Mrs Choong (Ms Yun)
Mrs Kerr
Mrs Leslie
Miss Repaci / Ms Lau
Miss Woloszuk

Name: __________________

Teacher: __________________

PYMBLE LADIES' COLLEGE

MATHEMATICS

TRIAL HSC EXAMINATION

2008

Reading Time: 5 minutes
Working Time: 3 hours

Instructions to students:

- Write using blue or black biro.
- All questions may be attempted.
- Diagrams are not to scale.
- All necessary working should be shown in every question.
- Your name and your teacher's name may be written before you begin the assessment.
- Start each question in a **new** booklet.
- Marks may be deducted for careless or untidy work.
- Board-approved calculators may be used.
- A table of standard integrals is provided.

Total marks – 120
Attempt Questions 1-10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

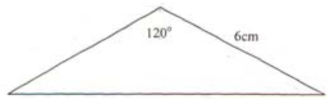
Question 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of $\log_e 12$ correct to three significant figures. **2**

(b) Factorise $xy - 3x + y^2 - 3y$. **2**

(c) Differentiate $2x^4 - \frac{1}{x}$. **2**

(d) Find the exact area of the isosceles triangle below. **2**

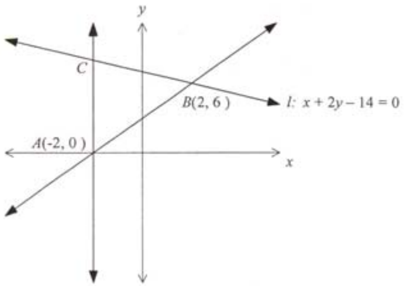


(e) Express $\frac{x}{3} - \frac{2-x}{4}$ as a single fraction in its simplest form. **2**

(f) Given that $\sqrt{3} + \sqrt{27} = \sqrt{a}$, find a. **2**

Question 2 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)



The diagram shows the points $A(-2,0)$, $B(2,6)$ and the line l with equation $x+2y-14=0$ in the cartesian plane. The line k is drawn parallel to the y axis through A. The lines k and l intersect at the point C.

(i) What is the equation of line k? **1**

(ii) Show that C has co-ordinates (-2, 8). **1**

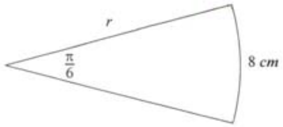
(iii) For the line AB find the size of the angle of inclination with the positive direction of the x axis to the nearest degree. **2**

(iv) Find the perpendicular distance from A to the line l. **2**

(v) A circle is drawn with its centre at A so that the line l does not intersect with the circle. Give a possible equation for the circle. **1**

(vi) Use inequalities to describe the region enclosed by triangle ABC. **3**

(b) Find the length of the radius of the sector of the circle shown in the diagram. Answer to the nearest millimetre. **2**



Question 3 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate:

(i) $\sqrt{1-x}$ **2**

(ii) $\dfrac{e^{2x}}{x+1}$ **2**

(b) Find:

(i) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sec^2 2x)\,dx$ **3**

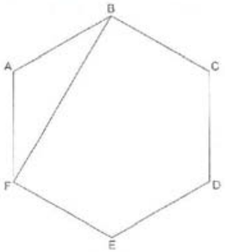
(ii) $\displaystyle\int\frac{x}{x^2-1}\,dx$ **2**

(c) Solve the inequality $|2x-1|<3$ and represent your solution on a number line. **3**

Question 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\sum_{n=1}^{4} n^3 - n$. 2

(b) The diagram shows the regular hexagon $ABCDEF$. FB is joined.

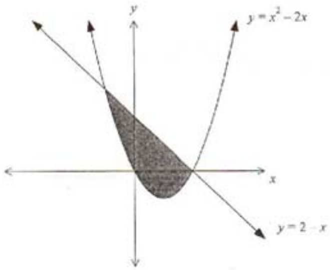


NOT TO SCALE

(i) Show that the size of each interior angle of the regular hexagon is $120°$. 1

(ii) Show that $FB \perp FE$. 2

(c) The graphs of $y = x^2 - 2x$ and $y = 2 - x$ intersect at (2, 0) and the point P.

