NAME:____________________________

Mrs Hickey
Mrs Choong
Mrs Gaensler
Mrs Quarles
Mrs Lee
Mrs Keuch
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Mrs Gibson
Mrs Leslie

PYMBLE LADIES' COLLEGE

YEAR 12

2 UNIT MATHEMATICS

HSC TRIAL EXAMINATION - 1999

Time Allowed: 3 hours
(Plus 5 minutes reading time)

INSTRUCTIONS TO CANDIDATES:

1. All questions must be attempted.
2. All necessary working must be shown
3. **Start each question on a new page.**
4. Put your name and your teacher's name on every sheet of paper.
5. Marks may be deducted for careless or untidy work.
6. Only approved calculators may be used.
7. Diagrams are not necessarily to scale.
8. **DO NOT** staple different questions together.
9. Hand this question paper in with your answers.
10. All rough working paper must be attached to the back of the last question.
11. There are ten (10) questions in this paper.
12. All questions are of equal value.

There are 12 pages in this paper.

QUESTION 1

Marks

(a) Find the value of $\dfrac{3\cdot 5^2}{\sqrt{9\cdot 4 \times 3\cdot 7}}$ correct to two significant figures. 2

(b) Factorize fully: $5a + 5b - a^2 + b^2$ 2

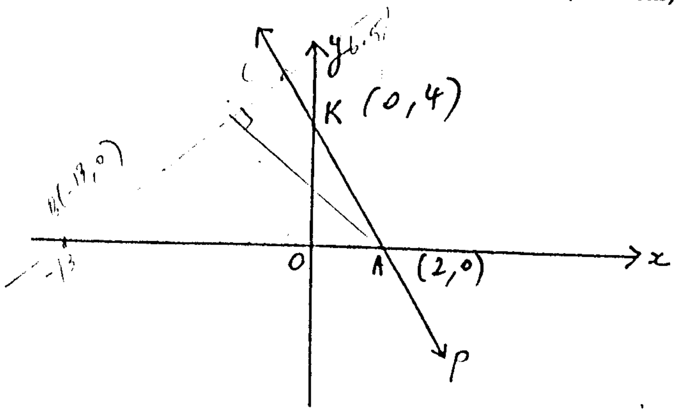
(c) Solve $|1-2x| < 5$ 2

(d) Express $\dfrac{1}{x} - x^2$ as an exact value, in simplest form, if $x = 1-\sqrt{2}$ 3

(e) Solve for x:

$$5^x \times 25^{x+1} = \frac{1}{5}$$ 3

QUESTION 2 **(Start a new page)** **(12 Marks)**



Line p has the equation $2x + y - 4 = 0$

(i) Find the co-ordinates of A, the x intercept of line p, and K, the y-intercept of line p.

(ii) What is the angle of inclination of line p with the positive direction of the x-axis? (to nearest degree).

(iii) Show that the equation of line t which is perpendicular to line p and passes through the point $D(3, 8)$, is given by $x - 2y + 13 = 0$.

(iv) Find the perpendicular distance of A from the line t.

(v) If line t meets the x-axis at B, find the co-ordinates of B.

(vi) Find C, the point of intersection of lines p and t.

(vii) Hence, or otherwise, determine the area of triangle ABC.

QUESTION 3 **(Start a new page)**

Marks

(a) Differentiate with respect to x. 5

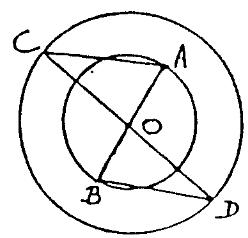
(i) $(2x-1)^3$

(ii) $e^{2x}+1$

(iii) $\dfrac{x}{\tan x}$

(iv) $\sec x$

(b) 3



In the above diagram, O is the centre of both circles, AB is the diameter of the smaller circle and CD is a diameter of the larger circle.

(i) Show that $\Delta AOC \equiv \Delta BOD$.

(ii) Hence show that $AC \parallel BD$.

(c) Evaluate, leaving answer in exact form. 4

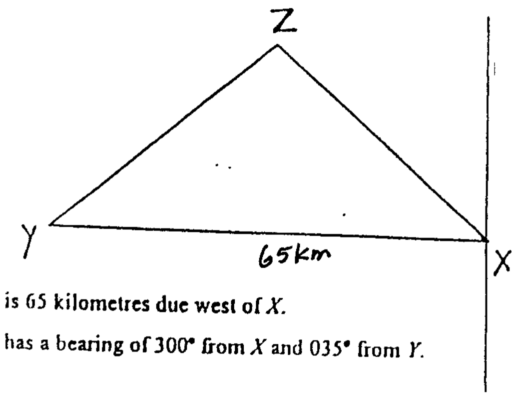
(i) $\displaystyle\int_6^9 \frac{3}{x-5}\,dx$

(ii) $\displaystyle\int_0^{\ln 3} e^{2x}\,dx$

QUESTION 4 **(Start a new page)**

Marks

(a) 4



Y is 65 kilometres due west of X.

Z has a bearing of 300° from X and 035° from Y.

(i) Copy the diagram onto your answer sheet, then complete by filling in all the relevant information.

