Question 1 **Marks**

a) Evaluate, correct to four significant figures, $\sqrt[4]{\dfrac{4^5 - 5^4}{17 + 7^3}}$. **1**

b) Solve $\dfrac{|5 - x|}{7} \geq 3$ Graph your solution on a number line. **3**

c) Solve $x(5 - x) = 8 - x$. **2**

d) Find a primitive of $5 - \dfrac{1}{e^x}$. **2**

e) Simplify $\dfrac{x^2 + x + 1}{5x^3 - 5} + \dfrac{7}{x - 1}$ **2**

f) Five 78 seater coaches, all full, can bus the entire population of a certain school to Homebush in 3 trips each. If a different bus company was engaged for the return journey, using seven small buses, each seating 28, how many trips would each bus need to travel? **2**

Question 2 **Start a new page** **Marks**

a) The graph of a parabolic function crosses the x-axis at the origin and at $x = 4$. If the minimum value of the function is -12, determine the parabolic function. **3**

b) A right isosceles triangle has one vertex at the origin O and another at the point $A(1,3)$. The base of the triangle has equation $x = 2y$.

(i) Show that the third vertex B has coordinates (4,2). **3**

(ii) Find the length of OA . **1**

(iii) Find the area of the triangle. **2**

(iv) Find the length of OB . **1**

(v) Find the perpendicular height of the triangle. from A to OB. **2**

Question 3 **Start a new page** **Marks**

a) Evaluate $\int_0^2 \frac{x}{5-x^2}\,dx$. **2**

b) Jesse loves to play tennis. One day Jesse hopes be a champion tennis player. Jesse's coach says that her skills improve by about 5% with every competition match. How many matches will Jesse need to play so that her game skills are at lease twice as good as they are right now? **2**

c) Differentiate with respect to x:

(i) $\sin(3x^2+4x)$. **2**

(ii) $\frac{\ln 5x}{e^{7x}}$. **2**

d) A particle moves in a straight line. At time t seconds its distance x metres from a fixed point O in the line is given by $x = 3\cos \pi t - 3$.

(i) Sketch the graph of x as a function of t for $0 \le t \le 2$. **1**

(ii) Show that the time when the particle first comes to rest is t = 1. **1**

(iii) In two or more sentences, describe the motion of the particle during the first second. **2**

Question 4 **Start a new page** **Marks**

a) Without sketching the function, determine the set

of x values for which $y = \dfrac{6}{x^2-1} - 3$ is defined

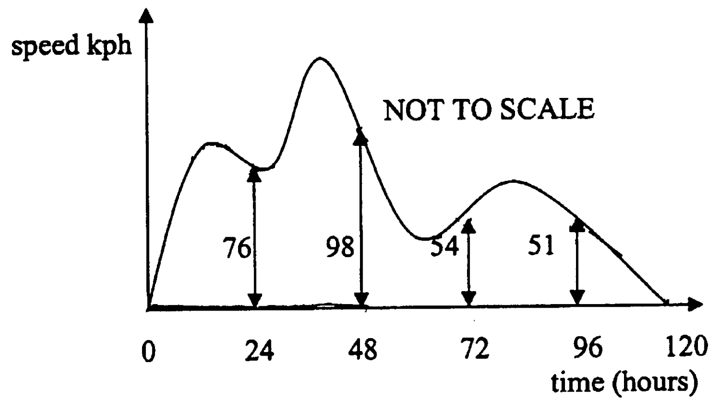
and write down any x and y intercepts. **3**

b) State the range and domain of the functions

(i) $g(x) = e^{3\sin 2x}$. **2**

(ii) $f(x) = \log_e(e^{3\sin 2x})$. **2**

c)



The curve graphed above represents the speed of a hot air balloon during a 5 day adventure. The area under the speed graph represents the total distance travelled during the flight.

(i) Use the trapezoidal rule to obtain an approximate value for the total distance travelled. **3**

(ii) State whether you believe that the actual distance travelled by the balloon is more or less than the answer obtained in part (i) above. Give reasons for your answer. You may include a diagram or sketch. **2**

Question 5 **Start a new page** **Marks**

a) Show that zero is the least integer value of k for which the quadratic equation $(k+1)x^2 - x + 1 = 0$ has no real roots. **2**

b) Consider the function $y = x^4 - 4x^3$.

(i) On a neat set of coordinate axes sketch the function showing points of inflection, intercepts on axes and turning points. **2**

(ii) On the same set of axes sketch the gradient function showing its points of inflection, turning points and intercepts. **2**

c) (i) Sketch the curves $y = 1 + \frac{1}{x}$ and $y = \sin(2x)$, for $0 \le x \le \pi$.
[Use the same coordinate axes for both] **2**

(ii) Shade the area represented by $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 + \frac{1}{x} - \sin(2x)\,dx$. **1**

(iii) Find the value of the shaded area as a simplified exact value. **3**

Question 6 **Start a new page.** **Marks**

a) The 3rd term of an arithmetic sequence is -6 and the 23rd term is 9.

