Student number ______________________________

2019

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
  Black pen is preferred
- NESA approved calculators may be used
- A reference sheet is provided
- No liquid paper or correction tape allowed in this examination

Total Marks – 100

Section I: 10 marks

Section II: 90 marks

- Attempt questions 11-16, on separate writing booklets.

	Preliminary	Probability	Trigonometry	Exponential and Logarithms	Series and sequences	Applications to Calculus	Rates of Change Growth/Decay	Quadratics	Total
Multiple Choice	1, 8 /2	3 /1	4 /1	7 /1	9 /1	5, 6,10 /3		2 /1	/10
Q11	b /2	e /2		a /2	f /2			c , d /7	/15
Q12	d /2		a (iii), c /4	a(i), a(ii) b /7				e /2	/15
Q13	b /7		a(i) a(ii) /3				c /5		/15
Q14			a, b, c /12			d /3			/15
Q15				a /11			b /4		/15
Q16		a /3			b /5	c, d /7			/15
Total	/13	/6	/20	/21	/8	/13	/9	/10	/100

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

Q1. What is value of the following expression correct to 4 significant figures?

$$\sqrt{\frac{3\pi + 4e^2}{1.8 + 4ln2}}$$

(A) 2.92

(B) 2.920

(C) 2.919

(D) 2.9198

Q2. What are the solutions of $3x^2 - 5x + 3 = 0$?

(A) $x = \frac{5 \pm \sqrt{37}}{6}$

(B) $x = \frac{-5 \pm \sqrt{37}}{6}$

(C) No Real Solution

(D) $x = \frac{5 \pm \sqrt{13}}{6}$

Q3. Two ordinary dice are rolled. The score "is the sum of the numbers on the top faces". What is the probability that the score is 7?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

Q4. What are the solutions of $2sinx = 1$ for $0 \leq x \leq 2\pi$?

(A) $\frac{11\pi}{6}$ and $\frac{5\pi}{6}$

(B) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

(C) $\frac{4\pi}{3}$ and $\frac{7\pi}{3}$

(D) $\frac{5\pi}{6}$ and $\frac{\pi}{6}$

Q5. A table of values are given below for the curve $y = f(x)$.

It is given that $f(x)$ is continuous over the domain $5 \leq x \leq 13$.

x	5	7	9	11	13
$f(x)$	12	15	20	10	5

Which of the following is an **estimate** of $\int_5^{13} f(x)dx$ using Simpson's rule with these five values?

(A) 105

(B) 209

(C) 131

(D) 263

Q6. The displacement of a particle is given by $x = \cos 2t$, where t is the time in seconds. Which of the following is a possible expression for its acceleration?

(A) $-2\ sin2t$

(B) $-4\cos 2t$

(C) $4\cos 2t$

(D) $4\sin 2t$

Q7. What is the value of $\int_0^1 e^{3x+1} dx$?

(A) $\frac{1}{3}(e^4 - 3)$

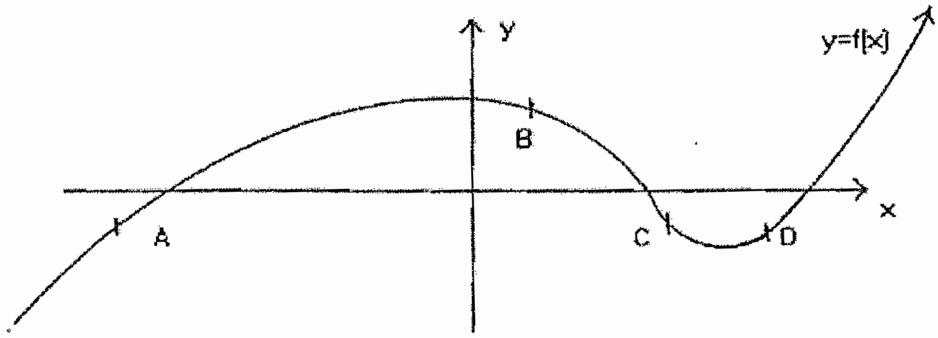
(B) $\frac{1}{3}(e^4 - 1)$

(C) $\frac{1}{3}e^4 - e$

(D) $\frac{1}{3}(e^4 - e)$

Q8. State which point on the sketch below fits the following description:

$$y < 0\,, \quad \frac{dy}{dx} > 0 \quad \text{and} \quad \frac{d^2y}{dx^2} < 0$$



(A) Point A

(B) Point B

(C) Point C

(D) Point D

Q9. For what range of values of x, does the geometric series

$1 + \frac{x}{2} + \frac{x^2}{2^2} + \frac{x^3}{2^3} + \ldots.\ \infty$, have a limiting sum?

