Penrith Selective High School

2014

Higher School Certificate
Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2–5

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–14

60 marks

- Attempt Questions 11–16
- Allow about 2 hours 45 minutes for this section

Student Number: ________________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2014 Higher School Certificate Examination.

Section I:

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

Q1. What is 6.04976 correct to 4 significant figures.

(A) 6.049

(B) 6.0497

(C) 6.050

(D) 6.0498

Q2. What are the solutions of $3x^2 - 7x - 3 = 0$?

(A) $x = \dfrac{-7 \pm \sqrt{85}}{6}$

(B) $x = \dfrac{7 \pm \sqrt{85}}{6}$

(C) $x = \dfrac{7 \pm \sqrt{13}}{6}$

(D) $x = \dfrac{-7 \pm \sqrt{13}}{6}$

Q3. $\frac{x}{3} - \frac{x-4}{6}$ is equal to

(A) $\dfrac{x - 4}{6}$

(B) $\dfrac{x + 4}{6}$

(C) $\dfrac{x + 2}{3}$

(D) $\dfrac{x + 4}{3}$

Q4. What are the solutions of $2cosx = -\sqrt{3}$ for $0 \leq x \leq 2\pi$?

(A) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(B) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

(C) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(D) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

Q5. The line which is perpendicular to $2x - y + 1 = 0$ with a y intercept of 4 has equation:

(A) $y = -2x + 4$

(B) $y = \frac{-x}{2} + 4$

(C) $y = 2x + 4$

(D) $y = \frac{x}{2} + 4$

Q6. What is the derivative of $\frac{x}{2x+3}$?

(A) $\frac{3}{(2x+3)^2}$

(B) $\frac{1}{2}$

(C) $\frac{4x+3}{(2x+3)^2}$

(D) $\frac{1}{4}$

Q7. Two six-sided dice are thrown. The probability that the sum of the uppermost faces is even is:

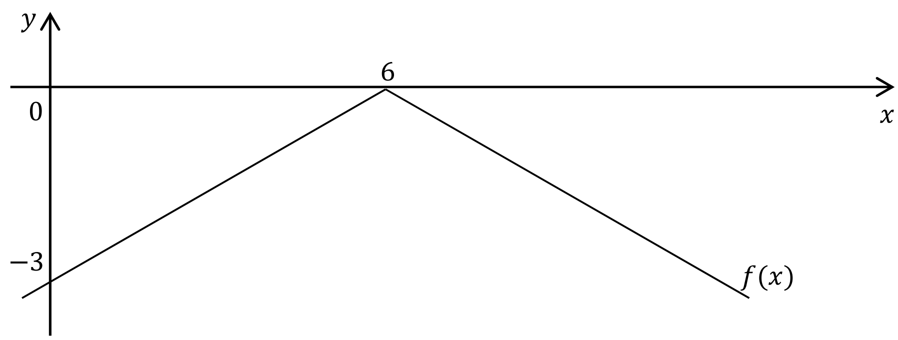
(A) 1

(B) even

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Q8.



The diagram shows the graph of $y = f(x)$. The equation of $y = f(x)$ is:

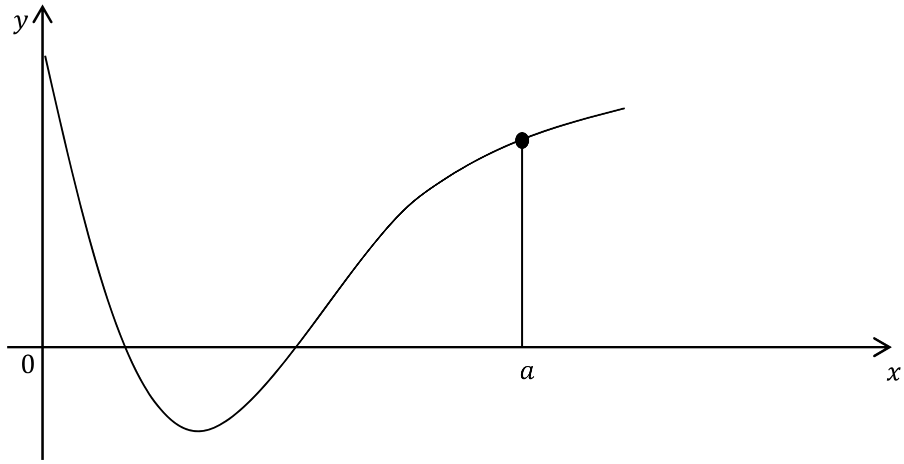
(A) $f(x) = \left|\frac{1}{2}x - 3\right|$

(B) $f(x) = -|2x - 3|$

(C) $f(x) = -\left|\frac{1}{2}x + 3\right|$

(D) $f(x) = -\left|\frac{1}{2}x - 3\right|$

Q9. The diagram shows the graph of $y = f(x)$



Which of the following statements is true?

