PYMBLE LADIES' COLLEGE

MATHEMATICS

TRIAL HSC EXAMINATION

2005

Reading Time: 5 minutes
Working Time: 3 hours

Instructions to students:

- Write using blue or black biro.
- All questions may be attempted.
- Diagrams are not to scale.
- All necessary working should be shown in every question.
- Your name and your teacher's name may be written before you begin the assessment.
- Start each question in a **new** booklet.
- Marks may be deducted for careless or untidy work.
- Board-approved calculators may be used.
- A table of standard integrals is provided.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks) Use a separate writing booklet

MARKS

(a) Evaluate $\sqrt{\dfrac{3.705^3}{93.87 \times 2.7}}$, correct to two decimal places. **2**

(b) Solve: $\dfrac{x+1}{4} - \dfrac{x-3}{5} = 1$ **2**

(c) Express $\dfrac{\sqrt{5}-\sqrt{3}}{2-\sqrt{3}}$ with a rational denominator. **2**

(d) If Patrick was given a discount of 25% followed by a discount of 4%, what percentage of the original price did he have to pay? **2**

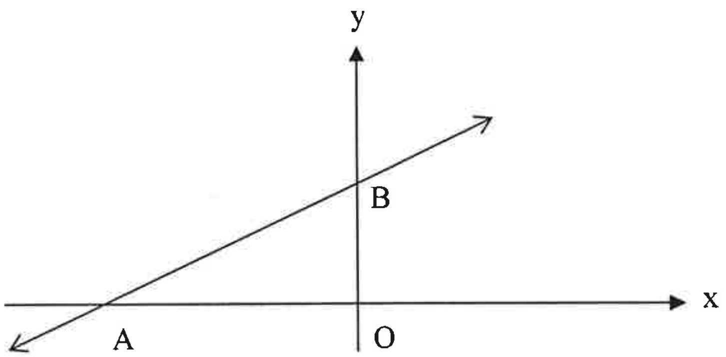
(e) Solve: $x^2 - 5x < 6$ **2**

(f) A function is defined as $f(x) = \begin{cases} 2x-1 & \text{for } 0 \le x \le 3 \\ \dfrac{1}{3}x+4 & \text{for } 3 < x \le 5 \end{cases}$ **2**

What is the range of this function?

Question 2 (12 marks) Use a separate writing booklet

MARKS



A is the point (-3, 0) and B is the point (0, 2).

(i) Calculate the length of the interval AB. **1**

(ii) Find the gradient of the line AB. **1**

(iii) Show that the equation of the line AB is $2x-3y+6=0$. **1**

(iv) Write down the co-ordinates of a point C such that $ABCO$ is a parallelogram. **1**

(v) Write down the co-ordinates of the point M where the diagonal AC cuts the y axis. **1**

(vi) What is the size of the acute angle (to the nearest degree) made by the line AB with the positive direction of the x axis? **1**

(vii) Hence determine the size of $< ABC$. **1**

(viii) Find the perpendicular distance from the origin to the line AB. **2**

(ix) Hence, or otherwise, find the area of the parallelogram $ABCO$. **1**

(x) Shade the area satisfied by the following simultaneously: **2**

$$x \leq 0 \text{ and } y \geq 0 \text{ and } 2x-3y+6 \leq 0$$

Question 3 (12 marks) Use a separate writing booklet

MARKS

(a) Differentiate with respect to x:

(i) $2\cos 3x$ **2**

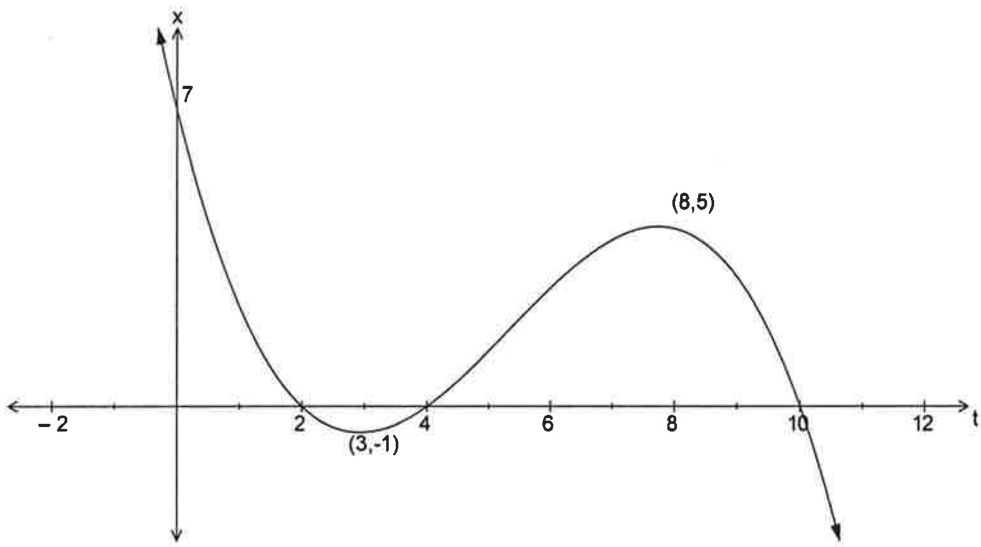
(ii) $5x^2(1+x)^3$ **2**

(b) Evaluate:

(i) $\int_1^2 \frac{2}{x}\,dx$ **2**

(ii) $\int_0^2 \left(e^{2x}+1\right)\,dx$ **2**

(c)