(ii) Find the length of XZ (correct to 1 decimal place)

(iii) What is the bearing of Y from Z?

(b) Given $f(x) = 2x^3 - 9x^2 + 15$ 8

(i) Find the stationary points of $f(x)$.

(ii) Determine their nature.

(iii) Find any point(s) of inflexion?

(iv) Sketch $f(x)$ showing all important features.

QUESTION 5 **(Start a new page)**

Marks

(a) If $g(x) = x^2 + 5$ 3

(i) evaluate $g(-2)$

(ii) for what value(s) of x is $g(x) = 6$?

(b) For what values of k does the equation $kx^2 + x + k = 0$ have no real roots? 2

(c) Evaluate $\ln 2 + \ln 4 + \ln 8 + ... + \ln 1024$
Leave the answer in exact form. 2

(d) (i) One the same number plane sketch $y = x^2 - 6$ and $y = x$, showing all important features. 5

(ii) Show that the points of intersection of $y = x^2 - 6$ and $y = x$ are $(3, 3)$ and $(-2, -2)$

(iii) Find the area bounded by $y = x^2 - 6$ and $y = x$.

QUESTION 6 (Start a new page)

Marks

(a) Express $0\cdot3\dot{6}\dot{5}$ as a fraction in simplest form. **2**

(b) Solve $2\sin 2x = \sqrt{3}$ for $0 \le x \le 2\pi$. **3**

(c) Show that $\dfrac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$. **2**

(d) (i) Sketch $y = 1 - 2\sin x$ for $0 \le x \le 2\pi$ showing all important features. **5**

(ii) Calculate the area bounded by the curve $y = 1 - 2\sin x$, the $x-axis$ and the lines $x = 0$ and $x = \dfrac{5\pi}{6}$.

QUESTION 7 (Start a new page)

Marks

(a) Solve $3(9^x) + 2(3^x) - 1 = 0$ **3**

(b) If α and β are the roots of the equation $2x^2 - x - 3 = 0$, find the value of **6**

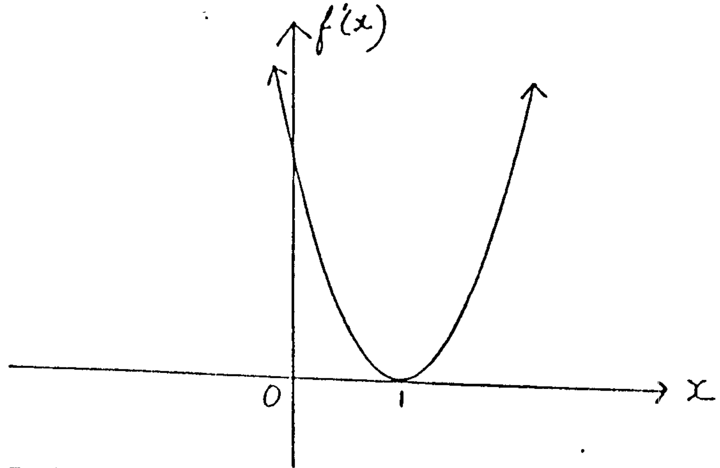
(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^3\beta^2 + \alpha^2\beta^3$

(iv) $\dfrac{\beta}{\alpha} + \dfrac{\alpha}{\beta}$

(c) **3**



Consider the function $f(x)$ whose gradient function is given in the sketch above.

(i) Comment on the sign of the gradient of $f(x)$ for all real x.

(ii) Sketch a possible graph of $f(x)$.

(iii) Sketch $f''(x)$.

QUESTION 8 (Start a new page) (12 Marks)

The point $P(x,y)$ moves so that its distance from the line $y = -4$ is equal to the distance from the point $S(0,0)$.

(a) Show that the locus of P is a parabola with equation $x^2 = 8(y+2)$.

(b) What is the vertex of this parabola?

(c) What is the focal length of this parabola?

(d) What are the x intercepts of this parabola?

(e) Sketch the parabola showing all the above information.

(f) Find the gradients of the parabola at these x-intercepts.

(g) Show that the equations of the normals at these x-intercepts are $x + y - 4 = 0$ and $x - y + 4 = 0$.

(h) Show that the point of intersection of these normals lies on the axis of symmetry of the parabola.

QUESTION 9 (Start a new page)

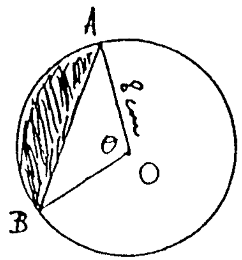
Marks

(a) 2

x	1	2	3
$f(x)$	5	25	125

Use the table given above and the trapezoidal rule with 3 function values to find an approximation to $\int_1^3 f(x)dx$.

(b) 4



O is the centre of the circle with radius 8cm and AB is a chord which subtends an angle of θ at the centre of the circle.
If the area of $\Delta ABO = 16cm^2$ find

(i) the angle θ, if θ is obtuse.

(ii) the area of the shaded region. (leave in exact form)

(c) To celebrate the new millennium a bank offers a superannuation fund which pays interest at 8% pa compounded half yearly. 6
Interest is paid on the 1st July and 31st December every year.
Michelle decided to invest \$5000 in this fund on the 1st January every year starting in the year 2000.

