

2018 TRIAL HSC EXAMINATION Mathematics

General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using blue or black pen Black pen is preferred
- NESA approved calculators may be used
- A reference sheet is provided
- No liquid paper or correction tape allowed in this examination

Total Marks – 100

Section I: 10 marks

Section II: 90 marks

• Attempt questions 11-16, on separate writing booklets.

TOPICS	M/C	11	12	13	14	15	16	TOTAL
Prelim	/4	/5	/4	/7		/2		/22
Geometry			/3				/6	/9
Calculus	/3	/4		/5	/4	/13		/29
Trig					/2		/9	/11
Exponential / logs	/1	/3		/3				/7
Series	/1		/4		/5			/10
Probability	/1		/4		/4			/9
Quadratics		/3						/3
TOTAL	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks Attempt Questions 1 to 10 Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

- 1. Simplify (2 3x) (5 4x)
 - (A) 7x 3
 - (B) 7x + 3
 - (C) x 3
 - (D) -x 3
- 2. Forty-five balls, numbered 1 to 45, are placed in a barrel, and one ball is drawn at random. What is the probability that the number on the ball drawn is even?
 - (A) $\frac{21}{45}$ (B) $\frac{22}{45}$ (C) $\frac{23}{45}$

 $(D)\frac{24}{45}$

- 3. What are the coordinates of the focus of the parabola with equation $x^2 = 4(y-1)$?
 - (A) (0,2)
 - (B) (0,−2)
 - (C) (0, 1)
 - (D) (0, -1)

4. Rationalise the denominator of $\frac{4}{\sqrt{5}-2}$

(A)
$$\frac{25-4\sqrt{2}}{27}$$

(B) $\frac{25+4\sqrt{2}}{27}$
(C) $\frac{4\sqrt{5}+8}{9}$
(D) $4\sqrt{5}+8$

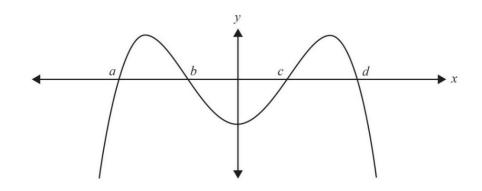
- 5. The third term of an arithmetic series is 32 and the sixth term is 17. What is the sum of the first ten terms of the series?
 - (A) 195
 - (B) 197
 - (C) 200
 - (D) 205
- 6. What is the value of f'(2) if $f(x) = \frac{1}{3x}$?
 - $(A) \frac{1}{12}$ $(B) \frac{1}{6}$
 - (C) $\frac{1}{3}$
 - (D) $-\frac{3}{4}$
- 7. The curve given by $y = 7 + 4x^3 3x^4$ has a stationary point at (0, 7). What is the nature of this stationary point?
 - (A) Relative maximum
 - (B) Relative minimum
 - (C) Horizontal point of inflexion
 - (D) Not a stationary point

8. What is the solution of $5^x = 4$?

(A)
$$x = \frac{\log_e 4}{5}$$

(B) $x = \frac{4}{\log_e 5}$
(C) $x = \frac{\log_e 4}{\log_e 5}$
(D) $x = \log_e \left(\frac{4}{5}\right)$

- 9. Evaluate $\lim_{x \to 2} \frac{x^3 8}{x^2 4}$ (A) 1 (B) 2 (C) 3 (D) 4
- 10. The graph of a function f, where f(-x) = f(x), is shown below.



The graph has x-intercepts at (a, 0), (b, 0), (c, 0) and (d, 0) only. The area bounded by the curve and the x-axis on the interval a to d is:

(A) $\int_{a}^{b} f(x)dx - \int_{c}^{b} f(x)dx + \int_{c}^{d} f(x)dx$ (B) $2\int_{a}^{b} f(x)dx - 2\int_{c}^{b+c} f(x)dx$ (C) $2\int_{a}^{b} f(x)dx + \int_{b}^{c} f(x)dx$ (D) $\int_{a}^{b} f(x)dx + \int_{c}^{b} f(x)dx + \int_{d}^{c} f(x)dx$

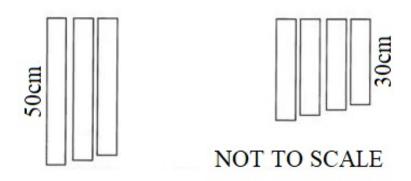
End of Section I

Section II 90 marks Attempt Questions 11 to 16 Allow about 2 hours 45 minutes for this section Answer each question on separate answer booklets. All necessary working should be shown in every question.

