PYMBLE LADIES' COLLEGE MATHEMATICS TRIAL 2004

2U

Question 1 (12 Marks)

Marks

(a) Evaluate $\sqrt{\dfrac{53.9\times17.8}{19.6+4.97}}$ correct to 2 decimal places. **1**

(b) Solve the following equations:

(i) $8^x=\dfrac{1}{4}$ **2**

(ii) $\dfrac{6}{x-1}-\dfrac{3}{x}=2$ **3**

(c) Find a primitive of $8+\dfrac{3}{e^x}$ **2**

(d) Simplify: $\dfrac{2\sqrt{3}}{\sqrt{3}-1}$ **2**

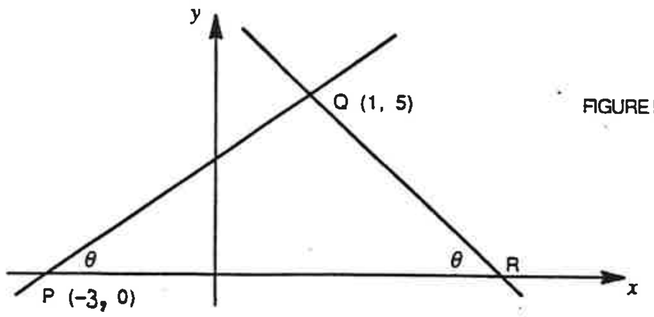
(e) A store makes a profit of 40% on the cost of all its sales. Find

(i) the selling price if the cost price is $84 **1**

(ii) the cost price if the selling price is $84. **1**

Question 2 (12 Marks)

Marks



In the diagram P and Q have coordinates (-3,0) and (1,5) respectively, and $\angle QPR=\angle QRP=\theta$.

Copy this diagram onto your ANSWER SHEET.

(a) Find the coordinates of the midpoint of PQ. **1**

(b) Show that the gradient of PQ is $\frac{5}{4}$. **1**

(c) Show that the equation of PQ is $5x-4y+15=0$. **2**

(d) Show that the gradient of QR is $-\frac{5}{4}$. **1**

(e) Show that the equation of QR is $5x+4y-25=0$. **2**

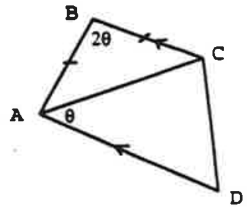
(f) Find the coordinates of R. **1**

(g) Hence find the perpendicular distance from R to PQ. **2**

(h) On your diagram, shade in the region satisfying both the inequalities: $5x-4y+15\le0$ and $5x+4y-25\ge0$ simultaneously. **2**

Question 3 (12 Marks) Start a new page. Marks

(a)

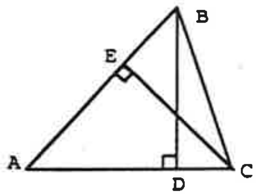


In the diagram, ABCD is a quadrilateral with AB = BC, $\angle ABC = 2\theta$ and $\angle CAD = \theta$.

Copy this diagram onto your ANSWER SHEET and find the value of θ, giving reasons.

3

(b)



In the diagram $BD \perp AC$ and $CE \perp AB$.

(i) Copy this diagram onto your ANSWER SHEET and prove that ΔECA is similar to ΔDBA. 2

(ii) If AB = 10cm, BD =7cm and AC =16cm, find the length of CE. 2

Question 3 continues over page

Question 3 continued

(c)

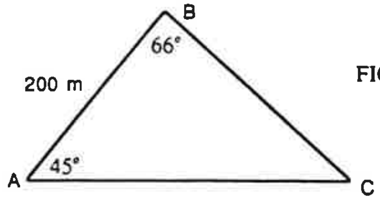


FIGURE NOT TO SCALE

Use the Sine Rule to calculate the length of the side BC to the nearest metre. 2

(d) Solve the equation: $2\sin\theta - 1 = 0$ for $0^\circ \le \theta \le 360^\circ$. 3

Question 4 (12 Marks) Start a new page. Marks

(a) Integrate with respect to x:

(i) $\int (2x-1)^6 \, dx$ 1

(ii) $\int \sin 2x dx$ 1

(b) Integrate and leave answer in exact form: $\int_0^2 e^{2x} \, dx$ 2

(c) Find the exact area of the region bounded by the curve $y = \frac{1}{x+4}$, the x-axis and the lines x = 0 and x = 4, giving your answer in the simplest form. 3

(d) (i) Differentiate $y = x^2 e^{-3x}$ with respect to x. 2

(ii) Find the equation of the tangent to the curve $y = x^2 e^{-3x}$ at $x = \frac{1}{3}$.

