Student's name ____________ Student's number ____________ Teacher's name ____________

PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2008

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

1	2	3	4	5	6	7	8	9	10	Total	Total
										/120	%

Question 1 (12 marks) **Start a new sheet of writing paper** **Marks**

(a) Calculate $\sqrt{\dfrac{135.2}{12.3\times 2.1}}$ correct to 2 decimal places. **2**

(b) Factorise completely $5x^3+135$. **2**

(c) Find integers a and b such that $\dfrac{3}{2\sqrt{2}+3}=a-\sqrt{b}$. **2**

(d) Solve $|x-3|\geq 5$. **2**

(e) Solve $x(2x+7)=4$. **2**

(f) Prove that **2**

$$\frac{1}{\sec x-1}-\frac{1}{\sec x+1}=2\cot^2 x.$$

End of Question 1

Question 2 (12 marks) Start a new sheet of writing paper

Marks

(a) Differentiate with respect to x

(i) $x^2 \tan x$ **2**

(ii) $\dfrac{\log_e x}{x}$ **2**

(b) (i) Find $\int \sin 4x \, dx$ **2**

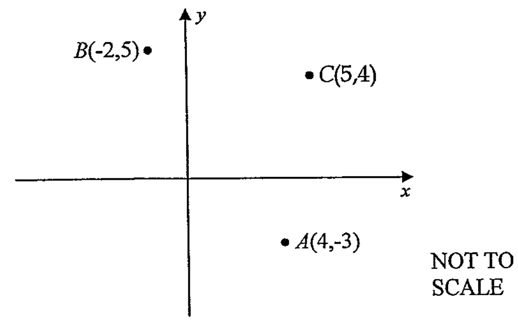
(ii) Evaluate $\displaystyle\int_0^1 \frac{x^2}{x^3+1} dx$ **3**

(c) Using Simpson's rule with five function values, find an approximation for $\displaystyle\int_0^{\pi} \cos^2 x \, dx$. **3**

End of Question 2

Question 3 (12 marks) Start a new sheet of writing paper

Marks



In the diagram, A, B, and C are the points (4, -3), (-2,5) and (5,4) respectively.

Copy or trace this diagram onto your writing paper.

(a) Show that triangle ABC is isosceles. **2**

(b) Use the cosine rule to find the size of $\angle ACB$. **2**

(c) Find the midpoint of AB. **1**

(d) Show that the perpendicular bisector of AB has equation $3x - 4y + 1 = 0$. **2**

(e) Show that $C(5,4)$ lies on the perpendicular bisector. **1**

(f) A circle is drawn through A and B with C as the centre.

(i) Find the equation of this circle. **1**

(ii) Find the length of the arc AB. **1**

(iii) Find the area of the segment cut off by the chord AB. **2**

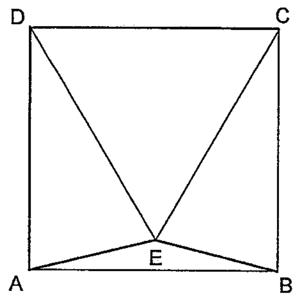
End of Question 3

Question 4 (12 marks) **Start a new sheet of writing paper** **Marks**

(a) Solve $3\tan^2\theta = 1$ for $0 \le \theta \le 2\pi$ **3**

(b) Solve $e^{2x} - 4e^x + 3 = 0$. **3**

(c)



In the diagram, $ABCD$ is a square and CDE is an equilateral triangle.

(i) Copy the diagram onto your answer sheet.

(ii) Prove that $\Delta AED \equiv \Delta BEC$. **3**

(iii) Hence, or otherwise, prove that $AE = BE$. **1**

(iv) Find the size of $\angle AEB$. Give reasons for your answer. **2**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper** **Marks**

(a) A function is defined as $f(x) = 12x - x^3$.

