Student's name | Student's number | Teacher's name

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2011

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

1	2	3	4	5	6	7	8	9	10	Total	Total
										/120	%

Question 1 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Find a correct to 3 significant figures, if **2**

$$\frac{a}{\sin 130°} = \frac{5}{\sin 22°}$$

b) A round clock face has a diameter of $25cm$. Find the distance the tip of the big hand moves in 25 minutes. Give your answer to the nearest mm. **2**

c) Factorise completely: **2**

$$8x^3 - 64$$

d) Solve $|2x+3| < 9$ **2**

e) Find the values of a and b if $\dfrac{2}{3-\sqrt{5}} = a + b\sqrt{5}$ **2**

f) Adult tickets to the PLC Production of 'A Mid-Summer Night's Dream' cost \$32 in 2011. Last year, the adult ticket price for 'A Chorus Line' was \$30. Find the percentage increase in price for an adult ticket? **2**

End of Question 1

Question 2 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Differentiate, with respect to x :

i) $\dfrac{e^{2x}}{2x}$ **2**

ii) $2\log_e \sqrt{x}$ **2**

iii) $\sin^2 3x$ **2**

b) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan x \, dx = \frac{1}{2}\log_e \frac{3}{2}$ **3**

c) Find $\int 3e^{1-x} \, dx$ **1**

d) Show that $x^2 + kx + (k-1) = 0$ has real roots for all values of k. **2**

End of Question 2

Question 3 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) i) Show that the equation of the locus of the point $P(x, y)$ that moves such that it is equidistant from $A(3,-1)$ and $B(-4,2)$ is $7x-3y+5=0$. **2**

ii) Describe this locus **geometrically**. **1**

iii) Show that $C(2,6\frac{1}{3})$ lies on $7x-3y+5=0$. **1**

iv) Find the co-ordinates of D such that $ABDC$ is a parallelogram. **1**

v) Show that the perpendicular distance from C to AB is $\frac{145}{3\sqrt{58}}$ units. **3**

vi) The distance of AB is $\sqrt{58}$ units, find the area of $ABDC$ in exact form. **1**

b) If α and β are the roots of $4kx^2+3(k-1)x-1=0$, find the value of k if:

i) 3 is a root. **1**

ii) The roots are reciprocals of each other. **2**

End of Question 3

Question 4 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) Find the equation of the normal to the curve $y=\left(x^2-2\right)^3$ at the point $\left(1\,,-1\right)$. Write your answer in general form. **3**

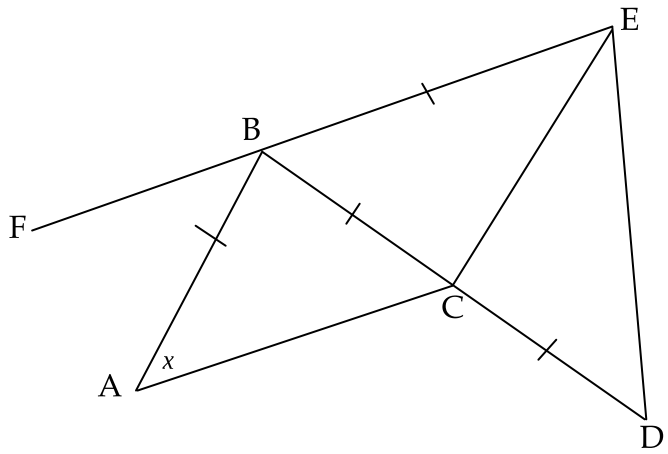
b) In the diagram below,

$AB=BC=BE=CD$, $FE \parallel AC$ and $\angle BAC+\angle CDE=90°$.

i) Copy or trace the diagram onto your answer sheet.

ii) By letting $\angle BAC=x$

prove that $BE\perp ED$, showing all working. **3**



NOT TO SCALE

c) i) On the same set of axes, sketch the graph of $y=3\sin\dfrac{x}{2}$ and $y=1.5$ for $0\le x\le 4\pi$. **2**

ii) Show that $x=\dfrac{\pi}{3}$ and $x=\dfrac{5\pi}{3}$ are two of the solutions of $3\sin\dfrac{x}{2}=1.5$ **1**

iii) Hence, find the area enclosed entirely between $y=3\sin\dfrac{x}{2}$ and $y=1.5$ in the first quadrant. **3**