Leave your answer in exact form. 3

Question 5 (12 Marks) Start a new page. Marks

(a) Given that α and β are the roots of the quadratic function $2x^2 - 3x - 5 = 0$ find

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $(\alpha + 1)(\beta + 1)$ 1

(b) Prove that the line $y = 4x - 4$ is a tangent to $y = x^2$. 2

(c) The mass M kg of a radioactive substance present after t years is given by $M = 10e^{-kt}$, where k is a positive constant. After 100 years the mass has reduced to 5 kg.

(i) What was the initial mass? 1

(ii) Find the value of k (leave answer in exact form). 2

(iii) What amount of the radioactive substance would remain after a period of 1000 years? Give answer correct to 4 significant figures. 2

(iv) How long would it take for the initial mass to reduce to 8 kg? Give answer in terms of years, correct to 2 decimal places. 2

Question 6 (12 Marks) Start a new page. Marks

(a) An office worker is employed at an initial salary of \$20 600 p.a. After each year this salary is increased by \$800.

(i) What is the salary for the seventh year of service? **2**

(ii) What is the worker's total salary for the first seven years? **2**

(b) Find the number "n" which when added to each of 2, 5 and 9 will give a set of three numbers in geometric progression.

2

(c) Show that the following represents an arithmetic series and hence evaluate

$$\sum_{k=1}^{9}(4k-1)$$ **2**

(d) A ball is dropped from a height of 20cm and continues to bounce $\frac{3}{4}$ of the height of the preceding bounce until it comes to rest. What is the total distance travelled by the ball?

4

Question 7 (12Marks) Start a new page. Marks

(a) The minute hand of a clock is 20cm long.

(i) Show that the arc length along which the tip of the hand travels in 16 minutes is $\frac{32}{3}\pi$ radians. **1**

(ii) Calculate the shortest distance between the initial and final positions of the tip of the hand. Give answer correct to 2 decimal places.

2

(b)

x	1	2	3	4	5
f(x)	0	0.3	0.5	0.6	0.7

Given the above table, where $f(x) = \log_{10} x$, use Simpson's Rule with the 5 function values to find the approximate area under the curve between $x = 1$ and $x = 5$ (correct to 1 decimal place).

2

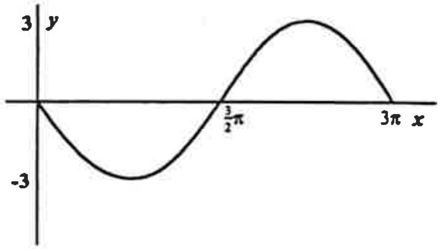
(c) A sheep, grazing in a paddock, is tethered to a stake by a rope 20m long. If the stake is 10m from a fence, find the area over which the sheep can graze. Give answer correct to 2 decimal places.

4

Question 7 continues over page

Question 7 continued

(d)



The above diagram represents a possible sine or cosine curve.

(i) Give the amplitude. **1**

(ii) Give the period. **1**

(iii) Write down a possible equation for the curve. **1**

Question 8 (12 Marks) Start a new page. Marks

(a) Given the function $y = x^3 + 3x^2 - 24x + 1$

(i) find any maximum or minimum turning points **4**

(ii) find any points of inflexion **2**

(iii) determine where the graph cuts the y-axis **1**

(iv) hence sketch the curve indicating its essential features.
Note: x intercepts not required. **2**

(b) Jane makes equal annual contributions of \$P into a retirement fund. This retirement fund pays compound interest of 8% per annum compounded annually. The first contribution was made on the 1st January 2001 and the last contribution is to be made on the 1st January 2025.

(i) How much does the first contribution amount to at the end of n years after interest is paid? **1**

(ii) If Jane wants to retire with a lump sum of \$200 000 on the 31st December 2025 after interest is paid for the year, find the amount of each equal annual contribution \$P. **2**

Question 9 (12 Marks) Start a new page. Marks

(a) Solve for x: $9^x - 10.3^x + 9 = 0$ **2**

(b) A particle initially at rest at the origin moves in a straight line with velocity v metres per second, such that $v = 3t(4-t)$ where t is the time elapsed in seconds. Find:

(i) the acceleration of the particle at the end of 1 second **1**

(ii) an expression for the displacement x of the particle in terms of t **2**

(iii) the particle's displacement when it is next at rest **2**

(iv) the velocity of the particle when it returns to the origin and state the direction in which the particle is travelling then? **2**