(i) Find the co-ordinates of the stationary point(s) of $y = f(x)$ and determine their nature. **3**

(ii) Find the co-ordinates of any point(s) of inflection of $y = f(x)$ **2**

(iii) Sketch the graph of $y = f(x)$, clearly showing intercept(s), stationary point(s) and point(s) of inflection. **2**

(iv) Find the value(s) of x for which the function, $y = f(x)$, is both increasing **and** concave up. **1**

(b) Let α and β be the roots of $2x^2 - 15x + 6 = 0$.
Find the values of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(2-\alpha)(2-\beta)$ **2**

End of Question 5

Question 6 (12 marks) **Start a new sheet of writing paper** **Marks**

(a) Hagrid predicts that the number of spiders in the Forbidden Forest, after t days, is given by $S = 45e^{0.26t}$.

(i) How many spiders were initially in the forest? **1**

(ii) At what rate is the spider population increasing after 10 days? Answer to the nearest spider. **2**

(iii) Centaurs also live in the Forest. The number of centaurs is given by $C = 18e^{0.53t}$. How long will it take for the number of spiders to be the same as the number of centaurs? Give your answer in days, correct to 1 decimal place. **2**

(b) Solve $2\ln(x+3) = \ln(x+7) + \ln(x+1)$. **2**

(c) Find the values of k for which **2**

$$x^2 - (k+4)x + 1 = 0 \text{ has no real roots.}$$

(d) The area bounded by the curve $y = \sec 2x$ between $x = \frac{\pi}{8}$ and $x = \frac{\pi}{6}$ is rotated around the *x-axis*. Find the volume of the solid of revolution formed. **3**

End of Question 6

Question 7 (12 marks) **Start a new sheet of writing paper** **Marks**

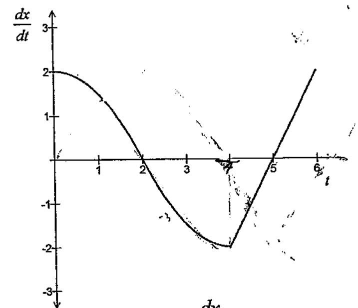
(a) A parabola has focus (5,-1) and directrix $y = 3$

(i) Find the co-ordinates of the vertex of the parabola **1**

(ii) Find the equation of the parabola. **2**

(iii) The equation of the tangent to the parabola at (-3,-7) has equation $2x - y - 1 = 0$. Find the exact perpendicular distance from the focus to this tangent. **2**

(b)



The graph shows the velocity, $\frac{dx}{dt}$, of a particle as a function of time, in seconds. Initially the particle is at the origin.

(i) At what time(s) is the particle at rest? **2**

(ii) At what time is the particle furthest from the origin? **1**

(iii) The particle travels a distance of $2\frac{2}{3}$ cm in the first 2 seconds. Find the total distance travelled by the particle during the first 6 seconds **and** its final position. **2**

(iv) Draw a sketch of the acceleration, $\frac{d^2x}{dt^2}$, as a function of time for $0 \le t < 4$. **2**

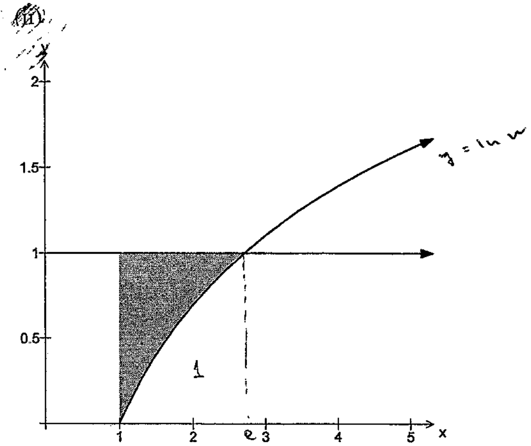
End of Question 7

Question 8 (12 marks) Start a new sheet of writing paper **Marks**

(a) Find the equation of the tangent to the curve $y = e^{3x}$ at $x = 0$. **3**

(b) (i) Show that $\frac{d}{dx}(x\ln x - x) = \ln x$ **2**

(ii) **3**



The shaded region in the diagram is bounded by the the curve $y = \ln x$ and the lines $y = 1$ and $x = 1$.
Using the result of part (i), or otherwise, find the area of the shaded region.

