Student's name **Student's number** **Teacher's name**

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2013
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen
  Black is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 100

Section I: Pages 3-5
10 marks

- Attempt questions 1-10, using the answer sheet on page 15.
- Allow about 15 minutes for this section

Section II: Pages 6-11
90 marks

- Attempt questions 11-16, using the booklets provided.
- Allow about 2 hours 45 minutes for this section

Multiple Choice	11	12	13	14	15	16	Total
							%

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

1. What is $\frac{4}{e}$ correct to 3 significant figures?

(A) 1.47
(B) 1.470
(C) 1.471
(D) 1.472

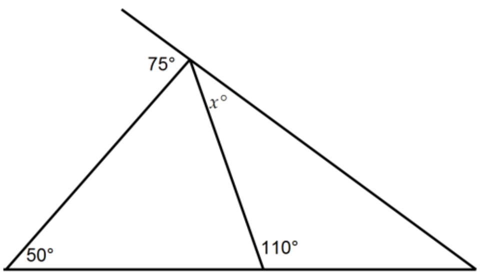
2. What is the gradient of any line perpendicular to the line $3x-2y+12=0$?

(A) $-\frac{3}{2}$
(B) $-\frac{2}{3}$
(C) $\frac{2}{3}$
(D) $\frac{3}{2}$

3. What is the value of y if $\frac{1}{3-\sqrt{2}}=x+y\sqrt{2}$

(A) -1
(B) $-\frac{1}{7}$
(C) $\frac{1}{7}$
(D) 1

4. In the diagram, what is the value of x?

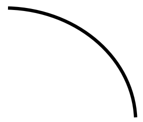


(A) 45
(B) 50
(C) 60
(D) 65

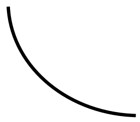
5. Which diagram below represents the following statements?

$f'(x) < 0$ and $f''(x) > 0$

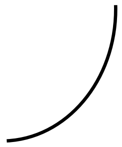
(A)



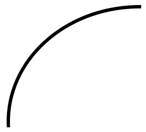
(B)



(C)



(D)



6. Solve $sin\,2x = -\frac{1}{2}$ for $0 \le x \le 2\pi$.

(A) $x = \frac{\pi}{12}, \frac{5\pi}{12}$

(B) $x = \frac{7\pi}{12}, \frac{11\pi}{12}$

(C) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

(D) $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

7. The number of ants in my Ant Farm after *t* days is given by $A = A_0 e^{kt}$, where *k* is a positive constant. Initially there are 50 ants and after 4 days the number has increased to 90. How many days, correct to 3 decimal places, will it take for the number of ants to reach 180?

(A) 0.147
(B) 0.545
(C) 8.000
(D) 8.717

8. A circular pizza with diameter of 28 centimetres is to be cut up into 12 equal sectors. What is the area of one sector, in square centimetres?

(A) $\dfrac{49}{3}(\pi - 3)$

(B) $\dfrac{49\pi}{3}$

(C) 294

(D) 2891

9. For the curve $y = x^3 - x^2 - x + 1$

Which of the following statements are true?

I: Stationary points occur at $x = \dfrac{-1}{3}$ and $x = 1$

II: The curve is concave down for $x < \dfrac{1}{3}$

(A) **I** only

(B) **II** only

(C) Both **I** and **II**

(D) Neither **I** nor **II**

10. Consider the function $y = \left(4 - x^2\right)^{\frac{1}{2}}$.

Use the **trapezoidal rule** with 3 function values to find the approximate area under the curve $y = \left(4 - x^2\right)^{\frac{1}{2}}$ for $0 \le x \le 2$, correct to 2 decimal places. What is this approximate area, in square units?

(A) 1.24

(B) 1.87

(C) 2.73

(D) 3.14

End of Section I

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section

Question 11: **15 Marks**

a) Differentiate with respect to *x*:

(i) $\dfrac{\log_e x}{x}$ **2**

(ii) $x\sin(3x-1)$ **2**

b) Evaluate $\int_1^{e^3} 4x^{-1}\ dx$ **2**

c) *ABC* is a triangle, right-angled at *A*. *AD* is perpendicular to *BC*. *P* is a point on the interval *BD* such that *CP*=*CA*. If $\angle APC = 80^0$, show that *PA* bisects $\angle BAD$, with reasons. **3**

d) Prove the following **2**

$(\sec\theta + \tan\theta)^2 = \dfrac{1+\sin\theta}{1-\sin\theta}$

e) Solve for *x*: $|2x-5| = 4x-1$ **2**

f) The vertex of a parabola $y = x^2 + bx + c$ is $(-4,-18)$. **2**
Find the values of *b* and *c*.