(i) Show that the amount in the fund on 2nd January, 2001 is $\$5000 \times 1{\cdot}04^2 + \$5000$

(ii) Find the amount in the fund on the 2nd January, 2005.

(iii) Michelle only checks her account on the 1st January every year just before she invests her next \$5000. On the 1st January of which year will Michelle, on checking her account, discover the fund has reached its first \$100000?

For n divide by 2 to get no. of years!!

Careful

QUESTION 10 (Start a new page)

Marks

(a) A particle moves with constant acceleration of $-3m/\sec^2$. If it is initially at rest 15m to the right of the origin, find: **4**

(i) its velocity function in terms of t.

(ii) its displacement function in terms of t.

(iii) its velocity when it returns to the origin again.

(b) (i) On the same axes draw careful sketches of the functions $y = x, y = e^x$ and $y = x + e^x$. **8**

(ii) Show that $\dfrac{d}{dx}(xe^x) = e^x + xe^x$.

(iii) Hence show that $\int xe^x dx = e^x(x-1)$.

(iv) The region which lies between the x-axis and the curve $y = x + e^x$ from $x = 0$ to $x = 2$ is rotated about the x-axis to form a solid. Find the volume of this solid. Leave answer in exact form.

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0.$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0.$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0.$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0.$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0.$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0.$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0.$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a.$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left[x + \sqrt{x^2 - a^2}\right],\ |x| > |a|.$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left[x + \sqrt{x^2 + a^2}\right].$$

NOTE: $\ln x = \log_e x,\ x > 0.$

2 Unit Trial Paper 1999

Question 1

a) $2\cdot 1$ (to 2s.f.) ②

b) $5a+5b-a^2+b^2$

$= 5(a+b) - (a^2-b^2)$ 1

$= 5(a+b) - (a-b)(a+b)$ ②

$= (a+b)(5-a+b)$ 1

c) $|1-2x| < 5$

$1-2x < 5$ or $-(1-2x) < 5$

$-2x < 4$ $\qquad -1+2x < 5$

$x > -2$ $\qquad 2x < 6$

1 $\qquad x < 3$ ②

$\therefore -2 < x < 3$ 1

d) $\frac{1}{x} - x^2$

$= \frac{1}{1-\sqrt{2}} - (1-\sqrt{2})^2$

$= \frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} - (1-2\sqrt{2}+2)$ 1 ③

$= \frac{1+\sqrt{2}}{1-2} - 3 + 2\sqrt{2}$ 1

$= -1-\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}-4$ 1

e) $5^x \times 25^{x+1} = \frac{1}{5}$

$5^x \times 5^{2x+2} = 5^{-1}$ $\qquad 1, \frac{1}{2}$

$5^{3x+2} = 5^{-1}$ $\qquad \frac{1}{2}$ ③

$\therefore 3x+2 = -1$

$\therefore x = -1$ 1

Question 2 ①

(i) $p: 2x+y-4=0$

When $x=0$, $y-4=0$

$y=4$

When $y=0$, $2x-4=0$ ②

$2x=4$

$x=2$

$\therefore A = (2,0)$ 1 and $K = (0,4)$ 1

(ii) $2x+y-4=0$

$y = 4-2x$

$\therefore m(\text{of } p) = -2$

$\therefore \tan\theta = -2$ 1 ②

$\therefore \theta = 116°34'$

$= 117°$ (to nearest degree) 1

(iii) $m(t) = \frac{1}{2}$ 1

Eqn of t: $y-8 = \frac{1}{2}(x-3)$ $\qquad \frac{1}{2}$ ②

$2y-16 = x-3$ $\qquad \frac{1}{2}$

$\therefore 0 = x-2y+13$

i.e. $x-2y+13=0$

(iv) $A = (2,0)$, $t: x-2y+13=0$

$\therefore x_1=2,\ y_1=0$, $a=1,\ b=-2,\ c=13$ ②

$\therefore d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|1\times 2+(-2)0+13|}{\sqrt{1+4}}$ 1

$= \frac{|15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$ units 1

Question 2 (contd)

(v) $t: x - 2y + 13 = 0$

When $y = 0$, $x + 13 = 0$ (1)

$x = -13$

$\therefore B = (-13, 0)$. 1

(vi) $x - 2y = -13$ --- (1)

$2x + y = 4$ --- (2)

× (1) by 2

$2x - 4y = -26$ --- (3)

(3) − (2) $\quad 2x + y = 4$ --- (2)

$-5y = -30$

$y = 6$ 1 (2)

Subst. for y in (1)

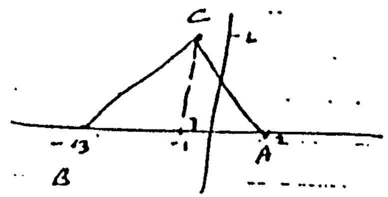
$\therefore x - 12 = -13$

$x = -1$ 1

$\therefore C = (-1, 6)$.

