Student's name ________________________

Student's number ___________

Teacher's name _____________________

PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2016

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using **black** pen
- Board-approved calculators may be used
- A reference sheet is attached to the examination paper
- A multiple choice answer sheet is attached to the examination paper
- All necessary working should be shown in every question

Total Marks – 100

Section I: Pages 3-7
10 marks

- Attempt questions 1-10, using the multiple choice answer sheet. Allow about 15 minutes for this section

Section II: Pages 8-20
90 marks

- Attempt questions 11-16, using the Answer Booklets provided.
- Allow about 2 hours 45 minutes for this section.

BLANK PAGE

Section I

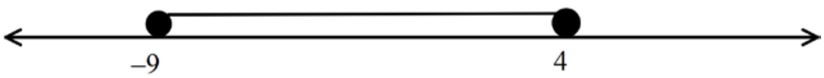
10 marks

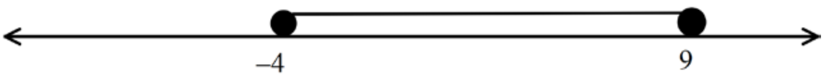
Attempt Questions 1 – 10.

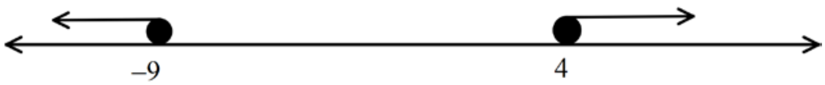
Allow about 15 minutes for this section.

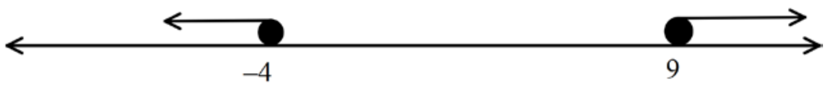
Use the multiple-choice answer sheet for Questions 1 – 10.

1. Which graph shows the solution to $|2x - 5| \leq 13$?

(A) 

(B) 

(C) 

(D) 

2. Given that $f(x) = \dfrac{4x^5 - 8x}{x^3}$, what is the value of $f'(2)$?

(A) 2

(B) 8

(C) 12

(D) 18

3. If $sin\, x = -\frac{1}{5}$ and $\pi \le x \le \frac{3\pi}{2}$, then $cot\, x$ equals:

(A) $-\frac{1}{2\sqrt{6}}$

(B) $2\sqrt{6}$

(C) $\frac{1}{2\sqrt{6}}$

(D) $-2\sqrt{6}$

4. What is the equation of the parabola with directrix $y = -3$ and focus $(0,3)$?

(A) $x^2 = 12y$

(B) $x^2 = -12(y-3)y$

(C) $x^2 = 24y$

(D) $x^2 = 24(y-3)$

5. Which of the following is the same as $\text{cosec}\,(\pi + \theta)$?

(A) $\frac{-1}{\sin\theta}$

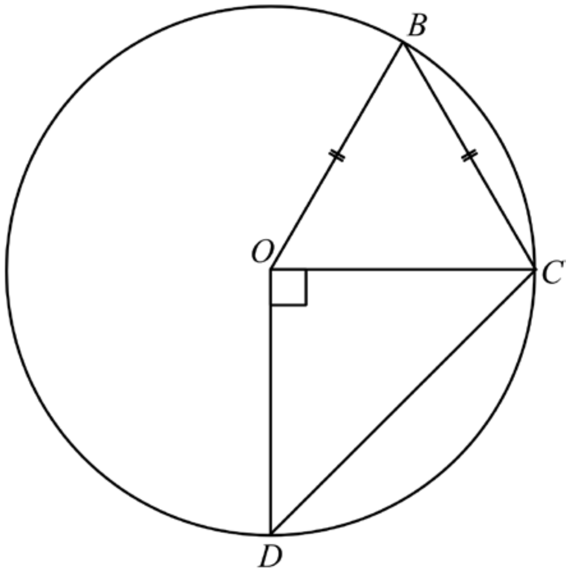
(B) $\frac{-1}{\cos\theta}$

(C) $\frac{1}{\cos\theta}$

(D) $\frac{1}{\sin\theta}$

6. In the diagram below, O is the centre of the circle, and B, C and D are points on the circumference.

$OB = BC$ and $\angle COD$ is a right angle.



What is the size of $\angle BCD$?

(A) $90°$

(B) $105°$

(C) $125°$

(D) $150°$

7. Which statement correctly describes the roots of $2x^2 + 4x - 5 = 0$?

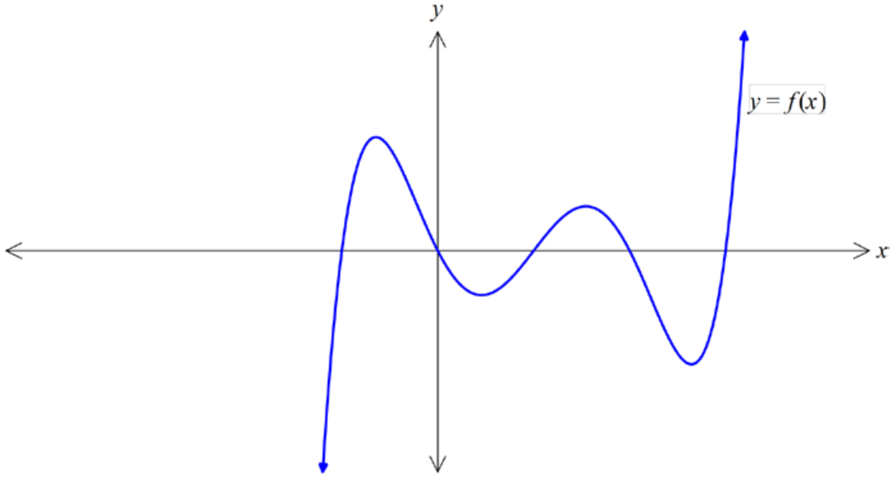
(A) The roots are equal, real and irrational.

(B) The roots are equal, real and rational.

(C) The roots are unequal, real and irrational.

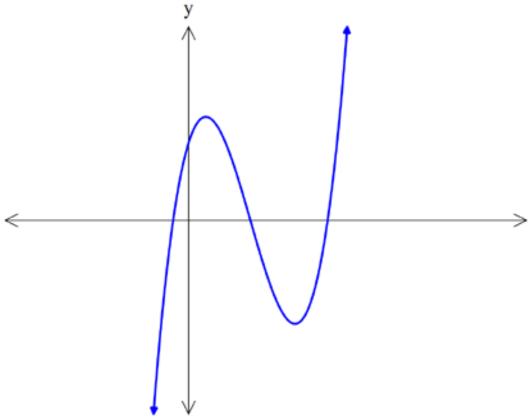
(D) The roots are unequal and unreal.

8. The graph of $y = f(x)$ is shown below.

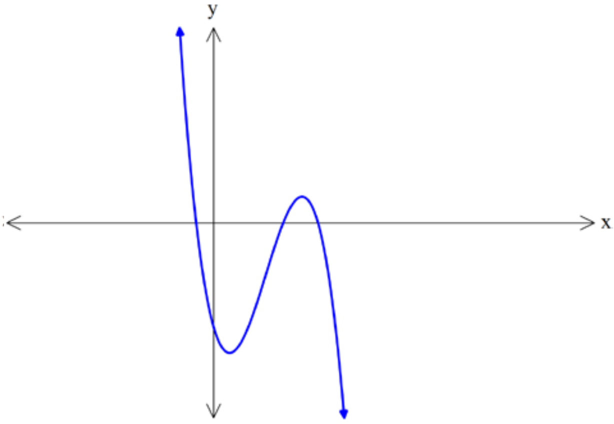


Which of these graphs could represent $y = f'(x)$?