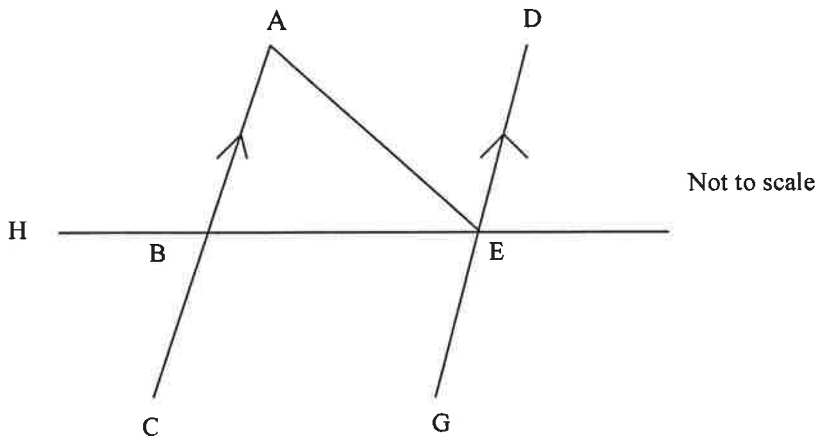


Copy the diagram of $x=f(t)$ and graph $x=f'(t)$ onto the same pair of axes. **1**

Question 3 (continued)

MARKS

(d)



In the diagram, $AB = AE$, $AC \parallel DG$, $\angle ABH = 146^\circ$ and $\angle AED = x^\circ$.

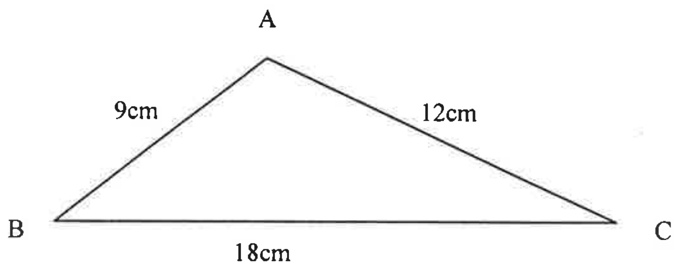
(i) Copy this diagram into your writing booklet and place all the information onto the diagram. **1**

(ii) Find the value of x, giving complete reasons. **2**

Question 4 (12 marks) Use a separate writing booklet

MARKS

(a)



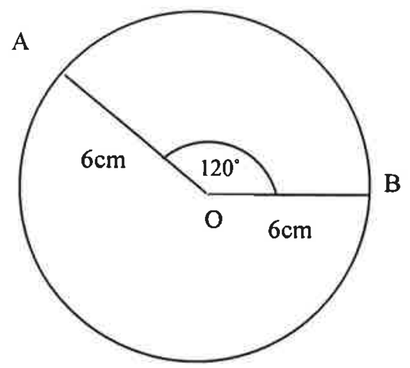
Given the sides of a triangle ABC are 9cm, 12cm and 18cm, find:

(i) the size of the smallest angle to the nearest degree; **2**

(ii) the area of ΔABC to the nearest square centimetre. **2**

(b)

Not to scale



O is the centre of the circle of radius 6cm. $\angle AOB = 120°$.

(i) Find the length of the arc AB, leaving answer in exact form. **2**

(ii) Find the area of the minor sector, leaving answer in exact form. **2**

(c) If p, q and 32 are the first three terms of a geometric series and q, 4 and p are the first three terms of another geometric series, find p and q. **4**

Question 5 (12 marks) Use a separate writing booklet

MARKS

(a) Given the curve $y = x^3 - x^2 - x + 1$ find:

(i) any stationary point(s) and determine their nature; **3**

(ii) any point(s) of inflexion. **2**

(iii) Sketch the curve, showing all essential features in the domain $-2 \le x \le 3$. **2**

(iv) Hence, find the minimum value of $x^3 - x^2 - x + 1$ for $-2 \le x \le 3$. **1**

(b) Find the equation of the tangent to $y = \ln(3x+1)$ at the point (2, 5). **2**

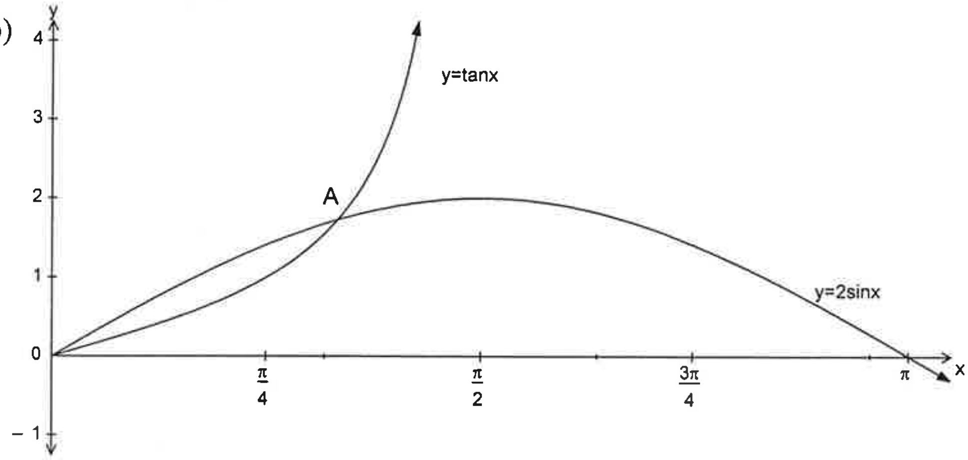
(c) Sketch $y = 3\sin 2x$ in the domain $0 \le x \le 2\pi$. **2**

Question 6 (12 marks) Use a separate writing booklet

MARKS

(a) Find the values of m for which the equation $2x^2 + mx + 8 = 0$ has no real roots. **2**

(b)



Consider the curves $y = \tan x$ and $y = 2\sin x$ in the domain $0 \leq x < \frac{\pi}{2}$.