End of Question 11

Question 12: **15 Marks**

a) (i) Sketch the curve $y = \sqrt{3} \tan x$ for $-\pi \le x \le \pi$. **1**

(ii) Hence, or otherwise, solve $\sqrt{3} \tan x > 1$ in this domain. **2**

b) For the parabola $x^2 = 16y$

(i) Find the co-ordinates of the focus, *S*. **1**

(ii) Show that the equation of the tangent to this parabola at the point $P(4,1)$ is $x - 2y - 2 = 0$. **2**

(iii) The straight line joining P to the focus, *S*, intersects the parabola again at *R*. Show that the co-ordinates of *R* are $(-16,16)$. **3**

(iv) The equation of the tangent at *R* is $2x + y + 16 = 0$. Show that the tangents at *P* and *R* intersect at T on the directrix. **2**

(v) Find the distance *PT*. **1**

(vi) Find the perpendicular distance from *R* to *PT*. **2**

(vii) Hence find the exact area of ΔTRP. **1**

End of Question 12

Question 13: **15 Marks**

a) Sketch on a Cartesian plane the region where $x^2 + y^2 \geq 9$ and $y \geq x - 3$ hold simultaneously. **2**

b) Give the exact solution of $2e^{-2x} - 5e^{-x} + 2 = 0$. **2**

c) (i) Sketch the line $2x - 3y = 6$, showing x and y intercepts. **1**

(ii) Find the co-ordinates of the point on the straight line $2x - 3y = 6$ which is closest to the origin. **3**

d) For the quadratic $kx^2 - (2k+1)x + 2 = 0$, find k if:

(i) $x = 1$ is a root of the quadratic **1**

(ii) Roots are real **3**

(iii) Roots are equal in magnitude but opposite in sign **2**

(iv) One root is the reciprocal of the other **1**

End of Question 13

Question 14: **15 Marks**

a) Find $\int e^{5x+2}\,dx$ **1**

b) The curve $y = x^3$ is rotated around the ***y-axis*** from $y = -1$ to $y = 1$. **3**
Find the volume of the solid of revolution.

c) Sketch $y = 1 - \cos\dfrac{2x}{3}$ for $0 \le x \le 3\pi$. **3**

d) If $y = \dfrac{x}{\sin x}$, show that $\sin x\dfrac{dy}{dx} + y\cos x = 1$. **2**

e) Two circles, with centres at *A* and *B*, intersect at *C* and *D* as shown in the diagram below. The radius of the circle centred at *A* is 2 metres, $CA \perp CB$ and AB=4 metres.

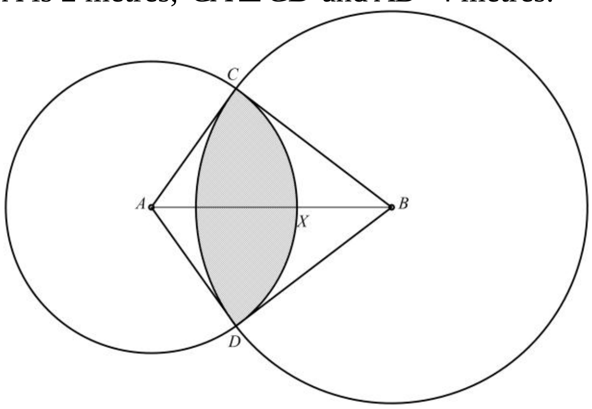


Diagram NOT drawn to scale

(i) Copy or trace the diagram into your answer booklet.

(ii) Prove $\Delta ABC \equiv \Delta ABD$. **2**

(iii) Given the arc length CXD is $\dfrac{2\pi}{3}$ *metres*, show that $\angle CAB = \dfrac{\pi}{6}$. **1**

(iv) Find the shaded area. **3**

End of Question 14

Question 15: **15 Marks**

a) A variable line with gradient m passes through the point $(1,2)$ and intersects the *x-axis* at $A(a,0)$ and the *y-axis* at $B(0,b)$, where both a and b are positive.

(i) Show that the equation of AB is $y = mx + 2 - m$ **1**

(ii) Show that the area of triangle ΔAOB is given by **2**

$$Area = \frac{4m - m^2 - 4}{2m}$$

(iii) Find the smallest possible area of ΔAOB. **2**

b) The displacement, x metres, of a particle moving along the *x-axis* is given at time t seconds by the equation $x = 12e^{-2t} - 12 + 24t$.