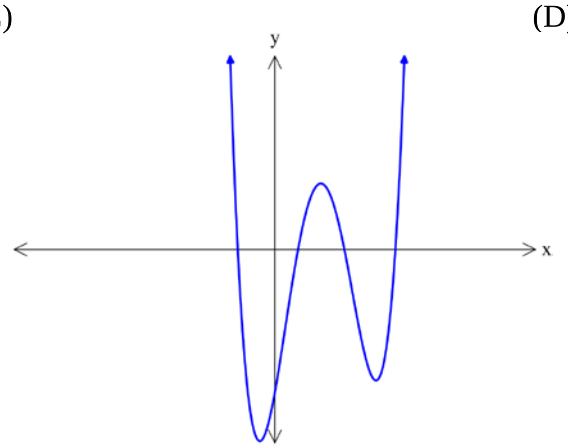
(A)



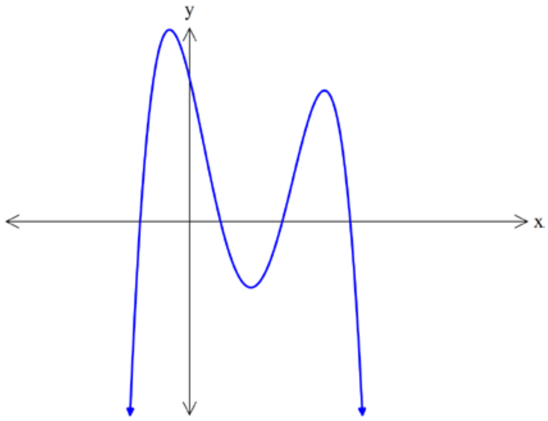
(B)



(C)



(D)

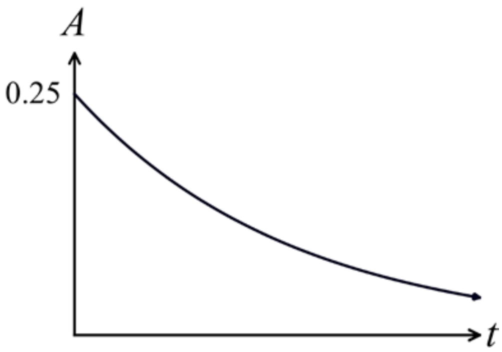


9. The amount of a substance (A) is initially 20 units.

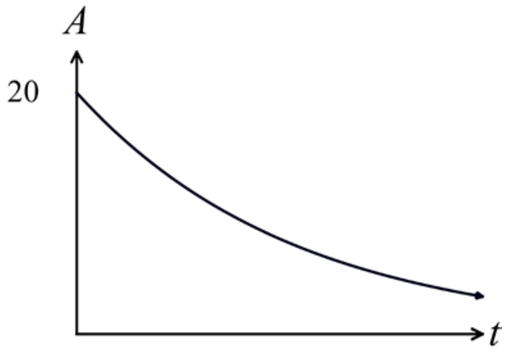
The rate of change in the amount is given by $\frac{dA}{dt} = 0.25A$.

Which graph shows the amount of the substance over time?

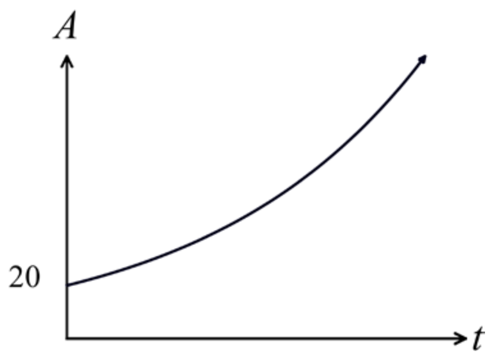
(A)



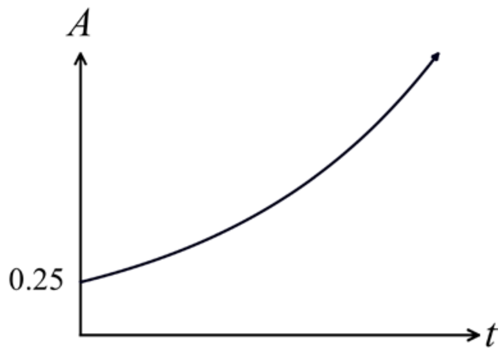
(B)



(C)



(D)



10. What is the approximate value of $\int_{e}^{3e} \ln x \, dx$,using Simpson's rule with three function values?

(A) $\frac{2}{3e}$

(B) $\frac{e(4\ln(5) + 3)}{6}$

(C) $\frac{e(4\ln(6) + 6)}{3}$

(D) $\frac{e(\ln(48) + 6)}{3}$

End of Section 1

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a NEW Writing booklet.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a new writing booklet.**

(a) Expand and simplify $\left(2\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)$. **2**

(b) Simplify $\dfrac{a^4 - ab^3}{a^4 - a^2b^2}$. **2**

(c) Find the gradient of the tangent to the curve $y=\left(e^x+1\right)^5$ at the point where $x = 0$. **2**

(d) Evaluate $\displaystyle\int_1^2 (3x+5)^4 \, dx$. **2**

(e) Show that $\dfrac{d}{dx}\left[(2x+3)^5(x+2)^6\right]=2(11x+19)(2x+3)^4(x+2)^5$ **3**

Question 11 continues on next page

Question 11 continued.

(f) Find the point of intersection of the lines $x+2y-3=0$ and $3x+5y+8=0$. **2**

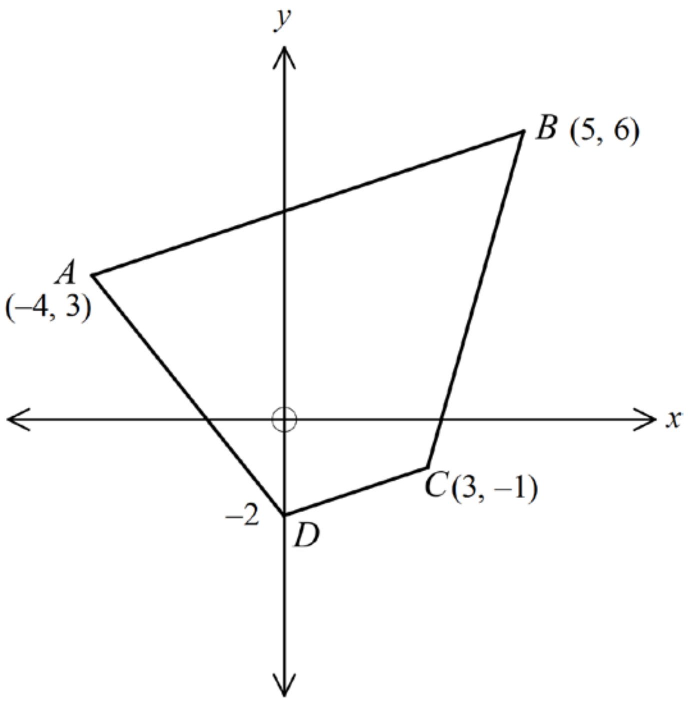
(g) A tangent to the curve $y=2x^2-3x+5$ is parallel to the line $y=5x-6$. Find the coordinates of the point of contact of the tangent to the curve. **2**

End of Question 11

Question 12 (15 marks) **Start a new writing booklet.**

(a) State the domain for the function $y = \sqrt{9 - 5x}$. **1**

(b) A quadrilateral is formed by the points $A(-4, 3)$, $B(5, 6)$, $C(3, -1)$ and $D(0, -2)$ as shown in the diagram.