(c) For Maths Week, Mr Palmer froze Mr Potato Head in a cylinder of ice. It melted in such a way that the block of ice remains a cylinder, similar to the original cylinder. Initially, the block of ice has a radius of 15cm and a height of 30cm. After 4 hours the ice has a volume of $5250\pi cm^3$. The rate of change of volume, in cm^3/h, is given by $\frac{dV}{dt} = -k$, for some constant $k > 0$.

(i) Show that $V = \pi(6750 - 375t)$. **3**

(ii) Find how long it took for the ice to reduce to half its original volume. **1**

End of Question 8

Question 9 (12 marks) Start a new sheet of writing paper **Marks**

(a) A new PLC Year 12 memento is to be made in the shape of a sector of a circle of radius, r, and containing an angle of θ radians at the centre. The perimeter of the memento is to be 60mm.

(i) Show that the area $A\ mm^2$ is given by **2**

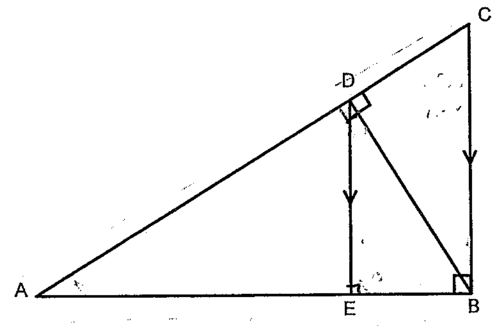
$$A = \frac{1800\theta}{(\theta+2)^2}$$

(ii) 1 Show that $\frac{dA}{d\theta} = \frac{1800(2-\theta)}{(\theta+2)^3}$ **1**

2 Find the maximum area of the memento. **2**

(b) Use the discriminant to show that $4x - y - 16 = 0$ is a tangent to the parabola $x^2 = 4y$. **2**

(c)



ΔABC is right-angled at B.
BD is drawn perpendicular to the hypotenuse.
DE is parallel to CB.
$BC = 15cm$ and $AB = 20cm$

Find the length of

(i) BD **2**

(ii) DE **3**

End of Question 9

Question 10 (12 marks) **Start a new sheet of writing paper** **Marks**

A particle, A, moves along a straight line starting from 2cm to the right of the origin with an initial velocity of 2 cm/s. Its acceleration as a function of time, t_1, is given by $a_A = -2\sin t_1$.

A second particle, B, is initially at rest and moves along the same line starting 1cm to the right of the origin. It is initially at rest. Its acceleration as a function of time, t_2, is given by $a_B = \cos t_2$

Particle B starts moving $\frac{\pi}{2}$ seconds after Particle A.

(a) Show that the expressions for velocity and displacement at any time, t_1, for particle A are given by $v_A = 2\cos t_1$ and $x_A = 2 + 2\sin t_1$. **2**

(b) Sketch the graph of the displacement of particle A for $0 \le t_1 \le 2\pi$. **2**

(c) Show that the expressions for velocity and displacement at any time, t_1, for particle B are given by $v_B = -\cos t_1$ and $x_B = 2 - \sin t_1$ **2**

(d) Find the distance between the particles, when $t_1 = \frac{\pi}{2}$. **2**

(e) At what times do the particles meet for $0 \le t_1 \le 2\pi$? **2**

(f) Find the difference between the distances travelled by the 2 particles between the first 2 times the particles meet. **2**

End of Paper

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Course	Mathematics 2U	Name of task/exam	Trial HSC

Question 1

(a) $\sqrt{\dfrac{135.2}{12.3\times 2.1}} = 2.28784\ldots$
$= 2.29$ (to 2dp)

(b) $5x^3+135 = 5(x^3+27)$
$= 5(x+3)(x^2-3x+9)$

(c) $\dfrac{3}{2\sqrt{2}+3} = \dfrac{3}{2\sqrt{2}+3}\times\dfrac{2\sqrt{2}-3}{2\sqrt{2}-3}$
$= \dfrac{6\sqrt{2}-9}{8-9}$
$= \dfrac{6\sqrt{2}-9}{-1}$
$= 9-6\sqrt{2}$
$= 9-\sqrt{72}$
$\therefore a=9 \quad b=72$