(i) Show that the coordinates of A are $\left(\frac{\pi}{3,}\sqrt{3}\right)$. **2**

(ii) Show that $\frac{d}{dx}[\ln \cos x] = -\tan x$. **1**

(iii) Hence find the area enclosed by these two curves in the above diagram. **3**

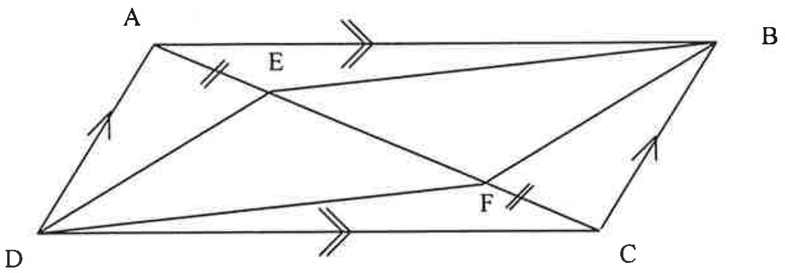
(c) (i) Show the locus of a point which moves so that it is equidistant from the point (0, 3) and the line $y = -3$ is a parabola $x^2 = 12y$. **2**

(ii) Find the vertex and the focal length of this parabola. **2**

Question 7 (12 marks) Use a separate writing booklet

MARKS

(a)



ABCD is a parallelogram.
On the diagonal AC, points E and F are chosen such that AE = FC.

(i) Prove that ΔADE is congruent to ΔCBF. **2**

(ii) Hence, or otherwise, show that the quadrilateral DEBF is a parallelogram. **2**

(b) The position x cm at time t seconds of a particle moving in a straight line is given by $x = 5t + e^{-5t}$.

(i) Find the position of the particle when $t = 1$. Give your answer correct to 3 significant figures. **1**

(ii) By finding an expression for the velocity of the particle, show that initially the particle is at rest. **2**

(iii) Find the limiting velocity of the particle as $t \to \infty$. **1**

Question 7 (continued)

MARKS

(c) The mass M, of a radioactive substance t years after it starts to decay is given by $M = M_0 e^{-kt}$ where M is the mass in kilograms of the substance present and M_0, k *are* constants.
If in three years, a mass of 12 kilograms will reduce to 8 kilograms, find:

(i) the decaying constant k (answer to three decimal places). **2**

(ii) the number of kilograms present after ten years (answer to the nearest gram). **1**

(iii) the number of years it takes for the mass to halve itself. **1**

Question 8 (12 marks) Use a separate writing booklet

MARKS

(a) A curve $y = f(x)$ has the following properties in the interval $a \le x \le b$: **2**

$f(x) > 0$; $f'(x) > 0$; $f''(x) < 0$.

Sketch a curve satisfying these conditions.

(b) Use the Trapezodial Rule with three function values to find an **3**

approximation for $\int_{0.5}^{1.5} \frac{\sin x}{x}\, dx$.

Give your answer correct to two decimal places.

(c) A pendulum is set swinging by lifting it to the right and releasing it. Its first swing (from right to left) is through an angle of 30^o. If the next swing (from left to right) is through 27^o, and each succeeding swing is through 90% of the angle of the previous swing.

(i) List the first four angles through which the pendulum swings. **2**

(ii) Explain why these four values form a geometric sequence. **1**

(iii) Find an expression for the n^{th} term of this sequence. **1**

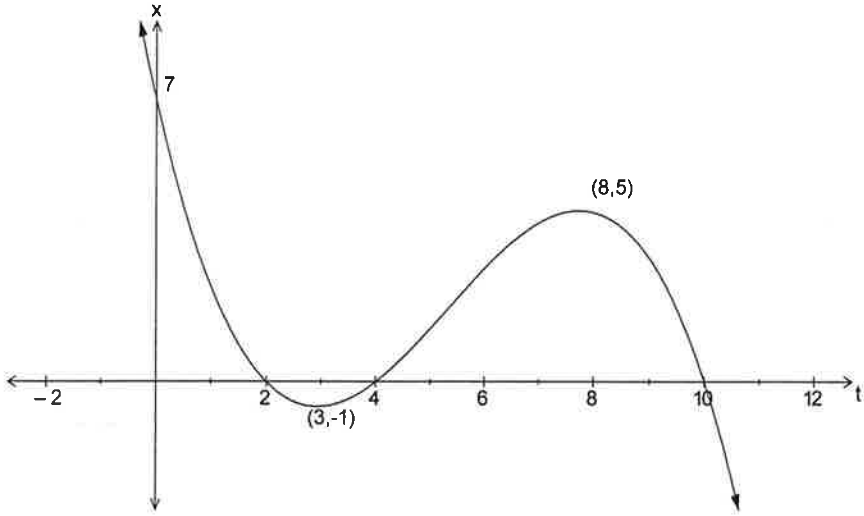
(iv) Which swing will be the first swing through less than 2^o? **2**

(v) Explain why it is impossible for the total angle the pendulum swings through to be greater than 300^o. **1**

Question 9 (12 marks) Use a separate writing booklet

MARKS

(a)



The graph shows the displacement, x metres from the origin, at any time t seconds, of a particle moving in a straight line.

(i) Where was the particle initially? **1**

(ii) When was the particle at the origin? **1**

(iii) When was the particle at rest? **2**

(iv) How far did the particle travel during the first 10 seconds? **2**

(b) A cylindrical can <u>without</u> a lid is to be made from 300π cm^2 of sheet metal.