(vii)

Area of $\triangle ABC$



$= \frac{b \times h}{2}$ (1)

$= \frac{15 \times 6}{2} = 45 \text{ units}^2$ 1

Question 3

(a) (i) $y = (2x-1)^3$

$y' = 3(2x-1)^2 . 2 = 6(2x-1)^2$ 1

(ii) $y = e^{2x} + 1$ 1

$y' = 2e^{2x}$

(iii) $y = \frac{x}{\tan x}$

$y' = \frac{\tan x . 1 - \sec^2 x . x}{\tan^2 x}$

$= \frac{\tan x - x \sec^2 x}{\tan^2 x}$ 2

(iv) $y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$y' = -1(\cos x)^{-2} . -\sin x$ 1

$y' = \sin x (\cos x)^{-2} = \frac{\sin x}{\cos^2 x} = \tan x . \sec x$

(b) (i) In $\triangle^s$ AOC, BOD ½

(1) AO = OB (equal radii) ½

(2) OC = OD (equal radii) ½

(3) $\angle AOC = \angle BOD$ (vertically opp. $\angle^s$) ½

$\therefore \triangle AOC \equiv \triangle BOD$ (SAS) ½

(ii) Hence $\angle CAO = \angle BDO$ (corr. $\angle^s$ of $\cong \triangle^s$ AOC, BOD)

But these angles are alternate ½

$\therefore$ AC ∥ BD

Question 3 (contd)

(c) (i) $\int_{6}^{9} \frac{3}{x-5}\,dx$

$= \left[3\ln(x-5)\right]_{6}^{9}$ 1

$= 3\ln 4 - 3\ln 1$

$= 3\ln 4 - 0$

$= 3\ln 4.$ 1

(ii) $\int_{0}^{\ln 5} e^{2x}\,dx$

$= \left[\frac{e^{2x}}{2}\right]_{0}^{\ln 5}$ 1 (4)

$= \frac{e^{2\ln 5}}{2} - \frac{e^{0}}{2}$

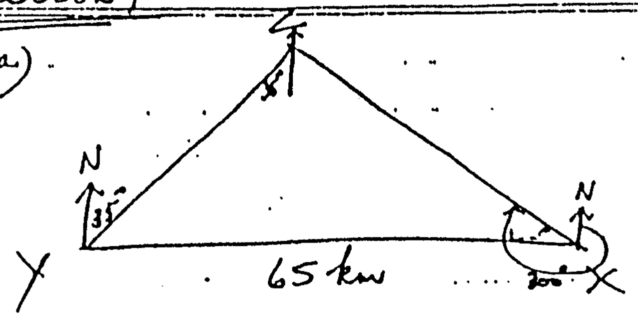
$= \frac{e^{\ln 5^2}}{2} - \frac{1}{2}$

$= \frac{5^2}{2} - \frac{1}{2} = \frac{24}{2} = 12$ 1

Question 4

(a)

(i)



1

(ii) $\angle ZYX = 90° - 35° = 55°$

$\angle ZXY = 30° \quad (270° + 30°)$

$\therefore \angle YZX = 180° - (55° + 30°)$

$= 95°$ 1

$\therefore \frac{XZ}{\sin 55°} = \frac{65}{\sin 95°}$

$XZ = \frac{65 \sin 55°}{\sin 95°}$ (4)

$= 53.4$ km. 1

(iii) Bearing of Y from Z

$= 180° + 35° = 215°$ 1

(b)

Question 4 (contd)

(b) (i) $f(x) = 2x^3 - 9x^2 + 15$

$f'(x) = 6x^2 - 18x$ $\frac{1}{2}$

Stat. pts when $f'(x) = 0$

i.e $6x^2 - 18x = 0$

$6x(x-3) = 0$ 1

$\therefore x = 0, 3$

When $x = 0$, $y = 15$

When $x = 3$, $y = 54 - 81 + 15 = -12$

$\therefore$ Stat pts are $(0, 15)$ 1, $(3, -12)$ 1

8

(ii) $f''(x) = 12x - 18$ $\frac{1}{2}$

When $x = 0$

$f''(x) = -18$ which is a max turning pt. $\frac{1}{2}$

When $x = 3$

$f''(x) = 36 - 18 = 18$ which is a min turning pt. $\frac{1}{2}$

(iii) Pt. of inflexion when $f''(x) = 0$ and there is a change in concavity about the point