(i) Show that the quadrilateral is a trapezium, with *AB* || *DC*. **2**

(ii) Show that the equation of *AB* is $x - 3y + 13 = 0$. **1**

(iii) Find the perpendicular distance from *D* to *AB*. **1**

(iv) Find the area of the trapezium *ABCD*. **2**

Question 12 continues on next page

Question 12 continued.

(c) Show that $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \dfrac{2}{\cos^2\theta-\sin^2\theta}$. **2**

(d) Show that the derivative of $f(x)=\dfrac{e^x}{\sqrt{x}}$ is $\dfrac{e^x(2x-1)}{2\sqrt{x^3}}$. **2**

(e) The displacement of a particle moving along the x-axis is given by

$$x=t^4+4t^3-20t^2$$

where x is the displacement from the origin in metres, t is the time in seconds and $t\geq 0$.

(i) What is the acceleration of the particle when t = 5. **2**

(ii) At what time(s) is the particle stationary, but accelerating toward the left? **2**

End of Question 12

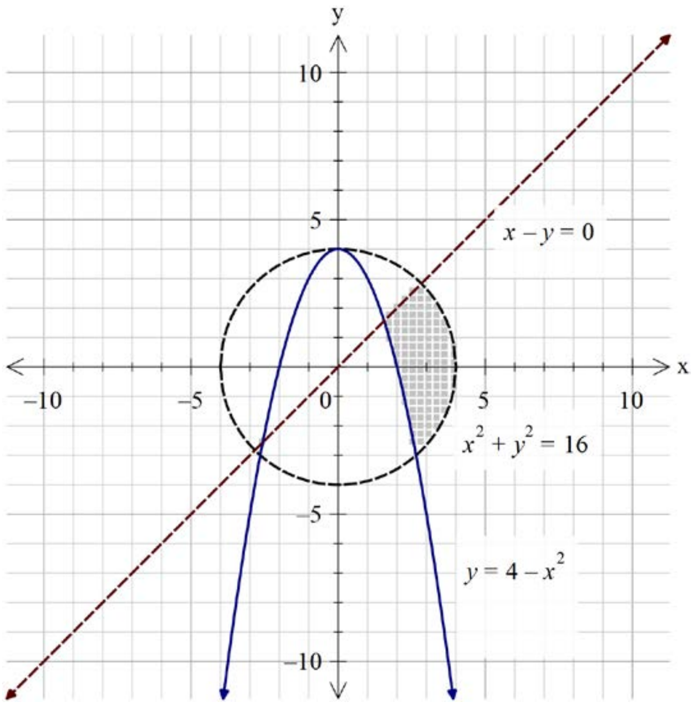
Question 13 (15 marks) **Start a new writing booklet.**

(a) Show that $\dfrac{d}{dx}\left[(\tan^2 x)\ (\cos x)\right] = \dfrac{2\sin x - \sin^3 x}{\cos^2 x}$. **2**

(b) The graphs of each of the following curves are drawn on the diagram below: **2**

$$y = 4 - x^2$$
$$x^2 + y^2 = 16$$
$$x - y = 0$$

Write the inequalities that define the shaded region shown in the diagram:



(c) (i) For what value of k is $\log_{10}\left(3x^2 - 2x\right) = \dfrac{\log_e\left(3x^2 - 2x\right)}{k}$? **1**

Give your answer as an exact value.

(ii) For $f(x) = \log_{10}(3x^2 - 2x)$, find $f'(2)$. **2**

Question 13 continued

(d) The acceleration of a particle moving along the x-axis is given by

$$\ddot{x} = 6t - 14$$

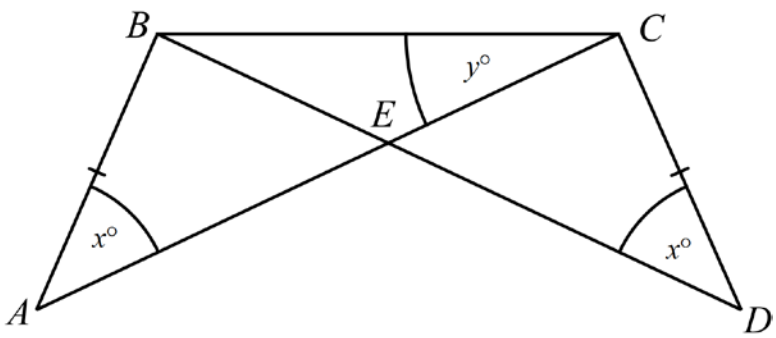
where x is the displacement from the origin in metres, t is the time in seconds and $t \geq 0$.

The particle is initially 2 m to the left of the origin, moving at 8 m/s toward the right.

(i) Find expressions for the velocity and displacement of the particle. **2**

(ii) At what times is the particle at rest? **2**

(e) In the diagram below, $AB = CD$ and $\angle BAC = \angle CDB = x°$. Also $\angle BCA = y°$.



(i) Prove that $\Delta ABE \equiv \Delta DCE$. **2**

(ii) Show that $\angle ABE = 180° - (x + 2y)°$. **2**

End of Question 13

Question 14 (15 marks) **Start a new writing booklet.**

(a) A particular curve passes through the point (2, 7). **2**

For this curve $\dfrac{dy}{dx} = 6e^{3x-6}$.

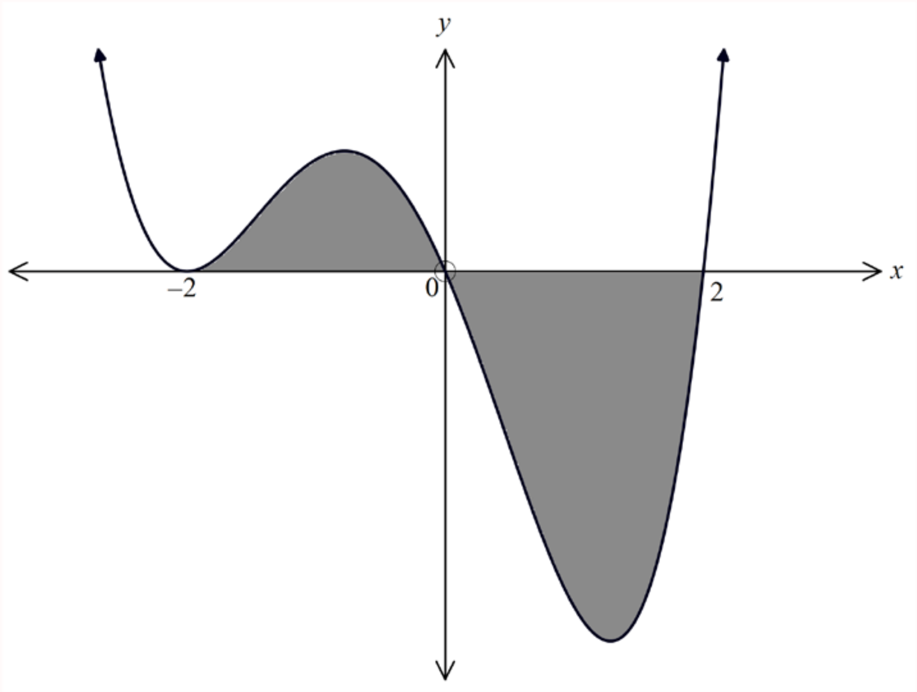
Find the equation of the curve.

(b) (i) Show that the exact solutions of $2u^2 + \sqrt{3}\,u - 3 = 0$ are $u = -\sqrt{3}$ and $\dfrac{\sqrt{3}}{2}$. **2**

(ii) Hence or otherwise solve $2\cos^2 x + \sqrt{3}\cos x - 3 = 0$ for $0 \leq x \leq 2\pi$. **2**

(c) The diagram below shows the curve $y = x^4 + 2x^3 - 4x^2 - 8x$. **2**

Calculate the shaded area.