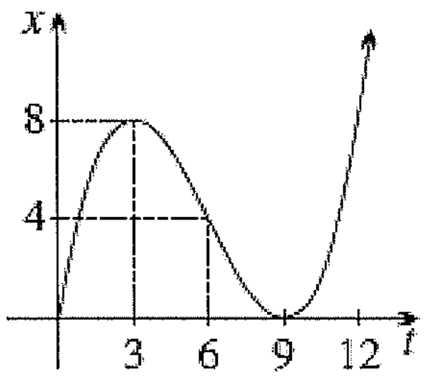
(A) $-1 \leq x \leq 1$

(B) $-2 \leq x \leq 2$

(C) $-2 < x < 2$

(D) It does not have a limiting sum.

Q10. A particle is moving horizontally so that its displacement, x metres, to the right of the origin at time t seconds is given by the graph shown below.



The time interval during which the particle's acceleration is negative, is given by

(A) $0 \leq t \leq 9$

(B) $0 \leq t < 6$

(C) $3 \leq t \leq 9$

(D) $9 \leq t < 12$

End of Section I

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Given that $log_a m = 3.62$ and $log_a n = 2.1$,

find the value, correct to two decimal places, of $log_a\left(\frac{m}{n}\right)$. *2*

b) Solve $|5x - 2| < 3$. *2*

c) The equation of the parabola is given by $x^2 + 2x = -4y + 3$.
Find the coordinates of its vertex and focus. 2

d) A quadratic function is given by $f(x) = 3x^2 + mx + 6$ and $f(-2) = 8$.

(i) Show that the value of m is given by 5. *1*

(ii) It is given that the roots of the equation $f(x) = 0$ are α and β.

Find the value of

a. $\alpha + \beta$ and $\alpha\beta$. *2*

b. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ *2*

Continues on next page ......

e) A red and a green die are thrown simultaneously.

Find the probability that the pair of dice show:

(i) at least one 6. *1*

(ii) a total of 5. *1*

(f) Find the value of *2*

$$\sum_{m=1}^{20} (2m+1)$$

End of Question 11

Continues on next page ......

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Differentiate the following:

(i) $y = ln(x^2 + 1)$ 2

(ii) $y = x^2 e^{-5x}$ 2

(iii) $y = \sin(x^2 - 3x)$ 2

b) Evaluate the following

$$\int_1^2 \frac{x}{x^2 + 3}\,dx$$

Leave your answer in exact form. 3

c) Explain, why the tangent to the curve $y = 2\sin x$ at $x = \frac{\pi}{2}$, is a horizontal tangent. 2

d) The graph of $y = f(x)$ passes through the point $(0, -1)$ and the gradient function of $f(x)$ is given by $f'(x) = 3x^2 - 5x + 4$.

Find $f(x)$. 2

e) For the quadratic expression $m^2x^2 - 6mx - 3$, show that there are no values of m for which the expression is positive definite. 2

End of Question 12

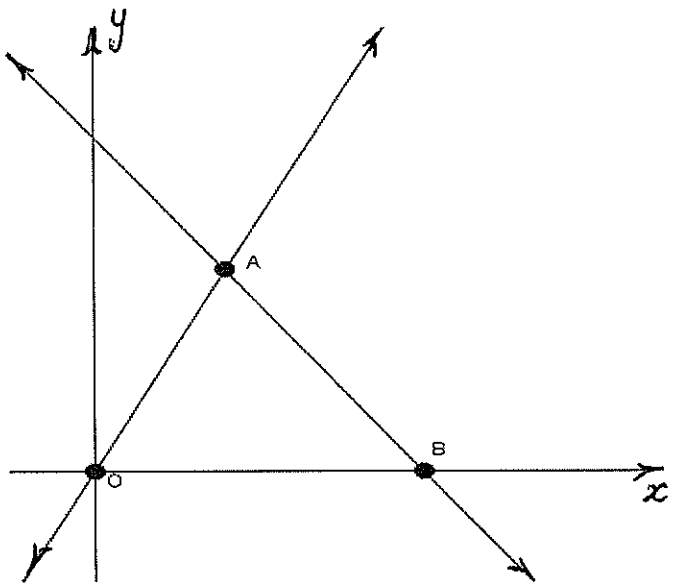
Continues on next page.......