(A) $f'(a) > 0$ and $f''(a) < 0$

(B) $f'(a) < 0$ and $f''(a) < 0$

(C) $f'(a) > 0$ and $f''(a) > 0$

(D) $f'(a) < 0$ and $f''(a) > 0$

Q10. A geometric series will have a limiting sum if:

(A) $|r| < 1$, where r is the common ratio

(B) $|r| > 1$, where r is the common ratio

(C) $r < 1$, where r is the common ratio

(D) $r > 1$, where r is the common ratio

Section II

60 Marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Simplify the expression $3x - 5(x - 2)$ **2**

b) Given that $S_n = \frac{a(r^n-1)}{r-1}$, find S_n when $n = 12$, $a = 3$ and $r = 2$ **2**

c) Differentiate $2x^3 + x^2 - 2$ **1**

d) Factorise $16a^2 - b^2$ **2**

e) Express $\frac{2}{4-\sqrt{7}}$ with a rational denominator **2**

f) James invests \$1000 at 7% per year compound interest, compounded quarterly. Calculate that value of the investment after 5 years. Give your answer correct to the nearest dollar. **3**

g) Given that $log_a b = 2.75$ and $log_a c = 0.25$, find the value of: **3**

(i) $log_a \left(\frac{b}{c}\right)$

(ii) $log_a (bc)^2$

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Differentiate and simplify where necessary.

(i) $xln(x-3)$ **2**

(ii) $\frac{3x^2-4x+7}{x}$ **2**

b) (i) Evaluate $\int_1^3 6e^{3x}+1\ dx$ **2**

(ii) Find $\int sin4x\ dx$ **2**

c) Find the equation of the tangent to the curve $y=\frac{1}{2}sinx$, at the point $(\pi, 0)$ **2**

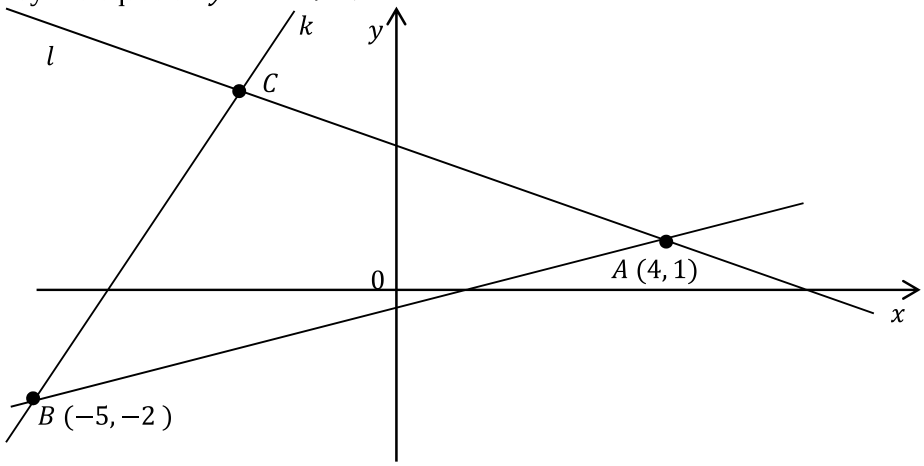
d) Sketch $y=3cos\frac{x}{2}$ for $-\pi \leq x \leq \pi$ showing all key features. **2**

e) A point P(x, y) moves so that it is always twice the distance from the point A(1, 4) as it is to point B(2, -8). Show that the equation of the path traced by P is $3x^2-14x+3y^2+72y+255=0$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) The diagram shows a triangle ABC. The point A$(4, 1)$ lies on l given by the equation $x + 2y = 6$, and the point B$(-5, -2)$ lies on the line k, given by the equation $y = 2x + 8$.



