PENRITH HIGH SCHOOL

MATHEMATICS
2012

HSC Trial

General Instructions:

- Reading time – 5 minutes
- Working time – **3 hours**
- Write using black or blue pen. Black pen is preferred
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- A multiple choice answer sheet is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Work on this question paper will not be marked.

Total marks – 100

SECTION 1 – Pages
10 marks
- Attempt Questions 1 – 10
- Allow about 15minutes for this section.

SECTION 2 – Pages
90 marks
- Attempt Questions 11 – 16
- Allow about 2 hours45 minutes for this section.

Section1

Question	Mark
1	
2	
3	
4	
5	

Question	Mark
6	
7	
8	
9	
10	
Total	/10

Section 2

Question	Mark
11α	/9
11β	/6
12α	/7
12β	/8
13α	/11
13β	/4

Question	Mark
14α	/8
14β	/7
15α	/5
15β	/10
16α	/7
16β	/8

Total	/100
%	

This paper MUST NOT be removed from the examination room

Student Name: ....

Section 2: Free Response

Start each question on a new page.
Answer these questions on the writing paper supplied.

Question 11.

α) i. If $a+\sqrt{b}=3(5+\sqrt{6})$ and a and b are integers, find the value of b. **(1)**

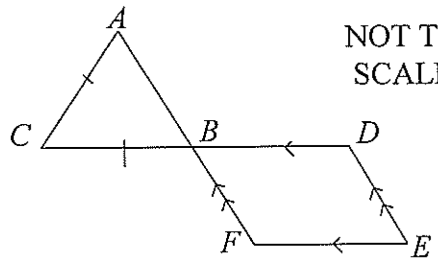
ii. Simplify $\dfrac{81}{3^{4x}\times 9^{1-2x}}$. **(2)**

iii. Solve the equation: $\dfrac{x}{4}-\dfrac{2x-1}{5}=3$. **(2)**

iv. Graph on a number line the solution set of: $|3x+1|\geq 8$. **(2)**

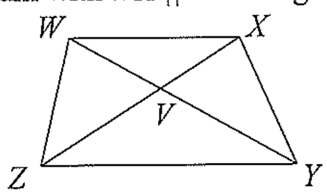
v. Find the values of x for which $x^2-2x-3\geq 0$. **(2)**

β) i.



In the figure, $AC = BC$, $BD \parallel FE$, $BF \parallel DE$ and $\angle ACB = 52°$.
Find the size of $\angle BFE$ giving reasons. **(2)**

ii. $WXYZ$ is a trapezium with $WX \parallel ZY$. Diagonals WY and XZ intersect at V.



a) Prove, giving reasons, that $\Delta WXV \,|||\, \Delta ZVY$. **(2)**

b) Hence, find ZY, given that $WX = 6$ cm, $WV = 4$ cm and $VY = 6$ cm. **(2)**

Question 12.

α) i. Show that $\sum_{k=-3}^{49}(4k-1)$ represents an arithmetic series and hence evaluate this sum. **(2)**

ii. Jamie was given cash gifts totalling $M for his birthday. He immediately deposited it into an account which paid interest at the rate of $\frac{1}{2}\%$ per month compounded monthly. For the next two years he withdrew *$15* each month from the account immediately after the interest was paid in order to subscribe to an online photography workshop.

a) Write down an expression in terms of M for the amount of money in Jamie's account immediately after making the first withdrawal? **(1)**

b) Show that after making the *n*th withdrawal, Jamie's balance in the account is given by the expression $\$\left[(M-3000)1.005^{n}+3000\right]$. **(2)**

c) After the final withdrawal at the end of two years the amount of money left in Jamie's account was *$31.56*. How much did Jamie initially deposit into this account? **(2)**

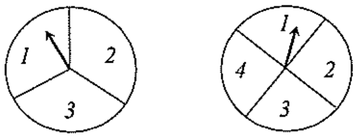
β) i. At the opening ceremony of the Olympic Games 75% of the seats were allocated to residents of the host country. At the beginning of the ceremony three lucky people were randomly selected from the crowd to participate in an activity in the main arena. Draw a tree diagram and use it to find the probability that:

a) all three are from the host country; **(1)**

b) the majority are from the host country; **(1)**

c) not more than *2* are from the host country. **(1)**

ii. The diagram shows two spinners which are spun simultaneously.



