NAME: ____________________

TEACHER'S NAME: ____________________

**Mrs Choong**
**Mrs Gibson**
**Mrs Hickey**
**Mrs Kench**
**Mrs Leslie**
**Mrs Quarles**
**Ms Slade**
**Mrs Stock**

# PYMBLE LADIES' COLLEGE

## 1997 TRIAL H.S.C. EXAMINATION

# MATHEMATICS
# 2 UNIT

**TIME ALLOWED: 3 HOURS**

**INSTRUCTIONS TO CANDIDATES:**

1. All questions must be attempted.
2. All necessary working must be shown.
3. **Start each question on a new page.**
4. Put you name and your teacher's name on every sheet of paper.
5. Marks may be deducted for careless or untidy work.
6. Only approved calculators may be used.
7. **DO NOT** staple different questions together.
8. Hand this question paper in with your answers.
9. All rough working paper must be attached to the back of the last question.
10. All questions are of equal value.

**There are ten (10) questions in this paper**



MARKS

**QUESTION 1**

2 (a) Evaluate $\dfrac{\sqrt{a^2 + b^2}}{c}$ if $a = 1.23$, $b = 0.85$ and $c = 4.81$

Answer correct to 2 decimal places.

1 (b) Factorise fully $3m^2 - 13m + 4$

2 (c) If $\dfrac{5}{2 + \sqrt{3}} = m + n\sqrt{3}$, rationalise the denominator to find $m$ and $n$
where $m$ and $n$ are integers

2 (d) Simplify $\dfrac{x - 2}{x^2 + x - 6}$

2 (e) Given $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, find $\log_a 18$.

3 (f) Solve $|2x - 1| > 5$ and graph the solution set on a number line.

MARKS

**QUESTION 2** (START A NEW PAGE)

1 (a) On a number plane mark the points A (- 4, 5) , B ( 0, 6) and C (1, 2).

1 (b) If ABCD forms a square, write down the co-ordinates of D.

1 (c) Show that the midpoint P of AC is (-1½, 3½).

2 (d) Show that the diagonals of the square ABCD bisect each other.

1 (e) Show that the gradient of the line AC = $\frac{-3}{5}$.

2 (f) Hence show that the diagonals are perpendicular to each other.

1 (g) Show that PB has a length of $\sqrt{\frac{17}{2}}$ units

1 (h) Find the area of the square ABCD

2 (i) If a circle is drawn to touch A, B, C and D, write down the equation of this circle. (Leave answer in unexpanded form.)



MARKS

**QUESTION 3** (START A NEW PAGE)

2 (a) Evaluate $\sum_{r=1}^{3} (2^r \times 3^{r-1})$

4 (b) Find: (i) $\frac{d}{dx}\left(\frac{1}{3x}\right)$

(ii) $\frac{d}{dx}\sin(2x-1)$

(iii) $\frac{d}{dx}\left(\frac{2x+1}{x-4}\right)$

1 (c) Show that $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

5 (d) If $\alpha$ *and* $\beta$ are roots of the equation $3x^2 + 2x - 6 = 0$, find the exact value of:

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2 + \beta^2$

(iv) $\alpha^3 + \beta^3$

MARKS

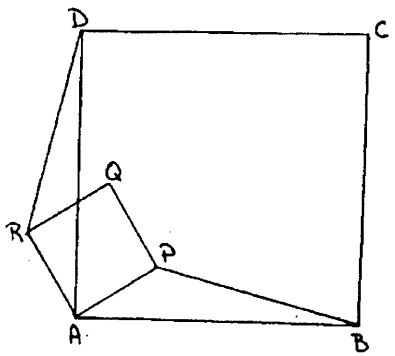
**QUESTION 4** (START A NEW PAGE)

2 (a) A parabola has its vertex at (2, 1) and focus at (2, -1), write down the equation of:

(i) its directrix

(ii) this parabola

5 (b) In the diagram below, both ABCD and APQR are squares




(i) Copy this diagram onto your answer sheet

(ii) Prove that $\Delta$ APB $\equiv$ $\Delta$ ARD.

(iii) Hence or otherwise prove that BP = DR.