When $f''(x) = 0$

$12x - 18 = 0$

$12x = 18$

$x = 1\frac{1}{2}$

When $x = 1\frac{1}{2}$

$y = \frac{27}{4} - \frac{81}{4} + 15$

$y = 6\frac{3}{4} - 20\frac{1}{4} + 15$

$y = 1\frac{1}{2}$

x	$1\frac{1}{2}^-$	$1\frac{1}{2}$	$1\frac{1}{2}^+$
$f''(x)$	$-$	0	$+$

1

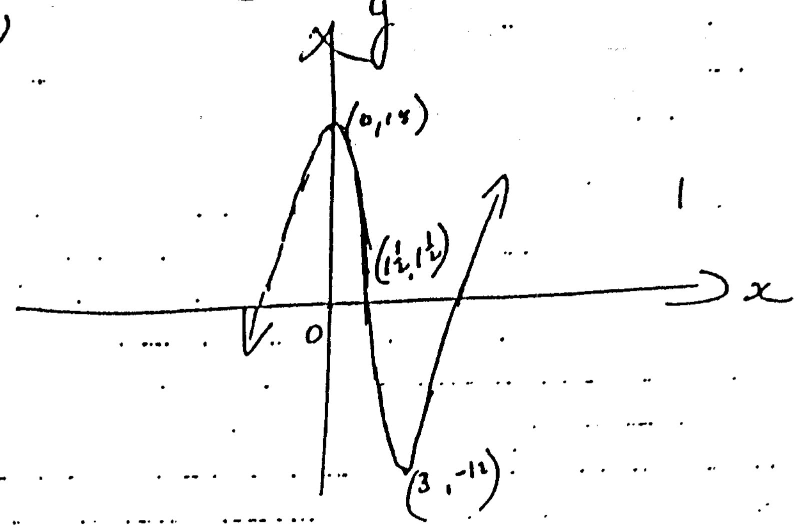
1

$\therefore$ a change in concavity about $(1\frac{1}{2}, 1\frac{1}{2})$

$\therefore (1\frac{1}{2}, 1\frac{1}{2})$ is a pt. of inflexion

Question 4 (contd) 4

(b) (iv)



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Question 5

(a) $g(x) = x^2 + 5$

(i) $g(-2) = (-2)^2 + 5$

$g(-2) = 4 + 5 = 9.$ 1

(ii) $6 = x^2 + 5$

$\therefore\ x^2 = 1$ (3)

$x = \pm 1$ 2

(b) $kx^2 + x + k = 0$

No real roots when $b^2 - 4ac < 0$

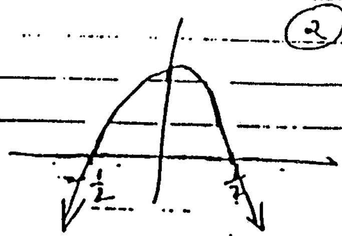
$1 - 4.k.k < 0$ 1

$1 - 4k^2 < 0$ (2)

$(1-2k)(1+2k) < 0$

$\therefore\ k < -\frac{1}{2}$ or $k > \frac{1}{2}$

$\frac{1}{2}$ $\frac{1}{2}$



(c) $\ln 2 + \ln 4 + \ln 8 + \dots + \ln 1024$

is an A.P. with $a = \ln 2$ and $d = \ln 2$ $\frac{1}{2}$

There are 10 terms as $2^{10} = 1024$ $\frac{1}{2}$

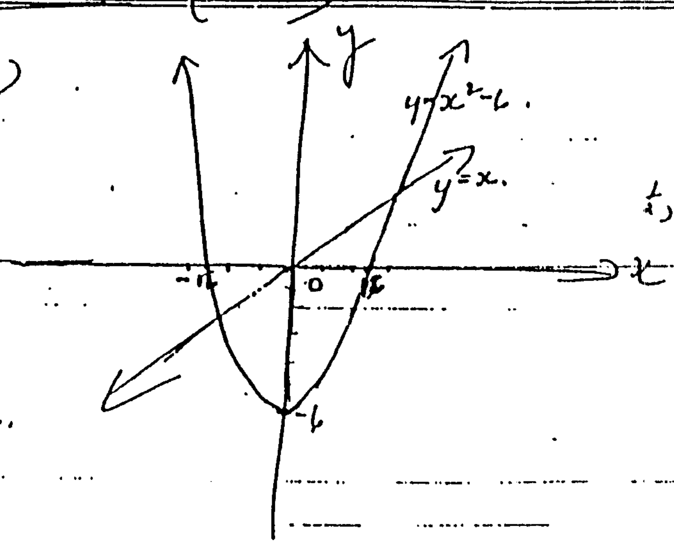
$\therefore\ S_{10} = \frac{10}{2}(\ln 2 + 10\ln 2)$ $\frac{1}{2}$

$= 5(11\ln 2)$ (2)

$= 55\ln 2.$ $\frac{1}{2}$

Question 5 (contd.)   (5)

(d) (i)



$\frac{1}{2}, \frac{1}{2}$

(ii) $y = x^2 - 6$

$y = x$

$\therefore\ x^2 - 6 = x$

$x^2 - x - 6 = 0$ (5)

$(x+2)(x-3) = 0$

$\therefore\ x = -2, 3.$

When $x = -2$, $y = -2$

When $x = 3$, $y = 3$ $\frac{1}{2}$ $\frac{1}{2}$

$\therefore$ Pts of intersection are: $(-2,-2), (3,3)$

(iii) Area $= \int_{-2}^{3} (x - x^2 + 6)\,dx$ 1

$= \left[\frac{x^2}{2} - \frac{x^3}{3} + 6x\right]_{-2}^{3}$ 1

$= (4\frac{1}{2} - 9 + 18) - (2 + 2\frac{2}{3} - 12)$

$= 13\frac{1}{2} + 7\frac{1}{3}$

$= 20\frac{5}{6}$ units^2 1

Question 6

(a) Let $x = 0.3656565\ldots$

Then $100x = 36.56565\ldots$ $\frac{1}{2}$

$-\quad x = 0.36565\ldots$

$99x = 36.2$ 1 (2)

