

$f''(-\frac{2}{3}) < 0 \therefore$ there is a maximum turning point at $(-\frac{2}{3}, \frac{206}{27})$

$f''(1) = 12 \times 1 - 2$

$= 10$

$f''(1) > 0 \therefore$ there is a minimum turning point at $(1, 3)$



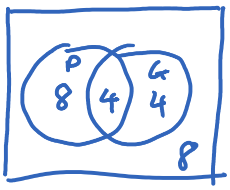
This was done very well. Almost all students were able to correctly determine the stationary points and sketch the curve. Students are reminded that they should have a consistent scale along each axis. Some students lost marks for this. Students should also read the question and be aware of what the question is asking them to do. Many students wasted time finding the point of inflexion even though the question did not ask for it.

Question 29 (4 marks)

A teacher in college asks her mathematics students what other subjects they are studying. She finds that, of her 24 students, 12 study physics, 8 study geography and 4 study both.

(a) A student is chosen at random from the class. Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent. **2**

$P(P \cap G) = P(P) \times P(G)$



$LHS = \frac{4}{24} = \frac{1}{6}$

$RHS = \frac{12}{24} \times \frac{8}{24}$

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

$= LHS \therefore$ independent

OR

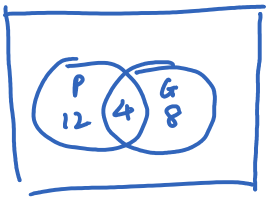
$LHS = \frac{4}{24} = \frac{1}{6}$

$RHS = \frac{16}{24} \times \frac{12}{24}$

$= \frac{2}{3} \times \frac{1}{2}$

$= \frac{1}{3}$

$\neq LHS \therefore$ not independent



This was not done very well. Many students need to go back and revise the meaning of mathematical independence as they did not know the formula even though it is on the reference sheet. The number of students doing each subject in this question was ambiguous so both interpretations were marked correct i.e. 12 physics, 8 geography, 4 both, 8 neither OR 16 physics, 12 geography, 4 both. Many students drew a Venn diagram in their working out and their probabilities needed to match the numbers in their Venn diagram to get the marks.

(b) It is known that for the whole college: **2**

- The probability of a student studying mathematics is $\frac{1}{5}$
- The probability of a student studying biology is $\frac{1}{6}$
- The probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

Calculate the probability that in the whole college a student studies mathematics or biology or both.

$P(M) = \frac{1}{5}$, $P(B) = \frac{1}{6}$, $P(B|M) = \frac{3}{8}$ $P(B \cup M) = ?$

$$P(B|M) = \frac{P(B \cap M)}{P(M)}$$

$$\frac{3}{8} = \frac{P(B \cap M)}{\frac{1}{5}}$$

$$P(B \cap M) = \frac{3}{40}$$

$$P(B \cup M) = P(M) + P(B) - P(B \cap M)$$

$$= \frac{1}{5} + \frac{1}{6} - \frac{3}{40}$$

$$= \frac{7}{24}$$

Overall done very well. Students were able to identify and use the conditional probability formula.

Question 30 (3 marks)

Consider the functions $f(x) = x + 1$ and $g(x) = \sqrt{1 - x^2}$.

(a) Find the value of $g(f(-0.5))$. **1**

$f(-0.5) = -0.5 + 1 = 0.5$

$g(0.5) = \sqrt{1 - 0.5^2} = \sqrt{\frac{3}{4}} =$

Well done.

(b) Find the domain of $g(f(x))$. **2**

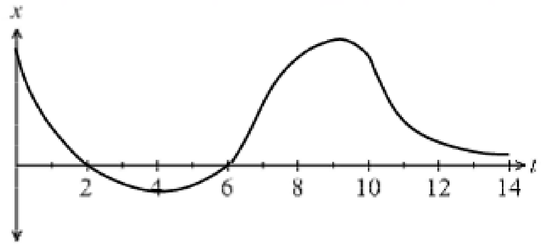
$-1 \leq x + 1 \leq 1$

$-2 \leq x \leq 0$

Generally well done. Most common mistake was finding the domain of both functions separately and not the combined function.

Question 31 (3 marks)

A particle P moves along a straight line. The graph of the particle's displacement $x(t)$ from the origin, is shown in the diagram below.