Ques	tion 1	1 (15 marks) Start a new writing booklet.	
(a)		Find the value of $\frac{1}{7+5\times3}$ correct to three significant figures.	1
(b)		Simplify $\frac{x}{3} + \frac{3x-1}{2}$	2
(c)	(i)	Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of: $\log_a \left(\frac{b}{c}\right)$	1
	(ii)	$\log_a(bc)^2$	2
(d)		Solve $5 - 3x < 7$	2
(e)		Differentiate $(3x^2 + 4)^5$	1
(f)		Find:	
	(i)	$\int \sec^2 6x dx$	1
		$\int_{1}^{e^{3}} \frac{5}{x} dx$	2
(g)		The roots of the equation $x^2 + 4x + 1 = 0$ are α and β . Find:	
	(i)	$\alpha + \beta$ and $\alpha\beta$	1
	(ii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$	2

End of Question 11

(a)



An instrument, similar to a xylophone has many bars, attached as shown in the diagram. The difference between the lengths of adjacent bars is a constant, so that the lengths of the bars are the terms of an arithmetic series.

The shortest bar is 30cm long and the longest bar is 50cm. The sum of the lengths of all the bars is 1240cm.

(i)	Find the number of bars.	2
(ii)	Find the difference in the length between adjacent bars.	2

2

2

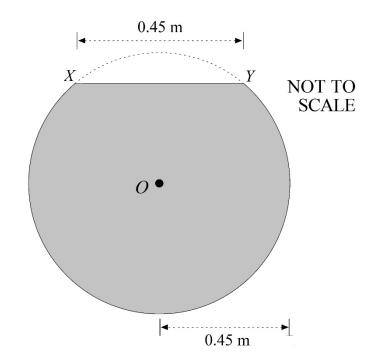
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- (b) (i) Draw the graphs of y = |x| and y = x + 4 on the same set of axes.
 - (ii) Find the coordinates of the point of intersection of these two graphs.

(c) Cameron and Jordan are playing golf. They will play two rounds and each has an equal chance of winning the first round.
 If Cameron wins the first round, his probability of winning the second round is increased to 0.6.
 If Cameron loses the first round, his probability of winning the second round is reduced to 0.3.

- (i) Draw a tree diagram for the two-round sequence. Label each branch of the diagram with the appropriate probability.
- (ii) Find the probability that Cameron wins exactly one round.

Question 12 continues on the next page



A table top is in the shape of a circle with a small segment removed as shown. The circle has centre O and radius 0.45 metres. The length of the straight edge is also 0.45 metres.

- (i) Explain why $\angle XOY = \frac{\pi}{3}$
- (ii) Find the area of the table-top.

End of Question 12

(d)

1 2 Start a new writing booklet.

(a) The graph of
$$y = f(x)$$
 passes through the point (1, 3) and $f'(x) = 3x^2 - 2$. Find $f(x)$. 2

(b) A layer of window tinting cuts out 15% of the light and lets through the remaining 85%.

(i) Show that two layers of the window tinting will let through 72.25% of the light.

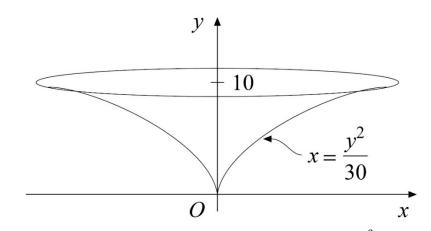
1

2

3

(ii) How many layers of window tinting is required to cut out at least 90% of the light?

(c)



A glass has a shape obtained by rotating part of the parabola $x = \frac{y^2}{30}$ about the y axis as shown. The glass is 10 cm deep. Find the volume of liquid which the glass will hold.