(v) the time taken for the particle to reach its greatest velocity **1**

(vi) the distance travelled by the particle in the first 5 seconds **2**

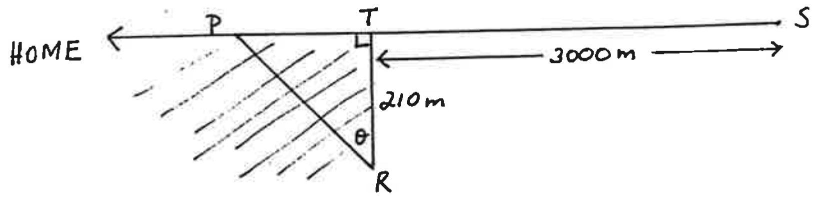
Question10 (12 Marks) Start a new page. Marks

(a)

(i) Sketch the curve $y = e^{-2x}$ and shade the region bounded by this curve, the x-axis, the y-axis and the line x = 2. **1**

(ii) Find the volume generated when this area is rotated about the x-axis (give your answer in terms of π). **3**

(b)



The diagram above shows a restaurant R which is 210 m along a road to a point T on the main street. There is a road TR and a grassed area to walk across at an angle θ from the restaurant to this street. Samantha must catch the last bus home after having dinner at this restaurant. The bus travels from S to T to P and beyond to home. The bus will leave the bus stop S at 10 pm sharp, travelling at a speed of 18 $m s^{-1}$. The distance from S to T is 3000 m. Samantha decides to walk across the grass at angle θ and reaches the street at the point P such that $\angle$ PRT = θ. She walks at a speed of 3 $m s^{-1}$.

(i) Find PR and PS in terms of θ. **2**

(ii) Find two expressions in terms of θ, one expression for the time taken, in seconds, for the bus to travel from S to P and the other expression for the time taken, in seconds, by Samantha to walk from R to P. **2**

(iii) What is the latest that Samantha can leave the restaurant in order to catch this bus? Do not test for maximum or minimum, as we are told that there is a maximum only. **4**

End of paper

2UNIT TRIAL Solutions

Question 1 (a) 6.25 (to 2dp) 1

(b) (i) $8^x = \frac{1}{4}$

$2^{3x} = 2^{-2}$ 1

$3x = -2$

$x = -\frac{2}{3}$ 1

(ii) $\frac{6}{x-1} - \frac{3}{x} = 2$

$6x - 3(x-1) = 2x(x-1)$ 1

$6x - 3x + 3 = 2x^2 - 2x$

$2x^2 - 5x - 3 = 0$ 1

$(2x+1)(x-3) = 0$

$x = -\frac{1}{2}$ or 3 1

(c) $\frac{2\sqrt{3}}{\sqrt{3}-1} = \frac{2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{1}{2}$

$= \frac{2\sqrt{3}(\sqrt{3}+1)}{3-1}$ $\frac{1}{2}$

$= \frac{\cancel{2}\sqrt{3}(\sqrt{3}+1)}{\cancel{2}}$ $\frac{1}{2}$

$= \sqrt{3}(\sqrt{3}+1)$

$= 3 + \sqrt{3}.$ $\frac{1}{2}.$

1 (e) (i) $S.P = 1.4 \times \$84$

$= \$117.60$

(ii) $140\% = \$84$

$1\% = \frac{84}{140}$

$100\% = \frac{\cancel{84}^{12}}{\cancel{140}_{7}} \times \cancel{100}^{5}$

$= \$60.$

Qu. 2 (contd)

(d) $m(QR) = ?$

$\tan\theta = \frac{5}{4}$

$\therefore \tan(180^\circ - \theta) = -\tan\theta$

$= -\frac{5}{4}$. 1

(e) Eqn. of QR: $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{5}{4}(x - 1)$ 1

$4y - 20 = -5x + 5$

$\therefore 5x + 4y - 25 = 0$ 1

(f) R lies on QR $\rightarrow$ $5x + 4y - 25 = 0$

When $y = 0$, $x = ?$

$5x - 25 = 0$. $\frac{1}{2}$

$5x = 25$

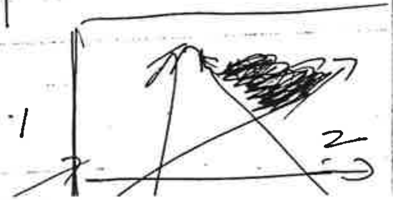
$x = 5$

$\therefore R = (5, 0)$. $\frac{1}{2}$.