$\therefore \quad x = \dfrac{36.2}{99} = \dfrac{362}{990} = \dfrac{181}{495}$ $\frac{1}{2}$

(b) $2\sin 2x = \sqrt{3}$ for $0 \le x \le 2\pi$

$\sin 2x = \dfrac{\sqrt{3}}{2}$ $\frac{1}{2}$ i.e. $0 \le 2x \le 4\pi$

$\therefore \quad 2x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{7\pi}{3}, \dfrac{8\pi}{3}$ $\frac{1}{2}$ each (3)

$\therefore \quad x = \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{7\pi}{6}, \dfrac{4\pi}{3}$ $\frac{1}{2}$

(c) $\dfrac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$

L.H.S. $= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \sec^2\theta - \sin^2\theta$

$= 1 + \tan^2\theta - \sin^2\theta$

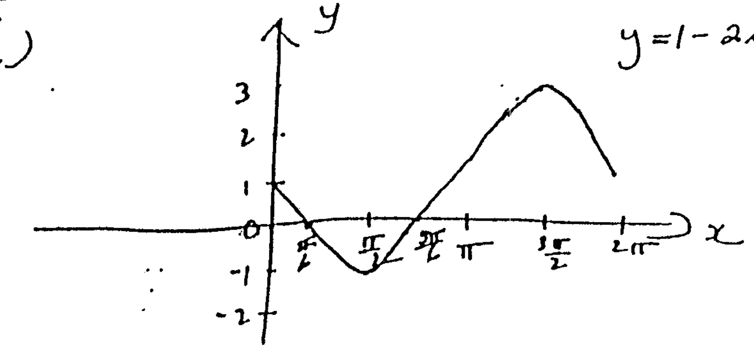
$= (1 - \sin^2\theta) + \tan^2\theta$ (2)

$= \cos^2\theta + \tan^2\theta$

$=$ R.H.S.

Question 6 (contd)

(d) (i)



1

When $y = 0$, 1 $\quad 1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$ 1

$\therefore \quad x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

(ii) Reqd area $= \displaystyle\int_0^{\frac{\pi}{6}} 1 - 2\sin x\,dx + \left|\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1 - 2\sin x\,dx\right|$

$= \Big[\underbrace{x + 2\cos x}_{1}\Big]_0^{\frac{\pi}{6}} + \left|\Big[x + 2\cos x\Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\right|$

$= \dfrac{\pi}{6} + \sqrt{3} - 2 + \left|\dfrac{5\pi}{6} + 2\cos\dfrac{5\pi}{6} - \dfrac{\pi}{6} - 2\cos\dfrac{\pi}{6}\right|$

2 $\quad = \dfrac{\pi}{6} + \sqrt{3} - 2 + \left|\dfrac{5\pi}{6} - \sqrt{3} - \dfrac{\pi}{6} - \sqrt{3}\right|$

$= \dfrac{\pi}{6} + \sqrt{3} - 2 + \left|\dfrac{4\pi}{6} - 2\sqrt{3}\right|$ units^2 (5)

Question 7

(a) $3(9^x) + 2(3^x) - 1 = 0$

Let $y = 3^x$

$3y^2 + 2y - 1 = 0$ 1

$(3y - 1)(y + 1) = 0$

$\therefore y = \frac{1}{3}$ or -1 1

i.e. $3^x = \frac{1}{3}$ or $3^x = -1$

$3^x = 3^{-1}$ invalid as 3^x cannot be negative

$\therefore x = -1$ 1

3

(b) $2x^2 - x - 3 = 0$

(i) $\alpha + \beta = -\frac{b}{a} = \frac{1}{2}$ 1

(ii) $\alpha\beta = \frac{c}{a} = \frac{-3}{2}$ 1

(iii) $\alpha^3\beta^2 + \alpha^2\beta^3$

$= \alpha^2\beta^2(\alpha + \beta)$

$= \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$ 2

6

(iv) $\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$= \frac{\beta^2 + \alpha^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ 2

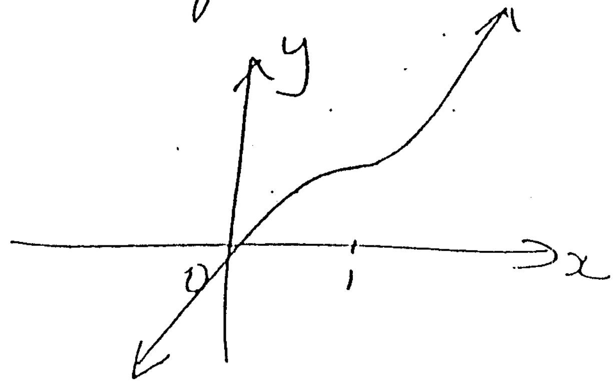
$= \frac{\frac{1}{4} - 2\left(\frac{-3}{2}\right)}{\frac{-3}{2}} = \frac{\frac{1}{4} + 3}{\frac{-3}{2}} = \frac{13}{4} \times -\frac{2}{3}$