End of Question 4

Question 5 **(12 marks)** **Start a new sheet of writing paper.**

Marks

Given $f(x)=\dfrac{x^2}{1-x^4}$

a) Find where the graph of the function cuts the x and y axes. **1**

b) Show that the function is even. **2**

c) Find all vertical asymptotes. **2**

d) Find the stationary point(s) and determine their nature. **4**

e) As x becomes very large, describe what will happen to $f(x)$. **1**

f) On a number plane (at least one-third of a page) sketch the curve $f(x)=\dfrac{x^2}{1-x^4}$ showing all of the above features. **2**

End of Question 5

Question 6 (12 marks) **Start a new sheet of writing paper.** **Marks**

a) Simplify $\dfrac{|1-x|}{x-1}$ **2**

b) The speed of a cyclist in a road race was recorded every half hour. The table below gives the time in hours and the speed in km/h. **2**

t	**0**	**0.5**	**1**	**1.5**	**2**
s(t)	**0**	**54**	**48**	**55**	**60**

The distance travelled by the cyclist in the first 2 hours is $\int_0^2 s(t)\, dt$.

Use the Trapezoidal Rule with these 5 function values to estimate the distance travelled.

c) Solve $3\tan^2 2\theta = 1$ for $-\pi \le \theta \le \pi$ **3**

d) The volume generated when an area under a certain curve is rotated about the *x*-axis is given by $V = \pi\int_{\frac{1}{4}}^{4} x\, dx$.

i) Find this volume **2**

ii) Write down the equation of the curve. **1**

iii) Find the size of the area being rotated. **2**

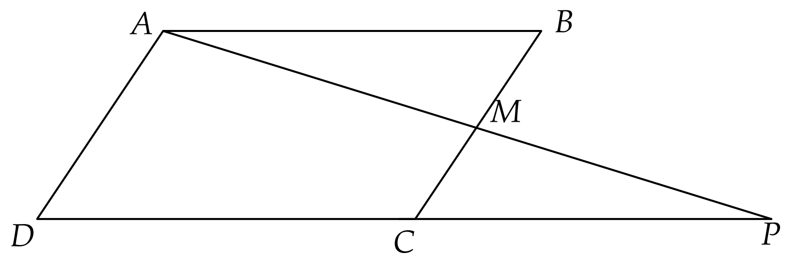
End of Question 6

Question 7 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) Assume that the population, P, of people in Sydney has been growing at a rate proportional to P. That is, $\frac{dP}{dt} = kP,$ where k is a positive constant. If the population of Sydney is going to double in 15 years time, find how long it will take for the population of Sydney to triple? **3**

b) Simplify $(1 - \cos^2 x + \sin^2 x)\cot^2 x$ **2**

c) The diagram below shows the parallelogram $ABCD$ with M the midpoint of BC. The intervals AM and DC are produced to meet at P.



i) Prove that $\Delta ABM \equiv \Delta PCM$ **2**

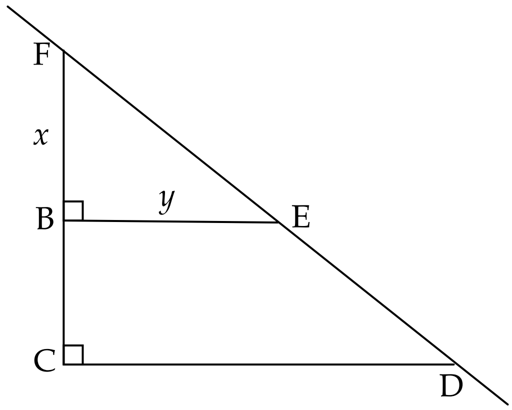
ii) Hence prove that $ABPC$ is a parallelogram. **2**

d) The line $y = mx + \frac{25}{4}$ is a tangent to $x^2 + y^2 = 25,$ find the value of m. **3**