Each of the three outcomes on the first spinner are equally likely, and each of the four outcomes on the second spinner are equally likely.

a) Show the probability that both spinners stop on the same number is $\frac{1}{4}$? **(1)**

b) Adithy and Boyd decide to play a game where they each take turns in spinning the two spinners. The winner is the first to have both spinners stop on the same number. Adithy spins first.
Show that the probability that Boyd wins on his first spin is $\frac{3}{16}$. **(1)**

c) Show that the probability that Boyd wins on his first or second spin is given by

$$\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^{3}\left(\frac{1}{4}\right).$$ **(1)**

d) What is the probability that Boyd wins the game? **(2)**

Question 13.

α) i. A parabola has equation $y = 8 - 4x - x^2$. Find:

a) the equation of its axis of symmetry; **(1)**

b) the coordinates of its vertex; **(1)**

c) the coordinates of its focus. **(2)**

ii. Find the value(s) of k in the equation $(2k + 1)x^2 + (k - 1)x - 1 = 0$ so that it has:

a) one root the reciprocal of the other; **(2)**

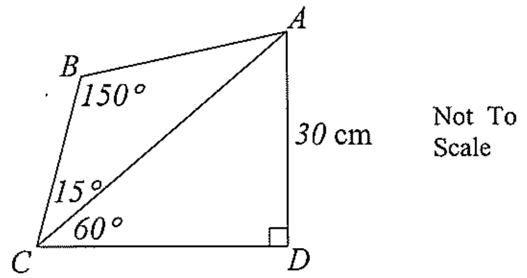
b) equal roots. **(2)**

iii. a) Show that $2 \times 4^x = 2^{2x+1}$. **(1)**

b) Use the result in part i. to solve for x: $2^{2x+1} - 17(2^x) + 8 = 0$. **(2)**

β) i. Show that $\sec(-30°) \sin 120° = 1$. **(1)**

ii. $BADC$ is a quadrilateral with $\angle ADC = 90°$, $\angle ABC = 150°$, $\angle BCA = 15°$, $\angle ACD = 60°$ and $AD = 30$ cm.



a) Show $AC = 20\sqrt{3}$ cm. **(1)**

b) Hence find the length of AB correct to 1 decimal place. **(2)**

Question 14.

α) Line n has equation $3x + y - 3 = 0$.

i. What is the value of its gradient? **(1)**

ii. Line m is perpendicular to line n and passes through the point $A(2, 2)$.
Show that the equation of line m is $x - 3y + 4 = 0$. **(2)**

iii. What acute angle, to the nearest minute, does the line m make with the x-axis? **(1)**

iv. Point B is the x-intercept of the line m. find the coordinates of B. **(1)**

v. Point C has coordinates $(2, 6)$. Find the area of triangle ABC. **(3)**

β) i. Sketch the curves in separate diagrams and state the domain and range:

a) $y = -\sqrt{25-x^2}$. **(2)**

b) $y = log(x-2)$. **(2)**

ii. Sketch on a number plane and shade in the region where the following inequalities hold simultaneously:

$$(x-2)^2 + y^2 \leq 4 \text{ and } y > \frac{1}{x-2}.$$ **(3)**

Question 15.