(i) Show that the 24th term is 9.75 **2**

(ii) Sum the terms from the 3rd to the 23rd inclusive. **2**

(iii) Show that 99 is a term of the sequence. **1**

b) The function $f(x) = 5xe^{\frac{-x}{2}} + 3$

has first derivative $f'(x) = -\frac{5x}{2}e^{\frac{-x}{2}} + 5e^{\frac{-x}{2}}$

and second derivative $f''(x) = \frac{5x}{4}e^{\frac{-x}{2}} - 5e^{\frac{-x}{2}}$.

(i) Find the coordinates of the stationary point. **1**

(ii) Find the values of x for which $f(x)$ is increasing. **1**

(iii) Find the values of x for which $f(x)$ is decreasing. **1**

(iv) Find the values of x for which $y=f(x)$ is concave up. **1**

(v) Find the values of x for which $y=f(x)$ is concave down. **1**

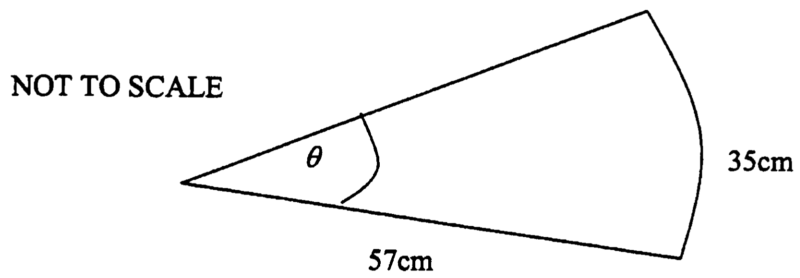
(vi) Find the y intercept and $f(6)$. **1**

(vii) Determine the absolute minimum of $f(x)$ for $0 \leq x \leq 6$ **1**

Question 7 **Start a new page** **Marks**

a) The region enclosed between the curves $y = -x^2$ and $x = y^2$ is rotated about the x axis. Find the volume of the solid of revolution. **3**

b)



Find the size of angle θ, shown in the diagram above.

Give your answer correct to the nearest minute. **2**

c) The time elapsed during the motion of a particle is given by $t = \dfrac{5}{(x+3)} - 1$.

(i) Explain why the particle can never move to a position where $x = 3$ **1**

(ii) Show that the displacement of the particle, in terms of t, is given by $x = \dfrac{5}{1+t} - 3$ **1**

(iii) Does the displacement approach a limiting position? Explain your answer. **2**

(iv) Find an expression for velocity in terms of time. **1**

(v) Does the particle ever appear to stop moving? Explain your answer. **2**

Question 8 **Start a new page** **Marks**

a) The nth term of the series $0+\frac{7}{5}-\frac{21}{50}+....$ is given by the formula $A(\frac{1}{5})^n+B(\frac{-1}{2})^n$.

(i) By writing $T_1=0$, find the values of the constants A and B and hence calculate the 4th term of the series. **3**

(ii) Show that the sum of the series to n terms is given by $\frac{5}{2}(1-(\frac{1}{5})^n)+\frac{-4}{3}(1-(\frac{-1}{2})^n)$ **2**

(iii) Find the sum to infinity. **2**

b) During the normal operation of a petrol driven engine, the volume V litres of petrol left in the tank reduces at a rate $\frac{dV}{dt}=-3e^{0.4t}$ where t is measured in minutes since the engine was switched on and the tank was full (100 Litres).

(i) At what rate is the petrol used, initially? **1**

(ii) Use integration to show that volume remaining can be expressed as $V=\frac{-30}{4}e^{0.4t}+107.5$ **2**

(iii) How long can the machine operate until the tank is only half full? Answer correct to the nearest second. **2**

Question 9 **Start a new Page** **Marks**

a) The number of bubbles appearing on the surface of a glass of lemonade decreases over time until eventually the lemonade is described as 'flat'. Marcus observed the lemonade when it was freshly poured. Initially, Marcus counted 72 bubbles. 16 seconds later there were only 24 bubbles. Assume that the number of bubbles satisfies the equation $N = N_0 e^{-kt}$ where N_0 and k are constants and t is measured in seconds.

(i) Find values of k and N_0 and predict when there will be only 2 bubbles observed. **3**

(ii) How many seconds will pass from there being 2 bubbles until there is only 1 bubble observed. **2**

b) Consider a **straight line** with equation $3y = mx + 6$, $m > 0$, and a **curve** with equation $y = \log_e(x+1)$.