(d) $|x-3| \geq 5$

$x-3 \geq 5$; $-(x-3) \geq 5$
$x \geq 8$; $-x+3 \geq 5$
$-x \geq 2$
$x \leq -2$

(e) $x(2x+7)=4$
$2x^2+7x-4=0$
$2x^2-x+8x-4=0$
$x(2x-1)+4(2x-1)=0$
$(2x-1)(x+4)=0$
$x=\frac{1}{2} \quad x=-4$

(f) $\dfrac{1}{\sec x-1} - \dfrac{1}{\sec x+1} = 2\cot^2 x$

LHS $= \dfrac{1}{\sec x-1} - \dfrac{1}{\sec x+1}$
$= \dfrac{\sec x+1-(\sec x-1)}{(\sec x-1)(\sec x+1)}$
$= \dfrac{2}{\sec^2 x-1}$
$= \dfrac{2}{\tan^2 x}$
$= 2\cot^2 x$
$=$ RHS as req'd

$\sin^2\theta+\cos^2\theta=1$
$\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}=\dfrac{1}{\cos^2\theta}$
$\tan^2\theta+1 = \sec^2\theta$
$\sec^2\theta \quad = \tan^2\theta$

Question 2

(a) (i) $\dfrac{d}{dx}(x^2\tan x)$
$= x^2\sec^2 x + 2x\tan x$
$= x(x\sec^2 x+2\tan x)$

(ii) $\dfrac{d}{dx}\left(\dfrac{\log_e x}{x}\right)$
$= \dfrac{x\cdot\frac{1}{x} - \log_e x\cdot 1}{x^2}$
$= \dfrac{1-\log_e x}{x^2}$

b) (i) $\int \sin 4x\, dx$
$= -\frac{1}{4}\cos 4x + C$

(ii) $\int_0^1 \dfrac{x^2}{x^3+1}\,dx$
$= \frac{1}{3}\left[\ln(x^3+1)\right]_0^1$
$= \frac{1}{3}\left[\ln 2 - \ln 1\right]$
$= \frac{1}{3}\ln 2$

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Solutions for exams and assessment tasks
Ver 1

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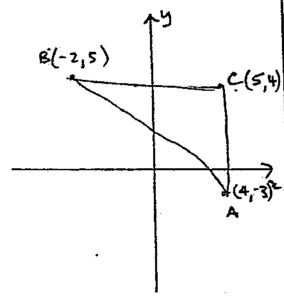
(c) $y=\cos^2 x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

$\therefore \int_0^{\pi}\cos^2 x\,dx$ $\quad h=\dfrac{\pi-0}{4} = \dfrac{\pi}{4}$

$\doteq \dfrac{\frac{\pi}{4}}{3}\left[1+4(\tfrac{1}{2})+2(0)+4(\tfrac{1}{2})+1\right]$
$= \frac{\pi}{12}[6]$
$= \frac{\pi}{2}$

Question 3



(a) $d_{AB} = \sqrt{(4+2)^2+(-3-5)^2}$
$= \sqrt{6^2+8^2}$
$= 10$

$d_{BC} = \sqrt{(5+2)^2+(5-4)^2}$
$= \sqrt{7^2+1^2}$
$= \sqrt{50}$

$d_{AC} = \sqrt{(5-4)^2+(4+3)^2}$
$= \sqrt{1^2+7^2}$
$= \sqrt{50}$

$\therefore \Delta ABC$ is isosceles, $AC=BC$

(b) $\cos\theta = \dfrac{50+50-100}{2\times 50\times 50}$
$= 0$
$\therefore \theta = \frac{\pi}{2}$ (90°)

(c) $\left(\dfrac{4-2}{2}, \dfrac{-3+5}{2}\right) = (1,1)$

(d) $m_{AB} = \dfrac{5+3}{-2-4}$
$= \dfrac{8}{-6}$
$= -\dfrac{4}{3}$
$\therefore m_{\perp} = \frac{3}{4}$