α) i. Differentiate with respect to x:

a) $2x^3 - \dfrac{3}{\sqrt{x}}$; **(1)**

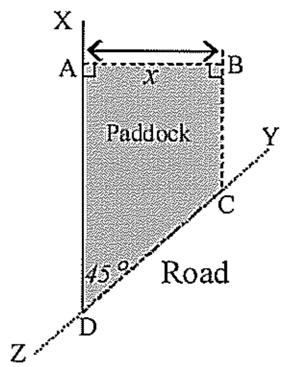
b) $x^3 \cot 3x$. **(2)**

ii. Find:

a) $\int (3x+5)^2\,dx$; **(1)**

b) $\int \dfrac{2e^x}{e^x - 3}\,dx$. **(1)**

β) i. $YCDZ$ is the edge of a straight road. XD is an existing straight fence which meets the road at $45°$. Farmer Bob has enough material to erect 800 metres of new fence. He plans to enclose a trapezoidal paddock with the maximum possible area by erecting the following straight fence.



AB perpendicular to XD;
BC parallel to XD:
CD along the edge of the road.

Let $AB = x$ metres.

a) Show that $BC = 800 - (1+\sqrt{2})x$. **(2)**

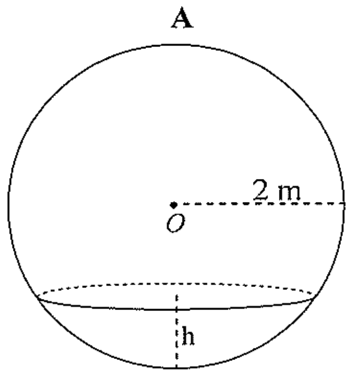
b) Show that the area of trapezium $ABCD$ is given by

$$A = 800x - \left(\frac{1+2\sqrt{2}}{2}\right)x^2.$$ **(2)**

c) Find the distance AB that will maximise the area of the paddock. **(3)**

Question 15 (continued).

ii.



A spherical container of radius *2* metres and centre at *O* has an opening, A, at the top through which liquid is poured.

Find the volume of the liquid in the container when its height, *h*, reaches *1* metre. **(3)**

Question 16.

α) Consider the function $y = x^2e^x$:

i. For what values of x is this function defined? **(1)**

ii. Describe the behaviour of the function as x:

a) approaches $+\infty$. **(1)**

b) approaches $-\infty$. **(1)**

iii. Find any stationary points and determine their nature. **(3)**

iv. Sketch the curve of this function on a number plane. **(1)**

β) i. For the function $f(x) = 3\cos 2x + 1$,

a) State the range of $f(x)$. **(1)**

b) Draw a neat sketch of $f(x) = 3\cos 2x + 1$ for $-\pi \le x \le \pi$. **(2)**

c) Calculate the exact area of the region in the first quadrant bounded by the curve $y = 3\cos 2x + 1$, the y- axis and the line $y = 1$. **(2)**

ii. Solve for θ, $\sqrt{3}\cos\theta + \sin\theta = 0$, $0 \le \theta \le 2\pi$. **(3)**

THE END

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \; \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

Student Name:____

Multiple Choice Answer Sheet

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.				
8.				
9.	A	B	C	D
10.	A	B	C	D

2012 Mathematics Trial HSC Solutions.

Question 11 (L.K.)

α. i) $a+\sqrt{b} = 3(5+\sqrt{6})$

$= 15+3\sqrt{6}$

$= 15+\sqrt{54}$

$\therefore b = 54$

ii) $\dfrac{81}{3^{4x}\times 9^{1-2x}} = \dfrac{3^4}{3^{4x}\times(3^2)^{1-2x}}$

$= \dfrac{3^4}{3^{4x}\times 3^{2-4x}} = 3^{4-4x-(2-4x)}$

$= 3^{4-4x-2+4x} = 3^2$

$= 9$

iii) $\dfrac{x}{4} - \dfrac{2x-1}{5} = 3$

$\dfrac{20x}{4} - \dfrac{\overset{4}{\cancel{20}}(2x-1)}{\cancel{5}} = 20\times 3$