(g) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ | $a = 5, b = -4, c = 15$, $x_1 = 5, y_1 = 0$

$= \frac{|5 \times 5 + (-4) \times 0 + 15|}{\sqrt{25+16}}$ 1

$= \frac{|40|}{\sqrt{41}} = \frac{40}{\sqrt{41}}$ units. 1



Question 3

(a) $\angle BAC = \angle ABC = \frac{180^\circ - 2\theta}{2}$ (base $\angle^s$ of isos $\triangle$ are equal and angle sum of $\triangle = 180^\circ$)

$\angle ABC + \angle BAD = 180^\circ$ (co. int $\angle^s$, $BC \parallel AD$)

$\therefore 2\theta + \frac{180^\circ - 2\theta}{2} + \theta = 180^\circ$ $\frac{1}{2}$

$4\theta + 180^\circ - 2\theta + 2\theta = 360^\circ$

$\therefore 4\theta = 180^\circ$

$\theta = 45^\circ$. 1

(b) (i) In $\triangle^s$ ECA, DBA

(i) $\angle AEC = \angle ADB$ (both 90°, given)

(ii) $\angle EAC = \angle BAD$ (common)

$\therefore \triangle ECA \sim \triangle DBA$ (equiangular)

(ii) $\frac{CE}{BD} = \frac{AC}{AB}$ (corr. sides of similar $\triangle^s$ are in the same ratio)

$\therefore \frac{CE}{7} = \frac{16}{10}$

$\therefore CE = \frac{16 \times 7}{10}$ 1

$= \frac{112}{10} = 11.2$.

Qu. 3 (contd)

(e) $\frac{BC}{\sin 45^\circ} = \frac{200}{\sin 69^\circ}$ ($\angle BCA = 180^\circ - (45^\circ + 66^\circ)$ angle sum of Δ) $\frac{1}{2}$

$\therefore BC = \frac{200 \times \sin 45^\circ}{\sin 69^\circ}$ 1

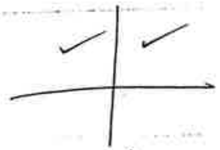
$= 151.148\ldots$

$= 151\text{ m}$ (to nearest m) $\frac{1}{2}$

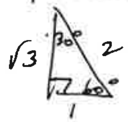
(d) $2\sin\theta - 1 = 0$ $0^\circ \le \theta \le 360^\circ$

$2\sin\theta = 1$ 1

$\sin\theta = \frac{1}{2}$



Acute $\theta = 30^\circ$



$\therefore \theta = 30^\circ, 150^\circ$.

1 1

Question 4

(a) (i) $\int (2x-1)^6\,dx$

$= \frac{(2x-1)^7}{7 \times 2} + C$

$= \frac{(2x-1)^7}{14} + C$ 1

(ii) $\int \sin 2x\,dx$

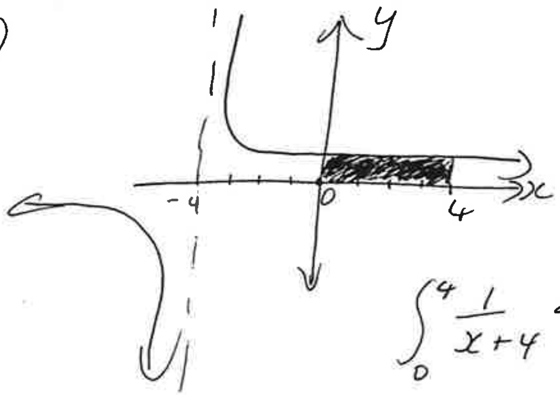
$= -\frac{\cos 2x}{2} + C$ 1

(b) $\int_0^2 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^2$ 1

$= \frac{e^4}{2} - \frac{e^0}{2}$

$= \frac{e^4}{2} - \frac{1}{2}$ 1

(c)



$\int_0^4 \frac{1}{x+4}\,dx = \left[\ln(x+4)\right]_0^4$

$= \ln 8 - \ln 4$

$= 3\ln 2 - 2\ln 2$

$= \ln 2$ 1

Qu. 4 (contd)

(d) (i)

$y = x^2 e^{-3x}$

$y' = x^2 \cdot -3e^{-3x} + 2x \cdot e^{-3x}$ 1

$y' = xe^{-3x}(2-3x)$ 1

(ii) $y = x^2 e^{-3x}$, $y' = xe^{-3x}(2-3x)$

When $x = \frac{1}{3}$, $y = \frac{1}{9}e^{-1} = \frac{1}{9e}$ $\frac{1}{2}$

and $y' = m = \frac{1}{3}e^{-1}(2-1) = \frac{1}{3e}$ $\frac{1}{2}$

$\therefore$ Eqn of tangent to curve at $x = \frac{1}{3}$ is:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{9e} = \frac{1}{3e}\left(x - \frac{1}{3}\right)$ $\frac{1}{2}$

$9ey \cancel{-1} = 3x \cancel{-1}$ 1

$y = \frac{3x}{9e}$

$\therefore y = \frac{x}{3e}$ $\frac{1}{2}$

Qu. 5 (contd)

(c) $M = 10e^{-kt}$

(i) When $t = 0$, $M = ?$

$M = 10e^0$

$M = 10$ 1

$\therefore$ initial mass is 10kg.