(d) An excavation site, initially free of any water, has been flooded due to recent wet weather. The water is pumped out so that building can commence.
The rate at which the water is being pumped out, in thousands of litres per hour, is given by

$$\frac{dV}{dt} = 5 - \frac{1}{1+2t} \quad \text{where } t \geq 0.$$

(i) Find the initial rate of water pumped out of the excavation site. **1**

(ii) Calculate the total amount of water pumped out during the first 2 hours. Give your answer correct to the nearest litre. **2**

(e) Atmospheric pressure decreases as the height above sea level increases and is given by

$$A = A_0 e^{kh}$$

where A is the amount of atmospheric pressure present, h is the height in metres above sea level, A_0 and k are constants.

(i) Show that $\frac{dA}{dh} = kA$ is a solution to $A = A_0 e^{kh}$ **1**

(ii) The atmospheric pressure decreases by 12% of its initial value at a height of 1000 m above sea level. Find the value of k. **2**

(iii) Mount Kosciuszko is the tallest mountain in Australia standing 2228 m above sea level. **1**
What percentage of the initial amount of atmospheric pressure will be present at the summit of Mount Kosciuszko?
Give your answer correct to 2 significant figures.

End of Question 14

Question 15 (15 marks) **Start a new writing booklet.**

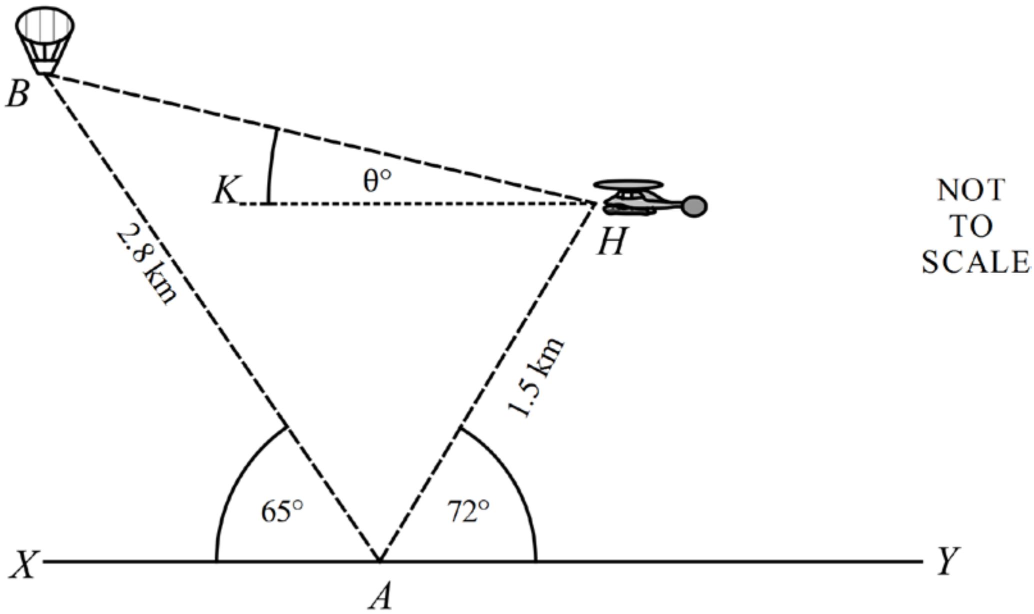
(a) The point $P(x, y)$, moves so that it is equidistant from the points $A\ (-2, 5)$ and $B\ (4, -7)$.

(i) Write an expression for $(AP)^2$. **1**

(ii) Write the equation that describes the locus of P. **2**

(b) From a point A on level ground an observer sees a balloon B and a helicopter H which are both, momentarily, stationary at the time.

The balloon is positioned due west of point A, at a distance of 2.8 km on an angle of elevation of $65°$ and the helicopter is positioned due east of point A, at a distance of 1.5 km on an angle of elevation of $72°$, as shown in the diagram.



(i) Show that the distance between the helicopter and the balloon is approximately 2.0 km, correct to two significant figures. **1**

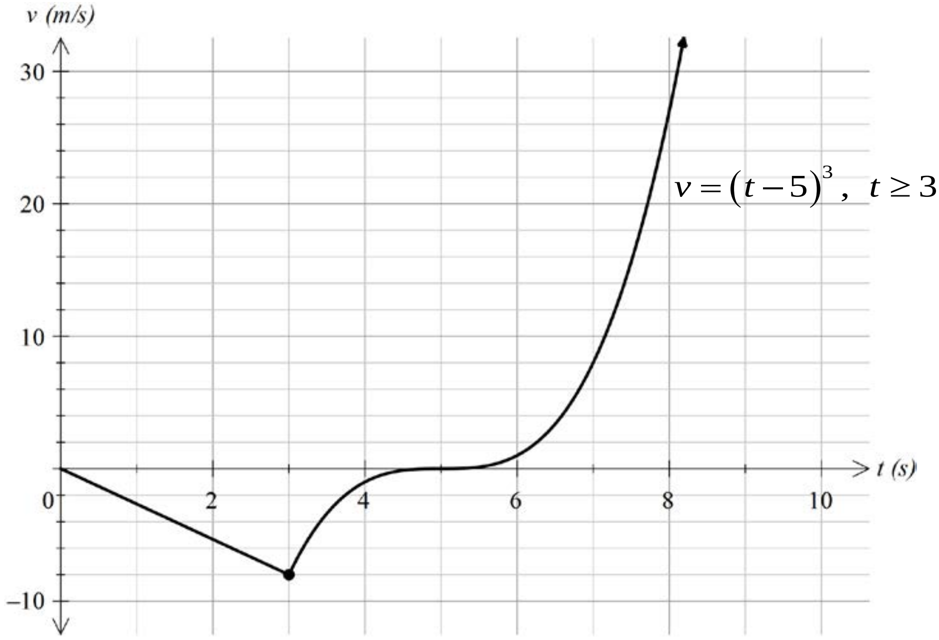
(ii) Calculate the angle of elevation of the balloon as seen from the helicopter (θ). **2**

Answer correct to the nearest degree.

Question 15 continues on next page

Question 15 continued.

(c) The graph shows the velocity, v m/s, of a particle moving on a straight line as a function of time, t seconds. Initially the particle is at the origin.



(i) Calculate the displacement after 5 seconds. **2**

(ii) At what approximate time does the particle return to the origin? Justify your answer with mathematical calculations, correct to 3 significant figures. **2**

Question 15 continues on next page

Question 15 continued.

(d) The curve $y = x^3 - 18x^2 + 60x$ passes through the origin and also has x intercepts at $x = 9 + \sqrt{21}$ and $x = 9 - \sqrt{21}$.

(i) Find the coordinates and nature of all the stationary points and find any inflexion points on the curve $y = x^3 - 18x^2 + 60x$. **3**

(ii) Draw a neat half page sketch of the curve $y = x^3 - 18x^2 + 60x$ showing all the features. **2**

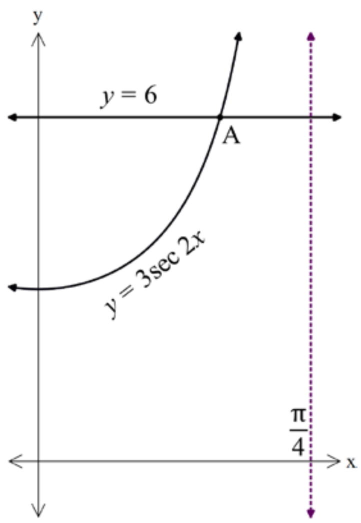
End of Question 15

Question 16 (15 marks) **Start a new writing booklet.**

(a) (i) Draw a neat sketch of $y = 3\cos\left(2x + \frac{\pi}{2}\right)$ for $0 \leq x \leq 2\pi$. **2**

(ii) Hence, or otherwise, solve $3\cos\left(2x+\frac{\pi}{2}\right)=0$ for $0 \leq x \leq 2\pi$. **2**

(b) The graph below shows the line $y = 6$ and the curve $y = 3\sec 2x$ for $0 \leq x \leq \frac{\pi}{4}$.