$5x - 8x + 4 = 60$

$-3x = 56$

$x = \dfrac{-56}{3}$

$x = -18\tfrac{2}{3}$

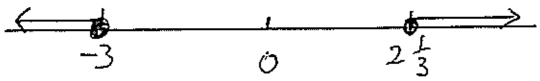
iv) $|3x+1| \geq 8$

$3x+1 \geq 8$ $\quad$ $-(3x+1)\geq 8$

$3x \geq 7$ $\quad$ $3x+1 \leq -8$

$x \geq 2\tfrac{1}{3}$ $\quad$ $3x \leq -9$

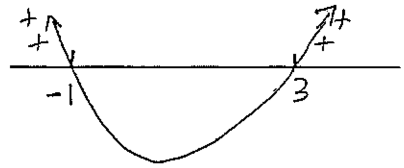
$x < -3$



1.	D
2	B
3	A
4	D
5	C
6	B
7.	165
8.	log 16
9.	D
10.	A

v) $x^2-2x-3 \geq 0$

$(x-3)(x+1) \geq 0$



$\therefore x \leq -1$ and $x \geq 3$

β. i) $\angle ABC = \dfrac{180-52}{2}$ ($\angle$ sum of isos. $\triangle ABC$ with $AC = CB$)

$= \dfrac{128}{2}$

$= 64°$

$\angle DBF = \angle ABC$ (vert. opp.)

$= 64°$

$\angle BFE = 180° - 64°$ (co-interior with $\angle DBF$, $BD \parallel FE$)

$= 116°$

ii) a) $\angle WVX = \angle YVZ$ (vert. opp)

$\angle WXV = \angle YZV$ (alt, $WX \parallel ZY$)

$\therefore \triangle WXV \parallel\!\parallel\!\parallel \triangle ZVY$ (equiangular)

b) $\dfrac{ZY}{WX} = \dfrac{VY}{WV}$ (ratios of corresp. sides of $|||\triangle$'s are equal)

$\therefore \dfrac{ZY}{6} = \dfrac{6}{4}$

$ZY = \dfrac{36}{4}$

$ZY = 9$cm

Question 12 (R.L.)

α. i) $\sum_{k=-3}^{49} (4k-1)$

$= -13 + {}^-9 + {}^-5 + \dots + 195$

A.P. with $a = -13$

$d = -9 - {}^-13 = -5 - {}^-9 = 4$.

$n = 53$

$S_{53} = \frac{53}{2}(-13 + 195)$

$= 4823$

ii) a) $A_1 = M + 0.005M - 15$

$= 1.005M - 15$

b) $A_2 = 1.005 A_1 - 15$

$= 1.005^2 M - 1.005(15) - 15$

$= 1.005^2 M - 15(1.005 + 1)$

$A_3 = 1.005 A_2 - 15$

$= 1.005^3 M - 15(1.005^2 + 1.005) - 15$

$= 1.005^3 M - 15(1.005^2 + 1.005 + 1)$

$\therefore A_n = 1.005^n M - 15(1.005^{n-1} + 1.005^{n-2} + \dots + 1)$

$= 1.005^n M - 15 \times \frac{1(1.005^n - 1)}{1.005 - 1}$

$= 1.005^n M - \frac{15(1.005^n - 1)}{0.005}$

$= 1.005^n M - 3000(1.005^n - 1)$

$= 1.005^n M - 1.005^n(3000) + 3000$

$= (M - 3000)\,1.005^n + 3000$

iii) $A_{24} = \$31.56 = (M-3000)1.005^{24} + 3000$

$31.56 - 3000 = (M - 3000)\,1.005^{24}$

$\frac{31.56 - 3000}{1.005^{24}} = M - 3000$

$3000 + \frac{31.56 - 3000}{1.005^{24}} = M$

$\$366.44 = M$

β. i) a) $Pr(\text{host}, \text{host}, \text{host})$

$= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$

b) $Pr(hhh \text{ or } 2h\tilde{h})$

$= \frac{27}{64} + 3 \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

$= \frac{27}{64} + \frac{27}{64}$

$= \frac{54}{64}$

$= \frac{27}{32}$

c) $Pr(\tilde{h}\tilde{h}\tilde{h} \text{ or } h\tilde{h}\tilde{h} \text{ or } hh\tilde{h})$