(ii) When $t = 100$, $M = 5$

$\therefore 5 = 10e^{-100k}$

$e^{-100k} = 0.5$ $\frac{1}{2}$

$\ln e^{-100k} = \ln 0.5$ $\frac{1}{2}$

$\therefore -100k = \ln 0.5$ $\frac{1}{2}$

$k = \frac{\ln 0.5}{-100}$ $\frac{1}{2}$

(iii) When $t = 1000$, $M = ?$

$M = 10e^{-1000k}$ $\frac{1}{2}$

$= 10e^{-1000 \times \frac{\ln 0.5}{-100}}$ $\frac{1}{2}$

$= 0.009765625$ (kg)

$= 9.766$ g (4 s.f.) $\frac{1}{2}$

(iv) When $M = 8$, $t = ?$

$8 = 10e^{-kt}$

$e^{-kt} = 0.8$ 1

$\ln e^{-kt} = \log 0.8$

$-kt = \log 0.8$

$t = \frac{\log 0.8}{-k}$

$= 32.19$ yrs

Question 5

(a) $2x^2 - 3x - 5 = 0$

(i) $\alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ 1

(ii) $\alpha\beta = \frac{c}{a} = -\frac{5}{2}$ 1

(iii) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$

$= \alpha\beta + (\alpha+\beta) + 1$ 1

$= -\frac{5}{2} + \frac{3}{2} + 1$

$= 0.$

(b) $y = 4x - 4$, $y = x^2$

Solve simultaneously

$4x - 4 = x^2$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$ 1

$\therefore x = 2$

only one root

$\therefore y = 4x - 4$ touches $y = x^2$

$\therefore y = 4x - 4$ is a tangent to $y = x^2$ 1

Qu 6

(a) This is an A.P. with $a = 20600$, $d = 800$, $t = ?$ ½

(i) $T_n = a + (n-1)d$ ½

$T_7 = 20600 + 6 \times 800$ 1

$= 25400$

(ii) $S_n = \frac{n}{2}(a + l)$

$S_7 = \frac{7}{2}(20600 + 25400)$ 1

$= \$161000$ 1

(b) G.P $\rightarrow$ $2+n,\ 5+n,\ 9+n$

$\therefore \frac{n+5}{n+2} = \frac{n+9}{n+5}$ 1

$(n+5)^2 = (n+2)(n+9)$

$\cancel{n^2} + 10n + 25 = \cancel{n^2} + 11n + 18$

$7 = n$ 1

$\therefore n = 7.$

(c) $\sum_{k=1}^{9}(4k-1) = 3 + 7 + 11 + 15 + \cdots$

This an AP with $a = 3$, $d = 4$, $n = 9$

$\therefore S_9 = \frac{9}{2}(6 + 8\times4)$

$= \frac{9}{2}(38) = 171$ 1

(d.) This a G.P. with $a = 15$, $r = \frac{3}{4}$

$\frac{3}{4} \times 20 = 15$

$\therefore a = 15,\ r = \frac{3}{4}$

$S_\infty = \frac{15}{\frac{1}{4}} = 60$

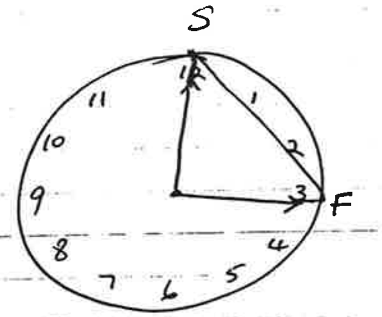
$\therefore$ Total distance $= 2 \times S_\infty + 20$

$= 2 \times 60 + 20$

$= 140\text{cm}.$

Question 7

(a)(i)



$60 \text{ min} = 360°$

$1 \text{ min} = \frac{360}{60} = 6°$

$\therefore 16 \text{ min} = 16 \times 6 = 96°$

Arc length $= \frac{96}{360} \times 2 \times \pi \times 20$

$= \frac{32\pi}{3}$ cm.

(ii) Shortest distance is SF.