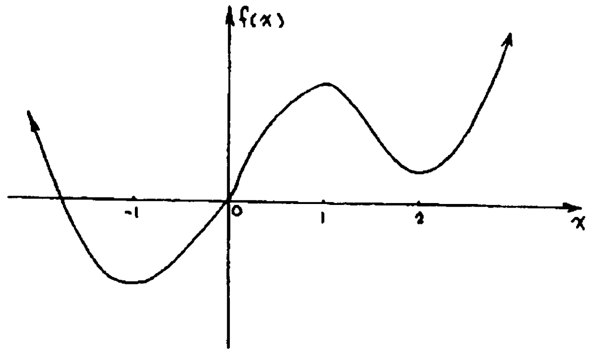
(QUESTION 4 CONTINUED OVER PAGE)



MARKS

**(QUESTION 4 CONTINUED)**

5 (c) The diagram below is a sketch of $y = f(x)$ where $f''(0) = 0$ and $f''(1\frac{1}{2}) = 0$.



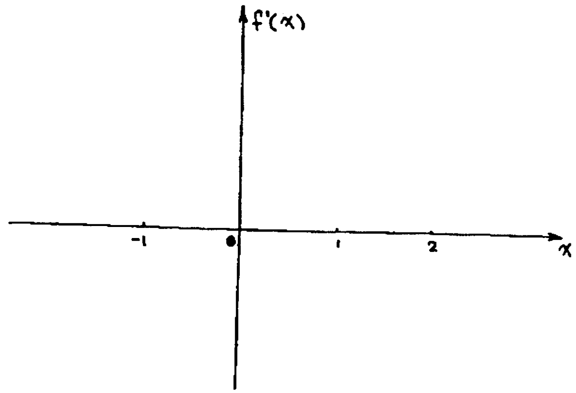

(1) For what values of $x$ is:

(i) $f'(x) = 0$

(ii) $f'(x) > 0$

(iii) $f''(x) < 0$

(2) Copy the axes below onto your answer sheet and sketch on it $y = f'(x)$, given that $f'(0) = 1$



MARKS

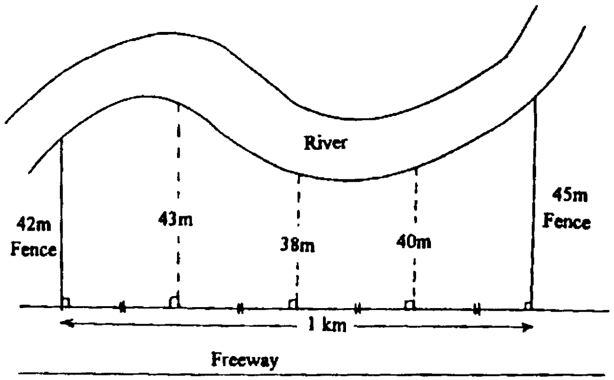
**QUESTION 5** (START A NEW PAGE)

3 (a) Find the equation of the line which makes an angle of $135°$ with the positive direction of the $x-$ axis and passes through the intersection of $2x+5y-10=0$ and $3x-y+19=0$

3 (b) (i) Show that $\frac{d}{dx} ln(\sin x) = \cot x$

(ii) Hence or otherwise find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} -\cot x \, dx$

3 (c)




The above diagram shows a field which is bounded by a river, a freeway and two fences.

Use Simpson's Rule with 5 function values to approximate the area of the field.

3 (d) (i) Find the points of intersection of $y = x+4$ and $y = 16-x^2$

(ii) On the same number plane, shade in the region where $y \geq 16-x^2$ and $y > x+4$ hold simultaneously.

Clearly show the points of intersection.



MARKS

**QUESTION 6** (START A NEW PAGE)

3 (a) Find:

(i) $\int (4x^3 + \frac{x}{5} - 1) \, dx$

(ii) $\int e^{\frac{x}{2}} \, dx$

3 (b) Find the values of A and B for which

$$\frac{1-x}{x^2-7x+12} = \frac{A}{x-3} - \frac{B}{x-4}$$

where $x \neq 3$ and $x \neq 4$.