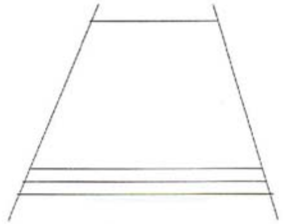


(i) Show that the co-ordinates of P are (-1,3). 2

(ii) Find the shaded area bounded by $y = x^2 - 2x$ and $y = 2 - x$. 3

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A ladder tapers in from the bottom to the top as shown in the diagram. The ladder has 20 steps. The bottom step is 1350 mm long. Each subsequent step is 26 mm shorter.



(i) Calculate the length of the top step. 2

(ii) Calculate the total length of all 20 steps. 2

(b) If α and β are the roots of the equation $3x^2 + 6x = 5$, evaluate:

(i) $\alpha\beta$ 1

(ii) $\alpha + \beta$ 1

(iii) $\alpha^{-1} + \beta^{-1}$ 2

(c) Find the equation of the normal to the curve $y = x \sin x$ at the point where $x = \frac{\pi}{2}$. 4

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Sketch the graph of $y = 1 + \cos 2x$ for $0 \le x \le 2\pi$. **2**

(ii) Use your graph to solve $1 + \cos 2x < 1$. **1**

(b) Consider the curve given by the equation $y = 9x(x-2)^2$.

(i) Show that $\frac{dy}{dx} = 27x^2 - 72x + 36$. **1**

(ii) Find the co-ordinates of the turning points and determine their nature. **3**

(iii) Find the co-ordinates of any point(s) of inflexion. **2**

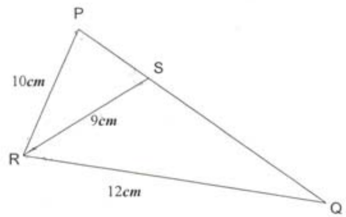
(iv) Sketch the curve in the domain $0 \le x \le 3$. **2**

(v) What is the maximum value of $9x(x-2)^2$ in the domain $0 \le x \le 3$? **1**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Given the equation of the parabola $x^2 - 4x - 8y = 8$, find the co-ordinates of the vertex and the focus. **3**

(b)



Given $\angle PRQ = \angle PSR$, find PQ giving reasons. **3**

(c) The position, x cm of a particle moving along the x axis is given by

$$x = 3t + e^{-2t}$$

where t is the time in seconds.

(i) Indicate the position (to the nearest millimetre) of the particle after half a second. **2**

(ii) What is the initial velocity of the particle? **2**

(iii) Show the initial acceleration of the particle is 4 cm/s^2. **1**

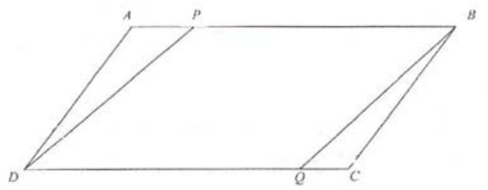
(v) Explain why the particle will never come to rest. **1**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate $y = \log_e\left(\dfrac{\sqrt{x}}{3x-1}\right)$. **3**

(b) Find the values of k for which the equation $9x^2 - kx + 1 = 0$ has real roots. **2**

(c) $ABCD$ is a parallelogram and $AP = QC$.



(i) Prove $\Delta APD \equiv \Delta CQB$. **2**

(ii) Hence prove that $PD \parallel QB$. **2**

(d) Find the volume of the solid formed when the region bounded by $y = 4 - 3x^2$ and the x axis, is rotated about the y axis. **3**

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the trapezoidal rule with three function values to approximate $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx$ to three decimal places. **2**

(b) When the engine of a plane is cut off and the plane continues flying at the same altitude, the acceleration $\dfrac{dv}{dt}$ is related to the velocity v by the equation $\dfrac{dv}{dt} = -kv$ where k is a constant.

(i) Show that $v = v_0 e^{-kt}$, where v_o is the initial velocity and t is time in minutes is a solution to the differential equation. **1**

(ii) The engine of a plane fails when it is flying at 500 km/h.
It cannot reduce its altitude because of mountain peaks.
After one minute the velocity has dropped to 400 km/h.