(d) (i) Prove that the line y = x + 2 is a tangent to the parabola $y = x^2 - 5x + 11$. 2

- (ii) Let *Q* be the point where the line y = x + 2 touches the parabola $y = x^2 5x + 11$. 2 Show that the normal to the parabola at *Q* is y = -x + 8.
- (iii) Find the area of the region enclosed between the parabola and the line y = -x + 8. 3

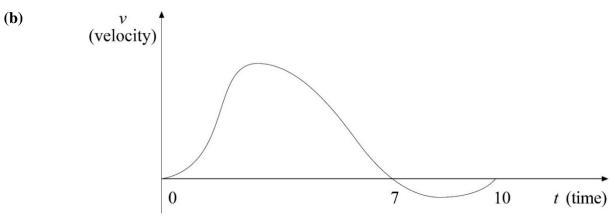
End of Question 13

Start a new writing booklet.

(a)		Iron is extracted from a mine at a rate that is proportional to the amount of coal remaining in the mine. Hence the amount <i>R</i> remaining after <i>t</i> years is given by $R = R_0 e^{-kt}$, where <i>k</i> is a constant and R_0 is the initial amount of coal.	
		After 20 years, 50% of the initial amount of coal remains.	
	(i)	Find the exact value of <i>k</i> .	2
	(ii)	How many more years will elapse before only 30% of the original amount remains? (Round answer to the nearest month)	2
(b)		By expressing $\sec \theta$ and $\tan \theta$ in terms of $\sin \theta$ and $\cos \theta$, show that $\sec^2 \theta - \tan^2 \theta = 1$	2
(c)		Craig has invented a game for one person. He rolls two ordinary dice repeatedly until the sum of the two numbers shown is either 7 or 9. If the sum is 9, Craig wins. If the sum is 7, Craig loses. If the sum is any other number, he continues to roll until it is 7 or 9. Given that the probability of Craig winning on his first roll of the dice is $\frac{1}{9}$.	
	(i)	What is the probability that Craig wins on his first, second or third roll? Leave your answer in unsimplified form.	2
	(ii)	Calculate the probability that Craig wins the game.	2
(d)		On the 1 st of July 2008, Aryan invested \$10 000 in a bank account that paid interest at a fixed rate of 3.2% per annum, compounded annually. Aryan also added \$1000 to his account on the 1 st of July each year, beginning on the 1 st of July 2009.	
	(i)	How much was in his account on the 1 st of July 2018 after the payment of interest and his deposit?	3
	(ii)	Aryan's friend, Raj, invested \$10 000 in an account at another bank on the 1 st of July 2008 and made no further deposits. On the 1 st of July 2018, the balance of Raj's account was \$13 857. If interest was compounded annually, calculate the annual rate of compound interest paid on Raj's account?	2

End of Question 14

(a) Graph the solution of $4x \le 15 \le -9x$ on a number line.



A particle is observed as it moves in a straight line in the period between t = 0 and t = 10. Its velocity v at time t is shown on the graph above. Copy this graph into your writing booklet.

- (i) On the time axis, mark and clearly label with the letter X the times when the acceleration 2 of the particle is zero.
- (ii) On the time axis, mark and clearly label with the letter H the time when the acceleration is 1 greatest.
- (iii) There are three occasions when the particle is at rest, i.e. t = 0, t = 7, and t = 10. The particle is furthest from its initial position on one of these occasions. Indicate which occasion, giving reasons for your answer.
- (c) Consider the function $y = \ln(x 2)$ for x > 2.
 - (i) Sketch the function, showing its essential features.

$$\int_3^5 \ln(x-2)\,dx$$

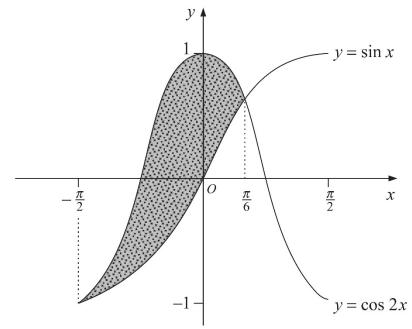
Question 15 continues on the next page

2

2

2

The diagram below shows the graphs of the functions $y = \cos 2x$ and $y = \sin x$ between $4x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. The two graphs intersect at $x = \frac{\pi}{6}$ and $x = -\frac{\pi}{2}$. Calculate the area of the shaded region.