$y-1 = \frac{3}{4}(x-1)$
$4y-4 = 3(x-1)$
$4y-4 = 3x-3$
$3x-4y+1=0$

(e) LHS $= 3(5)-4(4)+1$
$= 15-16+1$
$= 0$
$=$ RHS as req'd

(f)(i) $(x-5)^2+(y-4)^2=50$

(ii) $l = r\theta$
$= \sqrt{50}\left(\frac{\pi}{2}\right)$
$= \dfrac{5\sqrt{2}\,\pi}{2}$ units

(iii) $A = \frac{1}{2}r^2(\theta-\sin\theta)$
$= \frac{1}{2}\cdot 50\left(\frac{\pi}{2}-\sin\frac{\pi}{2}\right)$

$= 25\left(\frac{\pi}{2}-1\right) u^2$

Question 4.

(a) $3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}, \pi-\frac{\pi}{6}, \pi+\frac{\pi}{6}, 2\pi-\frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(b) $e^{2x}-4e^{x}+3=0$

let $m=e^x$

$m^2-4m+3=0$

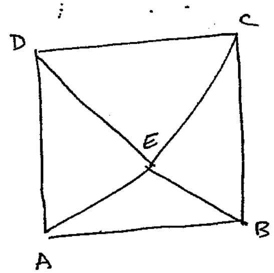
$(m-1)(m-3)=0$

$m=1 \qquad m=3$

i.e. $e^x=1 \qquad e^x=3$

$\therefore x=0 \qquad x=\ln 3$

(c)(i)



(ii) $\angle CDA = \angle DCB = 90°$ (property of square)

$\angle CDE = \angle DCE$ (property of equilateral triangle)

$\therefore \angle ADE = \angle BCE = 30°$ (adjacent complementary angles)

In $\triangle ADE$ & $\triangle BCE$

$\angle ADE = \angle BCE$ ($90°-60°=30°$)

$DE = CE$ (equal sides of equilateral triangle)

$AD = BC$ (equal sides of square)

$\therefore \triangle ADE \equiv \triangle BCE$ (SAS cong test)

(ii) $AE = BE$ (corresponding sides of congruent triangles)

(iii) $AD = ED$

Since $AD = DC$ &

$ED = DC$.

(properties of squares & equilateral triangles)

$\therefore \triangle ADE$ is isosceles

$\angle ADE = \angle ADC - \angle EDC$

$= 90 - 60$

$= 30°$

$\therefore 2\angle DAE + \angle ADE = 180°$

(angle sum of $\triangle$ is $180°$)

$\therefore 2\angle DAE = 180 - 30$

$= 150$

$\therefore \angle DAE = 75°$

$\angle AEB + \angle DAE + \angle CED + \angle CEB$

$= 360°$ (angles around a pt)

$\therefore \angle AEB = 360 - (2\times 75 + 60)$

$= 150°$

Question 5

(a) $= 12x - x^3$

(i) $= 12 - 3x^2$

$= 3(4-x^2)$

for stationary pt $=0$

$3(4-x^2)=0$

$3(2-x)(2+x)=0$

$x = \pm 2$

when $x=-2 \qquad x=2$

Test

x	-3	-2	0	2	3
$f'(x)$	-15	0	12	0	-15

$\therefore$ min at $(-2,-16)$

max at $(2,16)$

(ii) For pts of inflection

$f''(x)=0$

i.e. $f''(x) = -6x$

$-6x=0$

$x=0$

Test

x	-1	0	1
$f''(x)$	6	0	-6

$\therefore$ change of concavity

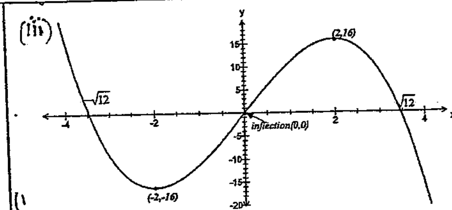
so pt of inflection

$f(0) = 12(0)-(0)^3$

$=0$

$\therefore$ Pt of inflection at $(0,0)$

(iii)