(i) Show the particle is initially at rest. **1**

(ii) Find the velocity when $t = \log_e\left(\frac{4}{3}\right)$ seconds. **2**

(iii) What is the limit approached by the velocity as t increases? **1**

(iv) Sketch the velocity-time graph for $0 \le t \le 3$. On your sketch clearly show the velocity at t=1.5. **2**

(v) Using Simpson's Rule with 3 function values, find the distance travelled in the first 3 seconds. Write your answer correct to 1 decimal place. **2**

(vi) Prove that the acceleration is 48-2v, where v is velocity. **2**

End of Question 15

Question 16 **15 Marks**

a) Find the values of A and B if $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \, dx = A \log_e B$ **2**

b) For the curve $y = f(x)$ where $f(x) = \dfrac{sec^2 x}{1 + tan\, x}$ for $0 \le x \le \pi$,

(i) Find the y-intercept and $f(\pi)$. **2**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{sec^2 x\left(tan^2 x + 2\,tan\, x - 1\right)}{\left(1 + tan\, x\right)^2}$ **3**

(iii) There are two stationary points on the curve $y = \dfrac{sec^2 x}{1 + tan\, x}$ in the interval $0 \le x \le \pi$. Given (0.39, 0.83) is a minimum stationary point, find the other stationary point and determine its nature. **3**

Do not try to find the points of inflexion.

(iv) Show there are vertical asymptotes at $x = \dfrac{\pi}{2}$ and $x = \dfrac{3\pi}{4}$ **1**

(v) Sketch the curve in the interval $0 \le x \le \pi$, clearly showing all the above information. **2**

(vi) Find the exact area under the curve $f(x) = \dfrac{sec^2 x}{1 + tan\, x}$ bounded by the *x-axis*, the *y-axis* and $x = \dfrac{\pi}{3}$ in exact form. **2**

End of Paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

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Mathematics: Multiple Choice Answer Sheet

Student Number__________

Completely fill the response oval representing the most correct answer.

1.	**A** ○	**B** ○	**C** ○	**D** ○
2.	**A** ○	**B** ○	**C** ○	**D** ○
3.	**A** ○	**B** ○	**C** ○	**D** ○
4.	**A** ○	**B** ○	**C** ○	**D** ○
5.	**A** ○	**B** ○	**C** ○	**D** ○
6.	**A** ○	**B** ○	**C** ○	**D** ○
7.	**A** ○	**B** ○	**C** ○	**D** ○
8.	**A** ○	**B** ○	**C** ○	**D** ○
9.	**A** ○	**B** ○	**C** ○	**D** ○
10.	**A** ○	**B** ○	**C** ○	**D** ○

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Mathematics: Multiple Choice Answer Sheet

Student Number ANSWERS

Completely fill the response oval representing the most correct answer.

1.	A ●	B ○	C ○	D ○
2.	A ○	B ●	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ●	B ○	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ○	B ○	C ○	D ●
7.	A ○	B ○	C ○	D ●
8.	A ○	B ●	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ○	B ○	C ●	D ○

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Question 11:

a i $\frac{d}{dx}\left(\frac{\log_e x}{x}\right)$

quotient rule

$u = \ln x$

$u' = \frac{1}{x}$

$v = x$

$v' = 1$

$= \frac{vu' - uv'}{v^2}$

$= \frac{x\left(\frac{1}{x}\right) - \ln x\,(1)}{x^2}$

$= \frac{1 - \ln x}{x^2}$

ii $\frac{d}{dx}\left[x \sin(3x-1)\right]$

product rule

$u = x$

$u' = 1$

$v = \sin(3x-1)$

$v' = 3\cos(3x-1)$

$= uv' + vu'$

$= x\left(3\cos(3x-1)\right) + \sin(3x-1)(1)$

$= 3x\cos(3x-1) + \sin(3x-1)$

b $\int_1^{e^3} 4x^{-1}\,dx$

$= 4\int_1^{e^3} \frac{1}{x}\,dx$

$= 4\left[\ln x\right]_1^{e^3}$

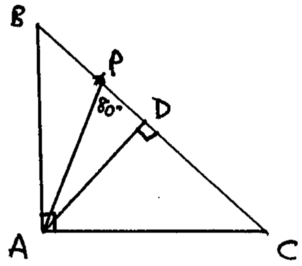
$= 4\left[\ln e^3 - \ln 1\right]$

$= 4\left[3\ln_e e - 0\right]$

$= 4(3)$

$= 12$

c



In $\triangle PCA$: $PC = CA$ (given)

$\angle APC = \angle PAC$ (angles opposite equal sides are equal).

$80° = \angle PAC$

$\therefore \angle PAC = 80°$

In $\triangle ABC$: $\angle BAC = 90°$ (given)

$\therefore \angle BAP = \angle BAC - \angle PAC$ (right angle)

$= 90° - 80°$

$= 10°$

In $\triangle PDA$: $\angle PDA = 90°$ ($AD \perp BC$)

$\therefore \angle PAD = 180 - \angle DPA - \angle PDA$ (angle sum of triangle)

$= 180° - 80° - 90°$

$= 10°$.