$= 1 - Pr(hhh)$

$= 1 - \frac{27}{64}$

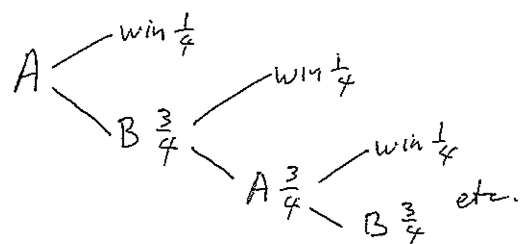
$= \frac{37}{64}$

ii) a) $Pr(\text{any}, \text{same}) = 1 \times \frac{1}{4} = \frac{1}{4}$

OR $Pr(1,1 \text{ or } 2,2 \text{ or } 3,3)$

$= \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4} = \frac{3}{12} = \frac{1}{4}$

b)



$Pr(A, B, \text{win}) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

c) $Pr(A, B, \text{win} \text{ or } A, B, A, B, \text{win})$

$= \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

$= \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4}$

Question 12. B ii) d).

Pr (Boyd wins)

$= \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^5 \times \frac{1}{4} + \cdots$

G.P. $a = \frac{3}{4} \times \frac{1}{4}$, $r = \left(\frac{3}{4}\right)^2$.

$S_\infty = \dfrac{\frac{3}{4} \times \frac{1}{4}}{1 - \left(\frac{3}{4}\right)^2} = \dfrac{\frac{3}{16}}{1 - \frac{9}{16}} = \dfrac{\frac{3}{16}}{\frac{7}{16}}$

$= \frac{3}{7}$

Question 13 (D.A)

α i) a) $x = \frac{-b}{2a} = \frac{-4}{-2} = -2$

b) $x = -2$, $y = 8 - 4(-2) - (-2)^2$

$= 8 + 8 - 4$

$= 12$

c) $y = 8 - 4x - x^2$

$x^2 + 4x = 8 - y$

$x^2 + 4x + 4 = 8 - y + 4$

$(x+2)^2 = 12 - y$

$(x+2)^2 = -4 \times \frac{1}{4}(y - 12)$

$\therefore V(-2, 12)$, $a = \frac{1}{4}$

$\therefore S(-2, 11\frac{3}{4})$

ii) $(2k+1)x^2 + (k-1)x - 1 = 0$

a) $\alpha, \frac{1}{\alpha}$ $\therefore \alpha \times \frac{1}{\alpha} = 1$

$\frac{c}{a} = 1 = \frac{-1}{2k+1}$

$2k + 1 = -1$

$2k = -2$

$k = -1$

b) $\Delta = 0$

$(k-1)^2 - 4(2k+1) \cdot -1 = 0$

$k^2 - 2k + 1 + 8k + 4 = 0$

$k^2 + 6k + 5 = 0$

$(k+5)(k+1) = 0$

$k = -1$ or -5

iii) a) $2 \times 4^x = 2 \times (2^2)^x$

$= 2 \times 2^{2x}$

$= 2^{2x+1}$

b) $2^{2x+1} - 17(2^x) + 8 = 0$

$2(2^x)^2 - 17(2^x) + 8 = 0$

Let $A = 2^x$

$2A^2 - 17A + 8 = 0$

$(2A - 1)(A - 8) = 0$

$\therefore A = \frac{1}{2}$ and $A = 8$

$\therefore 2^x = \frac{1}{2}$ and $2^x = 8$

$x = -1$ $\quad x = 3$

B i) $\sec(-30°) \sin 120°$

$= \frac{1}{\cos(-30°)} \times \sin 60°$

$= \frac{1}{\cos 30°} \times \sin 60°$

$= \frac{1}{\frac{\sqrt{3}}{2}} \times \frac{\sqrt{3}}{2}$

$= 1$

ii) a) In $\triangle ADC$

$\sin 60° = \frac{30}{AC}$

$AC = \frac{30}{\sin 60}$

$AC = \frac{30}{\frac{\sqrt{3}}{2}}$

$AC = 30 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 20\sqrt{3}$

b) $\frac{AB}{\sin 15°} = \frac{AC}{\sin 150°}$

$AB = \frac{20\sqrt{3} \sin 15°}{\sin 150°}$

$AB \doteqdot 17 \cdot 9$ cm

Question 14 (B.F.)