(i) Show that the point C, which is the point of intersection of l and k has coordinates (-2, 4). **1**

(ii) Find the gradient of the line joining A and B. **1**

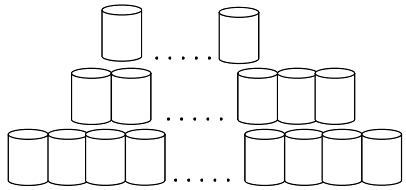
(iii) Hence, or otherwise, find the equation of the line AB. **1**

(iv) Find the perpendicular distance from the point A to the line k. **2**

(v) Hence, or otherwise, find the area of the triangle ABC. **2**

Question 13 continues on page 9

b) Food tins are stacked so there are 49 tins on the bottom row, 45 tins on the next row, 41 tins on the row after and so on until a total of 321 cans are stacked.



(i) Write down a formula for the number of cans in the n^{th} row. **1**

(ii) How many rows are in the stack in total? **2**

(iii) How many cans are in the top row of this stack? **1**

c) If α and β are the roots of the quadratic equation $3x^2 + 8x - 7 = 0$, find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

(iv) $\alpha^2 + \beta^2$ **1**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) A factory manufactures light bulbs. Testing showed that 1 out of 20 bulbs tested was faulty.
Three of these bulbs are selected at random and tested.
What is the probability that:

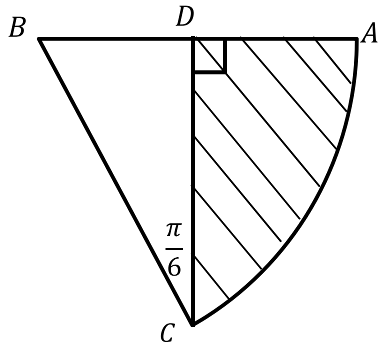
(i) All three bulbs tested are faulty? **1**

(ii) None of the bulbs are faulty? **1**

(iii) Exactly two bulbs are faulty? **2**

(iv) At most two bulbs are faulty? **2**

b) ABC is a sector. $\angle$ BCD = $\frac{\pi}{6}$, BA = BC = 9 cm and $DC \perp AB$.



(i) Calculate the area of sector BAC. **3**

(ii) Calculate the area of the shaded region. Leave your answer in exact form. **3**

c) Find the primitive of:

(i) $\dfrac{2x}{x^2+3}$ **1**

(ii) $\dfrac{e^{2x}}{e^{2x}+3}$ **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) Consider the curves $y = x^2$ and $y = 4x + 5$.

(i) Find any points of intersection. **1**

(ii) Sketch the graphs of the two equations on the same set of axes. **2**

(iii) Find the area of the region enclosed by these two equations. **2**

b) Consider the function $y = 2sinx + cosx$

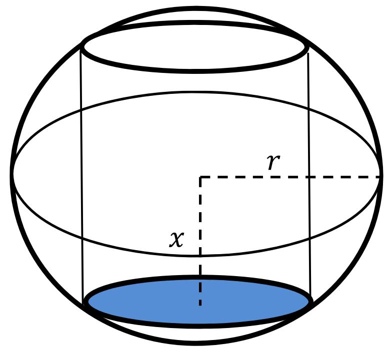
(i) Copy and complete the table of values correct to three decimal places where necessary. **1**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Use two applications of Simpson's rule to calculate the approximate area under the curve between $x = 0$ and $x = \pi$. Leave your answer correct to 2 decimal places. **3**

Question 15 continues on page 12

c) A cylinder is made to fit inside a sphere with fixed radius r as shown in the diagram.



Not to scale

Let x be the distance from the base of the cylinder to the centre of the sphere, as shown in the diagram. Let R be the radius of the circular base of the cylinder.

(i) Find an expression for, R, the radius of the base of the cylinder in terms of r and x. **1**

(ii) Show that the volume, V, of the cylinder is given by $V = 2\pi x(r^2 - x^2)$ **2**