3 (c) (i) For what values of $x$ can the geometric series

$1 + 2x + 4x^2 + 8x^3 + \ldots\ldots$ ~~be summed to infinity~~ have a limiting sum,.

(ii) For what value of $x$ does $\sum_{r=0}^{\infty} (2x)^r = \frac{5}{9}$

3 (d) Radioactive decomposition takes place according to the law $S = Pe^{-kt}$

where $t$ is in years.

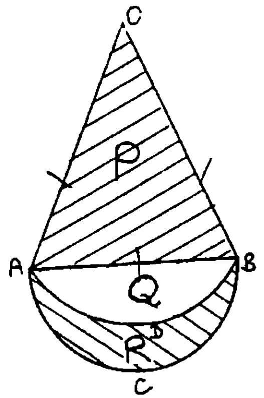
Given $k = 0.05$ and the initial mass of the radioactive material is 20mg.

Find the rate of radioactive decomposition after 15 years. (Answer correct to 2 significant figures.)

MARKS

**QUESTION 7** (START A NEW PAGE)

7 (a) AOBD is a sector of a circle with centre $O$ and radius $r$.
ACB is a semicircle drawn with AB as diameter and AB = OA = OB = $r$.
P and Q are the areas of the triangle and the segment ADB respectively.
R is the area enclosed by the arcs ADB and ACB, as shown in the diagram below.




(i) Show that:

(α) $P + Q = \dfrac{\pi r^2}{6}$

(β) $Q + R = \dfrac{\pi r^2}{8}$

and hence show that (γ) $P - R = \dfrac{\pi r^2}{24}$

(δ) $R = \dfrac{3P - Q}{4}$

(ii) Find the exact area of P in terms of r.

(QUESTION 7 continued over page)



MARKS

**(QUESTION 7 CONTINUED)**

5 (b) Given $f(x) = \sin 2x - 1$ and $g(x) = \cos 2x$;

(i) Write down the range of $f(x)$ and $g(x)$ for the domain

$0 \le x \le \dfrac{\pi}{2}$.

(ii) On the same number plane, draw the graphs of $f(x)$ and $g(x)$.

$\left(0 \le x \le \dfrac{\pi}{2}\right)$.

(iii) How many solutions does $\sin 2x = 1 + \cos 2x$ have?

$\left(0 \le x \le \dfrac{\pi}{2}\right)$.

MARKS

**QUESTION 8** (START A NEW PAGE)

7 (a) Consider the curve $y = x^4 + 2x^2 - 1$.

(i) Find any stationary points and determine their nature.

(ii) Sketch the curve. (There is no need to find the $x$ intercepts.)

(iii) Show that the points of intersection of

$y = x^4 + 2x^2 - 1$ and $y = 2x^2$ are (1, 2) and (-1, 2).

(iv) On the same number plane in (ii), sketch the curve $y = 2x^2$.

(v) Find the area bounded between the curves

$y = x^4 + 2x^2 - 1$ and $y = 2x^2$.

2 (b) Solve $2\sin^2\theta - \cos\theta = 1$ for $0 \le \theta \le 2\pi$.

3 (c) The captain of a submarine spots a freighter on the horizon. He knows that a single torpedo has a probability of ¼ of sinking the freighter, ½ of damaging it and ¼ of missing it. He also knows that 2 damaging shots will sink the freighter. If two torpedoes are fired independently, find the probability of:

(i) sinking the freighter with 2 damaging shots

(ii) sinking the freighter.



MARKS

**QUESTION 9** (START A NEW PAGE)

4 (a) A body moves in a straight line such that after $t$ seconds its acceleration is given by $(t + 2)\ ms^{-2}$. If it starts from rest, find:

(i) its velocity after 3 seconds

(ii) the distance travelled during the fourth second.

3 (b) During the last long weekend, the probability of a random breath test station picking a driver under the legal limit was 0.75. This station picked $n$ cars over the weekend.

(i) What is the probability that no driver was over the legal limit? (i.e. all $n$ drivers were under the limit).

(ii) How many cars must be picked to be at least 95% certain that at least one driver will be over the legal limit?

5 (c) A fund is set up with a single investment of \$2000 to provide an annual prize of \$150. The fund accrues interest at 5% p.a. paid yearly and the first prize is awarded one year after investment.