(α) Show that $k = \ln 1.25$. **2**

(β) How long will it be before the velocity drops to 150 km/h, its stalling speed? Give your answer to the nearest second. **2**

(c) Carmel invests \$1500 on the 1st January each year into a superannuation fund which pays interest at 6% pa, compounded on 30th June and 31st December each year.

(i) Show that the first \$1500 she invests will amount to \$4893.06 after 20 years, to the nearest cent. **1**

(ii) Hence, show that the total value of her investment at the end of the year in which she makes her 20th deposit of \$1500 is \$59 108.27. **3**

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\sec 2\theta = \frac{-2\sqrt{3}}{3}$, $0 \le \theta \le 2\pi$. **2**

(b) $V(t)$ is the volume of water in a bucket t seconds after it was filled. Water leaks through a hole in the bucket at a rate $\frac{dV}{dt} = 0.4t - 40$. (All volume units are in cubic centimetres.)

(i) How fast is the volume decreasing after 30 seconds? **1**

(ii) The bucket initially held 2.5 L of water. Express V as a function of time. **2**

(iii) For how many seconds does water leak from the bucket? Explain why the hole is in the side of the bucket. **2**

(c) An isosceles triangle is drawn with its vertex at the origin, its base parallel to and above the x-axis and the vertices of its base lie on the curve $12y = 36 - x^2$.

(i) Show that the area of any such triangle is given by $A = 3x - \frac{x^3}{12}$. **2**

(ii) Determine the exact area of the largest such triangle. **3**

2008 YEAR 12 TRIALS - MATHEMATICS SOLUTIONS AND MARKING GUIDELINES

QUESTION 1

(a) $\log_e 12$ — 1 mark: Correct calculator steps or correct calculator display

2 Marks

$= 2.48490665$ — 1 mark: Correct answer

$\doteqdot 2.48$ correct to 3 significant figures

(b) $xy - 3x + y^2 - 3y$

2 Marks

$= x(y-3) + y(y-3)$ — 1.

$= (y-3)(x+y)$ — 1.

(c) $\frac{d}{dx}\ 2x^4 - \frac{1}{x}$

2 Marks

$= \frac{d}{dx}\ 2x^4 - x^{-1}$ — $\frac{1}{2}$.

$= 8x^3 + x^{-2}$ — $1 + \frac{1}{2}$

$= 8x^3 + \frac{1}{x^2}$

(d) $A = \frac{1}{2} ab \sin C$

2 Marks

$= \frac{1}{2} \times 6 \times 6 \times \sin 120^\circ$ — 1

$= 18 \times \frac{\sqrt{3}}{2}$ — 1

$= 9\sqrt{3}$ units^2

(e) $\frac{x}{3} - \frac{2-x}{4}$

2 Marks

$= \frac{4x - 3(2-x)}{12}$ — 1.

$= \frac{4x - 6 + 3x}{12}$ — $\frac{1}{2}$

$= \frac{7x - 6}{12}$ — $\frac{1}{2}$

(f) $\sqrt{3} + \sqrt{27} = \sqrt{a}$

2 Marks

$\sqrt{a} = \sqrt{3} + 3\sqrt{3}$ — $\frac{1}{2}$

$= 4\sqrt{3}$ — $\frac{1}{2}$

$= \sqrt{16 \times 3}$

$= \sqrt{48}$ — $\frac{1}{2}$

$\therefore a = 48$ — $\frac{1}{2}$

QUESTION 2

(a) (i) $x = -2$ — 1 mark: Correct Answer Only

(1) Mark

(ii) $x + 2y - 14 = 0$

Sub $x = -2$

$-2 + 2y - 14 = 0$

$2y = 16$

$y = 8$

$\therefore$ C has co-ordinates $(-2, 8)$

1 mark: Correct Method used to establish C $(-2, 8)$ lies on the lines k and L.