(a) At what time(s) is the particle at rest? **1**

$t=4$, $t=9$

(b) What are the two times the acceleration of the particle is equal to 0? **2**

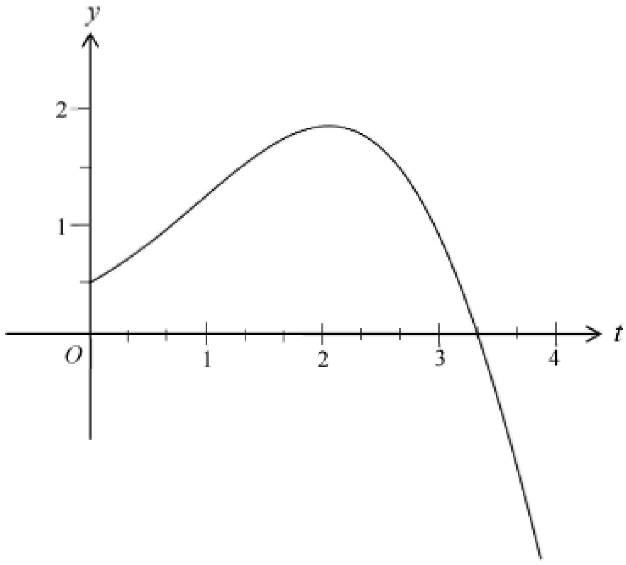
$t=6\cdot5$, $t=10\cdot5$

Generally well done. Some students found the values where $x=0$ for (a) and then the turning points for (b).

Question 32 (5 marks)

During a recent flood, the level of water in a river was measured at regular time intervals starting from midnight. The height h metres, by which the water level exceeded normal levels was recorded. The rate at which h increased at time t, is given by $R(t)$.

The graph of $y = R(t)$ is shown below.



(a) The equation of $R(t)$ is given by $R(t) = 0.5(1 + e^{0.5t}\sin t)$. Calculate the missing values in the table below to 1 decimal place. **1**

t	0	$\frac{5}{6}$	$\frac{5}{3}$	$\frac{5}{2}$	$\frac{10}{3}$
$R(t)$	0.5	1.1	1.6	1.5	0

Some students may wish to go back and revise the meaning of 1 decimal place.

Question 32 continues on page 25

Question 32 (continued)

(b) The water level was 0.25 m higher than the normal levels at midnight, and the town needs to be evacuated if the water rises to 4 m above the normal levels. Use the trapezoidal rule with five function values to determine if the town needed to be evacuated. Show all working and provide reasons. **3**

$A = \frac{\frac{5}{6}}{2}\left[0.5 + 2(1.1 + 1.6 + 1.5) + 0\right]$

$= \frac{89}{24}$

$= 3.71 \text{ (2dp)}$

∴ the total increase in the height of the water from midnight is 3.71 m

∴ peak water level = 3.71 + 0.25

= 3.96 m

Since this is less than 4m, the town will not have to be evacuated.

Two most common mistakes were 1 - inputting $n = 5$, rather than $n = 4$ into the formula, and 2 - not taking into account the initial value of 0.25m.

(c) If the water level returned to normal at time p, find the value of the definite integral **1**

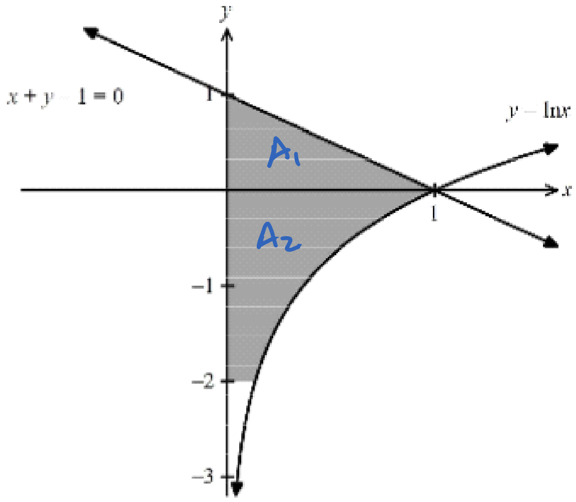
$$\int_0^p R(t)\, dt .$$

-0.25

Was tricky, only about 6 people got it. Some people put 0.25, without the minus. Some people said zero.

Question 33 (3 marks)

The curves $y = \ln x$ and $x + y - 1 = 0$ are shown in the diagram below. Find the exact shaded area bounded by the curves, the y-axis, and the line $y = -2$. **3**



$A_1 = \frac{1}{2} \times 1 \times 1$

$= \frac{1}{2}$

$A_2 = \int_{-2}^{0} e^y \, dy$ $\quad y = \ln x$, $e^y = x$

$= \left[e^y \right]_{-2}^{0}$

$= e^0 - e^{-2}$

$= 1 - \frac{1}{e^2}$

$\therefore$ shaded area $= \frac{1}{2} + 1 - \frac{1}{e^2}$

$= \frac{3}{2} - \frac{1}{e^2} \ u^2$

Most people got 3 easy marks. The most common mistake was not treating it as a piecewise function and attempting to integrate the whole range, either 0 to 1 *dx*, or −2 to 1 *dy*.

Question 34 (5 marks)

A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using $m = m_0 e^{-kt}$ where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

It takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

(a) The scientist drinks two strong cups of coffee at 8 am. By first finding the value of k, use the model to estimate the mass of caffeine in the scientist's body at midday to the nearest gram. **3**

$m = m_0 e^{-kt}$

$t = 5.7$

$m = \frac{m_0}{2}$

$\frac{1}{2} = e^{-k \times 5.7}$

$\ln\frac{1}{2} = -k \times 5.7$

$-\ln 2 = -k \times 5.7$

$k = \frac{\ln 2}{5.7}$

$m = 400e^{-k \times 4}$

$= 245.929...$

$= 246$ (nearest mg)

Well done here with most students scoring 3/3. Best answers used logs to simplify value of k to a manageable form for part (b).