(iii) Find, in terms of r, the maximum volume of the cylinder. Give your answer in exact form. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) Find the equation of the parabola whose axis is parallel to the y-axis, vertex is $(2, -1)$ and has a tangent with equation $y = 2x - 7$. **3**

b) A quantity Q of radium at time t in years is given by

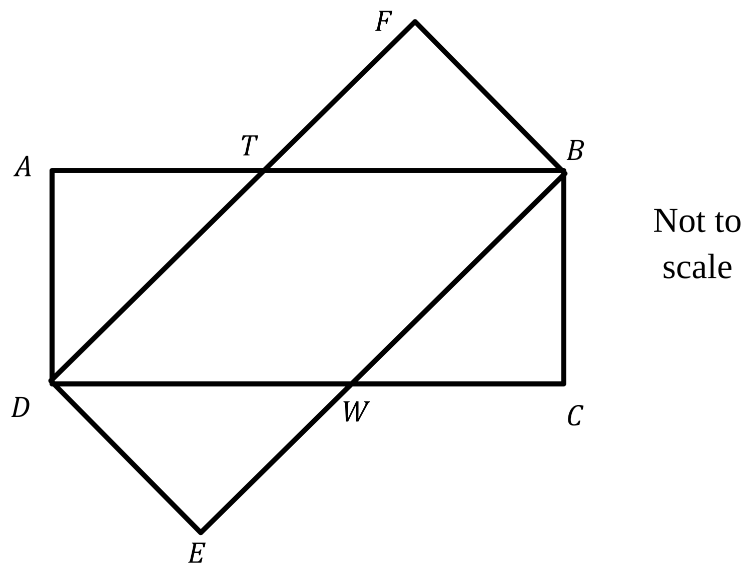
$$Q = Q_0 e^{-kt}$$

where k is a constant and Q_0 is the initial amount of radium at time $t = 0$.

(i) Given that $Q = \frac{1}{2}Q_0$ when $t = 1530$ years, calculate k, correct to three significant figures. **1**

(ii) After how many years does only 20% of the initial amount of radium remain, to the nearest whole number. **1**

c)



$ABCD$ and $DEBF$ are two congruent rectangles with sides 3 and 7 units as shown in the diagram. ($AB = DF = 7, AD = DE = 3$)

(i) Show that $AT = \frac{20}{7}$ **3**

(ii) Find the area of the figure $DWBT$. **2**

Question 16 continues on page 14

d) A truck is to travel 1000 kilometres at a constant speed of v km/h.
When travelling at v km/h, the truck consumes fuel at the rate of
$\left(60 + \frac{v^2}{50}\right)$ litres per hour.
The truck company pays \$1.40 for fuel and pays each of the two drivers \$40 per hour whilst the truck is travelling.

(i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that **3**

$$C = 28v + \frac{164000}{v}$$

(ii) The truck must take no longer than 12 hours to complete the trip, and speed limits require $v \leq 100$. **2**
At what speed v should the truck travel to minimise the cost C?

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Student Number:____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

2014

Mathematics Trial Solutions

Multiple Choice

1	C
2	B
3	B
4	B
5	B
6	A
7	D
8	D
9	A
10	A

Question	Marker
11	Lopez
12	Soth
13	Katyal
14	Antone
15	Young
16	Chirgwin

Exam 2 UNIT TRIAL MATHEMATICS : Question 11

Suggested Solutions	Marker's Comments
a) $3x - 5(x-2)$ $3x - 5x + 10$ $-2x + 10$ or $-2(x-5)$	- Errors was made when factorising after simplifying. - A few students went on to solve for x.
b) $S_n = \frac{3(2^{12}-1)}{2-1} = 12285$	
c) $y' = 6x^2 + 2x$	
d) $(4a-b)(4a+b)$	
e) $\frac{4+\sqrt{7}}{4+\sqrt{7}} \times \frac{2}{4-\sqrt{7}} = \frac{8+2\sqrt{7}}{9}$	
f) $A = P(1+\frac{r}{100})^n \quad r = 7/4 = 1.75 \quad n = 20$ $= 1000(1+\frac{1.75}{100})^n = \$1415.$	- 7% and term was not converted to quarters - students over-complicat this question by not using compound interest formula.
g) i) $\log_a b - \log_a c = 2.75 - 0.25 = 2.5$ ii) $2(\log_a b + \log_a c) = 2(3) = 6.$	- A large number of students had $(3)^2 = 9$

Exam MATHEMATICS : Question 12

Suggested Solutions | **Marker's Comments**

a) i) $y' = uv' + vu'$

$= x\left(\frac{1}{x-3}\right) + \ln(x-3) \cdot 1$

$= \frac{x}{x-3} + \ln(x-3)$

Some Students forgot the product rule

ii) $y' = \frac{vu' - uv'}{v^2}$

$= \frac{x(6x-4) - (3x^2 - 4x + 7)}{x^2}$

$= \frac{6x^2 - 4x - 3x^2 + 4x - 7}{x^2}$

$y' = \frac{3x^2 - 7}{x^2}$

Some students incorrectly wrote ~~the~~ quotient rule. students did not expand correctly

b)

i) $\int_1^3 6e^{3x} + 1 \; dx$

$= \frac{6}{3} e^{3x} + x \Big|_1^3$

$= 2e^{3x} + x \Big|_1^3$

$= (2e^9 + 3) - (2e^3 + 1)$

$= 2e^9 - 2e^3 + 2$

$= 16167.997$ (to 3dp)

Some students confused integration with differentiation multiplied instead of dividing by 3.