(i) Show that the volume, V cm^3, may be expressed in terms of the radius, r cm, by $V = 150\pi r - \dfrac{\pi r^3}{2}$. **2**

(ii) Hence, find the radius of the can which will give the can a maximum volume. **4**

Question 10 (12 marks) Use a separate writing booklet

MARKS

(a) Stephanie wishes to have \$50 000 capital in eight years time. She invests a fixed amount of money at the beginning of each month during this time. Interest is accumulated at 6% per annum, compounded monthly.

(i) Let \$$P$ be the monthly investment. Show that the total amount, \$$A$, after eight years is given by : **2**

$$A = \$P\,(1.005 + 1.005^2 + \ldots\ldots + 1.005^{96})$$

(ii) Find, to the nearest dollar, the amount to be invested each month in order to achieve her goal. **2**

(b) Given $f(x) = \dfrac{e^x - e^{-x}}{2}$.

(i) Find $f'(x)$. **1**

(ii) Show that the graph $y = f(x)$ is increasing for all values of x and that there is a point of inflexion at $x = 0$. **3**

(iii) Sketch the graph of $y = f(x)$. **1**

(iv) Let $y = f(x)$. **1**
Show that this equation can be written in the form
$e^{2x} - 2ye^x - 1 = 0$.

(v) Hence, deduce that $x = \ln\left(y + \sqrt{y^2+1}\right)$. **2**

END OF PAPER

2005 Mathematics Trial Soln.

Question 1

(a) $\sqrt{0.20066612\ldots}$

$= 0.44795\ldots$

$= 0.45$

(b) $\dfrac{x+1}{4} - \dfrac{x-3}{5} = 1$

$20\left(\dfrac{x+1}{4}\right) - 20\left(\dfrac{x-3}{5}\right) = 20$

$5x + 5 - 4x + 12 = 20$

$x + 17 = 20$

$x = 3$

(c) $\dfrac{\sqrt5 - \sqrt3}{2-\sqrt3} = \dfrac{\sqrt5-\sqrt3}{2-\sqrt3} \times \dfrac{2+\sqrt3}{2+\sqrt3}$

$= \dfrac{(\sqrt5-\sqrt3)(2+\sqrt3)}{4-3}$

$= 2\sqrt5 + \sqrt{15} - 2\sqrt3 - 3$

(d) $\left(\dfrac{96}{100} \times \dfrac{75}{100}\right)x = 0.72x$

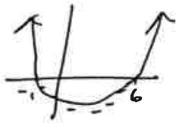
$\therefore$ He pays 72% of original price of $x.

(e) $x^2 - 5x < 6$

$x^2 - 5x - 6 < 0$

$(x+1)(x-6) < 0$

$\therefore -1 < x < 6$

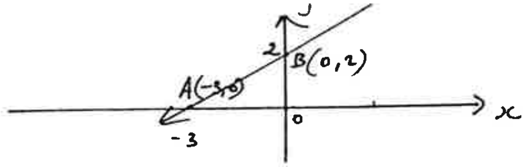


(f) When $x = 0$, $f(x) = (2\times0) - 1 = -1$

When $x = 5$, $f(x) = \frac{1}{3}\times5 + 4 = 5\frac{2}{3}$

$\therefore$ Range: $-1 \le f(x) \le 5\frac{2}{3}$

Question 2



(i) $AB^2 = 3^2 + 2^2$

$= 13$

$AB = \sqrt{13}$ units

(ii) $m(AB) = \dfrac{2-0}{0+3} = \dfrac{2}{3}$

(iii) Eqn of AB: $y - 2 = \frac{2}{3}(x-0)$ or $y = \frac{2}{3}x + 2$

$3y - 6 = 2x$ $\qquad 3y = 2x + 6$

$\therefore 2x - 3y + 6 = 0$

(iv) $C = (3,2)$

(v) $M = (0,1)$ Midpoint of AC

(vi) $m(AB) = \frac{2}{3}$

$\therefore \tan\theta = \frac{2}{3}$

$\therefore \theta = 34$

(vii) $\angle ABC = 180° - 34°$

$\therefore \angle ABC = 146°$

(viii) $x_1 = 0,\ y_1 = 0,\ a = 2,\ b = -3,\ c = 6$

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|0+0+6|}{\sqrt{4+9}}$

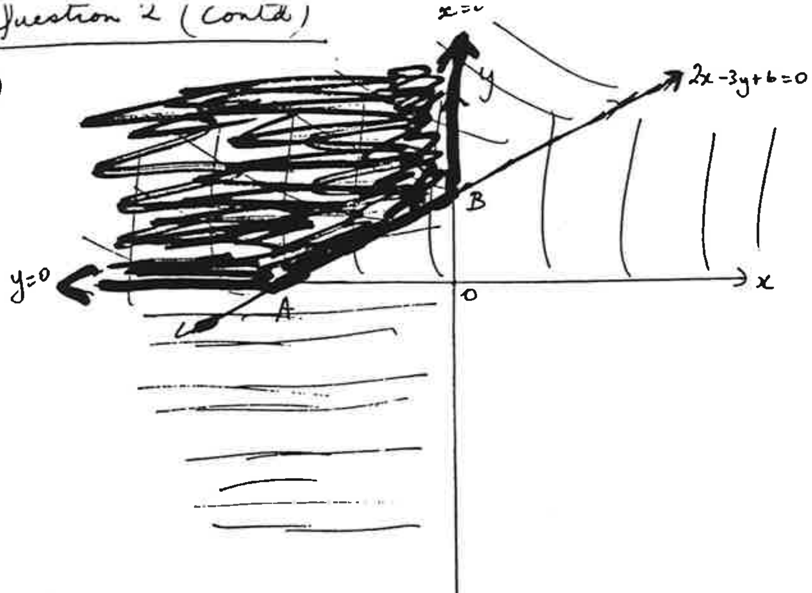
$= \dfrac{6}{\sqrt{13}}$ units

(ix) Area (parm ABCO) $= \sqrt{13} \times \dfrac{6}{\sqrt{13}}$

$= 6$ units^2

Question 2 (contd)

x)