(i) Show that the value of the fund after $n$ years is given by

$$2000\,(1.05)^n - 150[1 + 1.05 + (1.05)^2 + \ldots + (1.05)^{n-1}].$$

(ii) Find the number of years for which the full prize can be awarded.

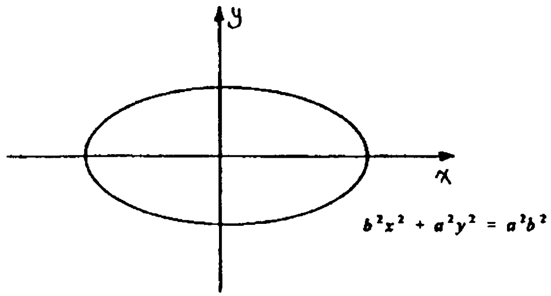
MARKS

**QUESTION 10** (START A NEW PAGE)

5 (a) A football has a volume that is approximately the same as the volume generated by revolving the area bounded by the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ (where $a$ and $b$ are constants) around the $x$ - axis.

Find: (i) the $x$ intercepts of the ellipse

(ii) the volume so generated




7 (b) The cost ($C) of running a jetcat at a constant speed of V km/h

is found to be $\left(30 + \dfrac{V^{\frac{5}{2}}}{50}\right)$ per hour.

(i) If the total length of the journey is 400km, show that the total cost of the journey is given by $C = \dfrac{12000}{V} + 8V^{\frac{3}{2}}$

(ii) The jetcat must take no longer than 18 hours to complete the journey and its speed limit is $V \le 25\,km/h$.

At what speed (V) should the jetcat travel to minimize the cost (C)?

Justify your answer.

**END OF PAPER**



**STANDARD INTEGRALS**

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \ne -1;\ x \ne 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0.$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \ne 0.$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \ne 0.$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \ne 0.$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \ne 0.$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \ne 0.$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \ne 0.$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a.$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left[x + \sqrt{x^2-a^2}\right],\ |x| > |a|.$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left[x + \sqrt{x^2+a^2}\right].$$

NOTE: $\ln x = \log_e x,\ x > 0.$

# 2U TRIAL SOLUTIONS 1994

## Q1.

a) $\dfrac{\sqrt{1.23^2 + 0.85^2}}{4.81}$

$= 0.3108\ldots$ 1

$= 0.31 \quad (2\text{ d.p.})$ 1

b) $3m^2 - 13m + 4$

$= (3m - 1)(m - 4)$ 1

c) $\dfrac{5}{2+\sqrt{3}} = \dfrac{5}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{10 - 5\sqrt{3}}{4-3}$ 1

$= 10 - 5\sqrt{3}$

$\therefore m = 10$ ½ & $n = -5$. ½

d) $\dfrac{x-2}{x^2+x-6} = \dfrac{x-2}{(x-2)(x+3)}$ 1

$= \dfrac{1}{x+3}$ 1

e) $\log_a 18 = \log_a 2 + 2\log_a 3$ 1

$= 0.4 + 2(0.6)$ ½

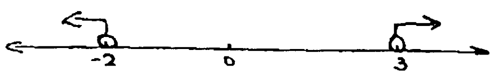
$= 1.6$ ½

f) $|2x - 1| > 5$

$2x - 1 > 5$ OR $-2x + 1 > 5$

$2x > 6 \qquad -2x > 4$

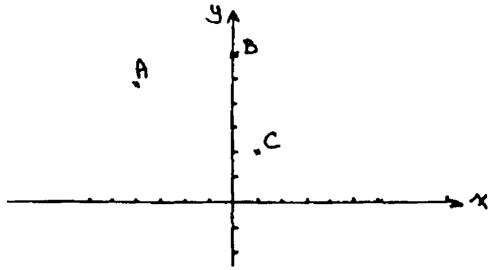
$x > 3$ 1 $\qquad x < -2$ 1



1

## Q2

a)