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) (i) Differentiate $y = \ln(\cos x)$ **2**

(ii) Hence, find $\int_0^{\frac{\pi}{4}} tanx\, dx$. Leave your answer in exact form. *1*

b) In the diagram, OA is the line $y = 2x$, and AB is the line $4x + 3y - 40 = 0$.



Copy the diagram and answer the following:

(i) Find the coordinates of A and B. *2*

(ii) Show that the angle of inclination of the line AB to the nearest degree is 127^o. *2*

(iii) Write the three inequalities that satisfy the points that lie inside ΔAOB. *3*

Continues on next page ......

c) The emission of a particular gas since 2010 have been recorded according to the differential equation

$$\frac{dC}{dt} = kC$$

where C is the number of gigatonnes (Gt) of the gas emitted by burning fossil fuels.

During 2010, 9.19 *Gt* was emitted throughout the entire world. In 2015, this had risen to 9.9 Gt.

(i) Show that $C = C_0 e^{kt}$ is a solution to this differential equation. *1*

(ii) Find the value of C_0 and k, correct to 2 decimal places. *2*

(iii) Find the number of gigatonnes of the gas emitted in 2050 if no action is taken and assuming that emissions continue to grow at this rate, correct to two decimal places. *1*

(iv) Find the rate of increase (correct to two decimal places), of the gas emitted during the year 2050. *1*

End of Question 13

Continues on next page ......

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) Covert $\frac{2\pi}{5}$ radians to degrees. *1*

b) An arc AB of a sector of a circle is of length $\frac{\pi}{3}$ metres and subtends an angle of 30^o at the centre, O, of the circle.

(i) Show that the radius of the circle is 2 metres. *1*

(ii) Find the **exact** area of the sector AOB. *2*

(iii) Find the length of the chord AB. Give your answer correct to two decimal places. *2*

c) Consider the curve given by $y = 2\cos(2x)$.

(i) State the amplitude and the period of the curve. *2*

(ii) Sketch the graph of $y = 2\cos(2x)$ for $0 \leq x \leq \pi$.
Your diagram should take up at least one-third of the page. *2*

(iii) On the same diagram, draw the line $y = \frac{2x}{\pi}$. *1*

(iv) Hence, find the total number of solutions to the equation $\pi \cos(2x) = x$. *1*

d) Consider the curve given by $y = \sqrt[4]{x+1}$.

Find the exact volume of the solid of revolution formed when the region bounded by the curve $y = \sqrt[4]{x+1}$ and the y –axis from $x = 0$ to $x = 15$ is rotated about the y –axis. *3*

End of Question 14

Continues on next page ……

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) Consider the curve given by $y = e^{-x}(1 - x)$.

(i) Find the x −intercept and y − intercept. *2*

(ii) Show that $y'(x) = -e^{-x}(2 - x)$ and $y''(x) = e^{-x}(3 - x)$. *3*

(iii) Find the stationary point of the curve and determine its nature. *2*

(iv) Also, find the point of inflexion, if any. *2*

(v) Hence, sketch the curve by labelling all the essential features including showing the behaviour of the curve as $x \to \pm\infty$. *2*

b) The value, V, of a machine which loses value over time, t years, is given by $V = V_0 e^{-kt}$, where V_0 was the value of the machine when new and k is a constant, $k > 0$.

(i) The new price of the machine was \$285 000 and it loses 13% of its value in the first year.

Use this information to show that $k = 0.139$. *1*

(ii) Find how much as a percentage the machine drops in value in five years' time? Answer to the nearest whole number. *1*

(iii) The value of the machine continues to fall by 13% each year. Once the machine's value is less than 10% of its original price, the company can write the machine off.

After how many years and months can the machine be written off? Round up to the nearest month. *2*

End of Question 15

Continues on next page ......

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) A certain soccer team has a probability of 0.5 of winning a match and a probability of 0.2 of drawing a match.

If the team plays two matches, find the probability that:

(i) both matches will be a draw. *1*

(ii) the team wins at least one match. *2*

b) Kevin has borrowed \$17 000 to buy a used car. The interest on the loan is 18% p.a. paid monthly. The loan is to be repaid in equal monthly instalments of \$M over a term of 5 years.

Let the amount owing on the loan after n months be $\$A_n$.

