Student Name: Teacher Name:

Penrith High School

Mathematics Department

Trial Higher School Certificate

2006
Mathematics 2U

Time Allowed: 3 hours
Reading Time: 5 minutes

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Start each section on a new page.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.
- This paper contains **11** sections in **8** pages.
- The page of **Standard Integrals** may be detached for your convenience.

Section	1	2	3	4	5	6	7	8	9	10	1
Mark	/12	/12	/6	/6	/6	/6	/24	/12	/12	/12	/1

Section 1. **Basic Arithmetic and Algebra** **Marks**

1. Express in scientific notation: $\dfrac{1\cdot 3\times 10^{7}}{5\times 10^{-2}}$. **(1)**

2. Evaluate $\dfrac{\log_2 32}{\log_2 4}$. **(1)**

3. Simplify $5\sqrt{2}+\sqrt{45}-\sqrt{32}+4\sqrt{5}$ **(2)**

4. Simplify $\dfrac{3x^2-6x-9}{x^2-1}$. **(2)**

5. Rationalise the denominator $\dfrac{7}{3+\sqrt{7}}$. **(2)**

6. Solve the equation: $\dfrac{x}{3}-\dfrac{2x-1}{5}=2$. **(2)**

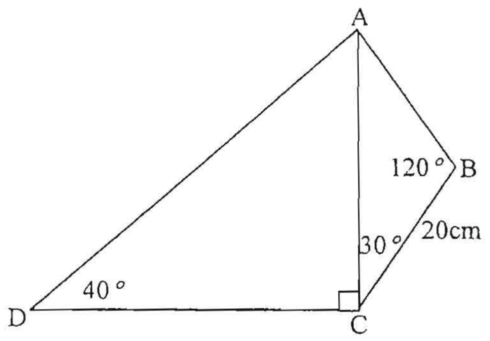
7. Find the value(s) of x for which $|4-3x|=5$. **(2)**

Section 2. Trigonometry *(Start this question a new page)*

8. Find the exact value of $\operatorname{cosec}(-60°)\sin 240°\cos 150°$ **(2)**

9. Given $90°\le\theta\le180°$ and $\sin\theta=\dfrac{8}{17}$, find the exact value of $\cos\theta$. **(2)**

10. $BADC$ is a quadrilateral with $\angle ACD=90°$, $\angle ABC=120°$, $\angle BCA=30°$, $\angle ADC=40°$ and $CB=20$ cm.



a. Show $AC=20\sqrt{3}$ cm. **(2)**

b. Hence find the length of CD correct to 1 decimal place. **(1)**

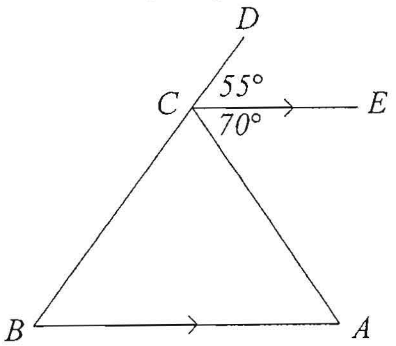
11. Solve the equation $4\sin^2 x=1$, where $0\le x\le 2\pi$. **(2)**

12. Simplify $\sec^2\theta-\tan^2\theta$. **(1)**

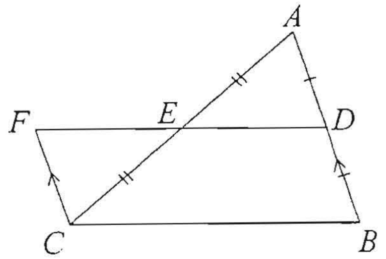
13. From a point on level ground the angle of elevation to the top of a vertical tower is 40°. What is the angle of elevation to the top of the same tower from a point halfway between the original position and the base of the tower? **(2)**

Section 3. Geometry *(Start this question a new page)*

14. Show that ΔABC is isosceles, giving reasons. **(2)**



15. In ΔABC, D and E are the midpoints of AB and AC, respectively. $CF \parallel AB$.



a. Prove that $\Delta ADE \equiv \Delta CFE$. (2)

b. Hence or otherwise, prove that $DBCF$ is a parallelogram. (2)