(i) By solving the equation $3\sec 2x = 6$, show that the point A where the line and curve intersect has coordinates $\left(\frac{\pi}{6}, 6\right)$. **2**

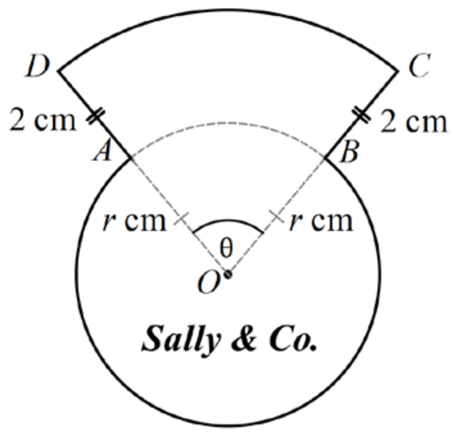
(ii) The region enclosed between the curve $y = 3\sec 2x$ and the x- axis between $x = 0$ and $x = \frac{\pi}{6}$ is rotated about the x- axis. **3**

Find the exact volume of the solid formed.

Question 16 continues on next page

Question 16 continued.

(c) The solid lines in the diagram below shows the new logo of a jewellery business called Sally & Co. The circle has a radius of r cm and centre at O. Sector ODC subtends an angle of θ radians from the centre O and has a radius of $(r+2)\text{cm}$.



NOT TO SCALE

(i) Given that the perimeter of the logo is 24 cm, show that θ is given by **2**

$$\theta = 10 - \pi r \,.$$

(ii) Show that the area (A) of the logo is given by **2**

$$A = 20r + 20 - \pi r^2 - 2\pi r \,.$$

(iii) Find the value of r that produces a maximum area for the logo. Give your answer correct to 2 decimal places. **2**

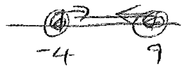
End of Examination

Academic Year	Yr 12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

1. $2x-5 \le 13$, $-2x+5 \le 13$

$2x \le 18$ $\quad$ $-2x \le 8$

$x \le 9$ $\quad$ $x \ge -4$



B.

2. $f(x) = \dfrac{4x^5}{x^3} - \dfrac{8x}{x^3}$

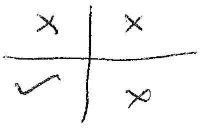
$= 4x^2 - 8x^{-2}$

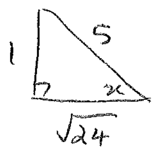
$f'(x) = 8x + 16x^{-3}$

$f'(2) = 8(2) + 16(2)^{-3}$

D

3. $\sin x = -\dfrac{1}{5}$





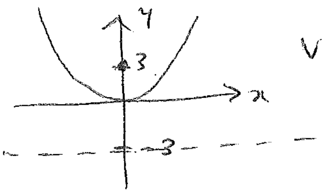
$\tan x = \dfrac{1}{\sqrt{24}}$

$\tan x = \dfrac{1}{2\sqrt{6}}$

$\cot x = 2\sqrt{6}$

B.

4.



vertex $(0,0)$

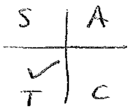
$a = 3$

$x^2 = 4ay$

$x^2 = 12y$

A.

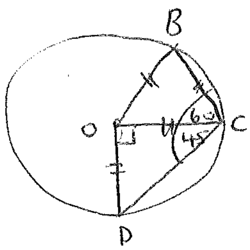
5. $\operatorname{cosec}(\pi+\theta) = \dfrac{1}{\sin(\pi+\theta)}$



$= \dfrac{1}{-\sin\theta}$

A.

6.



$60 + 45 = 105^\circ$

B.

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

7. $\Delta = (4)^2 - 4(2)(-5)$

$= 56 > 0$

a 2
b 4
c −5

C.

8. gradient L → R

C

9. $\frac{dA}{dt} = 0.25A.$ increasing function

initial amount $= 20.$

C

10.

x	e	$2e$	$3e$
$f(x)$	1	$\ln 2e$	$\ln 3e$

$\ln 2e = \ln 2 + \ln e$
$= \ln 2 + 1$

$\ln 3e = \ln 3 + \ln e$
$= \ln 3 + 1$

$A \doteq \frac{e}{3}\left[1 + 4(\ln 2 + 1) + \ln 3 + 1\right]$

$\doteq \frac{e}{3}\left[1 + 4\ln 2 + 4 + \ln 3 + 1\right]$

$\doteq \frac{e}{3}\left[6 + \ln 2^4 + \ln 3\right]$

$\doteq \frac{e}{3}\left[6 + \ln 16(3)\right]$

D.

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

Question 11

a. $(2\sqrt{2}-\sqrt{3})(\sqrt{2}-\sqrt{3})$

$4-2\sqrt{6}-\sqrt{6}+3$

$7-3\sqrt{6}$

b. $$\frac{a^4-ab^3}{a^4-a^2b^2} = \frac{a(a^3-b^3)}{a^2(a^2-b^2)}$$

$$= \frac{\cancel{a}(\cancel{a-b})(a^2+ab+b^2)}{a^{\cancel{2}}(a+b)(\cancel{a-b})}$$

$$= \frac{a^2+ab+b^2}{a(a+b)}$$

c. $y=(e^x+1)^5$

$$\frac{dy}{dx} = 5(e^x+1)^4 \cdot e^x$$

at $x=0$

$m_T = 5e^0(e^0+1)^4$

$= 5(2)^4$

$= 80$

d. $$\int_1^2 (3x+5)^4\,dx = \left[\frac{(3x+5)^5}{15}\right]_1^2$$

$$= \frac{(3(2)+5)^5}{15} - \frac{(3(1)+5)^5}{15}$$

$$= \frac{11^5-8^5}{15} = \frac{42761}{5} = 8552.2$$

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

e. $u = (2x+3)^5 \qquad v = (x+2)^6$

$u' = 10(2x+3)^4 \qquad v' = 6(x+2)^5$

$$\frac{dy}{dx} = 10(2x+3)^4(x+2)^6 + 6(2x+3)^5(x+2)^5$$

$$= 2(2x+3)^4(x+2)^5\left[5(x+2) + 3(2x+3)\right]$$

$$= 2(2x+3)^4(x+2)^5\left[5x+10+6x+9\right]$$

$$= 2(2x+3)^4(x+2)^5(11x+19)$$

f. $x + 2y - 3 = 0 \quad (1)$

$x = 3 - 2y$

$3x + 5y + 8 = 0 \quad (2)$

$3(3-2y) + 5y + 8 = 0$

$9 - 6y + 5y + 8 = 0$

$17 - y = 0$

$y = 17$

$x = 3 - 2(17) = -31 \qquad (-31, 17)$

g. $y = 2x^2 - 3x + 5$

$\frac{dy}{dx} = 4x - 3$

$m_T = 5$ (parallel)

$4x - 3 = 5$

$x = 2$

$y = 2(2)^2 - 3(2) + 5$

$y = 7$

$(2, 7)$

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

Question 12

a. $9 - 5x \geq 0$

$5x \leq 9$

$x \leq \frac{9}{5}$.

b. (i) $m_{AB} = \frac{6-3}{5+4}$, $m_{DC} = \frac{-1+2}{3-0}$

$= \frac{3}{9}$ $= \frac{1}{3}$

$= \frac{1}{3}$

Since $m_{AB} = m_{DC}$

$AB \parallel DC$

(ii) $m_{AB} = \frac{1}{3}$ $B(5, 6)$

$y - 6 = \frac{1}{3}(x-5)$

$3y - 18 = x - 5$

$x - 3y + 13 = 0$.