Some Students did not evaluate.

Exam MATHEMATICS : Question 12

Suggested Solutions | **Marker's Comments**

ii) $\int \sin 4x \; dx$

$= -\frac{1}{4} \cos 4x + C$

Some students forgot the $+C$.

c) $y' = \frac{1}{2} \cos x$

$y'(\pi) = \frac{1}{2} \cos \pi$

$= \frac{1}{2}(-1)$

$= -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

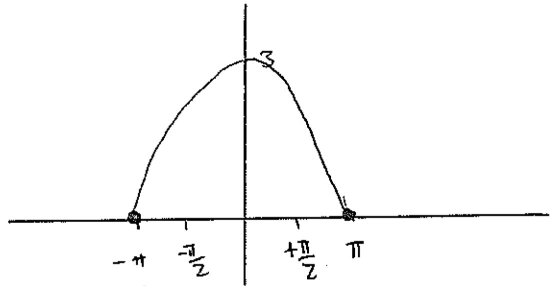
$y - 0 = -\frac{1}{2}(x - \pi)$

$y = -\frac{x}{2} + \frac{\pi}{2}$ or $x + 2y - \pi = 0$

Students did not substitute π.

Students did not write the general correctly

d)



Students did not have the correct period.

e) $PA = 2PB$

$PA^2 = (2PB)^2 = PA^2 = 4PB^2$

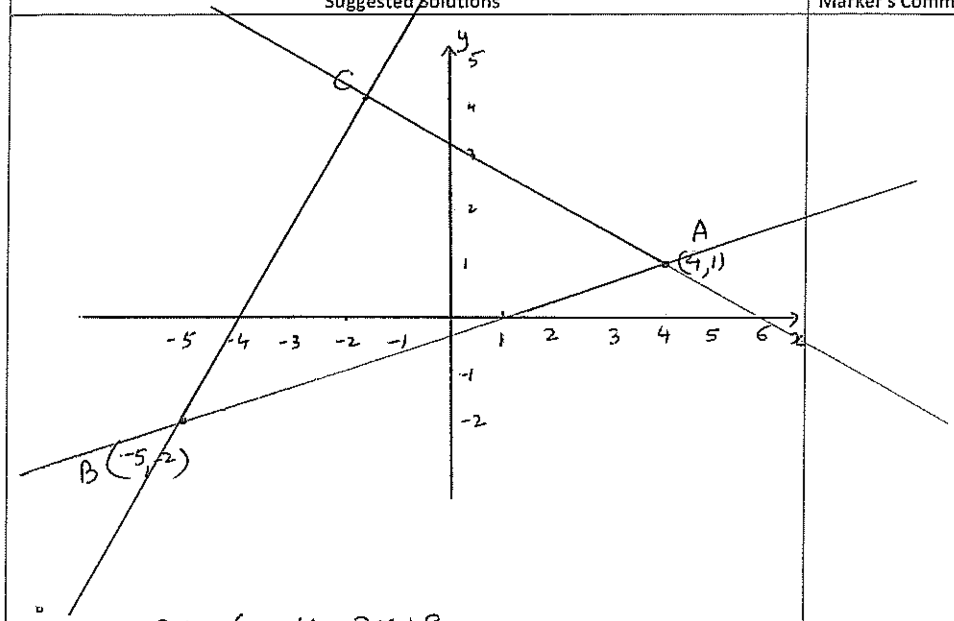
$(x-1)^2 + (y-4)^2 = 4[(x-2)^2 + (y+8)^2]$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 4[x^2 - 4x + 4 + y^2 + 16y + 64]$