A(-4, 5)
B(0, 6)
C(1, 2)

b) $D = (-3, 1)$

c) $P = \left(\dfrac{-4+1}{2}, \dfrac{5+2}{2}\right)$

$= (-1\tfrac{1}{2}, 3\tfrac{1}{2})$

d) Midpt. of BD $= \left(\dfrac{-3+0}{2}, \dfrac{1+6}{2}\right)$

$= (-1\tfrac{1}{2}, 3\tfrac{1}{2})$

Since the midpts. of the diagonals are the same,
$\therefore$ the diagonals of the square bisect each other.

e) $m_{AC} = \dfrac{5-2}{-4-1}$

$= \dfrac{-3}{5}$

f) $m_{BD} = \dfrac{6-1}{0+3}$

$= \dfrac{5}{3}$

$m_{AC} \times m_{BD} = -\tfrac{3}{5} \times \tfrac{5}{3}$

$= -1$

$\therefore$ Diagonals are perpendicular to each other.

g) $PB = \sqrt{(-1\tfrac{1}{2} - 0)^2 + (3\tfrac{1}{2} - 6)^2}$

$= \sqrt{\dfrac{9}{4} + \dfrac{25}{4}}$

$= \sqrt{\dfrac{17}{2}}$

h) Area of the square
$= \frac{1}{2} \times \left(\sqrt{\frac{17}{2}}\right)^2 \times 4$
$= 17$ sq. units

i) $\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{17}{2}$

## Q3

a) $\sum_{r=1}^{3} (2^r \times 3^{r-1})$

$= 2 \times 3^0 + 2^2 \times 3^1 + 2^3 \times 3^2$
$= 2 + 12 + 72$
$= 86$

b) i) $\frac{d}{dx}\left(\frac{1}{3x}\right)$
$= -\frac{1}{3} x^{-2}$
$= \frac{-1}{3x^2}$

ii) $\frac{d}{dx} \sin(2x - 1)$
$= 2\cos(2x - 1)$

iii) $\frac{d}{dx}\left(\frac{2x+1}{x-4}\right)$
$= \frac{2(x-4) - 1(2x+1)}{(x-4)^2}$
$= \frac{-9}{(x-4)^2}$

c) RHS $= (a+b)(a^2 - ab + b^2)$
$= a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3$
$= a^3 + b^3$
$=$ LHS

d) $3x^2 + 2x - 6 = 0$
i) $\alpha + \beta = -\frac{2}{3}$
ii) $\alpha\beta = -2$
iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= \frac{4}{9} - 2(-2)$
$= 4\frac{4}{9}$
iv) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$
$= \left(-\frac{2}{3}\right)\left(4\frac{4}{9} + 2\right)$
$= -4\frac{8}{27}$

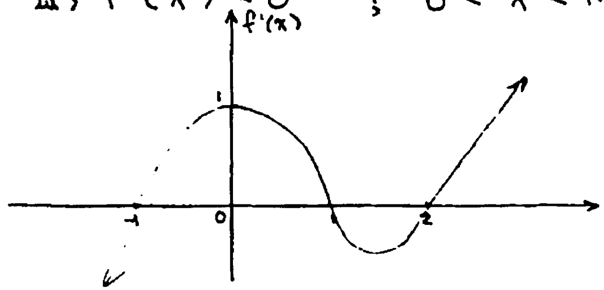
## Q4

a) i) $y = 3$
ii) $(x-2)^2 = -8(y-1)$

b) ii) $AP = AR$ (sides of square APQR)
$AB = AD$ ( " " " ABCD)
$\angle PAB = \angle DAB - \angle DAP$ ($\angle$s of squares APQR & ABCD)
$= \angle RAP - \angle DAP$
$= \angle RAD$
$\therefore \triangle APB \equiv \triangle ARD$ (S.A.S.)