(iii) $d = \left| \frac{x_1 - 3y_1 + 13}{\sqrt{1^2+3^2}} \right|$ $D(0, -2)$

$= \left| \frac{0 - 3(-2) + 13}{\sqrt{10}} \right|$

$d = \frac{19}{\sqrt{10}}$

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

(iv) $\text{Area} = \frac{h}{2}(a+b)$

$= \frac{19}{2\sqrt{10}}(a+b)$

AB: $a = \sqrt{(6-3)^2 + (5+4)^2}$

$= \sqrt{90}$

$= 3\sqrt{10}$

CD: $b = \sqrt{(-1+2)^2+(3-0)^2}$

$= \sqrt{10}$

$\text{Area} = \frac{19}{2\sqrt{10}}(3\sqrt{10}+\sqrt{10})$

$= \frac{19}{2\sqrt{10}}(4\sqrt{10})$

$= 38 \text{ unit}^2$.

c. $LHS = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$

$= \frac{\cos^2\theta + \cancel{2\sin\theta\cos\theta} + \sin^2\theta + \cos^2\theta - \cancel{2\sin\theta\cos\theta} + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + \cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \frac{2}{\cos^2\theta - \sin^2\theta}$

$= RHS$.

d. $f(x) = \dfrac{e^x}{\sqrt{x}}$

$u = e^x \qquad v = x^{\frac{1}{2}}$

$u' = e^x \qquad v' = \frac{1}{2}x^{-\frac{1}{2}}$

$$f'(x) = \frac{e^x x^{\frac{1}{2}} - \frac{1}{2}e^x x^{-\frac{1}{2}}}{\left((x)^{\frac{1}{2}}\right)^2}$$

$$= \frac{x^{-\frac{1}{2}} e^x \left(x - \frac{1}{2}\right)}{x}$$

$$= \frac{e^x \left(\frac{2x-1}{2}\right)}{x\, x^{\frac{1}{2}}}$$

$$= \frac{e^x (2x-1)}{2x^{3/2}}$$

$$= \frac{e^x (2x-1)}{2\sqrt{x^3}}$$

e. $x = t^4 + 4t^3 - 20t^2$

(i) $\dot{x} = 4t^3 + 12t^2 - 40t$

$\ddot{x} = 12t^2 + 24t - 40$

at $t = 5$ $\ddot{x} = 12(5)^2 + 24(5) - 40$

$= 380 \text{ m/s/s}$

(ii) $\dot{x} = 0$

$4t^3 + 12t^2 - 40t = 0$

$4t(t^2 - 3t - 10) = 0$

$t(t+5)(t-2) = 0$

$\therefore \quad t = 0, 2 \quad (t \neq -5)$

at $t = 0$ $\ddot{x} = -40$, accelerating left

$t = 2$ $\ddot{x} > 0$, accelerating right.

$\therefore$ <u>at $t = 0$.</u>

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

Question 13

a. $u = \tan^2 x \qquad v = \cos x$

$u' = 2\tan x \sec^2 x \qquad v' = -\sin x$

$$\frac{dy}{dx} = 2\cos x \tan x \sec^2 x - \sin x . \tan^2 x$$

$$= 2 \cancel{\cos x} . \frac{\sin x}{\cancel{\cos x}} . \frac{1}{\cos^2 x} - \sin x . \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{2\sin x}{\cos^2 x} - \frac{\sin^3 x}{\cos^2 x}$$

$$= \frac{2\sin x - \sin^3 x}{\cos^2 x}$$

b. $x^2 + y^2 < 16$

test $(3,0)$

$y \square 4 - x^2$

$0 \square 4 - 9$

$\therefore \leq$

test $(3,0)$

$x - y \square 0$

$3 \square 0$

$\therefore >$

$\therefore \quad x^2 + y^2 < 16$

$y \geq 4 - x^2$

$x - y > 0.$

c. (i) $\log_{10}(3x^2-2x) = \dfrac{\log_e(3x^2-2x)}{\log_e 10}$

$\therefore k = \log_e 10.$

(ii) $\dfrac{d}{dx}\left[\dfrac{\ln(3x^2-2x)}{\ln 10}\right] = \dfrac{1}{\ln 10} \times \dfrac{6x-2}{3x^2-2x}$

$f'(2) = \dfrac{1}{\ln 10} \times \left[\dfrac{6(2)-2}{3(2)^2-2(2)}\right]$

$= \dfrac{1}{\ln 10} \times \dfrac{10}{8} = \dfrac{5}{4\ln 10}.$

$\doteq 0.542868\ldots$

d. $\ddot{x} = 6t-14$

at $t=0$ $x=-2$ $\dot{x}=8$

(i) $\dot{x} = \int 6t-14\, dt$

$\dot{x} = 3t^2-14t+c$

at $t=0$ $\dot{x}=8$

$\therefore c = 8$

$\dot{x} = 3t^2-14t+8$

$x = \int 3t^2-14t+8\, dt$

$x = t^3-7t^2+8t+k$

at $t=0$ $x=-2$

$\therefore k = -2$

$\therefore \quad x = t^3 - 7t^2 + 8t - 2$

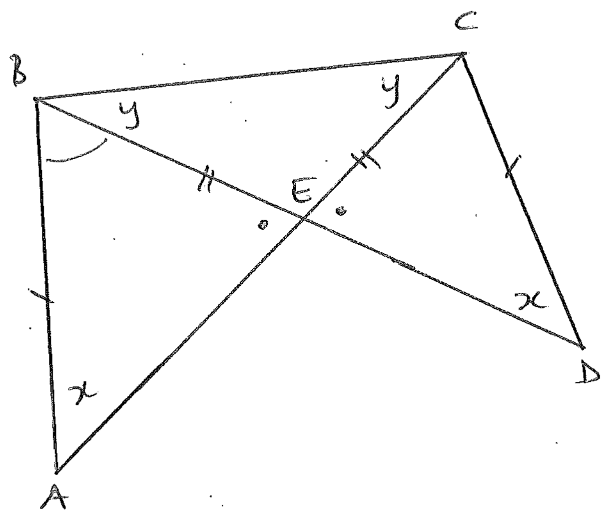
(ii) at rest $\dot{x} = 0$

$3t^2 - 14t + 8 = 0$

$(3t-2)(t-4) = 0$

$\therefore \quad t = \frac{2}{3}, 4.$

e.



