## Student number

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## 2018

## TRIAL HSC EXAMINATION

## Mathematics

## General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen Black pen is preferred
- NESA approved calculators may be used
- A reference sheet is provided
- No liquid paper or correction tape allowed in this examination


## Total Marks - 100

## Section I:

10 marks

## Section II:

90 marks

- Attempt questions 11-16, on separate writing booklets.

| TOPICS | M/C | 11 | 12 | 13 | 14 | 15 | 16 | TOTAL |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Prelim | $/ 4$ | $/ 5$ | $/ 4$ | $/ 7$ |  | $/ 2$ |  | $/ 22$ |
| Geometry |  |  | $/ 3$ |  |  |  | $/ 6$ | $/ 9$ |
| Calculus | $/ 3$ | $/ 4$ |  | $/ 5$ | $/ 4$ | $/ 13$ |  | $/ 29$ |
| Trig |  |  |  |  | $/ 2$ |  | $/ 9$ | $/ 11$ |
| Exponential / logs | $/ 1$ | $/ 3$ |  | $/ 3$ |  |  |  | $/ 7$ |
| Series | $/ 1$ |  | $/ 4$ |  | $/ 5$ |  |  | $/ 10$ |
| Probability | $/ 1$ |  | $/ 4$ |  | $/ 4$ |  |  | $/ 9$ |
| Quadratics |  | $/ 3$ |  |  |  |  |  | $/ 3$ |
| TOTAL | $/ 10$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 15$ | $/ 100$ |

## Section I

## 10 marks

Attempt Questions 1 to 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 to 10

1. Simplify $(2-3 x)-(5-4 x)$
(A) $-7 x-3$
(B) $7 x+3$
(C) $x-3$
(D) $-x-3$
2. Forty-five balls, numbered 1 to 45 , are placed in a barrel, and one ball is drawn at random. What is the probability that the number on the ball drawn is even?
(A) $\frac{21}{45}$
(B) $\frac{22}{45}$
(C) $\frac{23}{45}$
(D) $\frac{24}{45}$
3. What are the coordinates of the focus of the parabola with equation $x^{2}=4(y-1)$ ?
(A) $(0,2)$
(B) $(0,-2)$
(C) $(0,1)$
(D) $(0,-1)$
4. Rationalise the denominator of $\frac{4}{\sqrt{5}-2}$
(A) $\frac{25-4 \sqrt{2}}{27}$
(B) $\frac{25+4 \sqrt{2}}{27}$
(C) $\frac{4 \sqrt{5}+8}{9}$
(D) $4 \sqrt{5}+8$
5. The third term of an arithmetic series is 32 and the sixth term is 17 . What is the sum of the first ten terms of the series?
(A) 195
(B) 197
(C) 200
(D) 205
6. What is the value of $f^{\prime}(2)$ if $f(x)=\frac{1}{3 x}$ ?
(A) $-\frac{1}{12}$
(B) $-\frac{1}{6}$
(C) $\frac{1}{3}$
(D) $-\frac{3}{4}$
7. The curve given by $y=7+4 x^{3}-3 x^{4}$ has a stationary point at $(0,7)$. What is the nature of this stationary point?
(A) Relative maximum
(B) Relative minimum
(C) Horizontal point of inflexion
(D) Not a stationary point
8. What is the solution of $5^{x}=4$ ?
(A) $x=\frac{\log _{e} 4}{5}$
(B) $x=\frac{4}{\log _{e} 5}$
(C) $x=\frac{\log _{e} 4}{\log _{e} 5}$
(D) $x=\log _{e}\left(\frac{4}{5}\right)$
9. Evaluate $\lim _{x \rightarrow 2} \frac{x^{3}-8}{x^{2}-4}$
(A) 1
(B) 2
(C) 3
(D) 4
10. The graph of a function $f$, where $f(-x)=f(x)$, is shown below.