Section 4. Probability *(Start this question a new page)*

16. A die is made in the shape of a regular dodecahedron. Two of its opposing parallel faces are labelled with the letter T. The five faces adjoining one of these faces are labelled with the five vowels, A, E, I, O, and U. Two of the five remaining faces are labelled with the letter B and the three remaining faces with the letter M.

a. If the die is rolled once what is the probability that

i. the letter M is on the upper most face? **(1)**

ii. the letter B does not appear on the upper most face? **(1)**

b. If the die is rolled three times and the letter on the upper most face recorded what is the probability that

i. the name TIM is spelled out in order? **(2)**

ii. the names TIM or TOM are spelled out in order? **(2)**

Section 5. Functions *(Start this question a new page)*

17 For the function $y = 1 + \frac{1}{x}$

a. Complete the table below. **(2)**

x	-2	-1	$-0{\cdot}2$	$0{\cdot}2$	1	2
$1/x$						
$1 + 1/x$						

b. Can this function ever attain the value *1*? **(1)**

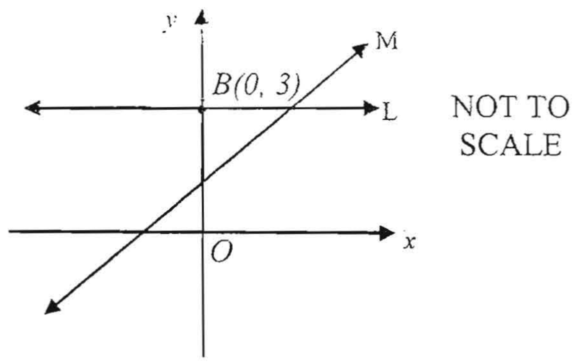
c. Describe the behaviour of $y = 1 + \frac{1}{x}$ as x tends to $\pm\infty$. **(1)**

d. Describe the behaviour of the function $y = 1 + \frac{1}{x}$ around $x = 0$. **(1)**

e. Sketch the function $y = 1 + \frac{1}{x}$ **(1)**

Section 6. Co-ordinate Geometry *(Start this question a new page)*

18.



The diagram shows the line L, parallel to the x-axis and passing through the point B(0, 3) and line M which has the equation $x - 2y + 2 = 0$.

a. What is the equation of line *L*? **(1)**

b. Find the co-ordinates *A*, the point of intersection of lines *L* and *M*. **(1)**

c. What is the angle of inclination of line *M* to the x-axis. **(2)**

d. What is the distance from point *B* to the line *M*.
(Leave your answer in exact form) **(2)**

Section 7. Differentiation and Applications *(Start this question a new page)*

19. Differentiate:

a. $2x^3 - \frac{5}{x} - 4$; **(2)**

b. $\frac{x^2}{x^3 - 2}$ **(2)**

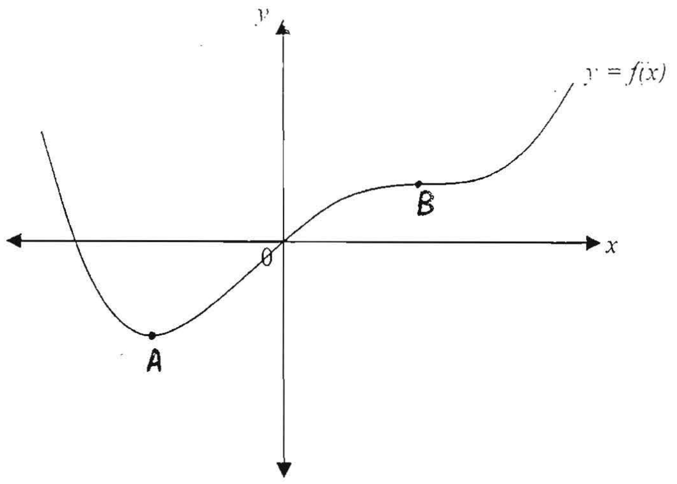
20. For the following function $f(x) = 2x^3 - 3x^2 - 12x + 6$

a. Find $f'(x)$. **(1)**

b. Evaluate $f'(-1)$. **(1)**

c. Find the equation of the normal to the curve $f(x) = 2x^3 - 3x^2 - 12x + 6$ at the point where $x = -1$. **(1)**