(iv) $-2<x<0$

b) $2x^2-15x+6=0$

(i) $\alpha+\beta = -\left(\frac{-15}{2}\right)$

$= \frac{15}{2}$

(ii) $\alpha\beta = \frac{6}{2}$

$=3$

(iii) $(2-\alpha)(2-\beta) = 4-2\alpha-2\beta+\alpha\beta$

$= 4-2(\alpha+\beta)+\alpha\beta$

$= 4-2\left(\frac{15}{2}\right)+3$

$= -8$

Question 6

(a) $S = 45e^{0.26t}$

(i) when $t=0$

$S = 45e^0$

$= 45$ spiders

(ii) $S = 45e^{0.26t}$

$\frac{dS}{dt} = 45(0.26)e^{0.26t}$

$= 11.7e^{0.26t}$

when $t=10$

$\frac{dS}{dt} = 11.7e^{2.6}$

$= 157.5257\ldots$

$\doteqdot 158$ spiders

(iii) $18e^{0.53t} = 45e^{0.26t}$

$\frac{45}{18} = \frac{e^{0.53t}}{e^{0.26t}}$

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$\frac{5}{2} = e^{0.27t}$

$0.27t = \ln\left(\frac{5}{2}\right)$

$t = \frac{\ln\left(\frac{5}{2}\right)}{0.27}$

$= 3.393669377$

$\doteqdot 3.4$ days

(b) $2\ln(x+3) = \ln(x+7) + \ln(x+1)$

$(x+3)^2 = (x+7)(x+1)$

$x^2+6x+9 = x^2+8x+7$

$2x = 2$

$x = 1$

(c) $x^2 - (k+4)x + 1 = 0$

for no real roots,

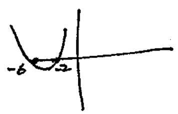
$\Delta < 0$

i.e. $(k+4)^2 - 4(1)(1) < 0$

$k^2+8k+16-4 < 0$

$k^2+8k+12 < 0$

$(k+6)(k+2) < 0$



$-6 < k < 2$

(d) $V = \pi\int y^2\,dx$

$= \pi\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \sec^2 2x\,dx$

$= \frac{\pi}{2}\left[\tan 2x\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$

$= \frac{\pi}{2}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{4}\right]$

$= \frac{\pi}{2}\left[\sqrt{3} - 1\right]\ u^3$

Question 7

(a) (i) Focal length = 2

$\therefore$ vertex $(5, 1)$

(ii) $(x-5)^2 = -4(2)(y-1)$

$(x-5)^2 = -8(y-1)$

(iii) $\perp d = \left|\frac{2(5) - 1(-1) - 1}{\sqrt{2^2+(-1)^2}}\right|$

$= \frac{10}{\sqrt{5}}$

$= \frac{10\sqrt{5}}{5}$

$= 2\sqrt{5}$ units

(b) (i) $t=2,\ t=5$

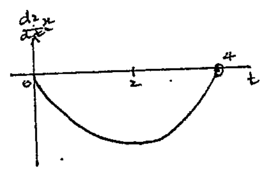
(ii) $t=2$

(iii) d = area under curve

$= 2\times\left(2\frac{2}{3}+1\right)$

$= 7\frac{1}{3}$

(iv)



Question 8

(a) $y = e^{3x}$

$\frac{dy}{dx} = 3e^{3x}$ at $x=0$

$= 3e^0$

$= 3$

When $x=0$

$y = e^{3\times 0}$

$= 1$

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$y - 1 = 3(x - 0)$

$y = 3x + 1$

(b) (i) $\frac{d}{dx}(x\ln x - x)$

$= x\cdot\frac{1}{x} + 1.\ln x - 1$

$= 1 + \ln x - 1$

$= \ln x$

(ii) $A = \text{rectangle} - \int_1^e \ln x\,dx$

$= e - 1 - \left[x\ln x - x\right]_1^e$

$= e - 1 - \left[e\ln e - e - (1\ln 1 - 1)\right]$

$= e - 1 - \left[e - e - \cancel{\ln 1}^{0} + 1\right]$

$= e - 2\ u^2$

(c) $\frac{dV}{dt} = -k$

(i) $V = -kt + c$

when $t=0$

$V = \pi(15)^2.30$

$= 6750\pi$

$\therefore c = 6750\pi$

$\therefore V = -kt + 6750\pi$

When $t=4$ $V = 5250\pi$

$5250\pi = -4k + 6750\pi$

$4k = 1500\pi$

$k = 375\pi$

$\therefore V = -375\pi t + 6750\pi$

$= \pi(6750 - 375t)$

(ii) $V = 3375\pi$

$3375\pi = \pi(6750 - 375t)$

$-375t = -3375$

$t = 9$ hours.