$SF^2 = 20^2 + 20^2 - 2 \times 20 \times 20 \times \cos 96°$

$= 883.62277\ldots$

$\therefore SF = 29.73$ cm.

7(b) $h = \frac{5-1}{4} = \frac{4}{4} = 1$ or $h = 1$ from table.

$\therefore A = \frac{1}{3}(0 + 4 \times 0.3 + 2 \times 0.5 + 4 \times 0.6 + 0.7$

$A = \frac{5.3}{3} = 1.777\ldots$

$= 1.8 \text{ unit}^2$

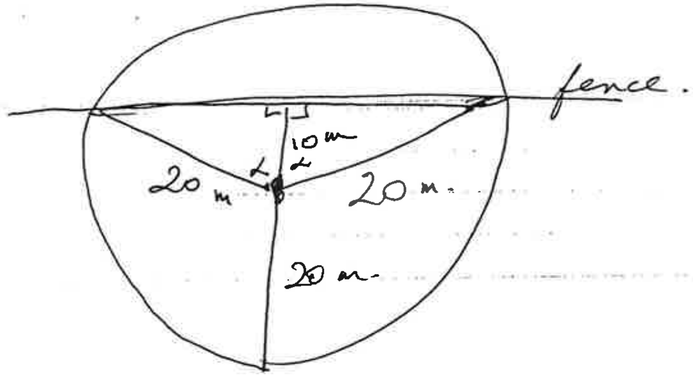
7(c) next sheet

7(d) (i) amplitude $= 3$

(ii) period $= 3\pi$

(iii) $-3 \sin 2x$

(c)



$\cos\alpha = \frac{10}{20} = \frac{1}{2}.$

$\alpha = 60°$

$2\alpha = 120° = \frac{2\pi}{3}.$ $\quad \frac{1}{2}$

Let $\theta = 2\alpha$.

A of sector $= \frac{1}{2}r^2\theta$ $\quad \frac{1}{2}$

$= \frac{1}{2} \times 20^2 \times \frac{2\pi}{3} = \frac{400\pi}{3}$

A of triangle $= \frac{1}{2}ab\sin\theta$

$= \frac{1}{2} \times 20 \times 20 \times \sin 120°$

$= \frac{1}{2} \times 20 \times 20 \times \frac{\sqrt{3}}{2}$ $\quad 1$

$= 100\sqrt{3}$

A of segment $= \frac{400\pi}{3} - 100\sqrt{3}$ $\quad \frac{1}{2}$

Area of circle $= \pi r^2 = \pi \times 400$

$= 400\pi$ $\quad \frac{1}{2}$

$\therefore$ grazing area $= 400\pi - \left(\frac{400\pi}{3} - 100\sqrt{3}\right)$

$= 1200\pi - 400\pi - 300\sqrt{3}/3$

$= 800\pi - 300\sqrt{3}/3$ $\quad 1$

Question 8

(a) (i) $y = x^3 + 3x^2 - 24x + 1$

$y' = 3x^2 + 6x - 24$ $\quad 1$

Stat pts when $y' = 0$

i.e. $3x^2 + 6x - 24 = 0$

$(\div 3)$ $\quad x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$

$\therefore x = 2$ or -4 $\quad 1$

$y'' = 6x + 6$

When $x = 2$, $y'' = 12 + 6$ (+ve) $\therefore$ a min. tp.

When $x = -4$, $y'' = -24 + 6$ (−ve) $\therefore$ a max tp

When $x = 2$, $y = -27$

When $x = -4$ $\quad y = 81$

$\therefore$ a max tp at $(-4, 81)$ and $\quad 1$

a min tp at $(2, -27)$. $\quad 1$

(ii) Pt. of inflexion when $y'' = 0$ and there is a change of concavity.