End of Question 15

(a)

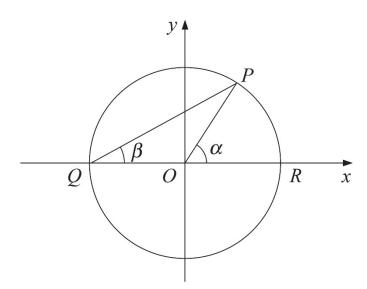
(b)

Start a new writing booklet.

A particle is moving along	g the x axis. Its position at time t is given by
	$x = t + \sin t$

- (i) At what times during the period $0 < t < 3\pi$ is the particle stationary? 2
- (ii) At what times during the period $0 < t < 3\pi$ is the acceleration equal to 0? 2
- (iii) Carefully sketch the graph of $x = t + \sin t$ for $0 < t < 3\pi$. 3

Clearly label any stationary points and any points of inflexion.



In the diagram, *Q* is the point (-1, 0), *R* is the point (1, 0), and *P* is another point on the circle with centre *O* and radius 1. Let $\angle POR = \alpha$ and $\angle PQR = \beta$, and let $\tan \beta = m$.

(i)	Given that $\triangle OPQ$ is isosceles, explain why $\alpha = 2\beta$.	1
(ii)	Find the equation of the line PQ.	1
(iii)	Show that the <i>x</i> -coordinates of <i>P</i> and <i>Q</i> are solutions of the equation $(1 + m^2)x^2 + 2m^2x + m^2 - 1 = 0$	2
(iv)	Using this equation, find the coordinates of P in terms of m .	2
(v)	Hence deduce that $\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta}$	2

End of Question 16

End of paper

 $(e, | A u = (3x^{2} + 4)^{5}$ $A = 5 \times (6x) \times (3x^{2} + 4)^{4}$ $A = 30 \times (3x^{2} + 4)^{4}$ (1) Examination: TRIAL HSC Level: 2unt Year: 2018 QUESTION: \((d) 5-3x<7 filijsec6xdx = Etmbx+c $\begin{array}{l} \text{(i)} \log_{n}(bc)^{2} = 2 \text{(log_{a}bc)} \\ = 2 \left(\log_{a}b + \log_{a}c\right) \\ = 2 \left(3\right) \\ = 6 \end{array}$ 1(m = 0, 0455 $(1) \int_{1}^{2} \frac{1}{2} dx = \left[5 \ln x \right]_{1}^{2} = 5 \left(\ln e^{3} - \ln 1 \right)$ $(G_{0}) \circ g_{a}(\frac{b}{2}) = \log_{a} b - \log_{a} c$ = 2.75 - Q.25 = 2.50 W=2× + 9×-3 $= \frac{1}{\sqrt{-3}}$ (\mathcal{G}) Э Θ ॥ ज 15×3 - 0 $\overline{\mathbb{C}}$ * st dets who wrote x < - 1/3 scored () Θ D got "9" as D they scored $\overline{\bigcirc}$ padly dane Markers Comments on answer * お

ت آر ن آر () y=n, y=n+4 Level: 2 Unit Maths Examination: Trially 2018 1) Two cases: (a) i) a= 30 1= 50 Ē Year: (2 QUESTION: 12 7 .. no solution ちょつ T3,= 30+(31-1)d = 50 カナン トン 1240= 40n 1240= <u>n</u> (80) $S_n = \frac{n}{2} (30+50)$ July Hand $h = \frac{1240}{40} = 31$... 30+300=50 300=20 _ '' Ч/ W ار م y= x and y=-x 5 = 1240 ;. Ø y=-n, y=×+4 -スニル+4 y= 2 : (-2,2) 3 -2 x = 4 2/3 cm. 1 1 bars ./ Dane - mostly well get d= 2% the 30 and Some students Done, swapped pretty Markers Comments <

1) 0.5 2 Year: 12 Level: 2 Unit Madus Examination: Tvid15 2018 .; Total area = 0.530+ 0.088 = 0.618m2 incomplete 11) Area of major sector X04 こ 1) P{cJ} + P{Jc] QUESTION: OR "even of minor segment, Total circle area = TT x 0.452 20.636 formula." = 1 (0.45) 2 x sin (1/3) 2 0.088 m2 .: Total area = 0.636-0.018 = 0.612m2 Area of minor segment = 1 (0.45)2 (The -sin The) Algeboro. Area of triangle XOY = 300 (T × 0.452) = 0.530 m2 · LXOY =TY3 .: XYO is an equilatered trimyte XY = XO = VY = 0.45m Since vaclius is o.45 m and = (0.5×0.4) + (0.5×0.3) = 0.35 y. 10.5 12 1.4 J 0:0 1 1 20.018 2/2 JC YY CJ CC Done well Students commen. the convect with choosing made mistakles Markers Comments formules. 5 5 5