(1) Mark

(iii) $A(-2, 0)$ $B(2, 6)$

$x_1\ y_1$ $x_2\ y_2$

(2) Marks

$M = \dfrac{y_2 - y_1}{x_2 - x_1}$

$M_{AB} = \dfrac{6 - 0}{2 + 2}$ — $\frac{1}{2}$

$= \dfrac{3}{2}$

As $\tan\theta = \dfrac{3}{2}$ — 1

$\therefore \theta \doteqdot 56^\circ$ to the nearest degree — $\frac{1}{2}$

2(a) (iv) L: $x + 2y - 14 = 0$ $A(-2, 0)$

$a = 1, b = 2, c = -14$ $x_1\ y_1$

(2) Marks

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|1\times -2 + 2\times 0 - 14|}{\sqrt{1^2 + 2^2}}$ — 1

$= \dfrac{16}{\sqrt{5}}$ — 1

$= \dfrac{16\sqrt{5}}{5}$ units

(v) $(x + 2)^2 + y^2 = \boxed{}$ — 1 mark: Correct Answer Only

(1) Mark

number must be between 0 and 51·2

(vi) Equation of AB : $A(-2, 0)$

(3) Marks

$M_{AB} = \dfrac{3}{2}$ from part (iii)

$y - y_1 = m(x - x_1)$

$y - 0 = \dfrac{3}{2}(x + 2)$ — 1 mark: Determining equation of AB

$y = \dfrac{3}{2}x + 3$ or $2y = 3x + 6$

$3x - 2y + 6 = 0$

Inequalities defining ΔABC are

$y \geqslant \dfrac{3x}{2} + 3$ — $\frac{1}{2}$

$x + 2y - 14 \leqslant 0$ — 1

$x \geqslant -2$ — $\frac{1}{2}$

2 (b) ② Marks

$L = r\theta$

$8 = r \cdot \frac{\pi}{6}$ 1.

$r = \frac{48}{\pi}$

$\therefore$ $r = 15.3$ cm correct to the nearest mm. $\frac{1}{2} + \frac{1}{2}$

QUESTION 3

(a) (i) ② Marks

$\frac{d}{dx} \sqrt{1-x}$

$= \frac{d}{dx} (1-x)^{1/2}$ $\frac{1}{2}$

$= \frac{1}{2} (1-x)^{-1/2} \cdot -1$ $\frac{1}{2} + \frac{1}{2}$

$= \frac{-1}{2\sqrt{1-x}}$ $\frac{1}{2}$

(ii) ② Marks

$\frac{d}{dx} \frac{e^{2x}}{x+1}$

Let $u = e^{2x}$ $v = x+1$

$\frac{du}{dx} = 2e^{2x}$ $\frac{dv}{dx} = 1$ $\frac{1}{2}$

$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(x+1) \cdot 2e^{2x} - e^{2x} \cdot 1}{(x+1)^2}$ 1

$= \frac{e^{2x}(2x+2-1)}{(x+1)^2}$

$= \frac{e^{2x}(2x+1)}{(x+1)^2}$ $\frac{1}{2}$

3(b) (i) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2 2x \, dx$

③ Marks

$= \left[\dfrac{\tan 2x}{2}\right]_{\pi/3}^{\pi/2}$ 1.

$= \dfrac{1}{2}\left\{\tan\left(2.\dfrac{\pi}{2}\right) - \tan\left(\dfrac{2\pi}{3}\right)\right\}$ 1

$= \dfrac{1}{2}\left(0 + \sqrt{3}\right)$ $\frac{1}{2} + \frac{1}{2}$

$= \dfrac{\sqrt{3}}{2}$

(ii) $\displaystyle\int \frac{x}{x^2-1} \, dx$

② Marks

$= \dfrac{1}{2}\displaystyle\int \frac{2x}{x^2-1} \, dx$

$= \dfrac{1}{2}\ln(x^2-1) + C$ $1 + \frac{1}{2} + \frac{1}{2}$

(c) $|2x-1| < 3$

③ Marks

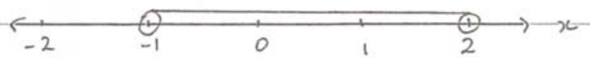
Either $2x - 1 < 3$ or $-(2x-1) < 3$ 1.

$2x < 4$ $\quad$ $2x - 1 > -3$

$x < 2$ $\quad$ $2x > -2$

$x > -1$ $\frac{1}{2}$

Ans: $-1 < x < 2$ $\frac{1}{2}$



1.