$x \le 0,\ y \ge 0,\ 2x-3y+6 \le 0.$

Question 3

a) (i) $y = 2\cos 3x$

$\frac{dy}{dx} = 2 \times -\sin 3x \times 3$

$= -6\sin 3x$

(ii) $y = 5x^2(1+x)^3$

$\frac{dy}{dx} = 10x(1+x)^3 + 5x^2 \times 3(1+x)^2 \times$

$= 10x(1+x)^3 + 15x^2(1+x)^2$

$= 5x(1+x)^2\left[2^{(1+x)} + 3x\right]$

$= 5x(1+x)^2(2+2x+3x)$

$= 5x(1+x)^2(2+5x).$

b) (i) $\int_1^2 \frac{2}{x}\,dx = 2\left[\ln x\right]_1^2$

$= 2(\ln 2 - \ln 1)$

$= 2\ln 2$

(ii) $\int_0^2 (e^{2x}+1)\,dx$

$= \left[\frac{e^{2x}}{2} + x\right]_0^2$

$= \left(\frac{e^4}{2}+2\right) - \left(\frac{e^0}{2}+0\right)$

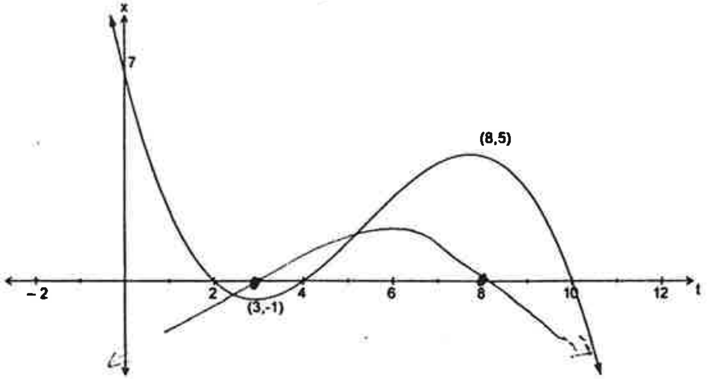
$= \frac{e^4}{2} + 2 - \frac{1}{2}$

$= \frac{e^4}{2} + 1\frac{1}{2}$

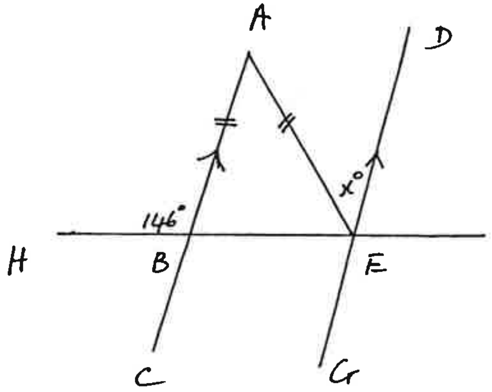
or $= \frac{e^4+3}{2}$

Question 3 (contd)

(c)



d)



$\angle ABE = 34°$ (supp. adj. to $\angle ABH$)

$\angle AEB = \angle ABE$ (base angles of isos. triangle)

$\therefore \angle ABE = 34°$

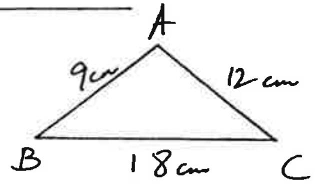
$\angle BED = \angle ABH$ (corr. angles, AC∥DG)

$\therefore x + 34 = 146$

$\therefore x = 146 - 34 = 112$

Question 4

(a)



(i) smallest angle opposite smallest side.

$\cos C = \frac{18^2 + 12^2 - 9^2}{2 \times 18 \times 12}$

$= 0.8958333...$

$\therefore C = 26°$

(ii) $\text{Area}(\triangle ABC) = \frac{1}{2} \times 18 \times 12 \times \sin C$

$= 108 \sin 26°$

$= 47 \text{ cm}^2$

(b) (i) $\text{arc } AB = \frac{120}{360} \times 2\pi \times 6$

$= 4\pi \text{ cm}$

or

$l = r\theta$

$= 6 \times \frac{2\pi}{3}$

$= 4\pi \text{ cm}.$

(ii) $\text{Area} = \frac{1}{2} r^2 \theta$ OR $\frac{\theta}{360} \times \pi r^2$

$= \frac{1}{2} \times 6^2 \times \frac{2\pi}{3}$ $= \frac{120}{360} \times \pi \times 6^2$

$= 12\pi \text{ cm}^2$

Question 4 (Contd)

(c) $p, q, 32$ . . . GP --- ①

$q, 4, p$ --- GP. --- ②

①
$$\frac{q}{p} = \frac{32}{q}$$
$$q^2 = 32p \quad \text{------} ①$$

②
$$\frac{4}{q} = \frac{p}{4}$$
$$pq = 16 \quad \text{------} ②$$

From ② $p = \frac{16}{q}$.

Subst in ①
$$q^2 = 32 \times \frac{16}{q}$$
$$q^3 = 512$$
$$q = 8$$
$$\therefore p = \frac{16}{8} = 2$$

Question 5

(a) $y = x^3 - x^2 - x + 1$

$y' = 3x^2 - 2x - 1$

$y'' = 6x - 2$

(i) Stat pts when $y' = 0$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\therefore x = -\frac{1}{3}, 1$

When $x = -\frac{1}{3}$, $y = 1\frac{5}{27}$

When $x = 1$, $y = 0$

When $x = -\frac{1}{3}$

$y'' = 6 \times -\frac{1}{3} - 2$

$= -4 \ (< 0)$ $\therefore$ a max t.p. at $\left(-\frac{1}{3}, 1\frac{5}{27}\right)$

When $x = 1$

$y'' = 6 - 2 = 4 \ (> 0)$ $\therefore$ a min tp at $(1, 0)$.