$\therefore \angle BAP = \angle PAD$

$\therefore PA$ bisects $\angle BAD$.

d Prove

$(\sec\theta + \tan\theta)^2 = \frac{1+\sin\theta}{1-\sin\theta}$

LHS $= (\sec\theta + \tan\theta)^2$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$

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$$LHS = \frac{1}{\cos^2\theta} + \frac{2 \times 1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1 + 2\sin\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}$$

$$= \frac{(1+\sin\theta)\cancel{(1+\sin\theta)}}{(1-\sin\theta)\cancel{(1+\sin\theta)}}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

$= RHS$

$\therefore$ proved

e $|2x-5| = 4x-1$

$2x - 5 = 4x - 1$

$-2x = 4$

$x = -2$

Check $LHS = |2(-2)-5|$

$= |-9|$

$= 9$

$RHS = 4(-2)-1$

$= -9$

$LHS \neq RHS$

$\therefore$ not a solution

OR $-(2x-5) = 4x-1$

$-2x+5 = 4x-1$

$-6x = -6$

$x = 1.$

$LHS = |2(1)-5|$

$= |-3|$

$= 3$

$RHS = 4(1)-1$

$= 3$

$LHS = RHS$

$\therefore x = 1$ is solution.

f $y = x^2 + bx + c$

Vertex: $x = \frac{-b}{2(1)}$

$-4 = \frac{-b}{2}$

$-8 = -b$

$\therefore b = 8$

$(-4, -18)$ satisfies eqn. of parabola

$-18 = (-4)^2 + 8(-4) + c$

$-18 = 16 - 32 + c$

$-18 = -16 + c$

$c = -2$

$\therefore b = 8 \quad c = -2$

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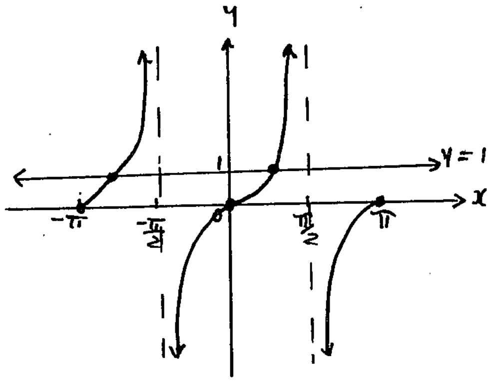
Question 12 :

a i



ii solve $\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, -\frac{5\pi}{6}$

$\therefore -\frac{5\pi}{6} < x < -\frac{\pi}{2}, \quad \frac{\pi}{6} < x < \frac{\pi}{2}$

b $x^2 = 16y$ vertex $(0,0)$

i $4a = 16$

$a = 4$

$\therefore S(0,4)$

ii $y = \frac{x^2}{16}$

$\frac{dy}{dx} = \frac{2x}{16}$

$\frac{dy}{dx} = \frac{x}{8}$

at $P(4,1)$

$m_{tang} = \frac{4}{8}$

$m = \frac{1}{2}$

$\therefore$ eqn tangent:

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x - 4)$

$2y - 2 = x - 4$

$0 = x - 2y - 4 + 2$

$\therefore x - 2y - 2 = 0$

iii equation of line joining P and S: $(4,1)$ $(0,4)$

$m = \frac{4-1}{0-4}$

$= \frac{3}{-4}$

eqn: $y - 4 = -\frac{3}{4}(x - 0)$

$4y - 16 = -3x$

$3x + 4y - 16 = 0$.

To find R: solve simult eqns

$x^2 = 16y$ ①

$3x + 4y - 16 = 0$ ②

① $y = \frac{x^2}{16}$

$3x + 4\left(\frac{x^2}{16}\right) - 16 = 0$

$3x + \frac{x^2}{4} = 16$

$x^2 + 12x - 64 = 0$

$(x-4)(x+16) = 0$

$x = 4 \quad x = -16$.

$\therefore (4,1) \quad (-16,16)$

$\therefore R(-16,16)$.

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Course		Name of task/exam	

iv eqn tangent R

$2x + y + 16 = 0$

eqn tangent P

$x - 2y - 2 = 0$

Solve to find y:
(eqn directrix $y = -4$).

$2x + y + 16 = 0$
$2x - 4y - 4 = 0$
$5y + 20 = 0$
$y = -4$.

$\therefore$ T lies on directrix

v coordinates of T $(-6, -4)$

$d_{PT} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - -6)^2 + (1 - -4)^2}$

$= \sqrt{100 + 25}$

$= \sqrt{125}$

$= 5\sqrt{5}$ units

vi R $(-16, 16)$

eqn PT is eqn of tangent at P

$x - 2y - 2 = 0$

$\therefore d_\perp = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{1(-16) - 2(16) - 2}{\sqrt{1^2 + (-2)^2}} \right|$