21. Below is the graph of $y = f(x)$ where A and B are stationary points on the curve.



a. Copy this graph onto your answer sheet.

b. On the same set of axes, sketch the graph of its derivative, $f'(x)$. **(2)**

22. Consider the curve given by the equation $y = 4x(1-x)^2$.

a. Find the co-ordinates of the stationary points and determine their nature. (3)

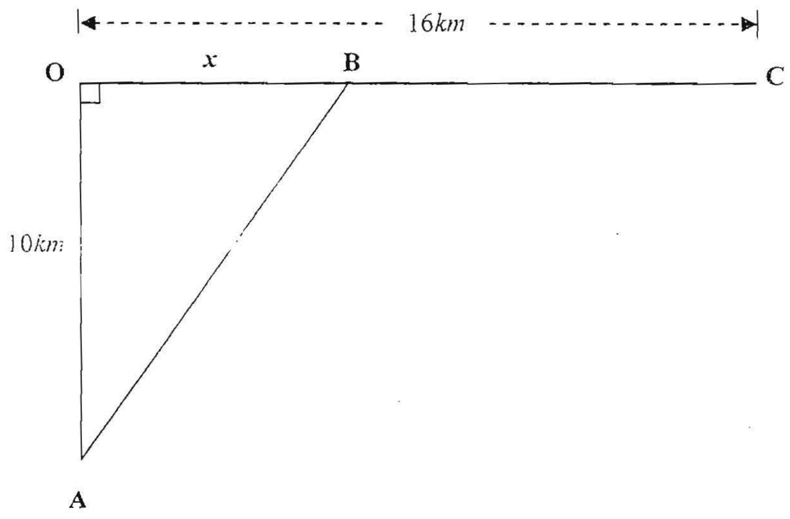
b. Find the co-ordinates of any points of inflexion. (1)

c. Sketch the curve in the domain $0 \le x \le 2$. (2)

d. What is the maximum value of $4x(1-x)^2$ in the domain $0 \le x \le 2$? (1)

d. For what values of x does the curve $y = 4x(1-x)^2$ rise with downward concavity? (1)

23. Two roads in the outback, $OA = 10km$ and $OC = 16km$, meet at right angles at O. A cyclist averages $30km/h$ riding along the roads but only $20km/h$ if he travels cross-country. To ride from A to C the cyclist decides to ride cross-country to B, where B is x kilometres from O, and then along the road to C as in the diagram below.



a. Show that the time taken to ride from A to B is given by $t = \dfrac{\sqrt{100+x^2}}{20}$. (2)

b. Show that the total time for the journey from A to C is given by

$$T = \frac{\sqrt{100+x^2}}{20} + \frac{16-x}{30}.$$ (1)

c. Show that $\dfrac{dT}{dx} = \dfrac{x}{20\sqrt{100+x^2}} - \dfrac{1}{30}$. (2)

d. Find the value of x such that the time taken for the journey is a minimum. (2)

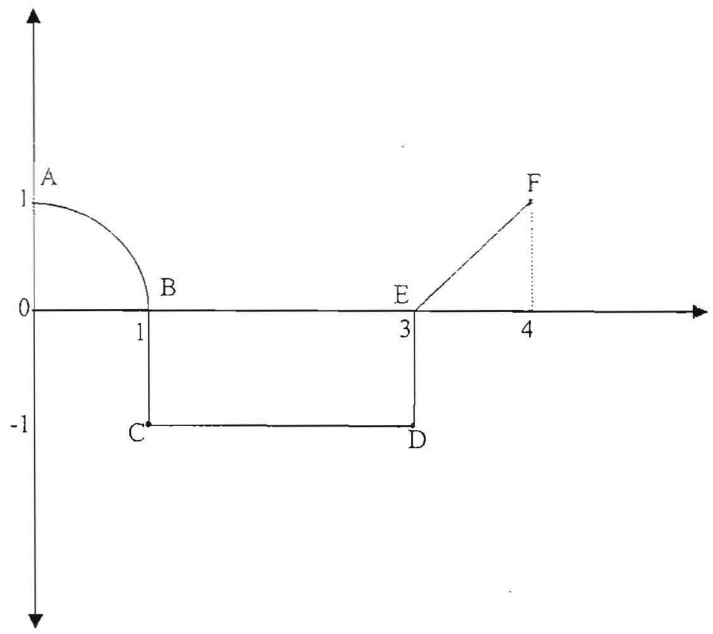
Section 8. **Integration and Applications** *(Start this question a new page)*