(i) By substituting m =2, sketch $3y = 2x + 6$ and $y = \log_e(x+1)$ together on the same diagram. **2**

(ii) Show that the vertical distance from the line to the curve is given by the expression $\frac{2x}{3} + 2 - \log_e(x+1)$. **1**

(iii) Now for the more general case where $3y = mx + 6$ and $y = \log_e(x+1)$, the **vertical** distance from the straight line to the curve is given by $\frac{mx}{3} + 2 - \log_e(x+1)$. Show that the <u>shortest</u> vertical distance is given the expression $3 - \frac{m}{3} - \log_e(\frac{3}{m})$ **3**

(iv) Find the value of m so that the shortest vertical distance coincides with the y-axis. **1**

Question 10 **Start a new page** **Marks**

a) Solve the equation $\tan \pi x = \frac{1}{\sqrt{3}}$ for $0 \le x \le 2$. **2**

b) Assume that a plant leaf will grow under suitable conditions until nutrients are in short supply, then the leaf will stop growing and it will maintain its size. A simple mathematical model for the growth of a certain type of plant leaf involves the following **split function**

$$\boxed{\left.\begin{array}{ll} W = W_0 e^{\mu t} & \text{for } 0 \le t < t_f \\ W = W_f & \text{for } \quad t \ge t_f \end{array}\right\}}$$

where

W *represents dry weight of the leaf at time* t

μ *represents 'growth rate',* $\mu > 0$

t_f *represents time when the leaf stops growing*

W_f *represents dry weight of the leaf at time* t_f

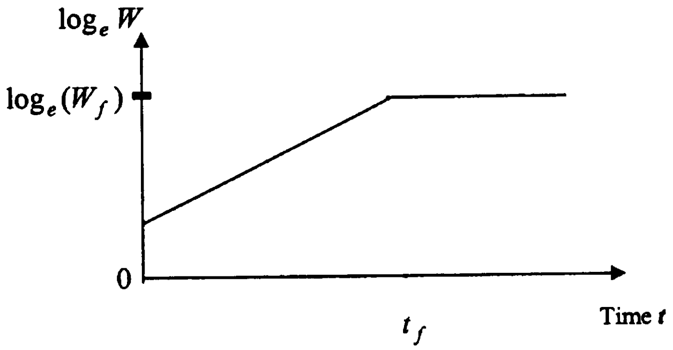
W_0 *represents initial dry weight of the leaf,* $W_0 > 1$

(i) Show by differentiation that the equation $W = W_0 e^{\mu t}$ may be written as $\frac{dW}{dt} = \mu W$ for $0 \le t < t_f$ **1**

(ii) Show that the **split function** is never decreasing **1**

(iii) Sketch the **split function** on a set of axes with W on the vertical axis and t on the horizontal axis. **2**

(iv) Study the graph of $\log_e W$ against time t, shown below.



For the sloping line segment above; show that the gradient is μ; determine the intercept on the vertical axis and write its equation as function of time. **3**

(v) Now, under experimental conditions using the same plant, additional nutrients are provided causing leaves to continue growing until they reach twice the usual size. Show that the additional time taken for the leaf to grow is given by $\frac{\log_e 2}{\mu}$ **3**

END OF PAPER.

Koshena

Q1 a) $\sqrt[4]{\frac{4^5-5^{-4}}{17+73}}$

$= 1.02604....$ ($\frac{1}{2}$)

$\doteq 1.026$ ($\frac{1}{2}$) for 4

1

b) $\left|\frac{5-x}{7}\right| \geq 3$

$5-x \geq 21$

$\therefore 5-x \geq 21$ or $5-x \leq -21$ $\frac{1}{2}$ $\frac{1}{2}$

3

$-x \geq 16$ or $-x \leq -26$

$x \leq -16$ ($\frac{1}{2}$) $x \geq 26$ ($\frac{1}{2}$)

(No $\frac{1}{2}$ m for incorrect direction of sign)

$-\frac{1}{2}$ m if $-21 \geq 5-x \geq 21$

-16 ($\frac{1}{2}$) 0 26 ($\frac{1}{2}$)

(no $\frac{1}{2}$ mark if not filled in for 1 of the answers)

c) $x(5-x) = 8-x$

$5x - x^2 = 8-x$

$-x^2+6x-8 = 0$

$-(x^2-6x+8) = 0$

$-(x-4)(x-2) = 0$ 1

$x = 2$ or 4 ($\frac{1}{2}$, $\frac{1}{2}$)

2

or $5x-x^2 = 8-x$.

$x^2-6x+8=0$

$(x-4)(x-2)=0$ 1

$x = 2$ or 4

$\frac{1}{2}$ $\frac{1}{2}$

d) $\int 5 - \frac{1}{e^x}$

$= \int 5 - e^{-x}$ $\frac{1}{2}$

2

$= 5x + e^{-x} + c$

$= 5x + \frac{1}{e^x} + c$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

or $\int 5 - \frac{1}{e^x}$

$= 5x + ...$ + c

1m

e) $\frac{x^2+x+1}{5x^3-5} + \frac{7}{x-1}$ or if

$\frac{x^2+x+1}{5(x-1)(x^2+x-1)} + \frac{7}{x-1}$ $\frac{1}{2}$

$= \frac{36}{5(x-1)}$ $\frac{1}{2}$ $\frac{1}{2}$

2

f) 1st trip: people $= 5 \times 78 \times 3$

$= 1170$ 1

2nd trip: $1170 \div (7 \times 28)$

$= 5.9693...$

$= 6$ TRIPS 1

($\frac{1}{2}$ for 5.6 buses)