α) i) $3x + y - 3 = 0$

$y = -3x + 3$

$\therefore m = -3$

ii) $m_2 = \frac{1}{3}$ + $A(2,2)$

$y - 2 = \frac{1}{3}(x-2)$

$3y - 6 = x - 2$

$0 = x - 3y + 4$

iii) $\tan\theta = \frac{1}{3}$

$\theta = \tan^{-1}(\frac{1}{3})$

$\theta \doteqdot 18°26'$

iv) $\underline{y=0}$, $x - 3y + 4 = 0$

$x - 0 + 4 = 0$

$x = -4.$

$\therefore B(-4, 0)$

v) ⊥ distance from C to m

$d = \left| \frac{2 - 3(6) + 4}{\sqrt{1^2 + (-3)^2}} \right|$

$= \left| \frac{2-18+4}{\sqrt{10}} \right|$

$= \frac{12}{\sqrt{10}}$

$d_{AB} = \sqrt{(2-4)^2 + (2-0)^2}$

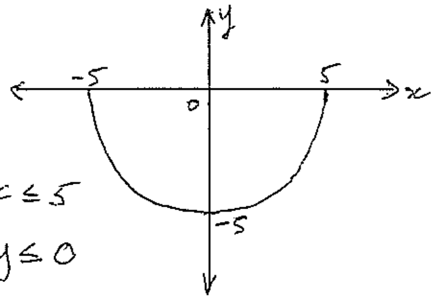
$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$A = \frac{1}{2} \times 2\sqrt{10} \times \frac{12}{\sqrt{10}} = 12u^2$

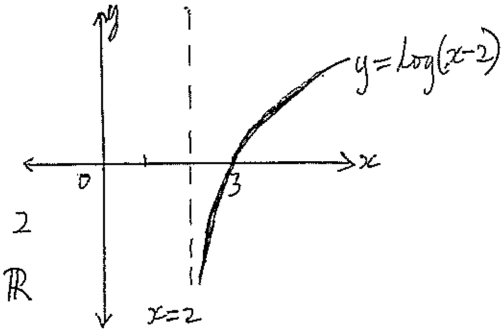
β. i) a)



D: $-5 \le x \le 5$

R: $-5 \le y \le 0$

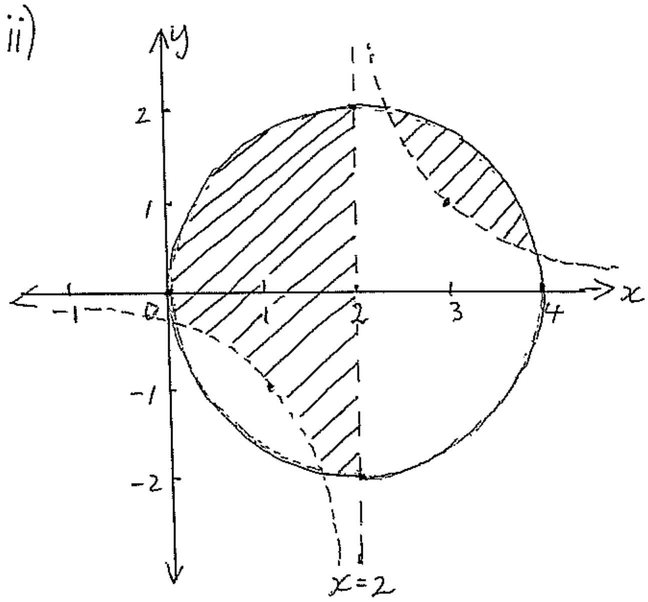
b)



D: $x > 2$

R: $y \in \mathbb{R}$

ii)



Question 15. (T.B.)