(i) Show that the amount

$$A_3 = \$\{(17\,000 \times 1.015^3) - M(1 + 1.015 + 1.015^2)\}$$ *2*

(ii) Calculate the monthly instalment $\$M$ paid on the loan. *2*

(iii) How much would Kevin have saved by paying cash for the car? *1*

c) A particle is moving in a straight line with its displacement x metres from a fixed point at time t seconds given by $x(t) = t^3 + t^2 - 5t - 2$, $t \geq 0$.

(i) Find the expression for the velocity and the acceleration of the particle. *2*

(ii) In what direction is the particle initially moving? *1*

(iii) Find the time when the particle comes to rest. *2*

Continues on next page.......

d) A particle moves in a straight line so that its displacement, in metres, is given by:

$$x = \frac{2t - 1}{t + 1}$$

where t is measured in seconds.

Is the particle ever at rest? Justify your response. *2*

END of Question 16

END OF EXAMINATION

2 Unit Trial – 2019

(1)

Multiple Choice

1) B 2) C 3) B Q4) D Q5) A

Q6) B Q7) D Q8) A Q9) C Q10) B

Q11

a) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$ ✓

$= 3.62 - 2.1$

$= 1.52$ ✓

Many students did it correctly. Only a very few divided the two quantities

b) $-3 < 5x - 2 < 3$ ✓

$-1 < 5x < 5$

$-\frac{1}{5} < x < 1$ ✓

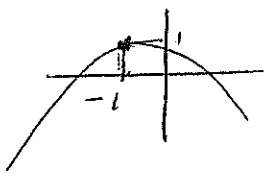
OR, deduct 1 if $x < 1$ ~~and~~ $x > -\frac{1}{5}$ (1) or

c) $x^2 + 2x + 1 = -4y + 3 + 1$

$(x+1)^2 = -4(y-1)$

vertex $(-1, 1)$, $4a = 4 \Rightarrow a = 1$

Focus $(-1, 0)$



Some students had difficulty completing the square. As a result had incorrectly found Vertex and Focus

d) $f(-2) = 8$

(i) $3(-2)^2 + m(-2) + 6 = 8$

$12 - 2m + 6 = 8$

$-2m = 8 - 18 = -10$

$\boxed{m = 5}$ ✓

well done

(ii) $f(x) = 3x^2 + 5x + 6$, α, β

a. $\alpha + \beta = -5/3$ ✓, $\alpha\beta = 2$ ✓

b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\frac{25}{9} - 4}{2}$ (1)

$= \frac{-11}{18}$ ✓

Some students did incorrectly $\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$.

11 e) $P(6,6) + P(6,\text{Not Six}) + P(\text{Not Six}, 6)$

(i) $= \frac{1}{36} + \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6}$

$= \frac{11}{36}$ ✓

(ii) $P(1,4) + P(4,1) + P(2,3) + P(3,2)$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$ ✓

$= \frac{1}{9}$

well done

f) $\sum_{m=1}^{20} (2m+1)$

$= 2(1+2+ - - + 20) + (20 \cancel{\times 1})$

$= \cancel{2}\left(\frac{1+2\cancel{0}}{\cancel{2}}\right) \times 20 + 20$

$= 2\cancel{1} \times 20 + 20$

$= 4\cancel{2}0 + 20$

$= 440$.

some students used the sum of an Arithmetic Progression incorrectly

—.

Question 12

a) $y = \ln(x^2+1)$

(i) $y' = \dfrac{1}{x^2+1} \times 2x$ ✓

$= \boxed{\dfrac{2x}{x^2+1}}$ (2)

(ii) $y = x^2 e^{-5x}$

$u = x^2 \quad v = e^{-5x}$

$u' = 2x \quad v' = -5e^{-5x}$

$\boxed{y' = (2x - 5x^2)e^{-5x}}$ (2) or $\boxed{y' = xe^{-5x}(2-5x)}$

(iii) $y = \sin(x^2 - 3x)$

$y' = \cos(x^2-3x) \times (2x-3)$

$= (2x-3)\cos(x^2-3x)$ (2) ✓ ✓

[Some students forget to put brackets]

b) $\dfrac{1}{2}\displaystyle\int_1^2 \frac{2 \times x}{x^2+3}\,dx$

$= \dfrac{1}{2}\left[\ln(x^2+3)\right]_1^2$

$= \dfrac{1}{2}[\ln 7 - \ln 4]$

$= \dfrac{1}{2}\left[\ln \dfrac{7}{4}\right]$

both acceptable (3)

c) $y = 2\sin x$

$y' = 2\cos x$ at $x = \frac{\pi}{2}$ ✓

$y' = 2\cos\frac{\pi}{2} = 0 \quad (m = 0)$

$\therefore$ the tangent to the curve at $y = \pi/2$ is a horizontal tangent. ✓ (2)