Question 34 continues on page 28

Question 34 (continued)

(b) The scientist wants the mass of caffeine in her body to stay below 480 mg. Use the model to find the earliest time that she could drink another cup of strong coffee. Give your answer to the nearest minute. **2**

She needs to have 480 - 200 = 280 mg of caffeine in her body.

$280 = 400e^{-kt}$

$\frac{280}{400} = e^{-kt}$

$\ln\frac{280}{400} = -kt$

$t = -\frac{1}{k}\ln\frac{280}{400}$

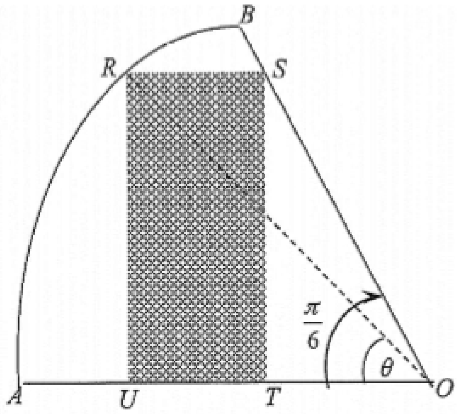
$= 2h\ 56'$

earliest time is 10:56 am.

The first part of this question was crucial, and a mark was awarded for identifying that $m = 280$. Many had $m = 480$ and this will yield negative time. A mark was allowed if the student did not fudge their calculation using $m = 480$, showing a negative time as this is a correct calculation from error. Note the function is modelling a decreasing function, hence only applicable when $m_0 < m$.

Question 35 (8 marks)

Consider the diagram below.



OAB is a sector of a circle with centre at O and radius r such that $\angle AOB = \frac{\pi}{6}$.

$RSTU$ is a rectangle drawn inside the sector and $\angle ROA = \theta$ as shown in the diagram where $0 < \theta < \frac{\pi}{6}$.

Question 35 continues on page 29

Question 35 (continued)

(a) Show that $UT = r\cos\theta - \sqrt{3}r\sin\theta$. **3**

UT = UO − OT

In ΔURO:

cosθ = UO/OR   sinθ = UR/OR

UO = OR cosθ   UR = OR sinθ

= r cosθ   = r sinθ

In ΔOTS

tan π/6 = ST/OT

OT = ST ÷ 1/√3

= √3 ST

But $ST = UR$ (opposite sides of a rectangle)

$DT = \sqrt{3}\, UR$

$= \sqrt{3}\, r\sin\theta$

$\therefore UT = r\cos\theta - \sqrt{3}\, r\sin\theta$

The logical steps and setting out for this question need attention. Students must give reasons and write a logical progression (stepwise) in their responses. Next time, reference the triangles that the trig ratios are relating to. Write down only the facts that relate to the logical progression of showing why the statement is true. Not stating RU = ST (sides of rectangle) was another issue but was not penalised so long as the solution was stepwise and logical. This type of question needs practice over the coming weeks so that marks are not thrown away on lack of diligence or attention to detail, in the HSC.

Question 35 (continued)

(b) Show that the area of the rectangle can be expressed as $A = r^2\left(\sin\theta\cos\theta - \sqrt{3}\sin^2\theta\right)$. Hence find the value of θ which will maximise the area of the rectangle. **5**

$\frac{dA}{d\theta} = r^2\left(\cos^2\theta - \sin^2\theta - 2\sqrt{3}\sin\theta\cos\theta\right)$

$0 = \cos^2\theta - \sin^2\theta - 2\sqrt{3}\sin\theta\cos\theta$

$0 = 1 - \tan^2\theta - 2\sqrt{3}\tan\theta$

$\tan^2\theta + 2\sqrt{3}\tan\theta - 1 = 0$

$\tan\theta = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4\times1\times-1}}{2}$

$= \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$= \frac{-2\sqrt{3} \pm 4}{2}$

$= -\sqrt{3} \pm 2$

$0 < \theta < \frac{\pi}{6}$ $\therefore \tan\theta = -\sqrt{3} + 2$ $(\tan\theta > 0)$

$\theta = 15^\circ$

θ	14°	15°	16°
A'	$0.07r^2$	0	$-0.07r^2$

Since $r^2 > 0$, $0.07r^2 > 0$ & $-0.07r^2 < 0$

∴ the maximum area occurs when $\theta = 15°$

Many students either forgot, missed or didn't know how to show the area of the rectangle expression and so missed out on the initial mark. If an error was made with the derivative, further marks relied on whether the error made the subsequent trig equation easier to solve or not. Dividing by $\cos^2\theta$ was crucial if solving for θ and this stopped many students from continuing the question. Also, the restriction of $\theta \leq \frac{\pi}{6}$ was not clearly stated in some responses in determining the solution. Showing that it's a maximum was generally well done.