e) $\frac{(x^2+x+1)(x-1) + 7(5x^3-5)}{(5x^3-5)(x-1)}$ $\frac{1}{2}$

$= \frac{x^3-x^2+x^2-x+x-1+35x^3-35}{(5x^3-5)(x-1)}$

$= \frac{36x^3-36}{(5x^3-5)(x-1)}$

$= \frac{36(x^3-1)}{(5x^3-5)(x-1)}$ $\frac{1}{2}$

$= \frac{36\cancel{(x-1)}(x^2+x-1)}{5(x^3-1)\cancel{(x-1)}}$ $\frac{1}{2}$

$= \frac{36(x^2+x-1)}{5(x^3-1)}$

$= \frac{36\cancel{(x^2+x-1)}}{5(x-1)\cancel{(x^2+x-1)}}$ $\frac{1}{2}$

$= \frac{36}{5(x-1)}$ ($\frac{1}{2}$)

or $\frac{36\cancel{(x^3-1)}}{5\cancel{(x^3-1)}(x-1)}$ ($\frac{1}{2}$)

$\frac{36}{5(x-1)}$ ($\frac{1}{2}$)

Question 2

Part a:

Method 1 $y = ax(x-4)$ ①

Sub in $(2,-12)$, $a=3$ ①

$\therefore$ eqn is

$y = 3x(x-4)$ // ①

Method 2

$y = a(x-2)^2 - 12$ No a ② only

Sub $(0,0)$, $a=3$

$\therefore$

$y = 3(x-2)^2 - 12$ // ③

Method 3

$(x-2)^2 = 4a(y+12)$ ②

or

$(x-2)^2 = 4(y+12)$ (No a)

Sub $(0,0)$, $a = \frac{1}{12}$

$\cancel{y = 3(x-2)^2 - 12}$

$(x-2)^2 = 4(\frac{1}{12})(y+12)$

$(x-2)^2 = \frac{1}{3}(y+12)$ // ③

2 (b)

(i) Prove $(4,2)$ lies on the line $x = 2y$ ①

Either prove $OA \perp AB$

or $OA = AB$ ②

(ii) $OA = \sqrt{3^2+1^2} = \sqrt{10}$ ①

(iii) Area $= \frac{1}{2}\, \overline{OA} \cdot \overline{OB} = \frac{1}{2}\sqrt{10} \times \sqrt{10} = 5$ s.u.

or

Area $= \frac{1}{2}\, \overline{OB} \times h = \frac{1}{2}(2\sqrt{5})(\sqrt{5}) = 5$ s.u. ②

(iv) $OB = \sqrt{4^2+2^2} = \sqrt{20}$ ①

(v) Method 1

Area $= \frac{1}{2} \cdot \overline{OB} \times h$

$5 = \frac{1}{2} \times 2\sqrt{5} \times h$

$h = \sqrt{5}$

Method 2. $x - 2y = 0$ (Eqn of OB)

Perp. dist from $A(1,3)$ to OB

$= \frac{|1\times1 - 2\times3 + 0|}{\sqrt{1^2+2^2}}$

$= \frac{|1-6|}{\sqrt{5}} = \frac{|-5|}{\sqrt{5}} = \sqrt{5}$

Method 3

Find mid-pt of OB : $M(2,1)$

Distance $AM = \sqrt{5}$

②

QUESTION 3.

a) $\int_0^2 \frac{x}{5-x^2}\,dx$

$= -\frac{1}{2}\int_0^2 \frac{-2x}{5-x^2}\,dx$

$= \left[-\frac{1}{2}\ln(5-x^2)\right]_0^2$

$= -\frac{1}{2}(\ln 1 - \ln 5)$

$= \frac{\ln 5}{2}$

b). G - game skills now

$G \times 1.05$ = after 1 match.

$G \times 1.05 \times 1.05$ = after 2 matches

$G \times 1.05^n = 2G$

$1.05^n = 2.$

$\ln 1.05^n = \ln 2$

$n = \frac{\ln 2}{\ln 1.05}$

$= 14.2$

$\therefore$ 15 matches.