$= -\frac{13}{6}$

Question 7 (contd) 7

(c) (i) The sign of $f'(x)$ is positive for all real x except $x = 1$ where the gradient is 0. 1

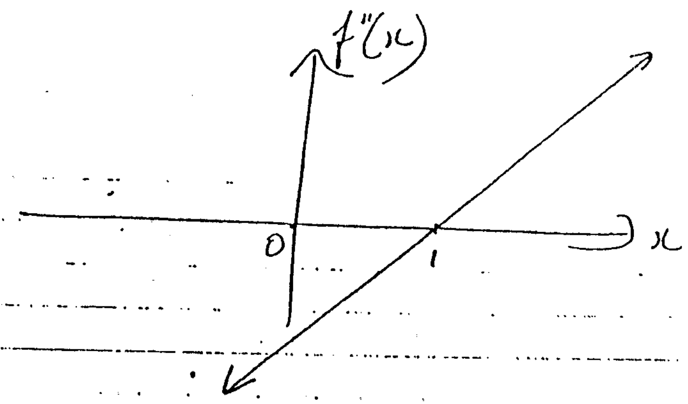
(ii)



1

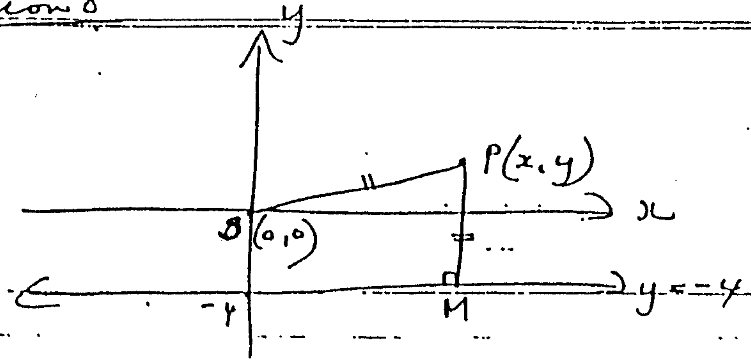
3

(iii)



1

Question 8



(a) $PS = PM$

$\sqrt{(x-0)^2+(y-0)^2} = y+4$ 1

$x^2+y^2 = (y+4)^2$

$x^2+y^2 = y^2+8y+16$ 1

$x^2 = 8y+16$

$x^2 = 8(y+2)$

(b) vertex $= (0,-2)$ 1

(c) $4a = 8$

$a = 2$

$\therefore$ focal length is 2 units. 1

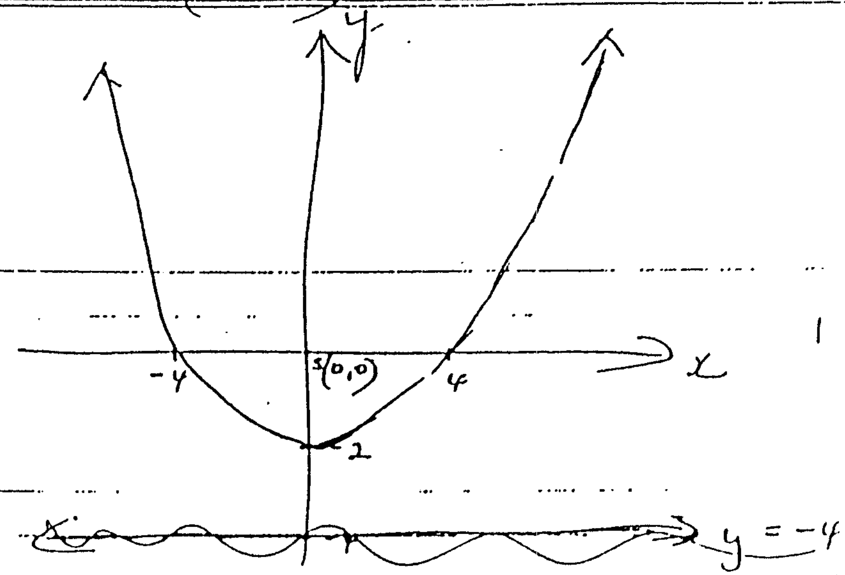
(d) When $y = 0$, $x^2 = 16$

$x = \pm 4.$ 1

(e)

Question 8 (cont'd)

(e)



1

(f) $x^2 = 8y+16$

$8y = x^2 - 16$

$y = \frac{x^2}{8} - 2$ $\frac{1}{2}$

$y' = \frac{x}{4}$ $\frac{1}{2}$

When $x = 4$, $y' = 1$ $\frac{1}{2}$

When $x = -4$, $y' = -1$ $\frac{1}{2}$

(g) m(normal at $x=4$) $= -1$

m(" " $x=-4$) $= 1$

Eqn. of normal at $x = 4$:

$y - 0 = -1(x-4)$

$y = -x+4$ 1

$\therefore x+y-4 = 0$

Eqn of normal at $x = -4$

$y - 0 = 1(x+4)$

$y = x+4$

$\therefore x - y + 4 = 0$ 1

Question 8 (contd)

(b)

$x + y = 4 \quad \text{--- (1)}$

$x - y = -4 \quad \text{--- (2)}$

(1) + (2) $\quad 2x = 0$

$x = 0$ 1

Subst. for x in (1)

$0 + y = 4$

$y = 4$ (12)

$\therefore$ pt. of intersection is $(0, 4)$

Axis of symmetry of parabola is $x = 0$ (y-axis).