(Comment is required)

d) $f'(x) = 3x^2 - 5x + 4$

$f(x) = \frac{\cancel{3}x^3}{\cancel{3}} - \frac{5x^2}{2} + 4x + c$ ✓

at $x = 0$, $f(0) = -1$

$-1 = (0) - \frac{5}{2}(0)^2 + 4(0) + c$

$c = -1$ ✓

2

$\therefore \boxed{f(x) = x^3 - \frac{5}{2}x^2 + 4x - 1}$

e) $m^2x^2 - 6mx - 3$;

$\Delta: b^2 - 4ac$

$= (6m)^2 - 4(m^2)(-3)$

2

$= 36m^2 + 12m^2$ ✓

$= 48m^2$

≥ 0 for all values of m (as m^2 is always positive or zero)

$\therefore$ there exists no value of m, so that the expression is a positive definite ($\Delta < 0$).

✓

(lose 1 mark for not indicating m^2 is always positive)

Question 13

a) (i) $y = \ln \cos x$, term 1

$y' = \frac{1}{\cos x} \times -\sin x$] (2)

$= -\tan x.$

Done well

(ii) integrate the above,

$\int_0^{\frac{\pi}{4}} \tan x \, dx = \int_0^{\pi/4} -y' \, dx$

$= -\left[\ln \cos x\right]_0^{\pi/4}$

$= -\left[\ln \frac{1}{\sqrt{2}}\right]$

$= \ln\sqrt{2}$ or $\frac{1}{2}\ln 2.$] (1)

b) $4x + 3y - 40 = 0$, $y = 2x$.

for point of intersection, A:

$4x + 6x - 40 = 0$

$10x - 40 = 0 \Rightarrow x = 4, \quad y = 8$

(i) $\boxed{A(4,8)}$ (1)

the line $4x + 3y - 40 = 0$ cuts the x-axis at point B, $(y = 0)$

Done well, good labelling

$4x - 40 = 0 \Rightarrow x = 10$

$\boxed{B(10,0)}$ (1)

(ii) $m_{AB} = \frac{0-8}{10-4} = \frac{-8}{6} = \frac{-4}{3}$ ✓

$\theta = \tan^{-1}\left(-\frac{4}{3}\right)$ $(53°) \rightarrow$ (1) or $-53°$

(obtuse angle with positive x-axis)

$= 180° - \tan^{-1}\frac{4}{3}$

$= 127°$ ✓ (1)

(iii) check points inside $\triangle AOB$, say $(4,4)$

① $\boxed{y < 2x}$ at $(4,4)$ $\quad (\because 4 < 2\times 4 = 8)$

① $\boxed{y > 0}$ as $(4 > 0)$

NOTE: No. of students doing $0 < x < 40$, $0 < y < 8$., no marks awarded because that's a rectangle

$3y = -4x + 40$

$y = \frac{-4}{3}x + \frac{40}{3}$ at $x = 4$, (RHS. $-\frac{4}{3}\times 4 + \frac{40}{3}$
$= \frac{24}{3} = 8 > y$.)

$y < -\frac{4}{3}x + \frac{40}{3}$

① $\boxed{3y + 4x < 40}$ or $y < -\frac{4}{3}x + \frac{40}{3}$

c) (i) $C = C_0 e^{kt}$

$\frac{dC}{dt} = k(C_0 e^{kt})$

$= kC$ ①

$\therefore C = C_0 e^{kt}$ satisfies the differential equation

$\frac{dC}{dt} = kC.$

(ii) $C_0 = 9.19$ ① $(t = 0)$

at $t = 5$, $C = 9.9$ Gt

$9.9 = 9.19\, e^{k\times 5}$

$\frac{9.9}{9.19} = e^{k5}$, $\ln\left(\frac{9.9}{9.19}\right) = 5k$

$k = \frac{1}{5}\ln\left(\frac{990}{919}\right) =$ (exact value) If not converted to decimal, -1.