OR

T_1	T_2	T_n
$1.05g$	$1.05(1.05)g.$	$2g.$

$T_n = ar^{n-1}$

$2g = 1.05g\,r^{n-1}.$

$2 = 1.05\,(1.05)^{n-1}.$

$\ln\left(\frac{2}{1.05}\right) = \ln 1.05^{n-1}$

$13.2 = n-1$

$\therefore n = 14.2$

$\therefore$ need 15 matches.

c). i) $y = \sin(3x^2+4x)$

$y' = 6x+4\ \cos(3x^2+4x)$

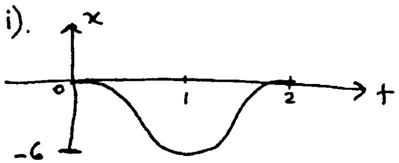
ii). $y = \frac{\ln 5x}{e^{7x}}$

$y' = \frac{e^{7x}\left(\frac{1}{x}\right) - \ln 5x\,(7e^{7x})}{(e^{7x})^2}$

$= \frac{\cancel{e^{7x}}\left(\frac{1}{x} - 7\ln 5x\right)}{(e^{7x})^{\cancel{2}}}$

$= \frac{1}{e^{7x}}\left(\frac{1}{x} - 7\ln 5x\right)$

d). i).



ii). $\dot{x} = -3\pi\sin\pi t$

$= 0$ for $t = 0, 1, 2, \ldots\ldots$

$\therefore$ 1st comes to rest at $t = 1$

iii). Particle starts from rest & moves in a negative direction. It slows down & comes to rest when $t=1$ at a negative distance of 6 units from its starting position.

Question 4

(a) all real x except | $x \neq \mp 1$ (1)

x intercept $y = 0$ $\quad \frac{6}{x^2-1} = 3$

$6 = 3x^2 - 3$ $\quad -\frac{1}{2}$ mistakes to here

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$ $\quad$ 3

y intercept $x = 0$ $\quad y = -6-3 = -9.$ $\quad$ 1

(b) (i) $g(x) = e^{3\sin x}$ $\quad$ D: all real x $\quad$ 1

$-1 \leq \sin x \leq 1$

$-3 \leq 3\sin x \leq 3$ $\quad$ 2

Range: $\frac{1}{e^3} \leq y \leq e^3$ $\quad \frac{1}{2}$

(ii) $f(x) = \log_e e^{3\sin 2x}$

$= 3\sin x$ $\quad \frac{1}{2}$

D: all real x $\quad$ 1 $\quad$ 2

R $\quad -3 \leq f(x) \leq 3$ $\quad$ 1

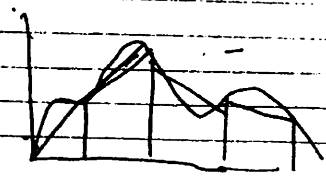
(c) $d = \int v\,dt$ $\quad -\frac{1}{2}$ each mistake

(i) $\int v\,dt = \frac{24}{2}\left[0 + 2\times 76 + 2\times 98 + 2\times 54 + 2\times 51 + 0\right]$

$= 12 \times 558$

$= 6696$ km $\quad$ (0 - Simpson's Rule) $\quad$ 3

(ii)



More, as curve concave down as trapezia are under curve.

(2 marks if explained) 2

Q5

a) $\Delta = (-1)^2 - 4(k+1)(1)$

$= 1 - 4k - 4$

$= -4k - 3$ 1

No real roots $\Rightarrow \Delta < 0$ $\frac{1}{2}$

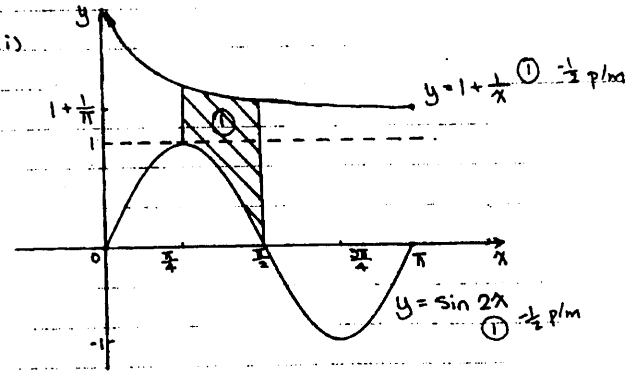
$-4k - 3 < 0$

$-4k < 3$

$k > \frac{-3}{4}$ $\frac{1}{2}$ (2)

Since $k > \frac{-3}{4}$, zero is the least integer value of k.

c) i) & ii)



iii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 + \frac{1}{x} - \sin 2x \, dx$

$= \left[x + \ln x + \frac{1}{2}\cos 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $1\frac{1}{2}$

$= \left(\frac{\pi}{2} + \ln\frac{\pi}{2} + \frac{1}{2}\cos\pi\right) - \left(\frac{\pi}{4} + \ln\frac{\pi}{4} + \frac{1}{2}\cos\frac{\pi}{2}\right)$ $\frac{1}{2}$

$= \frac{\pi}{4} + \ln\frac{\pi}{2} - \ln\frac{\pi}{4} - \frac{1}{2}$ 1

$= \ln 2 + \frac{\pi}{4} - \frac{1}{2}$ (3)

Q5

b) $y = x^4 - 4x^3$

$= x^3(x - 4)$

x - intercept = 0, 4.

$y' = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

$\left[\begin{array}{l} x = 0 \\ y = 0 \end{array}\right.$ OR $\left[\begin{array}{l} x = 3 \\ y = -27 \end{array}\right.$

$y'' = 12x^2 - 24x$

When $x = 3$, $y'' = 12 \times 3^2 - 24 \times 3 > 0$

$\therefore (3, -27)$ is a min. turning pt.