iii) $BP = DR$ (corresponding sides of congruent $\triangle$s APB & ARD)

c) (i) i) $f'(x) = 0$ ; $x = -1, 1$ & $2$
ii) $f'(x) > 0$ ; $-1 < x < 1$ & $x > 2$
iii) $f''(x) < 0$ ; $0 < x < 1\frac{1}{2}$



b

## Q5

a) $\begin{cases} 2x + 5y - 10 = 0 \\ 3x - y + 19 = 0 \end{cases}$

$6x + 15y - 30 = 0$

$- \; 6x - 2y + 38 = 0$

$17y - 68 = 0$

$\begin{cases} y = 4 \\ x = -5 \end{cases}$ 1½

$m = \tan 135^\circ$

$= -1$ ½

$-1 = \dfrac{y-4}{x+5}$ 1

$-x-5 = y-4$

$\therefore y = -x-1$

b) i) $\dfrac{d}{dx} \ln(\sin x) = \dfrac{\cos x}{\sin x}$ 1

$= \cot x$

ii) $\int -\cot x \, dx$

$= -\ln(\sin x) \Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= -\ln(\sin \frac{\pi}{2}) + \ln(\sin \frac{\pi}{4})$

$= -\ln 1 + \ln(\frac{1}{\sqrt{2}})$

$= \ln \frac{1}{\sqrt{2}}$

c) Area $\simeq \frac{250}{3}(42 + 38 + 4(43)) + \frac{250}{3}(38 + 45 + 4(40$

$= 41250 \text{ m}^2$

d) i) $\begin{cases} y = x + 4 \\ y = 16 - x^2 \end{cases}$

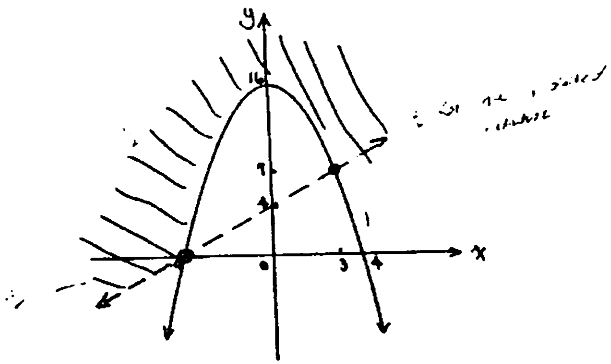
$x + 4 = 16 - x^2$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\begin{cases} x = -4 \text{ OR } x = 3 \\ y = 0 \text{ OR } y = 7 \end{cases}$

Int$^n$ pts. $= (-4, 0)$ & $(3, 7)$.



## Q6

a) i) $\int (4x^3 + \frac{x}{5} - 1)\, dx$

$= x^4 + \frac{x^2}{10} - x + C$

ii) $\int e^{\frac{x}{2}} dx = 2e^{\frac{x}{2}} + C$ 1

b) $\dfrac{A}{x-3} - \dfrac{B}{x-4}$

$= \dfrac{A(x-4) - B(x-3)}{x^2 - 7x + 12}$

$= \dfrac{Ax - 4A - Bx + 3B}{x^2 - 7x + 12}$

$\begin{cases} A - B = -1 \Rightarrow A = B - 1 \\ 3B - 4A = 1 \Rightarrow 3B - 4B + 4 = 1 \end{cases}$

A = 2, B = 3

c) i) $r = 2x$
$-1 < 2x < 1$
$-\frac{1}{2} < x < \frac{1}{2}$

ii) $\frac{1}{1-2x} = \frac{5}{9}$
$9 = 5 - 10x$
$10x = -4$
$\therefore x = -\frac{2}{5}$

d) $\frac{ds}{dt} = -kPe^{-kt}$
$= -0.05 \times 20 \times e^{-0.05\times15}$
$= -0.47$

Q7
a) i) (α) $P + Q = \pi r^2 \times \frac{\pi/3}{2\pi}$
$= \pi r^2 \times \frac{1}{6}$
$= \frac{\pi r^2}{6}$

(β) $Q + R = \frac{1}{2} \times \pi \times (\frac{1}{2}r)^2$
$= \frac{1}{2} \times \pi \times \frac{1}{4}r^2$
$= \frac{\pi r^2}{8}$

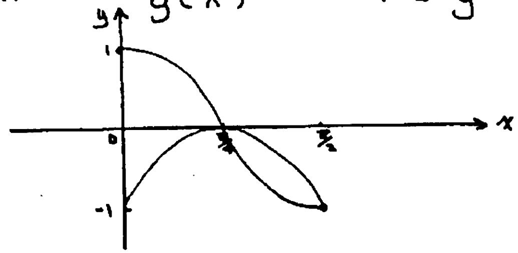
(γ) $P - R$
$= (P+Q) - (Q+R)$
$= \frac{\pi r^2}{6} - \frac{\pi r^2}{8}$
$= \frac{\pi r^2}{24}$