Level: Year 12 Examination: Year 12 Mathematics Trial HSC Year: 2018 13a) $f'(x) = 3x^2 - 2$ b) (i) 0.85 x 0.85 = 0.7225 QUESTION: 13 Sub (1,3) into x3-2x+C (i) $f(x) = \int 3x^2 - 2 \, dx$:. 15 layers required $f(x) = x^3 - 2x + 4$ n log 0.85 < log 0.1 $=\frac{3x^3}{3}-2x+c$ 10g 0.85n ≤ 10g 0.1 :. c = 3+2-1 $3 = 1^{3} - 2 + C$ 0.85 " < 0.1 1. 72.25% n 3 log 0.1 2 14.168 109 0.85 working out to gain I mark. when finding C. working out is substantial. · need to show made silly enors accepted as long used trial & error, instead of integrating which is also marks. most students differentiated · Students · Some students · Some students Markers Comments samed full · Over all,

69.8 cm	$= \frac{\pi}{4500} (10000 - 0)$ $= \frac{200}{9} \pi 0r$	$= \frac{\pi}{900} \int_{0}^{10} \left(\frac{y^2}{30}\right)^2 dy$ $= \frac{\pi}{900} \int_{0}^{10} y^4 dy$	Examination: Year 12 Mathematics Trial HSC Level: $y \in 12$ Year: 2018 CUESTION: 13 C) $V = \pi \int_{0}^{10} x^{2} dy$ Mar
many strango just uxd units Although it was not penahised, but admiced to ute appropriate units.	V= TJ_ grdx · poor notation · 10cm inplied rrevolume would be cm3	as some students had torgotten the tormula when volume is rotated about y-axis. Many used	*1 HSC Markers Comments • This question was poorly arswed
	:. x=3 Only one intersection point y=2c+2 must be tangent.	y = x + 2 $g = x^{2} - 5x + 11$ $x + \lambda = x^{2} - 5x + 11$ $0 = x^{2} - 6x + 9$ $0 = (x - 3)^{2}$	Examination: Year 12 Mathematics Trial HSC Level: Yr12 Year: 2018 QUESTION: 13 d)(i) Jolve Simuthaneously to find interection:
interstandig mat i intersection point meant me line in a tangent.	1 solution, that was also amarded 2 marks. • Marks was penalised when student	simplitying equation. Jonne students used A=0 to show that there was only	HSC Markers Comments • Alot of silly errors when

 $(d) (ii') \quad \begin{array}{l} y = \chi^2 - 5\chi + 11 \\ \frac{dy}{d\chi} = 2\chi - 5 \end{array}$ Year: 2018 Examination: Year 12 Mathematics Trial HSC Level: yr 12 (iii) $y = x^2 - x + 8$ $y = x^2 - 5x + 11$ QUESTION: 13 $= (-q + 1f - q) - (-\frac{1}{3} + 2 - 3)$ $= \left[\frac{-x^3}{3} + 2x^2 - 3x \right]_{1}^{3}$ $= \int_{1}^{3} (-x^{2} + 4x - 3) dx$:. $\chi = 1, 3$ $A = \int_{1}^{3} (-\chi + \delta) d\chi - \int_{1}^{3} (\chi^{2} - 5\chi + 11) d\chi$ $-x + \delta = x^2 - 5x + 11$ when x = 3, $\frac{dg}{dy} = 1$ 4 = 1 3 square units $: M_{N} = -1$ (3,5) $y-5=-1(\chi-3)$ y-5=-x+3 $\therefore q = -x + \delta$ O = (x-3)(x-1) $0 = \chi^2 - 4\chi + 3$ Markers Comments