α i. a) $\frac{d}{dx}\left(2x^3 - 3x^{-\frac{1}{2}}\right)$

$= 6x^2 + \frac{3}{2}x^{-\frac{3}{2}}$

$= 6x^2 + \frac{3}{2x\sqrt{x}}$

Question 15 (T.B.)

α i) b) $\frac{d}{dx} x^3 \cot 3x = \frac{d}{dx} \frac{x^3}{\tan 3x}$

$$= \frac{\tan 3x . 3x^2 - x^3 . \sec^2 3x . 3}{\tan^2 3x}$$

$$= 3x\left[x \tan 3x - 3x^3 \sec^2 3x\right] \cot^2 3x$$

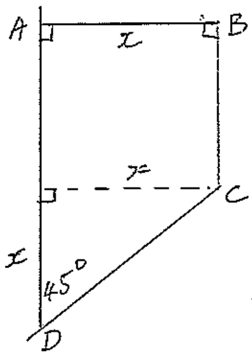
ii) a) $\int (3x+5)^2 \, dx$

$$= \frac{(3x+5)^3}{3.3} + c$$

$$= \frac{(3x+5)^3}{9} + c$$

b) $\int \frac{2e^x}{e^x - 3} \, dx = 2\ln(e^x - 3) + c$

β i) a)



$DC^2 = x^2 + x^2$

$DC = \sqrt{2x^2}$

$DC = x\sqrt{2}$

$$AB + BC + CD = 800$$
$$x + BC + \sqrt{2}x = 800$$
$$BC = 800 - x - \sqrt{2}x$$
$$BC = 800 - (1+\sqrt{2})x$$

b)

$$A = \tfrac{1}{2}[\ AD \qquad + \qquad BC\ ]\ AB$$
$$= \tfrac{1}{2}\left[x + 800 - (1+\sqrt{2})x + 800 - (1+\sqrt{2})x\right] x$$
$$= \tfrac{1}{2}\left[x^2 + 1600x - (2+2\sqrt{2})x^2\right]$$
$$= 800x + \frac{x^2}{2}(1 - 2 - 2\sqrt{2})$$
$$= 800x + \frac{x^2}{2}(-1 - 2\sqrt{2})$$
$$= 800x - \left(\frac{1+2\sqrt{2}}{2}\right)x^2$$

c) $\frac{dA}{dx} = 800 - 2\left(\frac{1+2\sqrt{2}}{2}\right)x$

$$0 = 800 - (1+2\sqrt{2})x.$$

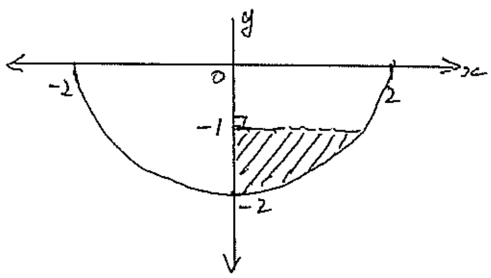
$$x = \frac{800}{1+2\sqrt{2}} \doteqdot 209\,m$$

$$\frac{d^2A}{dx^2} = -(1+2\sqrt{2}) < 0$$

∴ max^m

ii) Choose an appropriate circle in the Cartesian plane and rotate an ~~ar~~ appropriate area about an axis.