$= 0.01488\ldots$ ①

$= 0.01$ (2 dp)

(iii) $C = 9.19 \times e^{\frac{1}{5}\ln\left(\frac{990}{919}\right)\times 40}$ ①

$= 16.67$ (2dp)

Many students didn't use

(iv) $\frac{dC}{dt} = kC$ at $t = 40$ yea

$= 0.25$ (2dp)

① Gt per year.

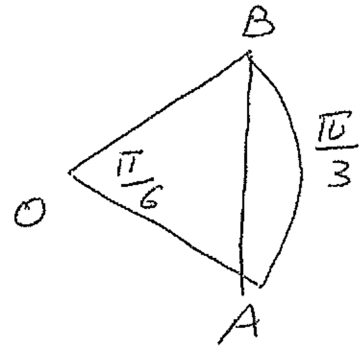
0.14 if rounded.

CFE ✓

Question 14 (7)

a) $\frac{2\pi}{5} = \frac{2 \times 180}{5} = 72^\circ$ 1

b) $l = r\theta$ $\quad 30^\circ = \frac{\pi}{6}$



(i) $\frac{\cancel{\pi}}{\cancel{3}} = r \times \frac{\cancel{\pi}}{\cancel{6}\,2}$ 1

$r = 2$ metres

(ii) Area of Sector AOB

$= \frac{1}{2} r^2 \theta$

$= \frac{1}{\cancel{2}} \times 2^{\cancel{2}} \times \frac{\pi}{6}$ 1

$= \frac{\pi}{3}\ m^2$ 1

(iii) using cosine rule in ΔOAB

$AB^2 = OA^2 + OB^2 - 2\,OA \cdot OB \cos\frac{\pi}{6}$

$= 2^2 + 2^2 - 2 \times 2 \times 2 \times \frac{\sqrt{3}}{2}$ ✓ 1

$AB^2 = 8\left(1 - \frac{\sqrt{3}}{2}\right)$

$|AB| = \sqrt{8\left(1 - \frac{\sqrt{3}}{2}\right)} = 1.07$ 1

$= 1.04$ m. (2 dp) ✓

(iv) $y = 2\cos 2x$ 1

2 (i) amplitude $= 2$, period $= \pi$ 1 radians 1

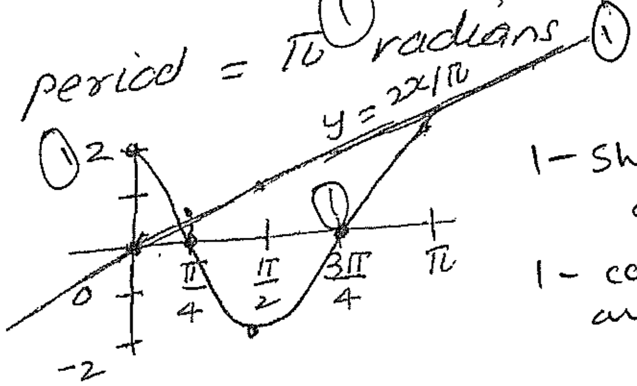
(ii) 2

(iii) 1

(iv) $2\cos 2x = \frac{2x}{\pi}$

$\pi \cos 2x = x$

$\boxed{3}$ 1



1 – Shape and domain

1 – correct A and period.

Many students only found 2.

d) $y = (x+1)^{1/4}$

$V = \pi \int x^2 \, dy$

$y^4 = x+1$, $x = 0,\ y^4 = 1 \Rightarrow y = 1$
$x = 15,\ y^4 = 16 \Rightarrow y = 2$

$x = y^4 - 1$

1 $x^2 = (y^4 - 1)^2 = y^8 - 2y^4 + 1$

$V = \pi \int_1^2 (y^8 - 2y^4 + 1) \, dy$ 1

$= \pi \left[\frac{y^9}{9} - 2\frac{y^5}{5} + y \right]_1^2$

$= \pi \left[\frac{2^9}{9} - \frac{2^6}{5} + 2 - \frac{1}{9} + \frac{2}{5} - 1 \right]$

$= \pi \cancel{\left[\frac{2^9}{9} - \frac{(2^6-1)}{5} + 1 \right]}$

$= \pi \left[\frac{2^9}{9} - \frac{2^6}{5} + \frac{58}{45} \right] u^3$ 1

$\left[\frac{2042}{45} \pi u^3 \right]$

$1 - \frac{1}{9} + \frac{2}{5}$

$\frac{45 - 5 + 18}{18}$

63
-5
58

Many algebra, integration and formula mistakes.
Some didn't use $\pi \int x^2 dy$ and some didn't find the real bounds.