The graph has $x$-intercepts at $(a, 0),(b, 0),(c, 0)$ and $(d, 0)$ only. The area bounded by the curve and the $x$-axis on the interval $a$ to $d$ is:
(A) $\int_{a}^{b} f(x) d x-\int_{c}^{b} f(x) d x+\int_{c}^{d} f(x) d x$
(B) $2 \int_{a}^{b} f(x) d x-2 \int_{c}^{b+c} f(x) d x$
(C) $2 \int_{a}^{b} f(x) d x+\int_{b}^{c} f(x) d x$
(D) $\int_{a}^{b} f(x) d x+\int_{c}^{b} f(x) d x+\int_{d}^{c} f(x) d x$

## Section II

90 marks
Attempt Questions 11 to 16
Allow about $\mathbf{2}$ hours $\mathbf{4 5}$ minutes for this section
Answer each question on separate answer booklets.
All necessary working should be shown in every question.
Question 11 (15 marks) Start a new writing booklet.
(a) Find the value of $\frac{1}{7+5 \times 3}$ correct to three significant figures.
(b) Simplify $\frac{x}{3}+\frac{3 x-1}{2}$
(c) Given that $\log _{a} b=2.75$ and $\log _{a} c=0.25$, find the value of:
(i) $\log _{a}\left(\frac{b}{c}\right)$
(ii) $\log _{a}(b c)^{2}$
(d) $\quad$ Solve $5-3 x<7$
(e) Differentiate $\left(3 x^{2}+4\right)^{5}$
(f)

## Find:

(i) $\int \sec ^{2} 6 x d x$
(ii) $\int_{1}^{e^{3}} \frac{5}{x} d x$
(g) $\quad$ The roots of the equation $x^{2}+4 x+1=0$ are $\alpha$ and $\beta$. Find:
(i) $\alpha+\beta$ and $\alpha \beta$
(ii) $\frac{1}{\alpha^{2}}+\frac{1}{\beta^{2}}$
(a)


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An instrument, similar to a xylophone has many bars, attached as shown in the diagram. The difference between the lengths of adjacent bars is a constant, so that the lengths of the bars are the terms of an arithmetic series.

The shortest bar is 30 cm long and the longest bar is 50 cm . The sum of the lengths of all the bars is 1240 cm .
(i) Find the number of bars.
(ii) Find the difference in the length between adjacent bars.
(b) (i) Draw the graphs of $y=|x|$ and $y=x+4$ on the same set of axes.
(ii) Find the coordinates of the point of intersection of these two graphs.
(c) Cameron and Jordan are playing golf. They will play two rounds and each has an equal chance of winning the first round.
If Cameron wins the first round, his probability of winning the second round is increased to 0.6 .
If Cameron loses the first round, his probability of winning the second round is reduced to 0.3.
(i) Draw a tree diagram for the two-round sequence. Label each branch of the diagram with the appropriate probability.
(ii) Find the probability that Cameron wins exactly one round.
(d)


A table top is in the shape of a circle with a small segment removed as shown. The circle has centre $O$ and radius 0.45 metres. The length of the straight edge is also 0.45 metres.
(i) Explain why $\angle X O Y=\frac{\pi}{3}$
(ii) Find the area of the table-top.

## End of Question 12

## Question 13 (15 marks)

(a) The graph of $y=f(x)$ passes through the point $(1,3)$ and $f^{\prime}(x)=3 x^{2}-2$. Find $f(x)$.
(b) A layer of window tinting cuts out $15 \%$ of the light and lets through the remaining $85 \%$.
(i) Show that two layers of the window tinting will let through $72.25 \%$ of the light.
(ii) How many layers of window tinting is required to cut out at least $90 \%$ of the light?
(c)


A glass has a shape obtained by rotating part of the parabola $x=\frac{y^{2}}{30}$ about the $y$ axis as shown. The glass is 10 cm deep.
Find the volume of liquid which the glass will hold.
(d) (i) Prove that the line $y=x+2$ is a tangent to the parabola $y=x^{2}-5 x+11$.
(ii) Let $Q$ be the point where the line $y=x+2$ touches the parabola $y=x^{2}-5 x+11$.

Show that the normal to the parabola at $Q$ is $y=-x+8$.
(iii) Find the area of the region enclosed between the parabola and the line $y=-x+8$.