Question 9

(a) (i) $P = 2r + r\theta$

$60 = 2r + r\theta$

$r(\theta+2) = 60$

$r = \frac{60}{(\theta+2)}$

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\left(\frac{60}{\theta+2}\right)^2.\theta$

$= \frac{1800\theta}{(\theta+2)^2}$

(ii) $\frac{dA}{d\theta} = \frac{(\theta+2)^2 1800 - 1800\theta.2(\theta+2)}{(\theta+2)^4}$

$= \frac{(\theta+2)1800\left[\theta+2-2\theta\right]}{(\theta+2)^4}$

$= \frac{1800(2-\theta)}{(\theta+2)^3}$

2. Maximum when

$\frac{dA}{d\theta} = 0$

ie. $\frac{1800(2-\theta)}{(\theta+2)^3} = 0$

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$\theta = 2$

Test:

θ	1	2	3
$\frac{dA}{d\theta}$	+	0	−

$\therefore$ max at $\theta = 2$

$r = \frac{60}{4}$

$= 15$

$A = \frac{1}{2} . r^2 \theta$

$= \frac{1}{2} 15^2 . 2$

$= 225 \text{ mm}^2$

(b) $x^2 = 4y$ --- (1)

$4x - y - 16 = 0$ ...(2)

from (1)

$y = \frac{x^2}{4}$

$\therefore 4x - \left(\frac{x^2}{4}\right) - 16 = 0$

$x^2 - 16x + 64 = 0.$

for tangent, $\Delta = 0$ (one

$\Delta = 16^2 - 4.1.64$

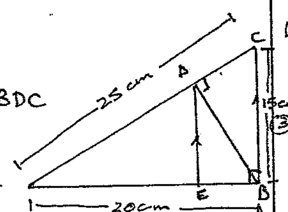
$= 0$

$\therefore 4x - y - 16 = 0$ is tangent to

$x^2 = 4y$

(c)

(i) In ΔABC & BDC



$\angle ABC = \angle BDC = 90°$ (given)

$\angle ACB = \angle DCB$ (common)

$\therefore \Delta ABC \parallel\!\mid \Delta BDC$ (equiangular)

so sides in ratio

$\frac{15}{25} = \frac{BD}{20}$

$\therefore BD = \frac{15 \times 20}{25}$

$= 12 \text{cm}$

(ii) $CD = \frac{3}{5} \times 15$

$= 9 \text{ cm}$

$\Delta BDE \parallel\!\mid \Delta BCD$ (equiangular)

Because $\angle DEB = CDB = 90°$ (

(CB||DE, co-int ∠'s equal

$\angle BDE = \angle CBD$ (alternate angles are equal; CB||DE)

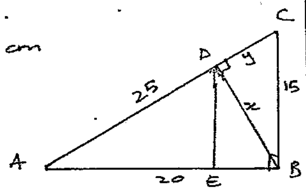
$\frac{DE}{12} = \frac{12}{15}$

$DE = \frac{144}{15}$

$= 9.6 \text{ cm}$

OR

(1)



In ΔDCB

$x^2 + y^2 = 225$ --- ①

In ΔADE

$x^2 + (25 - y)^2 = 400$ ... ②

from ① $x^2 = 225 - y^2$ ... ③

③ in ②

$225 - y^2 + 625 - 50y + y^2 = 400$

$850 - 50y = 400$

$-50y = -450$

$y = 9$

$x^2 = 225 - 81$

$= 144$

$x = 12$

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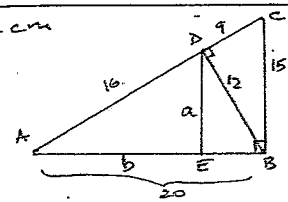
$\therefore BD = 12 \text{ cm}$