When $x = 0$, $y'' = 0$

x	-1	0	1
y''	+	0	-

$\therefore (0,0)$ is a horizontal pt. of inflexion

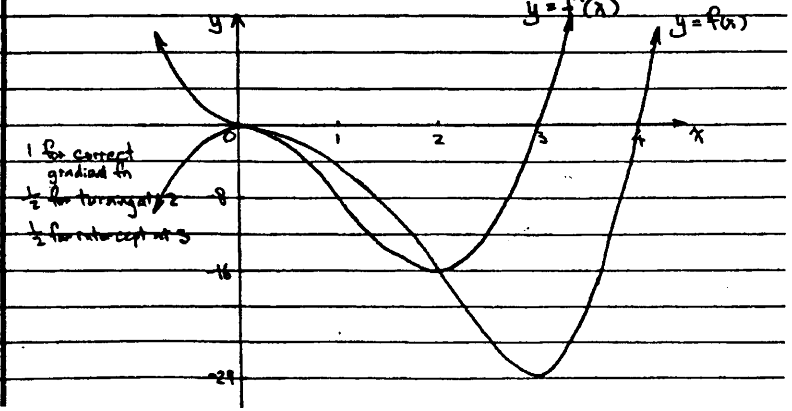
$y'' = 12x^2 - 24x$

$12x(x-2) = 0$

$x = 0$ OR $\left[\begin{array}{l} x = 2 \\ y = -16 \end{array}\right.$

x	1	2	3
y''	-	0	+

$\therefore (2, -16)$ is a pt. of inflexion



Q6

a) $T_3 = -6$
$T_{23} = 9$

$T_3 = a + 2d = -6$
$T_{23} = a + 22d = 9$
$\therefore 20d = 15$
$d = 3/4$ (½)

$\therefore a + 2 \times 3/4 = -6$
$a = -7\frac{1}{2}$ (½)

i) $T_{24} = a + 23d$
$= -7\frac{1}{2} + 23 \times 3/4$ (1)
$= 9.75$

ii) $S_{21} = \frac{21}{2}(-6 + 9)$

$= 31\frac{1}{2}$ (2)

iii) $99 = a + (n-1)d$
$99 = -7\frac{1}{2} + (n-1) \times 3/4$
$143 = n$ (½)

$\therefore$ since n is an integer, 99 is a term of the sequence (½)

b) i) stat pts occur when $f'(x) = 0$

$-\frac{5x}{2} e^{-\frac{x}{2}} + 5e^{-\frac{x}{2}} = 0$

$e^{-\frac{x}{2}}\left(-\frac{5}{2}x + 5\right) = 0$

$e^{-\frac{x}{2}} \neq 0$ $\quad -\frac{5}{2}x + 5 = 0$
$5x = 10$
$x = 2$ (½)

when $x = 2$, $f(x) = 5 \times 2 \times e^{-1} + 3$
$= \frac{10}{e} + 3$

$\left(2, \frac{10}{e} + 3\right)$ (½)

ii) $f(x)$ is increasing when $f'(x) > 0$

$e^{-\frac{x}{2}}\left(-\frac{5}{2}x + 5\right) > 0$

$-\frac{5}{2}x + 5 > 0$

$x < 2$ (1)

iii) $f(x)$ is decreasing when $f'(x) < 0$

$\therefore x > 2$ (1)

iv) concave up when $f''(x) > 0$

$5e^{-\frac{x}{2}}\left(\frac{1}{4}x - 1\right) > 0$

$\frac{1}{4}x - 1 > 0$

$x > 4$ (1)

v) concave down when $f''(x) < 0$

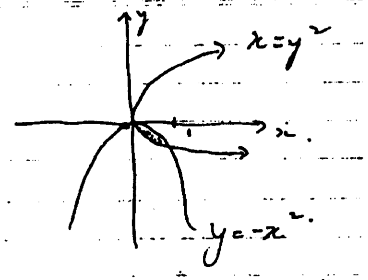
$\therefore x < 4$ (1)

vi) y-intercept
$f(0) = 3$ (½)

$f(6) = \frac{30}{e^3} + 3$
≈ 4.49 (2 dec. pl) (½)

vii) $\therefore$ absolute minimum for $0 \le x \le 6$ is 3. (1)

Q7. a)



$y^2 = x^4$

$\therefore x = x^4$

$x(\cancel{x}1 - x^3) = 0$

$x^3 = 1$ or $x = 0$

$x = 1$ or 0. $\tfrac{1}{2}$

$V = \pi \int y^2 dx$

$V = \pi \int_0^1 (x - x^4)\,dx$ $\tfrac{1}{2}$

$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$ 1

$= \pi \left[\frac{1}{2} - \frac{1}{5} - 0 \right]$ $\tfrac{1}{2}$

$= \frac{3\pi}{10}$ u^3 $\tfrac{1}{2}$

$\pi^c = 180°$

b) $35 = 57\theta$. $1^c = \frac{180}{\pi}$ c.