Question 15

a) $y = e^{-x}(1-x)$

i) x-intercept, $y = 0$ $1-x=0 \Rightarrow \boxed{x=1}$ ✓ e^{-x} never zero. (2)

✓ $\boxed{(1,0)}$

y-intercept, $x=0$, $y = e^{0}(1-0) = 1$ $\boxed{y=1}$ ✓

✓ $\boxed{(0,1)}$

(ii) $y = e^{-x}(1-x)$ $u = e^{-x}$ $v = 1-x$
$u' = -e^{-x}$ $v' = -1$

✓ $y' = -e^{-x} - e^{-x}(1-x)$

✓ $= e^{-x}(x-2)$ or $-(2-x)e^{-x}$.

Differentiate again, $u = 2-x$, $v = e^{-x}$
$u' = -1$ $v' = -e^{-x}$

✓ $y'' = -\left[-e^{-x} - e^{-x}(2-x)\right]$

$= -\left[-3e^{-x} + xe^{-x}\right]$ (3)

$= (3-x)e^{-x}$

(iii) For stationary point, $y' = 0$

$\Rightarrow \boxed{x=2}$ as e^{-x} never zero,

to determine its nature, (2)

y'' at $x=2$ $= (3-2)e^{-2} > 0$

$\therefore \boxed{(2, -e^{-2})}$ is a point of $\boxed{\text{minima}}$

(iv) For point of inflexion, $y'' = 0$

e^{-x} never zero. (2)

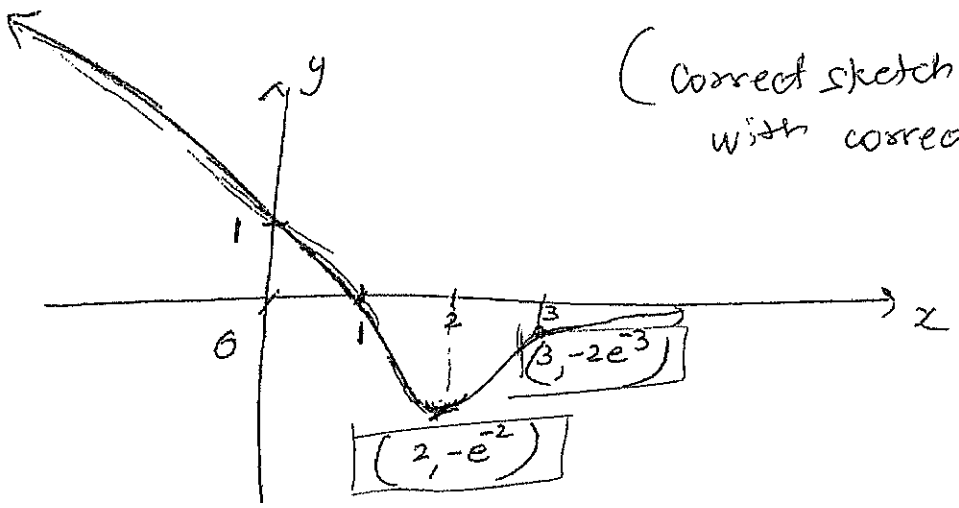
$\Rightarrow x = 3$

x	2.5	3	3.5
y''	0.04	0	−0.015
	>0		<0

$\therefore \boxed{(3, -2e^{-3})}$ is a point of inflexion

(Some students did not write numerical values. Lose 1 mark)

(10)



(Correct sketch required with correct label)

(2)

(b) $V = V_0 e^{-kt}$

$V_0 = 285000$, 13%

(i) $0.87 = e^{-k \times 1}$

$-\ln(0.87) = k$. (1)

$k = 0.139$

(ii) $V = 285000\, e^{-k \times 5}$

$= 142049.96$ (2dp)

$\boxed{\text{drops by } 50\%}$ (1)

(iii) $\frac{V}{V_0} = 0.1 = e^{-kt}$

$t = -\frac{1}{k} \ln(0.1)$ (2)

always use exact value $= \frac{1}{\ln(0.87)} \ln(0.1)$

$=$ 16 years 6 months (approx.) or 7 months

2u Trial – 2019

Q16. P(winning) = 0.5   P(drawing) = 0.2   P(losing) = 0.3

(i) P(Draw and Draw) = $0.2 \times 0.2 = 0.04$ (1)