QUESTION 4

(a) $\displaystyle\sum_{n=1}^{4} n^3 - n = (1^3 - 1) + (2^3 - 2) + (3^3 - 3) + (4^3 - 4)$

$= 0 + 6 + 24 + 60$

② Marks

$= 90$

1 mark: Summation of the 4 terms

1 mark: Correct Answer

(b)(i) Sum of Interior Angles $= (2n-4)$ Right Angles

① Mark

or

$(n-2)$ Triangles

Sum of Hexagon Angles $= (6-2) \times 180°$

$= 720°$

Interior Angle of Regular Hexagon $= \dfrac{720°}{6}$

$= 120°$

1 mark: Correct Proof

(ii) $\angle AFE = \angle A = 120°$ from Part (i) above

$AF = AB$ regular hexagon $\frac{1}{2}$

② Marks

$\therefore \triangle ABF$ is isosceles

$\angle AFB = \angle ABF$ Base angles of isosceles triangle ABF $\frac{1}{2}$

$= \dfrac{180° - 120°}{2}$ Angle sum of $\triangle ABF$ $\frac{1}{2}$.

$= 30°$

$\angle BFE = \angle AFE - \angle AFB$ Adjacent Angles $\frac{1}{2}$

$= 120° - 30°$

$= 90°$

$\therefore BF \perp FE$

4 (c) (i) $y = x^2 - 2x$ $\qquad y = 2 - x$

Sub $P(-1, 3)$

(2) Marks

$3 = (-1)^2 - 2\times -1$ $\qquad 3 = 2 - (-1)$ $\qquad 1 + 1$

$3 = 3$ $\qquad 3 = 2 + 1$

$3 = 3$

$\therefore$ The point P satisfies both equations and therefore lies on both lines i.e. must be a point of intersection

(ii)

(3) Marks

$$\text{Area} = \int_{-1}^{2} (2-x) - (x^2 - 2x)\ dx \qquad 1.$$

$$= \int_{-1}^{2} 2 + x - x^2\ dx$$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^{2} \qquad 1.$$

$$= \left\{ \left(2\times 2 + \frac{2^2}{2} - \frac{2^3}{3}\right) - \left(2\times -1 + \frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right) \right\} \qquad \frac{1}{2}$$

$$= 4 + 2 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3}$$

$$= 4\tfrac{1}{2} \text{ units}^2 \qquad \frac{1}{2}$$

(d) To show $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = 2$

(2) Marks

$LHS = \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$ $\quad 1.$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2 \times 1$ $\quad \frac{1}{2}.$

$= 2$

$= RHS$ $\quad$ $\frac{1}{2}$ mark: Proof setting out correct.

$\therefore (\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = 2$

QUESTION 5

(a) (i) $T_1 = 1350$, $d = -26$, $n = 20$ $\quad \frac{1}{2}$

a

(2) MARKS

$T_n = a + (n-1)d$

$T_{20} = 1350 + (20-1)\times -26$ $\quad 1$

$= 856$

The top step is 856 mm long. $\quad \frac{1}{2}$

(ii) $S_n = \frac{n}{2}(a + L)$

(2) MARKS

$S_{20} = \frac{20}{2}(1350 + 856)$ $\quad 1$

$= 22\,060$ $\quad \frac{1}{2}$

The total length of all 20 steps is 22.06 metres $\quad \frac{1}{2}$

(b) (i) $3x^2 + 6x = 5$

$3x^2 + 6x - 5 = 0$

(1) Mark

$a = 3,\ b = 6,\ c = -5$

$\alpha\beta = \frac{c}{a}$

$= \frac{-5}{3}$ $\quad$ 1 Mark: Correct Answer Only

(ii) $\alpha + \beta = -\frac{b}{a}$

(1) Mark

$= -\frac{6}{3}$ $\quad$ 1 mark: Correct Method used

$= -2$

5(b) (iii) $\alpha^{-1} + \beta^{-1}$

(2) Marks

$= \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\beta + \alpha}{\alpha\beta}$ 1.