(b) Pt. of inflexion

$6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

When $x = \frac{1}{3}$, $y = \frac{14}{27}$

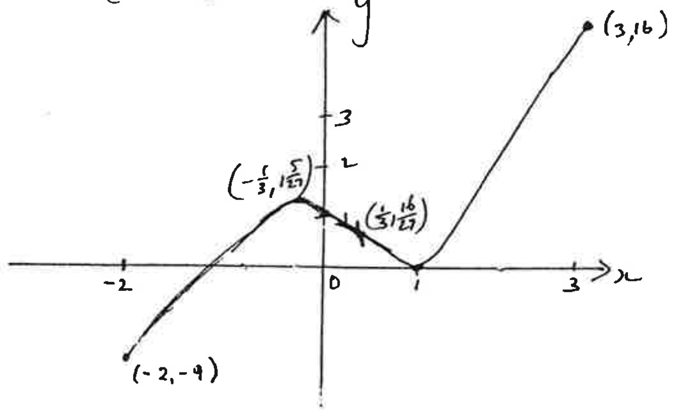
x	$\frac{1}{3}^-$	$\frac{1}{3}$	$\frac{1}{3}^+$
y''	$-$	0	$+$

$\therefore$ a change in concavity

$\therefore$ a point of inflexion at $\left(\frac{1}{3}, \frac{14}{27}\right)$

Question 5 (contd)

a)
iii)



When $x = -2$, $y = (-2)^3 - (2)^2 + 2 + 1$
$= -9$

When $x = 3$, $y = (3)^3 - (3)^2 - 3 + 1$
$= 16$

iv) ∴ Minimum (value) is -9 when $x = -2$

b) $y = \ln(3x + 1)$

$y' = \dfrac{3}{3x+1}$

$m(\text{tangent at } (2,5)) = \dfrac{3}{6+1} = \dfrac{3}{7}$

∴ Eqn. of tangent with $m = \frac{3}{7}$, thro. $(2,5)$

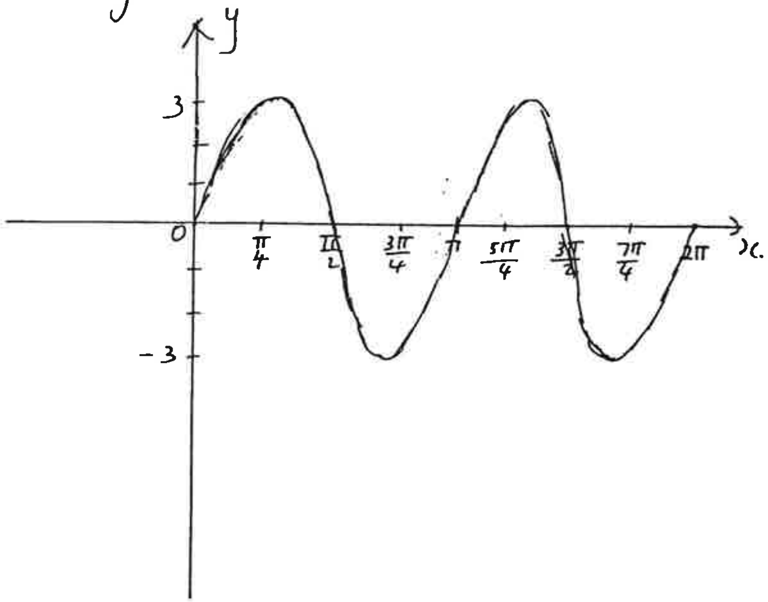
is: $y - 5 = \frac{3}{7}(x - 2)$

$7y - 35 = 3x - 6$

$3x - 7y + 29 = 0.$

Question 5 (contd)

c) $y = 3 \sin 2x \qquad 0 \le x \le 2\pi$



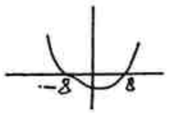
Question 6

(a) $2x^2 + mx + 8 = 0$

[2] No real roots $\Rightarrow b^2 - 4ac < 0$

$b^2 - 4ac = m^2 - 4 \times 8 \times 2$

$= m^2 - 64.$



For $m^2 - 64 < 0$ or $m^2 < 64$

$\therefore -8 < m < 8$ //.

(b) $\begin{cases} y = \tan x \text{ — ①} \quad 0 \le x < \frac{\pi}{2} \\ y = 2\sin x \text{ — ②} \end{cases}$

[2] (i) Sub $(\frac{\pi}{3}, \sqrt{3})$ and $(0,0)$ into both the equations or solve simultaneously

①, $\sqrt{3} = \tan\frac{\pi}{3}$

$\sqrt{3} = \sqrt{3}$ (True)

② $\sqrt{3} = 2\sin\frac{\pi}{3}$

$\sqrt{3} = 2(\frac{\sqrt{3}}{2})$

$\sqrt{3} = \sqrt{3}$ (True)

$\tan x = 2\sin x$

$\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x(\frac{1}{\cos x} - 2) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = 0$ or $x = \frac{\pi}{3}$

$\therefore y = 0$ or $y = \tan\frac{\pi}{3} = \sqrt{3}$.

Hence $(\frac{\pi}{3}, \sqrt{3})$ is a pt of intn on the 2 graphs

(ii) Show $\frac{d}{dx}[\ln\cos x] = -\tan x$.