i.e. $6x + 6 = 0$

$6x = -6$

$x = -1$ $\quad 1$

When $x = -1$

$y = 27$

x	-1^-	-1	-1^+
y''	$-$	0	$+$

$\therefore$ a change in concavity $\quad 1$

$\therefore$ a pt. of inflexion at $(-1, 27)$

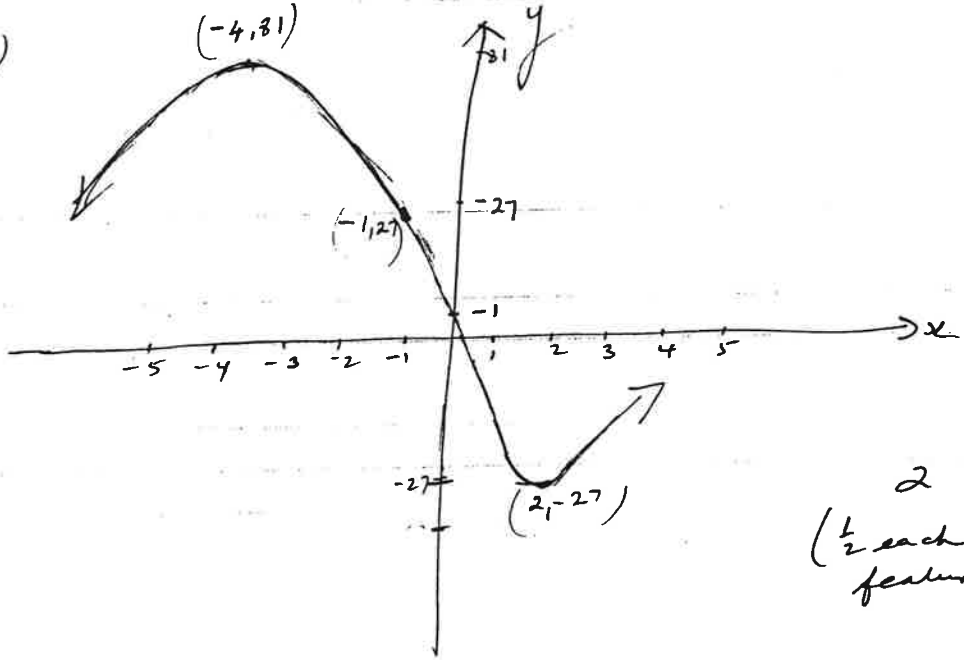
Qu. 8 contd.

(iii) When $x = 0$, $y = ?$

$y = 1$

$\therefore$ graph cuts y-axis at $(0, 1)$.

(iv)



2

($\frac{1}{2}$ each feature)

Qu 8 contd

8 (b)

$1^{st} = \$m\,(1.08)^{25}$

$2^{nd} = \$m\,(1.08)^{24}$

$\vdots$

$25^{th} = \$m\,(1.08)^{1}$ 1

$\therefore 200000 = m\left[1.08 + 1.08^2 + \ldots + 1.08^{25}\right]$

$$200\,000 = \frac{m\,(1.08)\left[1.08^{25} - 1\right]}{1.08 - 1}$$

$$\therefore m = \frac{200\,000 \times 0.8}{(1.08)(1.08^{25} - 1)}$$

$m = \$2533.11$ 1

Question 9

(a) $9^x - 10\cdot 3^x + 9 = 0$

Let $k = 3^x$

$\therefore k^2 - 10k + 9 = 0$

$(k-1)(k-9) = 0$

$\therefore k = 1$ or 9

$\therefore 3^x = 1$ or $3^x = 9$

$3^x = 3^0$, $3^x = 3^2$

$\therefore x = 0$, $x = 2$

$\therefore x = 0$ or 2.

(b) (i) $V = 3t(4-t)$

$V = 12t - 3t^2$

$\therefore a = 12 - 6t$

When $t = 1$, $a = 12 - 6 = 6\ m/s^2$

(ii) $x = \int (12t - 3t^2)\,dt$

$= \frac{12t^2}{2} - \frac{3t^3}{3} + c$

$= 6t^2 - t^3 + c$

When $t = 0$, $x = 0$, $\therefore c = 0$

$\therefore x = 6t^2 - t^3$

(iii) When $v = 0$

$3t(4-t) = 0$

$\therefore t = 0$ or 4

$\therefore$ it is next at rest after 4 sec.

$\therefore$ when $t = 4$, $x = 6.4^2 - 4^3$

$= 32\ m$

(iv) When $x = 0$, $6t^2 - t^3 = 0$

$t^2(6-t) = 0$

$t = 0$ or 6.

$\therefore$ When $t = 6$, $V = 18(4-6)$

$= -36\ m/s$

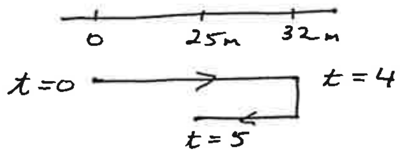
(v) Max velocity when $a = 0$

$\therefore 12 - 6t = 0$

$t = 2$

$\therefore$ it takes 2 secs for it to reach max. velocity.