$= \frac{-2}{-\frac{5}{3}}$

$= \frac{6}{5}$

$\frac{1}{2}$ mark: correct substitution using part (i) and (ii)

$\frac{1}{2}$ mark: correct calculation

(c) $y = x \sin x$

(4) Marks

Let $u = x$ $v = \sin x$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos x$ $\frac{1}{2}$.

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x\cos x + \sin x$ 1.

At $x = \pi/2$, $dy/dx = \frac{\pi}{2} + \sin\frac{\pi}{2}$, $y = \frac{\pi}{2}\sin\frac{\pi}{2}$

$= 1$ $\frac{1}{2}$. $= \pi/2$ $\frac{1}{2}$.

$\therefore$ Gradient of normal: $m = -1$ $\frac{1}{2}$.

Eqn of normal at $(\pi/2, \pi/2)$ is

$y - y_1 = m(x - x_1)$

$y - \frac{\pi}{2} = -(x - \frac{\pi}{2})$ $\frac{1}{2}$.

$y - \frac{\pi}{2} = -x + \pi/2$

or

$y = -x + \pi$ $x + y - \pi = 0$ $\frac{1}{2}$.

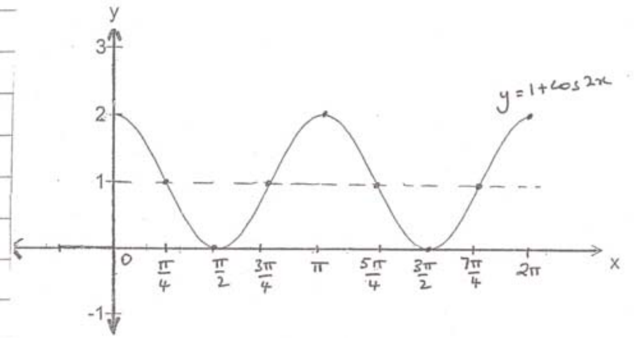
QUESTION 6

(a) (i) $y = 1 + \cos 2x$

(2) MARKS

Period $= \frac{2\pi}{2}$ Amplitude $= 1$ $\Rightarrow$ Range: $0 \le y \le 2$ $\frac{1}{2}$.

$= \pi$ 1.



$\frac{1}{2}$: shape + labelling of axes

(ii) $\frac{\pi}{4} < x < \frac{3\pi}{4}$ or $\frac{5\pi}{4} < x < \frac{7\pi}{4}$

(1) MARK

1 Mark: Correct Answer

(b) (i) $y = 9x(x-2)^2$ (A)

(1) MARK

Let $u = 9x$ $v = (x-2)^2$

$du/dx = 9$ $dv/dx = 2(x-2)$

$\frac{dy}{dx} = u\,dv/dx + v\frac{du}{dx}$

$= 18x(x-2) + 9(x-2)^2$

$= 18x^2 - 36x + 9(x^2 - 4x + 4)$ 1.

$= 18x^2 - 36x + 9x^2 - 36x + 36$

$= 27x^2 - 72x + 36$

6(b)(ii) Stationary points occur when $\frac{dy}{dx} = 0$

③ MARKS

i.e. $27x^2 - 72x + 36 = 0$

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$ $\begin{matrix} 3x & -2 \\ x & -2 \end{matrix}$

Either $3x - 2 = 0$ or $x - 2 = 0$

$x = \frac{2}{3}$ $x = 2$

Sub into Ⓐ

$y = 9 \cdot \frac{2}{3}\left(\frac{2}{3} - 2\right)^2$ $y = 9 \times 2 (2-2)^2$

$= 0$

$= 10^{2}/_{3}$

$f''(x) = 54x - 72$

$= 18(3x - 4)$

$f''(\frac{2}{3}) = 18\left(3 \times \frac{2}{3} - 4\right)$ $f''(2) = 18(3 \times 2 - 4)$

$= 36$

$= -36$ > 0

< 0

∴ Maximum Turning Point at $(\frac{2}{3}, 10\frac{2}{3})$

∴ Minimum Turning Point at $(2, 0)$

1 Mark: Find the x-values

$\frac{1}{2}$ mark: Find corresponding y-values

1 mark: Determine nature using either 1st Derivative or 2nd Derivative for each x-value.