$x^2 - 2x - 8y + 17 = 4x^2 - 16x + 16 + 4y^2 + 64y + 272$

$3x^2 - 14x + 3y^2 + 72y + 255 = 0$

Some Students did not expand correctly



(i) $x + 2y = 6$, $y = 2x + 8$,

substituting the value of y in the first equation,

$x + 4x + 16 = 6$

$5x = -10$, $x = -2$, $y = 4$

$C(-2, 4)$ – point of intersection ✓

(ii) $m_{AB} = \dfrac{1 - -2}{4 - -5}$

$= \dfrac{3}{9}$

$m_{AB} = \dfrac{1}{3}$ ✓

(iii) eqn of st. line AB,

$y - 1 = \frac{1}{3}(x - 4)$

$3y = x - 1$ ✓

(iv) Perpendicular distance from Point A to the line k:

$= \dfrac{|2(4) - 1 + 8|}{\sqrt{2^2 + 1^2}}$ ✓

$= \dfrac{15}{\sqrt{5}}$ / or $3\sqrt{5}$ units ✓

(v) Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$ ✓

$= \frac{1}{2} \times \sqrt{(4 - -2)^2 + (-2 - -5)^2} \times 3\sqrt{5}$

$= \frac{1}{2}\sqrt{45} \times 3\sqrt{5}$

$= 22\frac{1}{2}$ unit^2 ✓

(b) $T_1 = 49$, $T_2 = 45$, ...

$a = 49$, $d = -4$

$T_n = a + (n-1)d =$

$= 49 - 4(n-1)$ ✓

(i) $T_n = 53 - 4n$ OR ✓

(ii) $321 = \frac{n}{2}(2a + (n-1)d)$

$642 = n(98 + (n-1) \times -4)$

$4n^2 - 102n + 642 = 0$ ✓

$n = \dfrac{51 \pm \sqrt{33}}{4}$

$= 14.18$ or 11.3

$n = 14.18$ is not the valid answer as it exceeds 321 tiles.

$n = 11.3$ is the correct answer ✗

Therefore, $n = 14$ ✗

(iii) 2 tiles on the TOP row.

most of the students were getting $n = 11.3$, but they were rounding it off to $n = 11$.

Exam	MATHEMATICS : Question....13
Suggested Solutions	Marker's Comments

(c) (i) $\alpha+\beta = -\frac{8}{3}$

(ii) $\alpha\beta = -\frac{7}{3}$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= -\frac{8}{3} \div -\frac{7}{3}$

$= \frac{8}{7}$

(iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{8}{3}\right)^2 - 2\left(-\frac{7}{3}\right)$

$= \frac{106}{9}$

Some students were go putting incorrect signs in front of sum of roots and product of roots

Penrith SHS

2014 HSC Trial – 2U Maths

Qn(14): Total 15 Marks

$P(F) = \frac{1}{20}$ $\quad P(G) = \frac{19}{20}$

(a)(i) $P(FFF) = \left(\frac{1}{20}\right)^3 = \frac{1}{8000}$

(ii) $P(GGG) = \left(\frac{19}{20}\right)^3 = \frac{6859}{8000}$

(iii) $P(FFG) = 3\left(\frac{1}{20}\right)^2\left(\frac{19}{20}\right) = \frac{57}{8000}$

(iv) P(At most two bulbs are faulty) = 1 – P(FFF) = 1 - $\left(\frac{1}{20}\right)^3 = \frac{7999}{8000}$

(b)(i) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}\times 9^2 \times \frac{\pi}{3} = \frac{27\pi}{2}\text{ cm}^2$

(ii) $BD = 9\sin\frac{\pi}{6} = 4\cdot 5\ cm$ $\qquad CD = 9\cos\frac{\pi}{6} = 4\cdot 5\sqrt{3}\ cm$

Shaded Area $= \frac{27\pi}{2} - \frac{1}{2}BD \times CD = \frac{27\pi}{2} - \frac{1}{2}\times 4\cdot 5 \times 4\cdot 5\sqrt{3}$

$= \frac{27\pi}{2} - \frac{81\sqrt{3}}{8} = \frac{108\pi - 81\sqrt{3}}{8}\text{ cm}^2$

(c)(i) $\int \frac{2x}{x^2+3}dx = \ln(x^2+3) + C$

(ii) $\int \frac{e^{2x}}{e^{2x}+3}dx = \frac{1}{2}\ln(e^{2x}+3) + C$

Daniel Antone

Exam	MATHEMATICS : Question...... 15

Suggested Solutions	Marker's Comments

a) i $y = x^2 \quad y = 4x+5$

$x^2 = 4x+5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, -1$

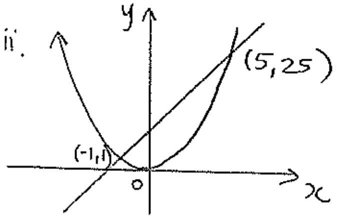
$x = 5 \quad y = 5^2 = 25$

$x = -1 \quad y = (-1)^2 = 1$

$(5,25)\ (-1,1)$ ✓

• remember to find the y value

ii.