Eg. $y = -\sqrt{4 - x^2}$



$$V = \pi \int_{-2}^{-1} x^2 \, dy$$

$$y = -\sqrt{4 - x^2}$$
$$y^2 = 4 - x^2$$
$$x^2 = 4 - y^2$$

$$\therefore V = \pi \int_{-2}^{-1} (4 - y^2)\, dy$$

$$= \pi\left[4y - \frac{y^3}{3}\right]_{-2}^{-1}$$

$$= \pi\left[\left(-4 + \tfrac{1}{3}\right) - \left(-8 + \tfrac{8}{3}\right)\right]$$

$$= \frac{5\pi}{3}\ m^3$$

Question 16

α. (B.F) $y = x^2 e^x$

i) $D: x \in \mathbb{R}$

ii) a) $x \Rightarrow \infty$, $y = \infty^2 . e^\infty \Rightarrow \infty$

b) $x \Rightarrow -\infty$, $y = \frac{\infty^2}{e^\infty} \Rightarrow 0^+$

iii) $y' = x^2 . e^x + e^x . 2x$

$0 = e^x . x(x+2)$

$\therefore e^x = 0$ (no sol^n), $x = 0$, $x = -2$

$y' = e^x(x^2 + 2x)$

$y'' = e^x(2x+2) + (x^2+2x)e^x$

$= e^x(x^2 + 4x + 2)$

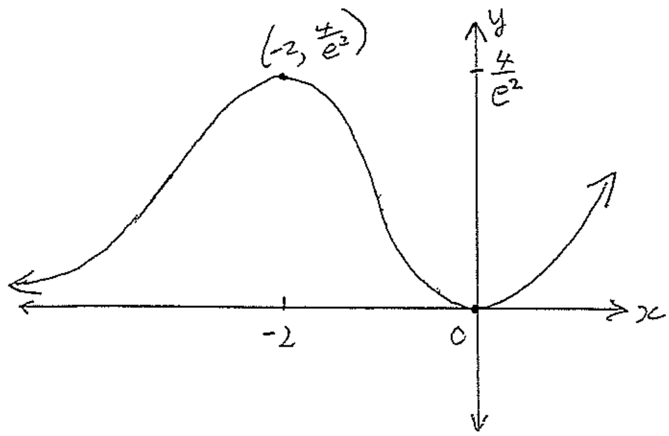
For $x = 0$, $y'' = 2 > 0$ $\therefore$ Min^m

$y = 0$ ie. $(0, 0)$

For $x = -2$, $y'' = \frac{-2}{e^2} < 0$ $\therefore$ Max^m

$y = \frac{4}{e^2}$ ie. $(-2, \frac{4}{e^2})$

iv)

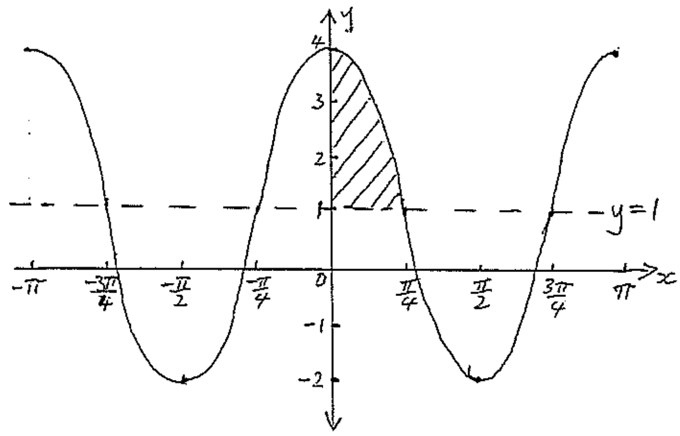


β (X.C.)

i) $f(x) = 3\cos 2x + 1$

a) $-2 \leq f(x) \leq 4$

b) $a = 3$, $T = \frac{2\pi}{2} = \pi$



c) The required area will be equal to the area enclosed by the curve $y = 3\cos 2x$ and the co-ordinate axes.

$A = \int_0^{\frac{\pi}{4}} 3\cos 2x \, dx$

$= \left[\frac{3\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{3}{2}\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= \frac{3}{2} u^2$

ii) $\sqrt{3}\cos\theta + \sin\theta = 0$

$\sin\theta = -\sqrt{3}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = -\sqrt{3}$, $\cos\theta \neq 0$

$\tan\theta = -\sqrt{3}$ (2nd & 4th)

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$