(i) $AB = CD$ (given)

$\angle BAE = \angle CDE$ (given)

$\angle AEB = \angle CED$ (vertically opposite angles)

$\therefore \quad \Delta ABE \equiv \Delta DCE$ (AAS)

(ii) Since $\Delta ABE \equiv \Delta DCE$

then $BE = EC$

ΔBCE is isosceles ($BE = EC$)

$\therefore \angle CBE = \angle BCE = y$

$\angle ABC = 180 - (x+y)$ angle sum of ΔABC

$\therefore \quad \angle ABE = 180 - x - y - y$ $(\angle ABC = \angle ABE + \angle CBE$

$= 180 - x - 2y$

$= 180 - (x + 2y)$

Academic Year	12	Calendar Year	2016
Course	2u	Name of task/exam	Trial

Question 14

a.

$$\frac{dy}{dx} = 6e^{3x-6}$$

$$y = \int 6e^{3x-6}\,dx$$

$$y = \frac{6e^{3x-6}}{3} + c$$

$$y = 2e^{3x-6} + c$$

$(2, 7)$

$$7 = 2e^{0} + c$$

$$\therefore c = 5$$

$$\therefore y = 2e^{3x-6} + 5$$

b. (i)

$$2u^2 + \sqrt{3}u - 3 = 0$$

$$u = \frac{-\sqrt{3} \pm \sqrt{(\sqrt{3})^2 - 4(2)(-3)}}{4}$$

$$u = \frac{-\sqrt{3} \pm \sqrt{3+24}}{4}$$

$$u = \frac{-\sqrt{3} \pm \sqrt{27}}{4}$$

$$u = \frac{-\sqrt{3} \pm 3\sqrt{3}}{4}$$

$$u = \frac{-\sqrt{3}+3\sqrt{3}}{4}, \frac{-\sqrt{3}-3\sqrt{3}}{4}$$

$$u = \frac{2\sqrt{3}}{4}, \frac{-4\sqrt{3}}{4}$$

$$u = \frac{\sqrt{3}}{2}, -\sqrt{3}$$

(ii) let $u = \cos x$

$\therefore \cos x = -\sqrt{3}$

$-1 \le \cos x \le 1$

$\therefore$ no solution

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

(c)

$$A_1 = \int_{-2}^{0} x^4 + 2x^3 - 4x^2 - 8x \, dx$$

$$= \left[\frac{x^5}{5} + \frac{2x^4}{4} - \frac{4x^3}{3} - \frac{8x^2}{2}\right]_{-2}^{0}$$

$$= \left[\frac{x^5}{5} + \frac{x^4}{2} - \frac{4x^3}{3} - 4x^2\right]_{-2}^{0}$$

$$= 0 - \left(\frac{(-2)^5}{5} + \frac{(-2)^4}{2} - \frac{4(-2)^3}{3} - 4(-2)^2\right.$$

$$= 0 - \left(-\frac{56}{15}\right)$$

$$= \frac{56}{15}$$

$$A_2 = \left| \left[\frac{x^5}{5} + \frac{x^4}{2} - \frac{4x^3}{3} - 4x^2\right]_{0}^{2} \right|$$ * Same integration as A_1

$$= \left| \frac{2^5}{5} + \frac{2^4}{2} - \frac{4}{3}(2)^3 - 4(2)^2 - 0 \right|$$

$$= \frac{184}{15}$$

$\therefore$ Total Area $= \frac{56 + 184}{15} = 16$ units^2.

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

d. $\dfrac{dV}{dt} = 5 - \dfrac{1}{1+2t}$

(i) at $t=0$ $\dfrac{dV}{dt} = 5 - \dfrac{1}{1+2(0)}$

$= 4$ (thousands of litres)

$\therefore$ rate at 4000L/h

(ii) $V = \int 5 - \dfrac{1}{1+2t}\, dt$

$= \int 5 - \dfrac{1}{2}\left(\dfrac{2}{1+2t}\right) dt$

$V = 5t - \dfrac{1}{2}\ln(1+2t) + c$

at $t=0$ $V=0$

$\therefore c = 0$

$V = 5t - \frac{1}{2}\ln(1+2t)$

at $t=2$

$V = 5(2) - \frac{1}{2}\ln(1+2(2))$

$V = 10 - \frac{1}{2}\ln 5$ (thousands of L)

$\therefore V = 9195.28104\ldots$ L

$\underline{V = 9195\text{ L}}$

Academic Year	12	Calendar Year	2016
Course	2u	Name of task/exam	Trial

e)

(i) $A = A_0 e^{kh}$

$\frac{dA}{dh} = A_0 e^{kh} \cdot k$

since $A = A_0 e^{kh}$

$\frac{dA}{dh} = kA$

(ii) at $h = 1000$ $A = (100\% - 12\%) A_0$

$= 0.88 A_0$

$A = A_0 e^{kh}$

$0.88 A_0 = A_0 e^{1000k}$

$e^{1000k} = 0.88$

$1000k = \ln(0.88)$

$k = \frac{\ln(0.88)}{1000}$

(iii) $h = 2228$

$A = A_0 e^{k \times 2228}$

$A = A_0 e^{\frac{\ln(0.88)}{1000} \times 2228}$

$A = A_0 (0.752155..)$

$A = A_0 (0.75)$

$\therefore 75\%$.

Question 15

a (i) $(AP)^2 = (x+2)^2 + (y-5)^2$

$= x^2 + 4x + 4 + y^2 - 10y + 25$

$= x^2 + 4x + y^2 - 10y + 29$

(ii) $(AP)^2 = (BP)^2$

$x^2 + 4x + y^2 - 10y + 29 = (x-4)^2 + (y+7)^2$

$x^2 + 4x + y^2 - 10y + 29 = x^2 - 8x + 16 + y^2 + 14y + 49$

$4x - 10y + 29 = -8x + 16 + 14y + 49$

$12x - 24y - 36 = 0$

$x - 2y - 3 = 0$

b (i) $\angle BAH = 180 - 65 - 72 = 43°$

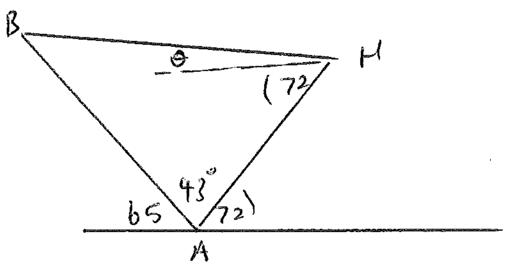
$BH^2 = 2.8^2 + 1.5^2 - 2(2.8)(1.5)\cos 43$

$= 3.9466289\ldots$

$BH = \sqrt{3.9466289\ldots}$

$= 1.9866\ldots$

$\doteqdot 2.0$ km



(ii) $\dfrac{\sin B}{1.5} = \dfrac{\sin 43}{1.9866\ldots}$

$B = \sin^{-1}\left[\dfrac{1.5 \sin 43}{1.9866\ldots}\right]$

$= 30°45'49''$

$\therefore \theta = 180 - (43° + 30°45'49'' + 72°)$

$= 34°14'$

c (i) $0 \le t \le 3$ $\quad$ Area $= \frac{1}{2} \times 8 \times 3 = 12$

$3 \le t \le 5$ $\quad$ Area $= \left| \int_3^5 (t-5)^3 \, dt \right|$

$$= \left| \left[\frac{(t-5)^4}{4} \right]_3^5 \right|$$

$$= \left| 0 - \left(\frac{(3-5)^4}{4} \right) \right|$$

$$= 4 \text{ m}$$

$\therefore$ total displacement $= 12 + 4 = 16$ m

(ii) $\int_5^k (t-5)^3 \, dt = 16$

$$\left[\frac{(t-5)^4}{4} \right]_5^k = 16$$

$$\frac{(k-5)^4}{4} - 0 = 16$$

$$\frac{(k-5)^4}{4} = 16$$

$$(k-5)^4 = 64$$

$$k - 5 = 64^{1/4}$$

$$\therefore k = 64^{1/4} + 5 = 7.83$$

d. $y = x^3 - 18x^2 + 60x$

$(0,0)$

$x = 9 \pm \sqrt{21}$

(i) $\frac{dy}{dx} = 3x^2 - 36x + 60$

$\frac{d^2y}{dx^2} = 6x - 36$

$\frac{dy}{dx} = 0$ $\quad 3x^2 - 36x + 60 = 0$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = 2, 10.$