✓ ✓

• label axes
• label points of intersection
• some students didn't show that lines intersect twice

iii $A = \int_{-1}^{5} 4x + 5 - x^2 \, dx$

$= \left[\frac{4x^2}{2} + 5x - \frac{x^3}{3}\right]_{-1}^{5}$

$= \left[2x^2 + 5x - \frac{x^3}{3}\right]_{-1}^{5}$

$= \left[\left(2(5)^2 + 5(5) - \frac{5^3}{3}\right) - \left(2(-1)^2 + 5(-1) - \frac{(-1)^3}{3}\right)\right]$

$= \left[\frac{100}{3} - -\frac{8}{3}\right]$

$= \frac{108}{3} = 36 \text{ units}^2$

b. i.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	$3/\sqrt{2}$	2	$\sqrt{2}/2$	-1

✓

answer in radians <u>not</u> degrees

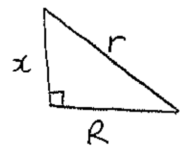
ii. $A = \frac{\frac{\pi}{2} - 0}{6}\left\{1 + 4\left(\frac{3}{\sqrt{2}}\right) + 2(2) + 4\left(\frac{\sqrt{2}}{2}\right) - 1\right\}$

$= \frac{\pi}{12} \cdot \left\{4 + 8\sqrt{2}\right\} = 4.009119\ldots = 4.01$

Exam	MATHEMATICS : Question......

Suggested Solutions	Marker's Comments

c i.



$R = \sqrt{r^2 - x^2}$

ii. $V = \pi R^2 h$ (cylinder)

✓ $h = 2x$

$V = \pi\left(r^2 - x^2\right) 2x$

✓ $= 2\pi x\left(r^2 - x^2\right)$

iii. $\frac{dV}{dx} = 2\pi\left(r^2 - x^2\right) + 2\pi x(-2x)$

$= 2\pi r^2 - 2\pi x^2 - 4\pi x^2$

$= 2\pi r^2 - 6\pi x^2 = 0$

$6\pi x^2 = 2\pi r^2$

$x^2 = \frac{r^2}{3}$

$x = \pm\frac{r}{\sqrt{3}}$ since $x > 0 \quad x = \frac{r}{\sqrt{3}}$

differentiate with respect to x

test for max volume

$\frac{d^2V}{dx^2} = -12\pi x = -\frac{12\pi r}{\sqrt{3}}$ when $x = \frac{r}{\sqrt{3}}$

$\therefore$ max volume since $\frac{d^2V}{dx^2} < 0$

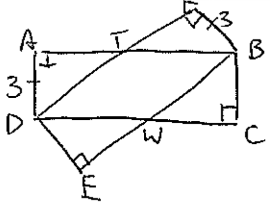
remember to test for max volume using $\frac{dV}{dx}$ or $\frac{d^2V}{dx^2}$

$V = 2\pi\left(\frac{r}{\sqrt{3}}\right)\left(r^2 - \left(\frac{r}{\sqrt{3}}\right)^2\right)$

$= \frac{2\pi r}{\sqrt{3}}\left(\frac{3r^2 - r^2}{3}\right) = \frac{2\pi r}{\sqrt{3}}\left(\frac{2r^2}{3}\right) = \frac{4\pi r^3}{3\sqrt{3}}$