$d_\perp = \left| \dfrac{-16 - 32 - 2}{\sqrt{5}} \right|$

$= \dfrac{50}{\sqrt{5}}$

$= 10\sqrt{5}$ units

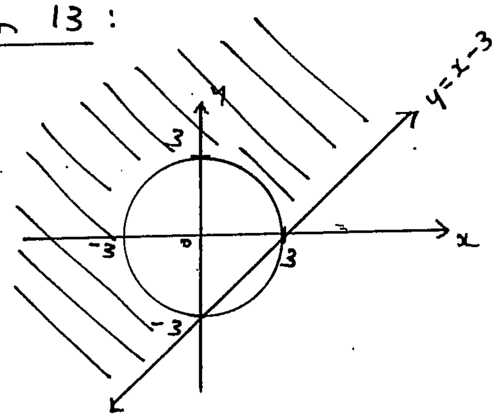
vii $A = \frac{1}{2} b h$

$= \frac{1}{2}(5\sqrt{5})(10\sqrt{5})$

$= 125$ units^2

Question 13:

a



$x^2 + y^2 \geq 9$
outside circle

$y \geq x - 3$
Test $(3,3)$
$3 \geq 0$
true

b $2e^{-2x} - 5e^{-x} + 2 = 0$

Let $u = e^{-x}$

$2u^2 - 5u + 2 = 0$

$(2u - 1)(u - 2) = 0$

$u = \frac{1}{2}$ $u = 2$

$e^{-x} = \frac{1}{2}$ $e^{-x} = 2$

$-x = \ln(\frac{1}{2})$ $-x = \ln 2$

$x = -\ln(\frac{1}{2})$ $x = -\ln 2$

or   or

$x = \ln 2$. $x = \ln(\frac{1}{2})$

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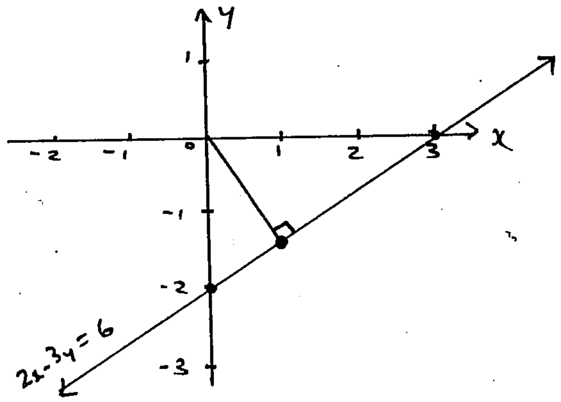
Course		Name of task/exam	

c i



$2x - 3y = 6$

x-intercept : set $y = 0$

$2x = 6$

$x = 3.$

y-intercept : set $x = 0$

$-3y = 6$

$y = -2$

ii Closest to origin.

we need to find eqn of line passing through origin and perpendicular to $2x-3y=6$.

$\therefore m_{line} = \frac{2}{3}$

$m_{\perp} = -\frac{3}{2}$ $(0,0)$

$y - 0 = -\frac{3}{2}(x - 0)$

$2y = -3x$

Need to solve

$2x - 3y = 6$ ①

$3x + 2y = 0$ ②

$6x - 9y = 18$ ①×3

$6x + 4y = 0$ ②×2

$-13y = 18$ ① − ②

$y = -18/13$

$x = 12/13$

$\therefore$ pt is $\left(\frac{12}{13}, -\frac{18}{13}\right)$

d $kx^2 - (2k+1)x + 2 = 0$

i $x = 1$ is root

$\therefore$ it satisfies quadratic

$k(1)^2 - (2k+1)(1) + 2 = 0$

$k - 2k - 1 + 2 = 0$

$-k + 1 = 0$

$k = 1$

ii for real roots $\Delta \geq 0$

$\therefore b^2 - 4ac \geq 0$

$[-(2k+1)]^2 - 4(k)(2) \geq 0$

$4k^2 + 4k + 1 - 8k \geq 0$

$4k^2 - 4k + 1 \geq 0$

$(2k-1)^2 \geq 0$

for real roots all k satisfy.

iii Let roots be $\alpha, -\alpha$

Sum of roots:

$\alpha + -\alpha = -\frac{b}{a}$

$0 = \frac{2k+1}{k}$

$2k + 1 = 0$

$k = -\frac{1}{2}$

iv Let roots be $\alpha, \frac{1}{\alpha}$

product of roots:

$\alpha\left(\frac{1}{\alpha}\right) = \frac{c}{a}$

$1 = \frac{2}{k}$

$\therefore k = 2$

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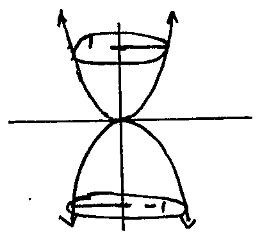
Question 14 :