24. Find $\int x - x^{\frac{1}{2}}\, dx$ **(2)**

25. The gradient function of a curve is given by $\frac{dy}{dx} = 4x - 6$. If this curve passes through $(-3, 2)$, find the equation of the curve. **(2)**

26. By first constructing a table of values, use Simpson's Rule with 5 function values (4 subintervals) to approximate the area enclosed between the curve $y = \frac{1}{(5 - x)}$, the x-axis and the lines $x = 0$ and $x = 4$. *(Answer correct to 2 significant figures)* **(2)**

27. A function, y = f(x), is drawn below and represented by the curve ABCDEF.
Note: AB is an arc of a circle, centre (0, 0) and radius 1 unit.



a. Show that the area enclosed between the curve $y = f(x)$, the x-axis and the ordinates $x = 0$ and $x = 4$ is $\frac{\pi + 10}{4}$. **(2)**

b. For the above function find the value of the integral $\int_4^0 f(x)dx$.

(Leave your answer in terms of π) **(1)**

28. Evaluate the area enclosed between the curve $y = x^2 - 4$, the x-axis and the ordinates $x = 1$ and $x = 3$. **(3)**

Section 9. Quadratic Polynomial and the Parabola *(Start this question a new page)*

29. Solve $(2x-3)^2 = 25$. **(2)**

30. If α and β are the roots of the equation $2x^2 - 5x - 3 = 0$, find:

a. $\alpha + \beta$ **(1)**

b. $\alpha\beta$ **(1)**

c. $\frac{1}{\alpha} + \frac{1}{\beta}$ **(2)**

31. Find the value(s) of k for which the expression $2x^2 + kx + k$ is positive definite. **(3)**

32. A parabola has its vertex at the point V(-1, 3) and its focus at S(-1, -1).

a. Sketch the parabola. **(1)**

b. What is the equation of its directrix? **(1)**

c. What is the equation of the parabola? **(1)**

Section 10. Series and Applications *(Start this question a new page)*

33. Find the twenty second term of the sequence $2, 3, 4.5, \ldots$, correct to two decimal places. (2)

34. What is the largest term of the sequence $-20, -16.5, -13, \ldots$ which is less than $10\,000$? **(3)**

35. Each cell of a particular virus is capable of dividing into two new cells every four seconds. Find the maximum number of virus cells that could be present after two minutes if the virus started as a single cell. **(2)**

36. Ishtisuq borrows \$3000 at 0.7% per month reducible interest. He agrees to repay the loan in 36 equal monthly instalments of \$M.

a. Write an expression in terms of M for the amount owing immediately after the third instalment. **(2)**

b. Show that the equation required to find the value of M contains a sum of terms of a geometric progression. (1)

c. Find the value of M. **(2)**

Section 11. Logarithmic and Exponential Functions *(Start this question a new page)*

37. Differentiate the following:

a. $4xe^{3x}$; **(2)**

b. $log_e(x^2 - x)$; **(2)**

38. Find

a. $\int (e^{0.5x} + \frac{1}{x})\,dx$ **(2)**

b. $\int \frac{x^2}{x^3 - 2}\,dx$. **(2)**

39. Find the gradient of the tangent to the curve $y = log_e(2x - 5)$ at the point $(3, 0)$. **(2)**

40. Find the volume when the area under the curve $y = e^{2x}$, between $x = 1$ and $x = 2$, is rotated about the x-axis. *Leave your answer in terms of π and* **e**. **(2)**

THE END