[1] $\frac{d}{dx}[\ln\cos x] = \frac{1}{\cos x}(-\sin x)$

$= \frac{-\sin x}{\cos x} = -\tan x.$

[3] (iii) area =

$= \int_0^{\frac{\pi}{3}} (2\sin x - \tan x)\,dx$

$= [-2\cos x + \ln\cos x]_0^{\frac{\pi}{3}}$

$= (-2\cos\frac{\pi}{3} + \ln\cos\frac{\pi}{3}) - (-2\cos 0 + \ln\cos 0)$

$= (-2 \times \frac{1}{2} + \ln\frac{1}{2}) - (-2 + \ln 1)$

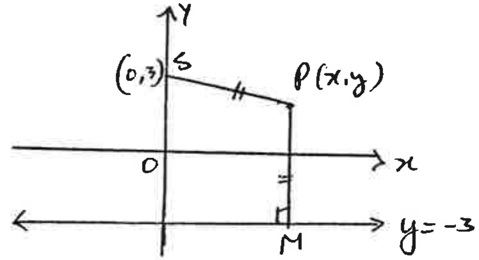
$= -1 + \ln\frac{1}{2} + 2$

$= \underline{\underline{(1 + \ln\frac{1}{2})}}$ s.u.

or $= 1 - \ln 2$

(c) (i)

[2]



given $PM = PS$

ie. $y + 3 = \sqrt{x^2 + (y-3)^2}$

$y^2 + 6y + 9 = x^2 + y^2 - 6y + 9$

Hence $x^2 = 12y$ — locus of P.

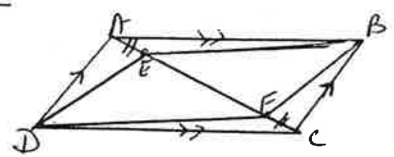
[2] (ii) Vertex is $(0,0)$

focal length = 3 units.

Question 1

(a)

[2]



(i) In $\triangle ADE$ and $\triangle CBF$

$AE = FC$ (given)

$AD = BC$ (Opp. sides of parall. ABCD)

$\angle DAE = \angle BCF$ (Alt. angles, $AD \parallel BC$)

$\therefore \triangle ADE \equiv \triangle CBF$ (SAS)

[2] (ii) Let $\angle AED = \angle BFC = x$ (Corr. $\angle$s of cong $\triangle$s ADE and CBF)

$\therefore \angle DEF = 180^\circ - x$ (Straight $\angle$)

& $\angle BFE = 180^\circ - x$ (")

Hence $\angle DEF = \angle BFE$

$\therefore ED \parallel BF$ (Since alt $\angle$s DEF & BFE are equal.)

Also, $ED = BF$ (Corr. sides of cong. $\triangle$s ADE & CBF)

$\therefore$ DEBF is a parallelogram. (1 pair of opp side equal and $\parallel$).

(b) $x = 5t + e^{-5t}$

[4] (i) When $t = 1$,

$x = 5 + e^{-5} = 5.0067$

$= 5.01$ (3 sig fig)

(ii) $V = \frac{dx}{dt} = 5 - 5e^{-5t}$

When $t = 0$, $V = 5 - 5e^0$

$= 5 - 5$

$= 0$

Hence initially the particle is at rest.

(iii) As $t \to \infty$, $V = 5 - \frac{5}{e^{5t}}$ as e^{5t} becomes very large.

$\to 5.$

$\therefore$ limiting vel $= 5$ cm/sec

) $M = M_0 e^{-kt}$

(i) When $t = 3$, $M_0 = 12$, $M = 8$

[2] $8 = 12e^{-3k}$

$e^{-3k} = \frac{8}{12} = \frac{2}{3}$

$-3k = \ln\frac{2}{3}$

$\therefore k = \frac{\ln\frac{2}{3}}{-3} = 0.135$ (3 d.p.)

(ii) When $t = 10$, find M.

[1] $M = 12e^{-0.135 \times 10}$

$= 3.106$ kg OR 3106g

(iii) When $M = 6$, find t.

[1] $6 = 12e^{-0.135t}$

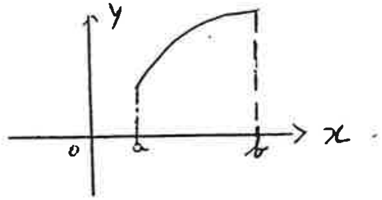
$e^{-0.135t} = \frac{1}{2}$

$-0.135t = \ln\frac{1}{2}$

$t = \frac{\ln\frac{1}{2}}{-0.135} = 5$ yrs. (to nearest whole no.)

Question 4

(a) [2]



(b) $\int_{0.5}^{1.5} \frac{\sin x}{x}\, dx$

[3]

x	0.5	1	1.5
$f(x)$	0.9589	0.8415	0.6650

$$\int_{0.5}^{1.5} \frac{\sin x}{x}\, dx \doteqdot \frac{0.5}{2}\left[0.9589 + 2 \times 0.8415 + 0.6650\right]$$

$$= \underline{\underline{0.83}} \quad (2\ d.p.)$$

(c) [7]

(i) 30°, 27°, 24.3°, 21.87°

(ii) It has a common ratio of 90% (or 0.9).

(iii) $T_n = ar^{n-1}$

$= 30 \times 0.9^{n-1}$

(iv) Let $T_n < 2$, find n.

$30 \times 0.9^{n-1} < 2$

$0.9^{n-1} < \frac{1}{15}$

$n - 1 > \frac{\ln \frac{1}{15}}{\ln 0.9} \doteqdot 25.70$

$\therefore \quad n > 25.70 + 1$

$n > 26.70$ OR use Trial and Error

$\therefore$ The 27^{th} swing //.

(v) $S_\infty = \frac{a}{1-r} = \frac{30^\circ}{1-0.9} = 300^\circ$

This implies the total angle the pendulum swings thro is always less than 300°

Question 1

(a) [6]

(i) 7 m to the right of the origin.