6 (a) $|i\rangle$ $k = k_0 e^{-\kappa t}$ Year: 2018 Level: 2 UNIT Examination: 2018 TRIAL HSC MATHEMATICS QUESTION: 14 in the when t = 20, $0.5 R_0 = R_0 e^{-20k}$ · LAS = RAS. LHS= Sec28 - tan28 . 14 years and months more $\begin{array}{c}
0.3 = e^{-kt} \\
e^{-kt} = 0.3 \\
-kt = lm(0.3) \\
t = lm(0.3) \\
t = -lm(0.3) \\
\frac{lm(1)}{20}
\end{array}$ 34.7-20 = 14.7 $0.3k_0 = R_0 e^{-kt}$ - cos26 2 00520 - Sin 20 t= 34 years 9 months $lm\left(\frac{1}{2}\right) = -20k$ 1 = e -20K $E = ln\left(\frac{1}{2}\right)$ 1/3 2 S did not shalat Markers Comments Was done students, well. Most This question well

	J.	(\tilde{J})	C)	
limiting own since $r = \frac{12}{18}$ where r	··	$P(n) = \frac{4}{36} = \frac{1}{9}$ $P(not a \ 7 \ or \ 9) = \frac{26}{36} = \frac{13}{18}$ $P(mns \ on \ first \ second \ or \ thind \ roll)$ $= \frac{1}{2} + \frac{13}{18} \times \frac{1}{7} + \frac{13}{18} \times \frac{13}{18} \times \frac{13}{18} \times \frac{1}{18}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Examination: TIRIAL HSC EXAM Level: 2 UNIF IMATIHEMATICS Year: 2018 $2/3$ QUESTION: 14 DIE 1 2/3
Sum exits but fixed to state 11/21 for limiting sum.	} Most students realised a limiting	This quarter		Markers Comments
3 2 6	in t			
	·		(L	
(1+r) = 15057 $(1+r)^{10} = (.3857) + (1+r)^{10} = (.3857) + (1+r)^{10} = (.3857) + (1+r)^{10} = (.0331587) + (1+r)^{10} = (.033157) + (1+r$	ount	$S_{10} = \frac{1}{(1.032)^{10}} - \frac{1}{(1.032)^{10}} $)	
(+r) =	$f_{mount} = [0000 (1.032)^{10} + 1000 (11.7700327) = 13702.41046 + 11570.0327 = $$25 272.44 = $$25 272.44$	$S_{10} = 1(1.032)^{10} - 1$ = 0.3762 41 046 = 1.032 1 = 1.032 2 = 11.5700327	$A = 10000 (1.032)^{10}$ $= 13.41046$ $A_{mount} = 10000 (1.032)^{10} + 10000 (1.032)^{10} + 10000 (1.032)^{10} + $	Examination: TRIAL ITSC Level: 2 UN IT Year: 2018 QUESTION: 14

Level: 2 Unit Marths Examination: Trial 2018 Year: 12. QUESTION: (\vec{x}) G (a) $4\chi \le 15 \le -9\infty$ (iii) Between t=0 & t=7, the particle is moving away from its initial position because its velocity is positive. $4x \le 15$ $x \le \frac{15}{4}$ $-\frac{15}{7} \ge x$ $x \le \frac{15}{5}$: the particle is furthest from its initial position at t=7. moving towards its initial position 0r x - - 15 0r x - - 15 because its relacity is negative. Between t=7 & t= 10, the particule is <> 5 0 ww 0 1/2 X 10,1 Markers Comments Level: 2 Unith Mathi Examination: Tricus 2018 Year: 12. (d) Area= $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos 2\pi - \sin \pi) d\pi \sqrt{\frac{\pi}{2}}$ = $\left[\frac{1}{2} \sin 2\pi + \cos \pi\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ QUESTION: $\binom{i}{i} \int_{3}^{3} \ln(\pi 2) d\pi = \frac{5-3}{6} \left[\ln 1 + 4 \ln 2 + \ln 3 \right]$ - 3V3 units ≠ 1.299 units $= \left(\frac{1}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - \left(\frac{1}{2}\sin(-\pi) + \cos(\frac{\pi}{2})\right)$ $= \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(0+0\right)$ (c) y= ln (x-2) for x>2 = 1.29 w/ or $\frac{1148}{3}$ $=\frac{1}{3}\left[0+4\ln 2+\ln 3\right]$ 5 4 2/2 y=(n(x-2) Markers Comments