<u>at $x = 2$:</u>

$y = 2^3 - 18(2)^2 + 60(2) = 56$

$\frac{d^2y}{dx^2} = 6(2) - 36 < 0 \quad \therefore$

$\therefore$ max $(2, 56)$

<u>at $x = 10$:</u>

$y = 10^3 - 18(10)^2 + 60(10) = -200$

$\frac{d^2y}{dx^2} = 6(10) - 36 > 0 \quad \therefore$

$\therefore$ min $(10, -200)$

Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

inflexion points:

$$\frac{d^2y}{dx^2} = 6x - 36 = 0$$

$$6x = 36$$

$$x = 6$$

at $x = 6$, $y = 6^3 - 18(6)^2 + 60(6)$

$= -72$

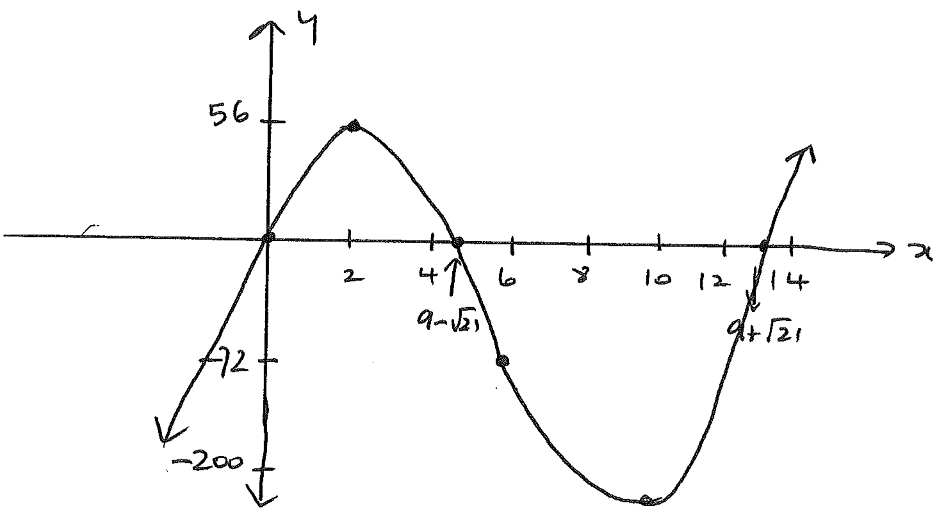
check concavity:

x	5	6	7
$\frac{d^2y}{dx^2}$	-6	0	7
	$<$		$>$

$\therefore$ change in concavity

$\therefore$ point of inflexion $(6, -72)$

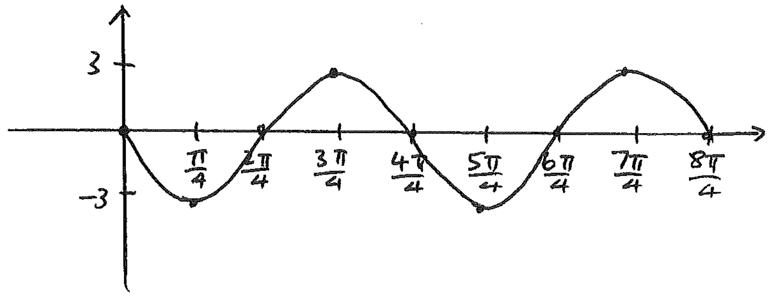
(e)



Academic Year	12	Calendar Year	2016
Course	2U	Name of task/exam	Trial

Question 16.

a. (i)



(ii) $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi.$

b. (i) $3\sec 2x = 6$

$\sec 2x = 2$

$\cos 2x = \frac{1}{2}$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$

at $x = \frac{\pi}{6}$ $\quad$ $\left.\begin{array}{l} y = 3\sec 2\left(\frac{\pi}{6}\right) \\ y = 3\sec\frac{\pi}{3} \\ y = 3\,\frac{1}{\cos\frac{\pi}{3}} \end{array}\right\}$ unnecessary

$y = 3.(2) = 6$

(ii) $V = \pi\int y^2\,dx$ $\qquad$ $y = 3\sec 2x$

$V = \pi\int_0^{\frac{\pi}{6}} 9\sec^2 2x\,dx$ $\qquad$ $y^2 = 9\sec^2 2x$

$V = 9\pi\int_0^{\frac{\pi}{6}} \sec^2 2x\,dx$

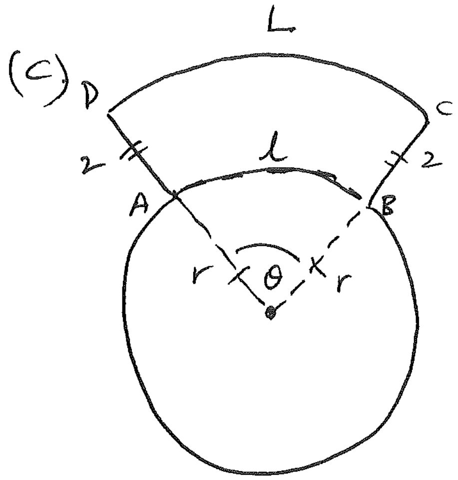
$V = 9\pi\left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{6}}$

$V = \frac{9\pi}{2}\left[\tan\frac{2\pi}{6} - \tan 0\right]$

$V = \frac{9\pi}{2}\left[\sqrt{3}\right] = \frac{9\sqrt{3}\,\pi}{2}$

Academic Year	12	Calendar Year	2016
Course	2u	Name of task/exam	Trial

(c)



(i) $L = r\theta$, $l = r\theta$

$r = r+2$

$\therefore L = (r+2)\theta$

$\therefore$ Perimeter $= 4 + (r+2)\theta + (2\pi r - r\theta)$

$P = 4 + r\theta + 2\theta + 2\pi r - r\theta$

$P = 4 + 2\theta + 2\pi r$

$P = 24$

$24 = 4 + 2\theta + 2\pi r$

$20 = 2\theta + 2\pi r$

$10 = \theta + \pi r$

$\theta = 10 - \pi r$

(ii) Area $= \frac{1}{2}(r+2)^2\theta - \frac{1}{2}r^2\theta + \pi r^2$

$A = \frac{1}{2}(r^2+4r+4)\theta - \frac{1}{2}r^2\theta + \pi r^2$

$A = \frac{\theta}{2}(r^2+4r+4) - \frac{1}{2}r^2\theta + \pi r^2$

$A = \frac{r^2\theta}{2} + \frac{4r\theta}{2} + \frac{4\theta}{2} - \frac{1}{2}r^2\theta + \frac{2\pi r^2}{2}$

$$A = 2r\theta + 2\theta + \pi r^2$$

$$\theta = 10 - \pi r \quad (i)$$

$$A = 2r(10 - \pi r) + 2(10 - \pi r) + \pi r^2$$

$$A = 20r - 2\pi r^2 + 20 - 2\pi r + \pi r^2$$

$$A = 20r - \pi r^2 + 20 - 2\pi r$$

(iii) $\frac{dA}{dr} = 20 - 2\pi r - 2\pi$

$= 0$ for stat. pt.

$$20 - 2\pi r - 2\pi = 0$$

$$2\pi r = 20 - 2\pi$$

$$\pi r = 10 - \pi$$

$$r = \frac{10 - \pi}{\pi} \doteq 2.183\ldots$$

$$\frac{d^2A}{dr^2} = -2\pi < 0 \quad \therefore \text{ max.}$$

$$\therefore r \doteq 2.18 \text{ cm.}$$