a $\int e^{5x+2}\,dx = \dfrac{e^{5x+2}}{5} + c$

b $y = x^3$



$V = \pi \times 2\int_0^1 x^2\,dy$

$x = y^{\frac{1}{3}}$

$\therefore V = 2\pi\int_0^1 (y^{\frac{1}{3}})^2\,dy$

$= 2\pi\left[\dfrac{3y^{\frac{5}{3}}}{5}\right]_0^1$

$= 2\pi\left[\frac{3}{5}(1)^{\frac{5}{3}} - \frac{3}{5}(0)^{\frac{5}{3}}\right]$

$= 2\pi \times \frac{3}{5}$

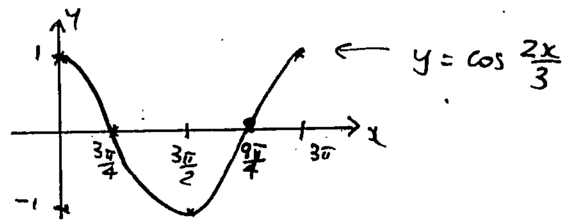
$= \dfrac{6\pi}{5}$ units^3

c $y = 1 - \cos\dfrac{2x}{3}$ $\quad 0 \leq x \leq 3\pi$

Consider $y = \cos\dfrac{2x}{3}$

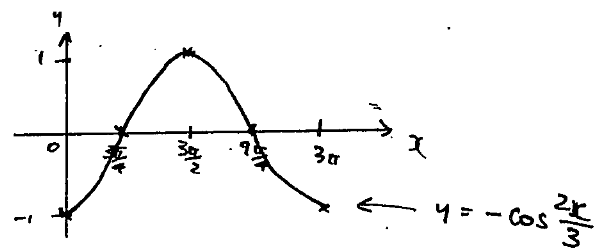
period $= \dfrac{2\pi}{2/3} = 3\pi$

amplitude $= 1$.



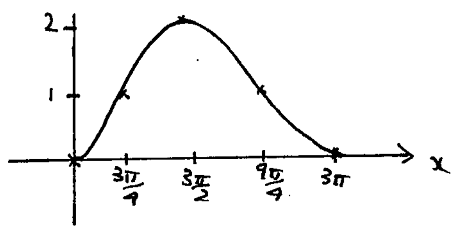
Consider $y = -\cos\dfrac{2x}{3}$

reflection in x-axis



Consider $y = 1 - \cos\dfrac{2x}{3}$

shifted up 1 unit



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d $y = \dfrac{x}{\sin x}$

$u = x$
$u' = 1$
$v = \sin x$
$v' = \cos x$

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$\dfrac{dy}{dx} = \dfrac{\sin x(1) - x(\cos x)}{(\sin x)^2}$

$= \dfrac{\sin x - x\cos x}{\sin^2 x}$.

RTS $\sin x \dfrac{dy}{dx} + y\cos x = 1$

$LHS = \sin x\left[\dfrac{\sin x - x\cos x}{\sin^2 x}\right] + \dfrac{x}{\sin x}(\cos x)$

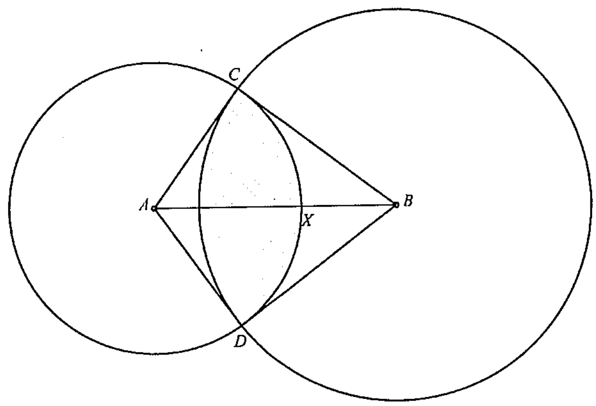
$= \dfrac{\sin x - x\cos x}{\sin x} + \dfrac{x\cos x}{\sin x}$

$= \dfrac{\sin x}{\sin x}$

$= 1$

$= RHS$

e i



ii In $\triangle ABC$ and $\triangle ABD$

$AC = AD$ (equal radii)
$BC = BD$ (equal radii)
AB common

$\therefore \triangle ABC \equiv \triangle ABD$ (SSS)

iii $l = r\theta$

$\dfrac{2\pi}{3} = 2\theta$

$\theta = \dfrac{\pi}{3}$

$\therefore \angle CAD = \dfrac{\pi}{3}$

$\therefore \angle CAB = \dfrac{1}{2}\angle CAD$

[since $\angle CAB = \angle DAB$ corresponding angles in congruent triangles are equal].

$\therefore \angle CAB = \dfrac{\pi}{6}$

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iv In $\triangle ACB$

$CA = 2m$, $AB = 4m$

$CA \perp CB$

$\therefore$ right angled triangle.