$\theta = \frac{35}{57}$ radians (1)

$= \frac{35}{57} \times \frac{180}{\pi}$ degrees $\tfrac{1}{2}$

$= 35° 11'$ nearest minute. $\tfrac{1}{2}$

c) $t = \frac{5}{x+3} - 1$

(i) $x = 3$ $t = \frac{5}{6} - 1$

$t = -\frac{1}{6}$

but $t \geq 0 \therefore x \neq 3$ (1)

(ii) $t + 1 = \frac{5}{x+3}$

$x + 3 = \frac{5}{t+1}$ (1)

$x = \frac{5}{t+1} - 3$

(iii) $t \to \infty$ $x \to -3$

$\therefore$ approaches limiting posn. of $x = -3$ (2)

(iv) $v = \dot{x} = \frac{-5}{(t+1)^2}$ (1)

(v) $\dot{x} = \frac{-5}{(t+1)^2} \neq 0$ 1 (2)

but as $t \to \infty$ $\dot{x} \to 0$. 1

Q 8

$T_1 = 0 = A\left(\frac{1}{5}\right) + B\left(-\frac{1}{2}\right)$

$0 = \frac{1}{5}A - \frac{1}{2}B$

$0 = 2A - 5B$ ① 1

$T_2 = \frac{7}{5} = \frac{1}{25}A + \frac{1}{4}B$ $\frac{1}{2}$

$140 = 4A + 25B$ ②

② − 2×① $140 = 35B$

$4 = B$ $\frac{1}{2}$

and $2A = 5\times4 = 20$

$A = 10$ $\frac{1}{2}$

$T_4 = 10\times\left(\frac{1}{5}\right)^4 + 4\times\left(\frac{-1}{2}\right)^4$

$= \frac{2}{125} + \frac{1}{4}$ $\frac{1}{2}$

$= \frac{133}{500}$ or 0.266

(ii) Series is $10\left(\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\dots+\frac{1}{5^n}\right) + 4\left(-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}\dots+\left(-\frac{1}{2}\right)^n\right)$

$S_n = 10\left(\frac{1}{5}\left(\frac{1-\left(\frac{1}{5}\right)^n}{1-\frac{1}{5}}\right)\right) + 4\left(-\frac{1}{2}\frac{\left(1-\left(-\frac{1}{2}\right)^n\right)}{1+\frac{1}{2}}\right)$ 1 mark for each portion

$= 10\left(\frac{1}{4}\left(1-\left(\frac{1}{5}\right)^n\right)\right) + 4\times\left(-\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)\right)$ 2

$= \frac{10}{4}\left(1-\left(\frac{1}{5}\right)^n\right) - \frac{4}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)$

$= \frac{5}{2}\left(1-\left(\frac{1}{5}\right)^n\right) - \frac{4}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)$

$S_\infty = \frac{5}{2}(1-0) - \frac{4}{3}(1-0)$

$= \frac{5}{2} - \frac{4}{3} = 1\frac{1}{6}$

OR

$S_\infty = 10\left(\frac{\frac{1}{5}}{1-\frac{1}{5}}\right) + 4\left(\frac{-\frac{1}{2}}{1+\frac{1}{2}}\right)$

$= 10\times\frac{1}{4} + 4\times-\frac{1}{3}$ 2

$= 1\frac{1}{6}$

b) (i) $\frac{dV}{dt} = -3e^{0.4t}$

when $t = 0$ ①

$\frac{dV}{dt} = -3$ l/min petrol reducing at rate of 3 l/min

(ii) $V = -3\int e^{0.4t}\,dt$

$= \frac{-3\times5}{2}e^{0.4t} + c$ — with or without c. $\frac{1}{2}$

when $t=0$ $V = 100$ ②

$100 = -\frac{15}{2}e^0 + c$ $\frac{1}{2}$

$\therefore c = 107.5$ $\frac{1}{2}$

$V = -\frac{30}{4}e^{0.4t} + 107.5$ $\frac{1}{2}$

(iii) $50 = -\frac{30}{4}e^{0.4t} + 107.5$ $\frac{1}{2}$

$57.5\times\frac{4}{30} = e^{0.4t}$ ②

$7.\dot{6} = e^{0.4t}$

$e^{0.4t} = 7\frac{2}{3}$ $\frac{1}{2}$

$0.4t\ \ln_e e = \ln 7\frac{2}{3}$

$t = \frac{1}{0.4}\ln\left(7\frac{2}{3}\right)$ $\frac{1}{2}$

$t = 5.09220418$ mins $\frac{1}{2}$

t = 5mins 6secs

Q9 $N_{(0)} = 72$.