## End of Question 13

(a) Iron is extracted from a mine at a rate that is proportional to the amount of coal remaining in the mine. Hence the amount $R$ remaining after $t$ years is given by

$$
R=R_{0} e^{-k t}
$$

where $k$ is a constant and $R_{0}$ is the initial amount of coal.
After 20 years, $50 \%$ of the initial amount of coal remains.
(i) Find the exact value of $k$.
(ii) How many more years will elapse before only $30 \%$ of the original amount remains?
(Round answer to the nearest month)
(b) By expressing $\sec \theta$ and $\tan \theta$ in terms of $\sin \theta$ and $\cos \theta$, show that

$$
\sec ^{2} \theta-\tan ^{2} \theta=1
$$

(c) Craig has invented a game for one person. He rolls two ordinary dice repeatedly until the sum of the two numbers shown is either 7 or 9 . If the sum is 9 , Craig wins. If the sum is 7 , Craig loses. If the sum is any other number, he continues to roll until it is 7 or 9 . Given that the probability of Craig winning on his first roll of the dice is $\frac{1}{9}$.
(i) What is the probability that Craig wins on his first, second or third roll? Leave your answer in unsimplified form.
(ii) Calculate the probability that Craig wins the game.
(d) On the $1^{\text {st }}$ of July 2008, Aryan invested $\$ 10000$ in a bank account that paid interest at a fixed rate of $3.2 \%$ per annum, compounded annually.
Aryan also added $\$ 1000$ to his account on the $1^{\text {st }}$ of July each year, beginning on the $1^{\text {st }}$ of July 2009.
(i) How much was in his account on the $1^{\text {st }}$ of July 2018 after the payment of interest and his deposit?
(ii) Aryan's friend, Raj, invested \$10 000 in an account at another bank on the $1^{\text {st }}$ of July 2008 and made no further deposits. On the $1^{\text {st }}$ of July 2018, the balance of Raj's account was \$13 857.
If interest was compounded annually, calculate the annual rate of compound interest paid on Raj's account?

## End of Question 14

## Question 15 (15 marks)

(a) Graph the solution of $4 x \leq 15 \leq-9 x$ on a number line.
(b)


A particle is observed as it moves in a straight line in the period between $t=0$ and $t=10$. Its velocity $v$ at time $t$ is shown on the graph above.
Copy this graph into your writing booklet.
(i) On the time axis, mark and clearly label with the letter $\boldsymbol{X}$ the times when the acceleration of the particle is zero.
(ii) On the time axis, mark and clearly label with the letter $\boldsymbol{H}$ the time when the acceleration is greatest.
(iii) There are three occasions when the particle is at rest, i.e. $t=0, t=7$, and $t=10$.

The particle is furthest from its initial position on one of these occasions. Indicate which occasion, giving reasons for your answer.
(c) Consider the function $y=\ln (x-2)$ for $x>2$.
(i) Sketch the function, showing its essential features.
(ii) Use Simpson's Rule with three function values to find an approximation to

$$
\int_{3}^{5} \ln (x-2) d x
$$

## Question 15 continues on the next page

(d) The diagram below shows the graphs of the functions $y=\cos 2 x$ and $y=\sin x$ between $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$. The two graphs intersect at $x=\frac{\pi}{6}$ and $x=-\frac{\pi}{2}$.
Calculate the area of the shaded region.


## End of Question 15

(a) A particle is moving along the $x$ axis. Its position at time $t$ is given by

$$
x=t+\sin t
$$

(i) At what times during the period $0<t<3 \pi$ is the particle stationary?
(ii) At what times during the period $0<t<3 \pi$ is the acceleration equal to 0 ?
(iii) Carefully sketch the graph of $x=t+\sin t$ for $0<t<3 \pi$.

Clearly label any stationary points and any points of inflexion.
(b)


In the diagram, $Q$ is the point $(-1,0), R$ is the point $(1,0)$, and $P$ is another point on the circle with centre $O$ and radius 1 . Let $\angle P O R=\alpha$ and $\angle P Q R=\beta$, and let $\tan \beta=m$.
(i) Given that $\triangle O P Q$ is isosceles, explain why $\alpha=2 \beta$.
(ii) Find the equation of the line $P Q$.
(iii) Show that the $x$-coordinates of $P$ and $Q$ are solutions of the equation

$$
\left(1+m^{2}\right) x^{2}+2 m^{2} x+m^{2}-1=0
$$

(iv) Using this equation, find the coordinates of $P$ in terms of $m$.
(v) Hence deduce that $\tan 2 \beta=\frac{2 \tan \beta}{1-\tan ^{2} \beta}$

## End of Question 16

## End of paper






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## (2)



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