(ii) At 2, 4 and 10 seconds.

(iii) At $t = 3$ sec and 8 sec.

(iv) $8 + 6 + 5 = 19$ m. //

($\frac{1}{2}$) (1) ($\frac{1}{2}$)

OR

$7 + 1 + 1 + 5 + 5 = 19$ m

($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$)

9.b) (i) Given surface area $= 300\pi$.

[2] $A = \pi r^2 + 2\pi r h$

$\therefore 300\pi = \pi(r^2 + 2rh)$

$h = \frac{300 - r^2}{2r}$

$V = \pi r^2 h$

$V = \pi r^2 \cdot \frac{300 - r^2}{2r}$

$= 150\pi r - \frac{\pi r^3}{2}$ cm^3.

(ii) $\frac{dV}{dr} = 150\pi - \frac{3\pi r^2}{2}$

[4] For S.P, let $\frac{dV}{dr} = 0$.

i.e. $150\pi - \frac{3\pi r^2}{2} = 0$.

$\frac{3\pi r^2}{2} = 150\pi$

$r^2 = \frac{300}{3} = 100$

i.e. $r = 10$ since $r > 0$.

$\frac{d^2V}{dr^2} = -\frac{3\pi}{2} \cdot 2r = -3\pi r$

When $r = 10$, $\frac{d^2V}{dr^2} = -30\pi < 0$.

$\therefore$ Max V occurs when $r = 10$ cm.

Question 10

(a) (i) Interest $= \frac{6\%}{12}$ per month $= 0.005$ / month.

[2] Let A_n be the amount accumulated after n months

$A_1 = \$P(1.005)$

$A_2 = \$P(1.005)^2 + \$P(1.005) = \$P(1.005^2 + 1.005)$

$A_3 = \$P(1.005^3 + 1.005^2 + 1.005)$

$\vdots$

$A_{96} = \$P(1.005^{96} + 1.005^{95} + \dots + 1.005)$

If A is the total amount accumulated, then

$A_{96} = A = \$P(1.005 + 1.005^2 + \dots + 1.005^{96})$

(ii) Let $A = \$50000$

[2] i.e. $50000 = \$P(1.005 + 1.005^2 + \dots + 1.005^{96})$

GP. with $a = 1.005$, $r = 1.005$, $n = 96$

$\therefore 50000 = P \cdot \frac{1.005[1 + 1.005^{96}]}{1.005 - 1}$, $S_n = \frac{a(1-r^n)}{1-r}$

$50000 = P \cdot \frac{0.6172\dots}{0.005} \doteqdot 123.44P$

$\therefore P = \$405$ to the nearest dollar.

Alternative solution

a i) Let A_n = the final amount of the n^{th} investment.

$\therefore A_1 = P(1.005)^{96}$

$A_2 = P(1.005)^{95}$

$\vdots$

$A_{96} = P(1.005)^1$

$\therefore A = A_1 + A_2 + \dots + A_{96}$

$= P(1.005)^{96} + \dots + P(1.005)^1$

$= P(1.005^{96} + \dots + 1.005^1)$

10 (b) $f(x) = \frac{e^x - e^{-x}}{2}$

[1] (i) $f'(x) = \frac{e^x + e^{-x}}{2}$

[3] (ii) Since $e^x > 0$ for all x
& $e^{-x} > 0$ for all x

$\therefore$ $f'(x) = \frac{e^x + e^{-x}}{2} > 0$ for all x

$\therefore$ $y = f(x)$ is an increasing fn.

$f''(x) = \frac{e^x - e^{-x}}{2}$

For pt of inflexion, let $f''(x) = 0$.

ie. $\frac{e^x - e^{-x}}{2} = 0$

$e^x - e^{-x} = 0$

$e^x = e^{-x}$

$x = -x$

$2x = 0$

$x = 0$, $y = f(0) = \frac{e^0 - e^0}{2} = 0$.

OR

$f''(0) = \frac{e^0 - e^0}{2}$

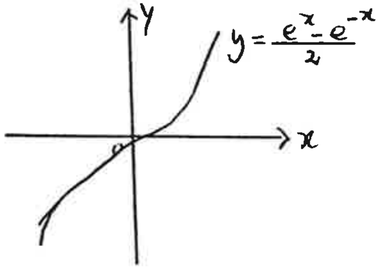
$= 0$

Checking:

x	0^-	0	0^+
$f''(x)$	$-$	0	$+$

Since there is a change of concavity
$\therefore$ $(0,0)$ is a pt of inflexion.

(iii)
[1]



(iv)
[1] Let $y = \frac{e^x - e^{-x}}{2}$

$2y = e^x - e^{-x}$

Multiply by e^x,

$2ye^x = e^{2x} - e^0$

$2ye^x = e^{2x} - 1$

$\therefore$ $e^{2x} - 2ye^x - 1 = 0$

OR $2y = e^x - \frac{1}{e^x}$

$2y = \frac{e^{2x} - 1}{e^x}$

(v) Solving $(e^x)^2 - 2y(e^x) - 1 = 0$
[2] using quadratic formula,

$a = 1$, $b = -2y$, $c = -1$

$e^x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2y \pm \sqrt{4y^2 + 4}}{2}$

$= \frac{2y \pm 2\sqrt{y^2+1}}{2} = y \pm \sqrt{y^2+1}$.

But $e^x > 0$ for all x

$\therefore$ $e^x = y + \sqrt{y^2+1}$ only

Taking log to base e on both sides,

$\ln e^x = \ln[y + \sqrt{y^2+1}]$

$x \ln e = \ln[y + \sqrt{y^2+1}]$

$x = \ln[y + \sqrt{y^2+1}]$.