$\therefore 4^2 = 2^2 + (CB)^2$

$16 = 4 + (CB)^2$

$CB = \sqrt{12}$

$CB = 2\sqrt{3}$

Using circle centre A

$A_{segment} = \frac{1}{2} 2^2 \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= 2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Using circle centre B

$A_{segment} = \frac{1}{2}(2\sqrt{3})^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

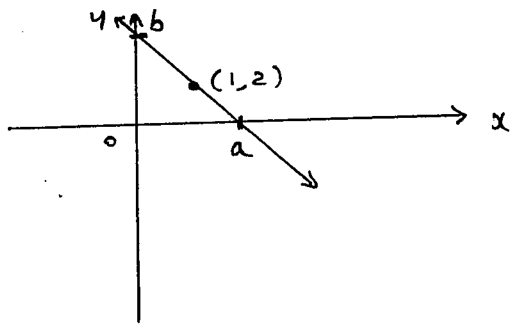
$= \frac{1}{2}(12)\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$\therefore A_{shaded} = \left(\frac{2\pi}{3} - \sqrt{3}\right) + 6\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{14\pi}{3} - 4\sqrt{3}$ units^2

Question 15:

a



i eqn of AB

$y = mx + b$.

we know $(1,2)$ lies on line with gradient m

$2 = m + b$

$\therefore b = 2 - m$.

$\therefore$ eqn of AB is

$y = mx + 2 - m$

ii $A = \frac{1}{2}bh$

$= \frac{1}{2} \times a \times b$

for the line $y = mx + 2 - m$

$(a, 0)$ satisfies

$0 = ma + 2 - m$

$m - 2 = ma$

$a = \frac{m-2}{m}$

for the line $y = mx + 2 - m$

$(0, b)$ satisfies

$b = 2 - m$.

$\therefore A = \frac{1}{2}\left(\frac{m-2}{m}\right)(2-m)$

$= \frac{(m-2)(2-m)}{2m}$

$= \frac{2m - m^2 - 4 + 2m}{2m}$

$= \frac{4m - m^2 - 4}{2m}$

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iii $\frac{dA}{dm} = \frac{vu' - uv'}{v^2}$

$u = 4m - m^2 - 4$

$u' = 4 - 2m$

$v = 2m$

$v' = 2$

$= \frac{2m(4-2m) - (4m - m^2 - 4)(2)}{(2m)^2}$

$= \frac{8m - 4m^2 - 8m + 2m^2 + 8}{(2m)^2}$

$= \frac{-2m^2 + 8}{(2m)^2}$

for max/min $\frac{dA}{dm} = 0$

$\therefore -2m^2 + 8 = 0$

$2m^2 = 8$

$m^2 = 4$

$m = \pm 2$

Since a and b both positive line AB is decreasing $\therefore m = -2$.

m	-3	-2	-1
$\frac{dA}{dm}$	$\frac{-10}{36}$	0	$\frac{6}{4}$

$\therefore$ min.

$\therefore$ min Area $= \frac{4(-2) - (-2)^2 - 4}{2(-2)}$

$= \frac{-8 - 4 - 4}{-4}$

$= 4$ units^2

b $x = 12e^{-2t} - 12 + 24t$

i initially $t = 0$

$v = -24e^{-2t} + 24$

$v = -24e^{0} + 24$

$= -24 + 24$

$\therefore v = 0$

$\therefore$ initially at rest.

ii $v = -24e^{-2t} + 24$

when $t = \ln\left(\frac{4}{3}\right)$

$v = -24e^{-2\ln\frac{4}{3}} + 24$

$= -24e^{\ln\left(\frac{4}{3}\right)^{-2}} + 24$

$= -24e^{\ln\left(\frac{3}{4}\right)^{2}} + 24$

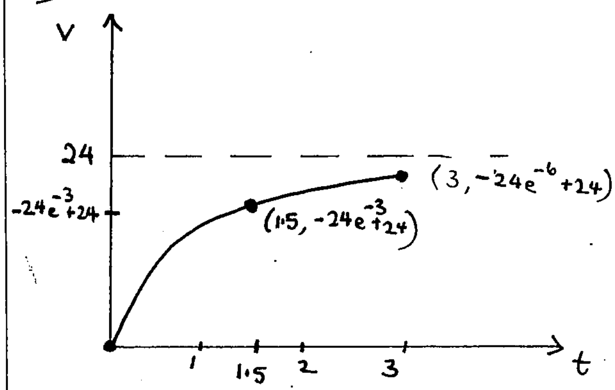
$= -24e^{\ln\frac{9}{16}} + 24$

$= -24\left(\frac{9}{16}\right) + 24$

$v = 10\frac{1}{2}$ m/s.