End of Question 7

Question 8 **(12 marks)** **Start a new sheet of writing paper.** **Marks**

a) A farmer wishes to fence some of her land as shown in the diagram below. Fences are to be erected at FC, CD and BE. The side FD is a river and no fence is needed there. CD is twice the length of BE.
$\angle FBE = \angle FCD = 90°$



i) If FB=x *metres* and BE=y *metres* prove by similar triangles that BC=x. **2**

ii) Write an expression, in terms of x and y for the:

(1) Area of ΔFCD **1**

(2) Length, L, of fencing the farmer would need. **1**

iii) If the total area of land to be enclosed is 1200 m^2, show that the length of fencing L is given by $L = 2x + \dfrac{1800}{x}$ *metres* **2**

iv) Hence, find the values of x and y for which the length of fencing required will be a minimum. **3**

b) For the parabola $y^2 = 12(x-2)$, find the coordinates of the vertex, the coordinates of the focus and the equation of the directrix. **3**

End of Question 8

Question 9 (12 marks) **Start a new sheet of writing paper.** **Marks**

a) Given that $\frac{dy}{dx} = \frac{x}{x^2 - 4}$

i) Find y in terms of x, given that $y=0$ when $x=3$ **3**

ii) State the set of x values for which y exists. **2**

b) When a tap is open, water flows into a large tank that is initially empty. The volume V litres, of water in the tank increases at the rate $\frac{dV}{dt} = 2e^{t} + 2e^{-t}$ where t is measured in hours from the time the tap is opened.

i) At what rate does the water enter the tank initially? **1**

ii) Find an expression for V in terms of t **2**

iii) Show that $2e^{2t} - 3e^{t} - 2 = 0$ when $V=3$ **2**

iv) Find the exact value of t when $V=3$ **2**

End of Question 9

Question 10 **(12 marks)** **Start a new sheet of writing paper.**

Marks

a) A ball falls from rest with acceleration given by $\ddot{x} = 10e^{-\frac{1}{3}t}\ cm/s^2$, where x metres is the distance below the origin at time t seconds.

i) Find the velocity-time function for the motion of the ball. **2**

ii) Sketch the velocity-time function for the motion of the ball. **1**

iii) What is the limiting velocity of the ball? **1**

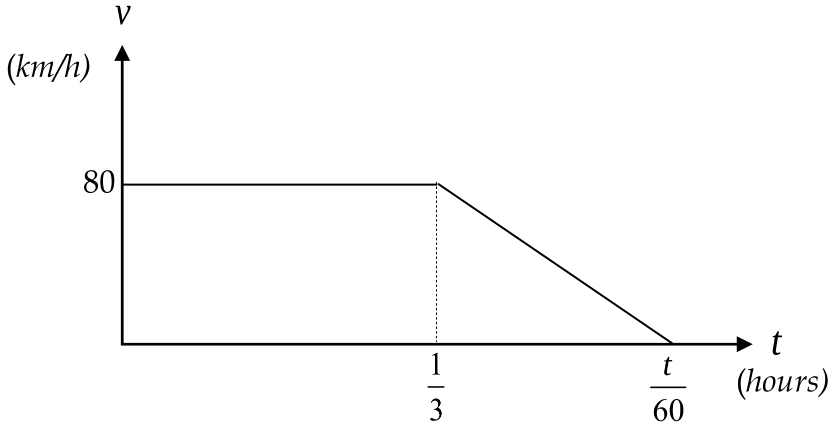
iv) How far does the ball travel in the first 3 seconds? **3**

Question 10 continues on the next page

b) A train is travelling at a constant velocity of $80 km/h$ as it passes through Croydon railway station. At the same time, a second train commences its journey from rest at Croydon station, travelling in the same direction as the first train. The second train accelerated for 15 minutes at a constant rate until it reaches $100 km/h$ and maintains this velocity for a further 5 minutes.

At this time each of the trains then begin to slow down at a constant rate, arriving at the next station, *X*, at the same time.

i) The graph of velocity versus time for the first train has been drawn below:



Copy or trace this diagram on your answer sheet.

ii) On the same sketch as in part i) draw in the velocity/time graph for the second train. **1**

iii) Calculate the time taken for the trains to travel between the two stations. **3**

iv) How far apart are the two stations? **1**

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$