(δ) Since $Q + R = \frac{\pi r^2}{8}$ & $P - R = \frac{\pi r^2}{24}$;
$3(P-R) = Q + R$
$3P - 3R = Q + R$
$3P - Q = 4R$
$\therefore R = \frac{3P-Q}{4}$

ii) Area of $P = \frac{1}{2}(AO)(BO)\sin\angle AOB$
$= \frac{1}{2}r^2 \sin(\frac{\pi}{3})$
$= \frac{\sqrt{3}}{4}r^2$

b) $f(x) = \sin 2x - 1$
$g(x) = \cos 2x$
i) Range of $f(x)$ : $-1 \le y \le 0$
" $g(x)$ : $-1 \le y \le 1$

ii)

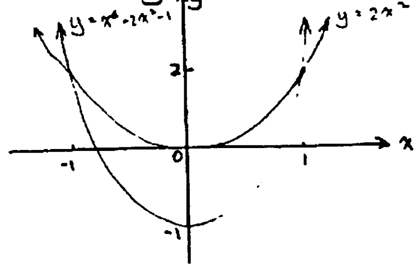



iii) $\sin 2x - 1 = \cos 2x$
$\sin 2x = \cos 2x + 1$
$\therefore$ 2 sol$^n$.

Q8
a) $y = x^4 + 2x^2 - 1$
i) $y' = 4x^3 + 4x = 0$
$4x(x^2+1) = 0$
$\left[\begin{array}{l} x = 0 \\ y = -1 \end{array}\right.$, $x^2 \neq -1$

$y'' = 12x^2 + 4$
When $x = 0$, $y'' = 4 > 0 \Rightarrow$ min.

ii)



iii) $\begin{cases} y = x^4 + 2x^2 - 1 \\ y = 2x^2 \end{cases}$

$2x^2 = x^4 + 2x^2 - 1$

$x^4 = 1$

$\begin{cases} x = 1 \\ y = 2 \end{cases}$ OR $\begin{cases} x = -1 \\ y = 2 \end{cases}$

$\therefore$ Int$^n$ pts are $(1, 2)$ & $(-1, 2)$.

v) Area $= 2\int_0^1 2x^2 - (x^4 + 2x^2 - 1)\ dx$

$= 2\int_0^1 (1 - x^4)\ dx$

$= 2\left[x - \frac{1}{5}x^5\right]_0^1$

$= 2\left[1 - \frac{1}{5} - 0\right]$

$= \frac{8}{5}$

b) $2\sin^2\theta - \cos\theta = 1 \quad ; 0 \leq \theta \leq 2\pi$

$2(1 - \cos^2\theta) - \cos\theta - 1 = 0$

$-2\cos^2\theta - \cos\theta + 1 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ OR $\cos\theta = -1$

$\theta = \frac{\pi}{3}$ OR $\frac{5\pi}{3}$ $\qquad \theta = \pi$

$\therefore \theta = \frac{\pi}{3}, \pi$ OR $\frac{5\pi}{3}$.

c) i) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

ii) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{11}{16}$

## Q9

a) $a = t + 2$

i) $v = \int (t + 2)\ dt$

$= \frac{1}{2}t^2 + 2t + c$

When $t = 0, v = 0 \Rightarrow c = 0$

$v = \frac{1}{2}t^2 + 2t$

~~When~~ $t = 3, v = \frac{9}{2} + 6$

ii) $s = \int_3^4 (\frac{1}{2}t^2 + 2t)\ dt$

$= \left[\frac{1}{6}t^3 + t^2\right]_3^4$

$= 13\frac{1}{6}$ m

b) i) $(0.75)^n$

ii) $1 - (0.75)^n = 0.95$

$(0.75)^n = 0.05$

$n \ln 0.75 = \ln 0.05$

$n = \ln 0.05 \div \ln 0.75$

$= 10.4\cdots$

$\therefore$ 11 cars must be picked.

c) See end of solutions