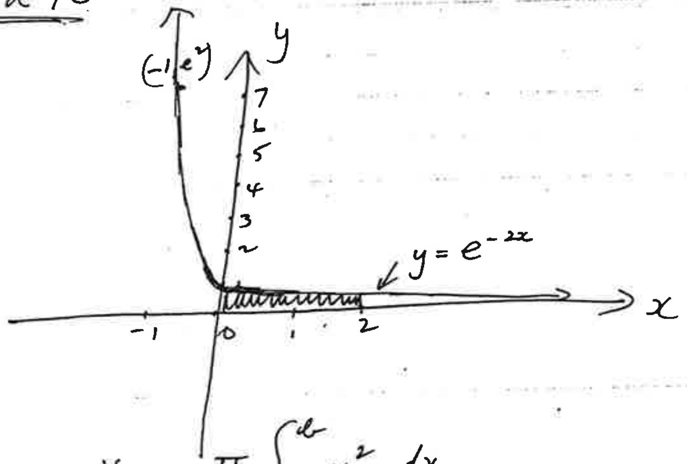
(vi) When $t = 5$, $x = 150 - 125 = 25\ m$.



$\therefore$ it travels $32 + (32 - 25) = 39m$ in the first 5 secs.

Question 10

(a) (i)



2

(ii) $V = \pi \int_a^b y^2 \, dx$

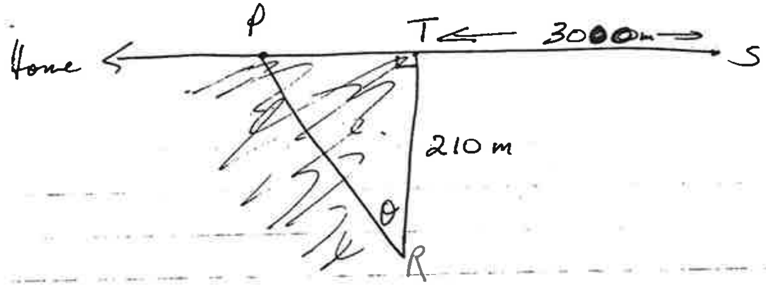
$V = \pi \int_0^2 \left(e^{-2x}\right)^2 dx$ 1

$= \pi \int_0^2 e^{-4x} \, dx$

$= \pi \left[\frac{e^{-4x}}{-4} \right]_0^2$ 1

$= \pi \left(\frac{e^{-8}}{-4} + \frac{e^0}{4} \right)$ 1

$= \frac{\pi}{4}\left(1 - \frac{1}{e^8}\right)$ units^3 1



$S(\text{Samantha}) = 3 \text{ m/s}$
$S(\text{Bus}) = 18 \text{ m/s}$

(i) $\cos\theta = \frac{210}{PR}$

$\therefore PR = \frac{210}{\cos\theta}$ or $210\sec\theta$ 1

$\tan\theta = \frac{PT}{210}$

$\therefore PT = 210\tan\theta$

$\therefore PS = 210\tan\theta + 3000$ 1

(ii) $t(\text{bus}) = \frac{d}{s} = \frac{210\tan\theta + 3000}{18}$ 1

$t(\text{Samantha}) = \frac{d}{s} = \frac{\frac{210}{\cos\theta}}{3} = \frac{210\sec\theta}{3}$ 1

(iii) $T = t(\text{bus}) - t(\text{Samantha})$

$= \frac{210\tan\theta + 3000}{18} - \frac{210\sec\theta}{3}$

$= \frac{210\tan\theta + 3000 - 1260\sec\theta}{18}$

$$\frac{dT}{d\theta} = \frac{1}{18}\left(210\sec^2\theta - 1260\sec\theta\tan\theta\right)$$

$$= \frac{35\sec^2\theta}{3} - 70\sec\theta\tan\theta$$

$$= 35\sec\theta\left(\frac{\sec\theta}{3} - 2\cancel{\sec\theta}\tan\theta\right)$$

When $\frac{dT}{d\theta} = 0$,

$35\sec\theta = 0$

i.e. $\sec\theta = 0$ (no soln)

or $\frac{\sec\theta}{3} - 2\cancel{\sec\theta}\tan\theta = 0$

$$\frac{\sec}{3} = 2\tan\theta$$

$$\frac{1}{3\cos\theta} = \frac{2\sin\theta}{\cos\theta}$$

$\therefore \sin\theta = \frac{1}{6}$

$\therefore \theta = 9^\circ 36'$ (to nearest min)

or $\theta = 0\cdot1674480$ radians

$$\therefore T = \frac{210\tan\theta + 3000 - 1260\sec\theta}{18}$$

$$= \frac{210\tan 0\cdot16745 + 3000 - 1260\sec 0\cdot1674}{18}$$

$= 97\cdot64514$ secs after 10pm.

$=$ 1 min 38 secs. after 10pm (using radians)

or 1 min 39 secs after 10 pm (using degrees)