$N_{(16)} = 24$

i) $24 = 72e^{-16k}$ $\frac{1}{2}$

$\frac{1}{3} = e^{-16k}$

$3 = e^{16k}$

$k = \frac{1}{16}\ln 3$

$2 = 72e^{-\ln 3^{1/16}\cdot t}$

$\frac{1}{36} = e^{-\ln 3^{1/16}\cdot t}$

$36 = e^{\ln 3^{1/16}\cdot t}$ $\frac{1}{2}$

$\frac{\ln 36}{\ln 3^{1/16}} = t$

$t = 52.189$ sec $\frac{1}{2}$

Q9 ctd

part a) ctd.

$1 = 72e^{-\ln 3^{1/6} t}$

$72 = e^{\ln 3^{1/6}\cdot t}$

$\frac{\ln 72}{\ln 3^{1/16}} = t$ $1\frac{1}{2}$

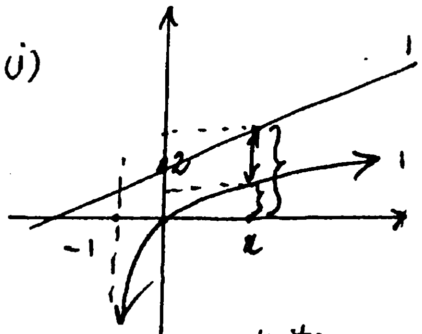
$-\frac{1}{2}$ for each error

$t = 62.28\ldots$

≈ 62 seconds

$\therefore$ 10 more seconds $\frac{1}{2}$

b/ (i)



ie. by subtracting ordinates.

(ii) $y = \frac{2x}{3} + 2 - \ln(x+1)$

(iii) $D = \frac{mx}{3} + 2 - \ln(x+1)$

$\frac{dD}{dx} = \frac{m}{3} - \frac{1}{x+1}$ $\frac{1}{2}$

$= 0$ for $x + 1 = \frac{3}{m}$

$x = \left(\frac{3}{m} - 1\right)$ $\frac{1}{2}$

$\frac{d^2D}{dx^2} = \frac{1}{(x+1)^2}$, $\frac{1}{2}$

> 0 for all x, $(x \neq -1)$

$\therefore$ minima at

$x = \frac{3}{m} - 1$ $\frac{1}{2}$

shortest vertical distance is

$\frac{m}{3}\left(\frac{3-m}{m}\right) + 2 - \ln\left(\frac{3}{m} - 1 + 1\right)$ $\frac{1}{2}$

$= 1 - \frac{m}{3} + 2 - \ln\frac{3}{m}$

$= 3 - \frac{m}{3} - \ln\frac{3}{m}$. $\frac{1}{2}$

(iv) from $x = \frac{3}{m} - 1$

$m = 3$ for $x = 0$ 1

to coincide with the y-axis.

Q 10

Q10 $\tan \pi x = \frac{1}{\sqrt{3}}$ $\quad 0 \le x \le 2$

$\pi x = \frac{\pi}{6}, \frac{7\pi}{6}$ $\quad 0 \le \pi x \le 2\pi$ 1

$= \frac{1}{6}, \frac{7}{6}.$ 1

b) $W = W_0 e^{\mu t}$

$\frac{dW}{dt} = \mu W_0 e^{\mu t}$ 1

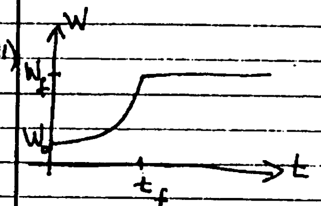
$= \mu W$ for $0 < t < t_f$

(ii) for $0 \le t < t_f$ $\quad \frac{dW}{dt} = \mu W > 0$ as $\mu > 0$ and $W > 0$

for $t \ge t_f$ $\quad \frac{dW}{dt} = 0$ $\frac{1}{2}$

Hence $\frac{dW}{dt} \not< 0$ for all values of t. i.e. it is never decreasing $\frac{1}{2}$

(iii)



5 things
$-\frac{1}{2}$ for anything missing 2

iv $\log W = \log W_0 e^{\mu t}$

$= \log W_0 + \mu t$ 1

$= \mu t + \log W_0$.

gradient $= \mu$ 1

OR

$\frac{d}{dt} \log W = \frac{W'}{W} = \frac{\mu W}{W} = \mu$

OR

gradient $= \frac{\log W_f - \log W_0}{t_f}$ $\quad$ now $W_f = W_0 e^{\mu t_f}$

$= \frac{\log\left(\frac{W_f}{W_0}\right)}{t_f}$

$= \frac{\log\left(\frac{W_0 e^{\mu t_f}}{W_0}\right)}{t_f}$

$= \frac{\mu t_f}{t_f}$

$= \mu$

y intercept $= \log W_0$. 1

(v) Let additional time be t $\quad$ i.e. time to get from $W_f \to 2W_f$.

$2W_f = W_f e^{\mu t}$

$2 = e^{\mu t}$

$\ln 2 = \mu t$

$\frac{\ln 2}{\mu} = t$

3.