iii as $t \to \infty$ $v \to 24$

iv



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v

t	0	1.5	3
v	0	$-24e^{-3}+24$	$-24e^{-6}+24$

distance = area under curve

$= \int_0^3 f(x)\,dx$

$= \frac{h}{3}\left[y_0 + y_n + 4y_1\right]$

$= \frac{1.5}{3}\left[0 + (-24e^{-6}+24) + 4(-24e^{-3}+24)\right]$

$= 57.58047\ldots$

$= 57.6$ m

vi $a = \frac{dv}{dt}$

$= 48e^{-2t}$

we know $v = -24e^{-2t} + 24$

$\therefore 24e^{-2t} = 24 - v \ldots (1)$

$\therefore a = 2 \times 24e^{-2t}$

$a = 2\left[24 - v\right]$ from (1)

$\therefore a = 48 - 2v$

Question 16:

a $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,dx = A\log_e B$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$

$= \left[\ln(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{4}\right)$

$= \ln 1 - \ln\frac{1}{\sqrt{2}}$

$= 0 - \ln\left(\frac{1}{\sqrt{2}}\right)$

Alternative Solution: $A = -1$, $B = \frac{1}{\sqrt{2}}$.

$= \ln\left(\frac{1}{\sqrt{2}}\right)^{-1}$

$= \ln\sqrt{2}$

$= \ln 2^{\frac{1}{2}}$

$= \frac{1}{2}\ln 2.$

$\therefore A = \frac{1}{2} \quad B = 2.$

b $f(x) = \frac{\sec^2 x}{1 + \tan x}$

i y-intercept: set $x = 0$

$y = \frac{1}{\cos^2(0)\left[1 + \tan 0\right]}$

$= \frac{1}{1(1)}$

$\therefore y = 1$

$f(\pi) = \frac{1}{(\cos\pi)^2\left[1 + \tan\pi\right]}$

$= \frac{1}{(-1)^2(1)}$

$\therefore f(\pi) = \frac{1}{1}$

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ii) $\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$u = \sec^2 x$
$u' = 2\sec x(\sec x \tan x)$
$u' = 2\sec^2 x \tan x$
$v = 1 + \tan x$
$v' = \sec^2 x$

$= \dfrac{(1+\tan x)2\sec^2 x \tan x - \sec^4 x}{(1+\tan x)^2}$

$= \dfrac{2\sec^2 x \tan x + 2\sec^2 x \tan^2 x - \sec^4 x}{(1+\tan x)^2}$

$= \dfrac{\sec^2 x(2\tan x + 2\tan^2 x - \sec^2 x)}{(1+\tan x)^2}$

$= \dfrac{\sec^2 x(2\tan x + 2\tan^2 x - (1+\tan^2 x))}{(1+\tan x)^2}$

$= \dfrac{\sec^2 x(2\tan x + 2\tan^2 x - 1 - \tan^2 x)}{(1+\tan x)^2}$

$= \dfrac{\sec^2 x(\tan^2 x + 2\tan x - 1)}{(1+\tan x)^2}$

iii) For stationary points $\dfrac{dy}{dx} = 0$

$\therefore \sec^2 x(\tan^2 x + 2\tan x - 1) = 0$

$\sec^2 x = 0$
no sol^n

$\tan x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2 \pm \sqrt{4+4}}{2}$

$= \dfrac{-2 \pm 2\sqrt{2}}{2}$

$\tan x = -1 \pm \sqrt{2}$

$\therefore x = 0.39, \ y = 0.83 \ ; \quad 1.96, \ -4.83$

$(0.39, 0.83) \quad (1.96, -4.83)$

x	0.3	0.39	0.4
$\frac{dy}{dx}$	-0.18	0	0.01

$\therefore$ min $(0.39, 0.83)$

x	1.9	1.96	2
$\frac{dy}{dx}$	4.4	0	-2.4

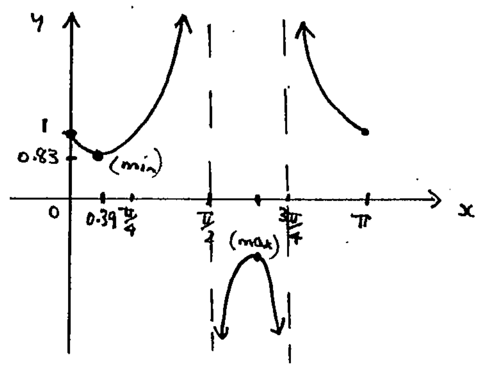
$\therefore$ max $(1.96, -4.83)$

iv) Vertical asymptotes where den $= 0$.

$\therefore 1 + \tan x = 0$
$\tan x = -1$ ✓4
$x = \frac{3\pi}{4}$

Also $\cos^2 x = 0$
$\cos x = 0$
$x = \frac{\pi}{2}$.

v)



vi) $A = \displaystyle\int_0^{\pi/3} y \, dx$

$= \displaystyle\int_0^{\pi/3} \frac{\sec^2 x}{1+\tan x} \, dx$

$= \left[\ln(1+\tan x)\right]_0^{\pi/3}$

$= \ln(1+\sqrt{3}) - \ln(1)$

$= \ln(1+\sqrt{3})$ units^2