$\therefore$ pt. of intersection lies on it 1

Question 9 (9)

(a ii) $\text{Area} \doteq \frac{h}{2}\left[5 + 125 + 2(25)\right]$

$\left(h = \frac{3-1}{2} = 1\right)$ (2)

$\therefore \text{Area} = \frac{1}{2}(180) = 90 \text{ units}^2$ 1

(b)

(i) $\text{Area of } \Delta = \frac{1}{2} ab \sin C$

$16 = \frac{1}{2} \times 8 \times 8 \sin\theta$ 1

$16 = 32 \sin\theta$

$\frac{1}{2} = \sin\theta$

$\therefore \theta = \frac{5\pi}{6}$ (as θ is obtuse) 1

(ii) Area of sector $ABO = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 64 \times \frac{5\pi}{6}$ $\frac{1}{2}$

$= \frac{80\pi}{3}$ 1 (4)

$\therefore$ Area of shaded region

$= \left(\frac{80\pi}{3} - 16\right) \text{cm}^2$ $\frac{1}{2}$

Question 1 (Contd.)

(c) (i) $r = 8\%$ p.a. $= 4\%$ per 6 months.

$P = \$5000$

A_1 (on 1st Jan 2001) $= P(1+r)^n$

$= \$5000(1+0.04)^2$

$= \$5000(1.04)^2$ (1)

$\therefore$ Amount on 2nd January 2001

$= \$5000(1.04)^2 + \$5000$

(ii) A_1 is in fund for 5 yrs.

$\therefore A_1 = 5000(1.04)^{2\times5} = 5000(1.04)^{10}$

$A_2 = 5000(1.04)^{2\times4} = 5000(1.04)^8$

$A_3 = 5000(1.04)^{2\times3} = 5000(1.04)^6$

$A_4 = 5000(1.04)^{2\times2} = 5000(1.04)^4$ (3)

$A_5 = 5000(1.04)^{2\times1} = 5000(1.04)^2$

(amount to 1st Jan 2005)

Total amount (1st Jan 2005) $= A_1 + A_2 + A_3 + A_4 + A_5$

$= 5000(1.04)^{10} + 5000(1.04)^8 + \dots + 5000(1.04)^2$

$= 5000(1.04)^2\left[\underbrace{1 + (1.04)^2 + (1.04)^4 + (1.04)^6 + (1.04)^8}_{G.P}\right]$

G.P $S_5 = \dfrac{1\left[(1.04)^{2\times5} - 1\right]}{1.04^2 - 1}$ $\quad r = \frac{T_2}{T_1} = (1.04)^2$

$S_5 = \dfrac{(1.04)^{10} - 1}{1.04^2 - 1}$

$\therefore$ Amount (1st Jan 2005) $= 5000(1.04)^2\left[\dfrac{1.04^{10} - 1}{1.04^2 - 1}\right]$

$= \$31827.95$

$\therefore$ Amount (2nd Jan 2005) $= \$31827.95 + \$5000$

$= \$36827.95$

(iii) $A = \$100\,000$ $\quad n = ?$

$r = 0.04$ $\quad P = \$5000$

$A_n = 5000(1.04)^2\left[\dfrac{1\left((1.04)^{2n} - 1\right)}{1.04^2 - 1}\right]$

$100\,000 = 5000(1.04)^2\left(\dfrac{1.04^{2n} - 1}{1.04^2 - 1}\right)$

$1 + \dfrac{100\,000 \times (1.04^2 - 1)}{5000\,(1.04)^2} = 1.04^{2n}$

$\log(1 + \dots) = 2n \log 1.04$

$\dfrac{\log(1 + \dots)}{\log 1.04} = 2n$ (2)

$\therefore n = 11.726\dots$

$\therefore$ it will be the year 2012.

Question 10 (contd)

12

(b) (iv) $V = \pi \int_0^2 (x+e^x)^2 \, dx$. $\frac{1}{2}$

$= \pi \int_0^2 (x^2 + 2xe^x + e^{2x}) \, dx$ $\frac{1}{2}$

$= \pi \left[\frac{x^3}{3} + 2e^x(x-1) + \frac{e^{2x}}{2} \right]_0^2$

$= \pi \left[\left(2\frac{2}{3} + 2e^2(2-1) + \frac{e^4}{2} \right) \right.$ 1

3

$\left. - \left(0 + 2(-1) + \frac{1}{2} \right) \right]$

$= \pi \left(2\frac{2}{3} + 2e^2 + \frac{e^4}{2} + 2 - \frac{1}{2} \right)$ 1

$= \pi \left(4\frac{1}{6} + 2e^2 + \frac{e^4}{2} \right)$ units^3

1 out of 3
if no middle bit