$= \frac{4\sqrt{3}\pi r^3}{9}$

Suggested Solutions	Marker's Comments
Q16 a) Vertex $(2,-1)$ $(x-2)^2 = 4a(y+1)$ $\quad y = 2x-7$ ✓ $x^2-4x+4 = 4a(2x-6)$ $x^2-4x+4 = 8ax-24a$ $x^2-(4+8a)x+(4+24a)=0$ tangent, so $\Delta = 0$ $(4+8a)^2 - 4\times1\times(4+24a) = 0$ ✓ $16+64a+64a^2-16-96a=0$ $64a^2-32a=0$ $a(2a-1)=0$ $a=0,\ a=\frac{1}{2}$ $\therefore (x-2)^2 = 2(y+1)$ ✓	Students unfamiliar with the general form of the parabola $(x-h)^2=4a(y-k)$, issue with how $y=2x-7$ the tangent fit in with the parabola.
b) i) $Q = Q_0e^{-kt}$ $\frac{1}{2}Q_0 = Q_0e^{-kt}$, $t=1530$ $\ln(\frac{1}{2}) = -kt$ $k = \frac{-\ln(\frac{1}{2})}{1530}$ $k = 0.000453$ (3 sig. fig) ✓ or 4.53×10^{-4}	i) incorrect rounding for significant figures is a common error.
ii) $0.2 = e^{-kt}$ $\ln 0.2 = -kt$ $t = \frac{\ln 0.2}{-k}$ $t = \frac{-\ln 0.2}{\frac{-\ln(\frac{1}{2})}{1530}}$ $t = 3553$ ✓	ii) students should use exact value of k for further calculation.

Suggested Solutions	Marker's Comments
Q16 c)  i) in ΔATD and ΔBFT $\angle DAT = \angle TFB = 90°$ (angles of a rectangle) $DA = FB = 3$ (width of congruent rectangles) $\angle ATD = \angle FTB$ (vertically opposite angles) $\therefore \Delta ATD \equiv \Delta BFT$ (AAS) ✓ $\therefore AT = TF$ (corresponding sides of congruent Δs) $TD = TB$ (corresponding sides of congruent Δs) Let $AT = x$, $TB = 7-x = TD$ $3^2+x^2 = (7-x)^2$ ✓ $9+x^2 = 49-14x+x^2$ $14x = 40$ $x = \frac{20}{7}$ $\therefore AT = \frac{20}{7}$ ✓	Common Errors: i) * assumptions made without proof were common * pronumerals introduced without showing/explaining what they represent * invalid circular logic, ie. assuming $AT = \frac{20}{7}$, then workout DT, then go back to stating AT is true.
ii) $BT = 7-AT = 7-\frac{20}{7}$ $= \frac{29}{7}$ ✓ Area $= (7-\frac{20}{7})\times3$ $= \frac{87}{7}$ $= 12\frac{3}{7}$ ✓ (area of a parallelogram) OR Area $= 3\times7 - 2\times\frac{1}{2}(\frac{20}{7}\times3)$ $= 12\frac{3}{7}$ (area of rectangle ABCD − area of ΔADT and ΔBCW)	ii) * some students forgot to take area of 2 triangles. * incorrect area formula used is a common error

Exam Yr 12 2U TRIAL MATHEMATICS : Question 16 CHIRGWIN

Suggested Solutions	Marker's Comments
Q16 d) i) Driver cost $= 2\times 40 \times \frac{1000}{v}$ $= \frac{80000}{v}$ ✓ Petrol cost $= 1.40 \times (60 + \frac{v^2}{50}) \times \frac{1000}{v}$ $= \frac{84000}{v} + 28v$ ✓ Total cost $C = \frac{80000}{v} + \frac{84000}{v} + 28v$ $= 28v + \frac{164000}{v}$ ✓	i) students didn't draw the connecting time $= \frac{1000}{v}$, or forgetting that there are 2 drivers.
ii) $\frac{dC}{dv} = 28 - 164000v^{-2}$ $28 = 164000v^{-2}$ $v^2 = \frac{41000}{7}$ $v = \pm\sqrt{\frac{41000}{7}}$ ✓ $v = \pm 76.53\ldots$ $\frac{d^2C}{dv^2} = 328000v^{-3}$ at $v = +\sqrt{\frac{41000}{7}}$ $\frac{d^2C}{dv^2} = 0.73\ldots$ $v = -\sqrt{\frac{41000}{7}}$ $\frac{d^2C}{dv^2} = -0.73\ldots$ $\therefore$ minimum cost when $v = +\sqrt{\frac{41000}{7}}$ However time has to be less than 12 hours, $t = \frac{1000}{\sqrt{\frac{41000}{7}}} \doteqdot 13$ hours. $\therefore v = \frac{1000}{12}$ $v = 83\frac{1}{3}$ km/h ✓	Common Errors ii) ✗ incorrect differentiation ✗ wrote down 16400 instead of 164000, transcript error ✗ majority of students forgot to check whether the v (speed) they found satisfied all conditions given by the question.