(ii) P(wins at least one match)

$= P(WW) + P(WD) + P(DW) + P(WL) + P(LW)$

$= (0.5 \times 0.5) + 2(0.5 \times 0.2) + 2(0.5 \times 0.3)$ (1)

$= 0.25 + 0.2 + 0.3$

$= 0.75$ (1)

or P(wins at least 1 match) = 1 − P(wins no matches)

$= 1 - \left(\frac{1}{2} \times \frac{1}{2}\right)$ (1)

$= 1 - \frac{1}{4}$

$= 3/4$ (1)

(b) (i) let A_n = amount owing after n^{th} repayment

$\therefore A_1 = 17000(1 + 0.015) - M$

$A_1 = 17000(1.015) - M$

$A_2 = A_1 \times 1.015 - M$

$\therefore A_2 = [17000(1.015) - M] \times 1.015 - M$

(1) $= 17000(1.015)^2 - M(1.015) - M$

$A_3 = A_2(1.015) - M$

had to have more than 3 lines overall for (1)

$= [17000(1.015)^2 - M(1.015) - M] \times (1.015) - M$

$= 17000(1.015)^3 - M(1.015)^2 - M(1.015) - M$

$= 17000(1.015)^3 - M[(1.015)^2 + 1.015 + 1]$

(ii) 5 years = 60 months

$A_{60} = 17000(1.015)^{60} - M(1 + 1.015 + 1.015^2 + \dots + 1.01$

but $A_{60} = 0$ (loan is paid off)

$\therefore 0 = 17000(1.015)^{60} - M(1 + 1.015 + \dots + 1.015^{59})$

G.P. where $a = 1$, $r = 1.015$, $n = 60$

(1) $17000(1.015)^{60} = M\left(\frac{1.015^{60} - 1}{1.015 - 1}\right)$

$\frac{41534.73619}{96.21} = M$

(1) $\therefore M = \$431.69$ (2 dp as its money!!).

(iii) Total money paid $= \$431.69 \times 60$

$= \$25901.~~4~~0$

Money saved $= \$25901.40 - 17000$

$= \$8901.40$ (1)

(c) (i) $x = t^3 + t^2 - 5t - 2$

$v = 3t^2 + 2t - 5$ (1)

$a = 6t + 2$ (1)

(ii) at $t = 0$, $v = 0 + 0 - 5 = -5$ (1)

$\therefore$ the particle is moving left.

(A lot of students subbed into x, and found where it was initially. Or they stated it was moving backwards)

They all scored zero marks.

(iii) velocity $= 0$ when particle is at rest.

ie $3t^2 + 2t - 5 = 0$

$(t-1)(3t+5) = 0$

$t = 1$ or $-5/3$ 1

but $t \geq 0 \therefore t = 1$ only 1

as time is positive

* If students didn't find 2 answers and explain why they eliminated one solution, they only scored a maximum of 1 mk.)

d) $x = \dfrac{2t-1}{t+1}$

$$\frac{dx}{dt} = \frac{(t+1)2 - (2t-1)\times 1}{(t+1)^2}$$

$$= \frac{2t+2-2t+1}{(t+1)^2}$$

$$= \frac{3}{(t+1)^2}$$ 1

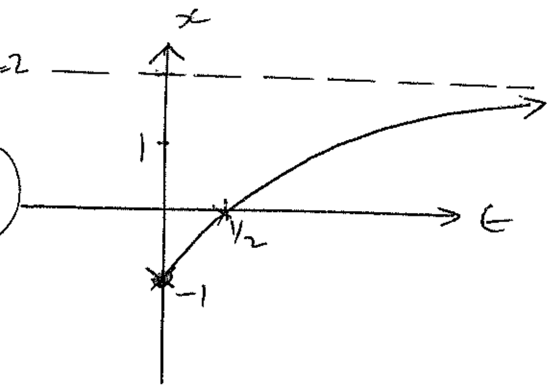
comes to rest when $\frac{dx}{dt} = 0$

ie $0 = \dfrac{3}{(t+1)^2}$

1 $0 \neq 3$

since the velocity is never zero, the particle is always moving, so it's never at rest.

or

1 

1 From the displacement time graph, we can see the particle is approaching $x=2$, but never reaches it, slowing down but never stopping.