(ii)



In ΔADE

$a^2 + b^2 = 256$ ----①

In ΔDEB

$a^2 + (20 - b)^2 = 144$ ---②

from ①

$a^2 = 256 - b^2$ ....③

③ into ②

$256 - b^2 + 400 - 40b + b^2 = 144$

$656 - 40b = 144$

$40b = 512$

$b = 12.8$

$a^2 = 256 - 12.8^2$

$= 92.16$

$a = 9.6$

$\therefore DE = 9.6 \text{cm}$

Question 10

$a_A = -2\sin t_1$, $t = 0$, $v = 2$, $x = 2$

(a) $a_A = -2\sin t_1$

$v_A = 2\cos t_1 + c$

when $t = 0$ $v = 2$

$2 = 2\cos 0 + c$

$2 = 2 + c$

$\therefore c = 0$

So $v_A = 2\cos t_1$

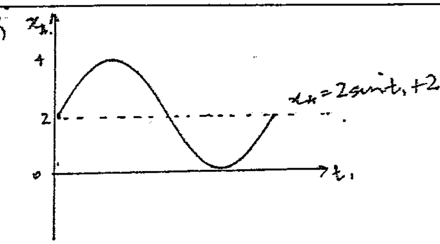
$x_A = 2\sin t_1 + c$

$2 = 2\sin 0 + c$

$c = 2$

$\therefore x_A = 2\sin t_1 + 2$

(b)



(c) $a_B = \cos t_2$ $\quad t_2 = t_1 - \frac{\pi}{2}$

$a_B = \cos\left(t_1 - \frac{\pi}{2}\right)$

$= \cos\left(-\left(\frac{\pi}{2} - t_1\right)\right)$

$= \cos\left(\frac{\pi}{2} - t_1\right)$

$= \sin t_1$ $\quad t_1 = \frac{\pi}{2}$ $v = 0$, $x = 1$.

$v_B = -\cos t_1 + c$

$0 = -\cos\frac{\pi}{2} + c$

$0 = +c$

$c = 0$

$\therefore v_B = -\cos t_1$

$x_B = -\sin t_1 + c$

$1 = -\sin\frac{\pi}{2} + c$

$1 = -1 + c$

$c = 2$

$\therefore x_B = 2 - \sin t_1$

(d) when $t_1 = \frac{\pi}{2}$

$x_A = 2\sin\frac{\pi}{2} + 2$

$= 4$

$x_B = 2 - \sin\frac{\pi}{2}$

$= 2 - 1$

$= 1$

$\therefore$ Distance b/w = 3cm

e) $2 + 2\sin t_1 = 2 - \sin t_1$

$3\sin t_1 = 0$

$\sin t_1 = 0$

$t_1 = 0, \pi, 2\pi$

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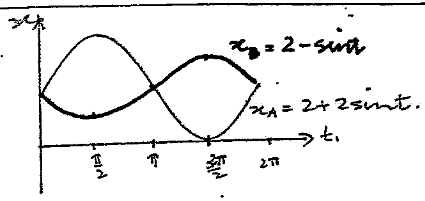

from $t = \pi \rightarrow t = 2\pi$

$d_A = 4$cm

$d_B = 2$cm

$\therefore$ Difference $= 2$cm.

Alternate (c)

$a_B = \cos t_2$

$\therefore v_B = \sin t_2 + c$

when $t_2 = 0$

$v_B = 0$

$0 = \sin 0 + c$

$c = 0$

$\therefore v_B = \sin t_2$ $\quad t_2 = t_1 - \frac{\pi}{2}$

$= \sin\left(t_1 - \frac{\pi}{2}\right)$

$= \sin\left(-\left(\frac{\pi}{2} - t_1\right)\right)$

$= -\sin\left(\frac{\pi}{2} - t_1\right)$

$v_B = -\cos t_1$