$\frac{1}{2}$ mark: Correctly stating conclusion regarding type.

6(b)(iii) Points of inflexion occur when $f''(x) = 0$

② MARKS

i.e. $54x - 72 = 0$ $\frac{1}{2}$

$x = \frac{4}{3}$

$y = 9 \cdot \frac{4}{3}\left(\frac{4}{3} - 2\right)^2 = 5\frac{1}{3}$ $\frac{1}{2}$.

x	1	$\frac{4}{3}$	2
$f''(x)$	−18	0	36

1.

As there is a CHANGE IN SIGN of $f''(x)$ then a point of inflexion exists at $(\frac{4}{3}, 5\frac{1}{3})$

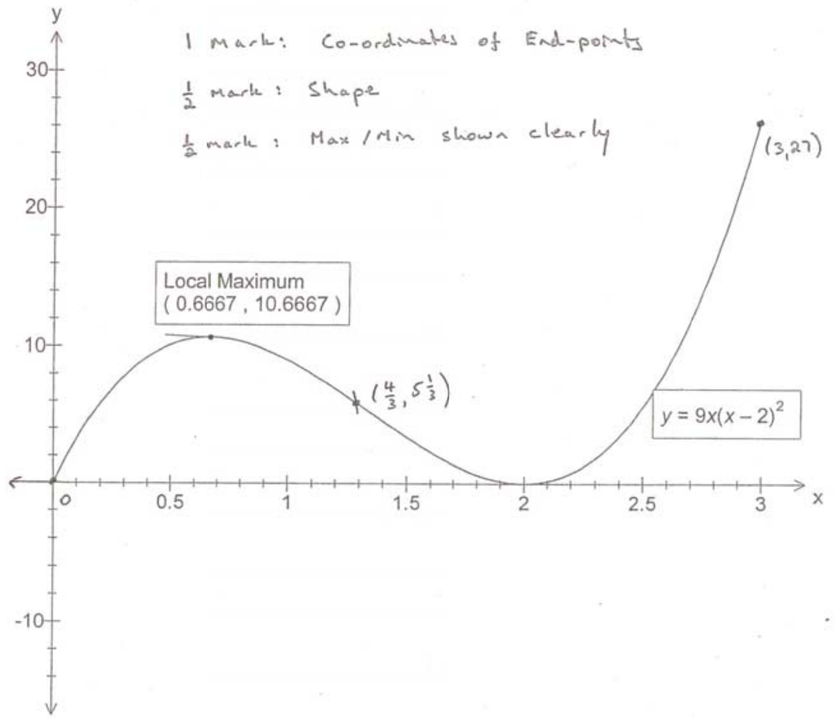
(iv)

② MARKS

1 Mark: Co-ordinates of End-points

$\frac{1}{2}$ mark: Shape

$\frac{1}{2}$ mark: Max / Min shown clearly



(v) Maximum Value = 27

① MARK

QUESTION 7

(a) $x^2-4x-8y=8$

$x^2-4x+2^2 = 8y+8+2^2$ $\frac{1}{2}$

$(x-2)^2 = 8y+12$

$(x-2)^2 = 8(y+\frac{3}{2})$ $\frac{1}{2}$

③ MARKS

$\therefore 4a=8$ conc. up

$a=2$ $\frac{1}{2}$

Vertex is $(2, \frac{-3}{2})$ 1

Focus is $(2, \frac{1}{2})$ $\frac{1}{2}$

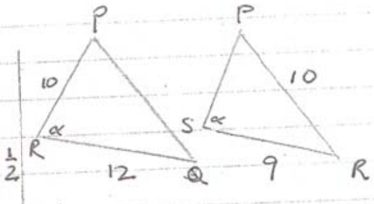
b) In ΔPRQ and ΔPSR

$\angle P$ is common

$\angle PRQ = \angle PSR$ given $\Big\}\frac{1}{2}$

$\therefore \Delta PRQ \,|||\, \Delta PSR$ (equiangular) $\frac{1}{2}$

③ MARKS



$\frac{PQ}{PR} = \frac{RQ}{SR}$ corresponding sides of similar Δ's shown above $\frac{1}{2}$

$\frac{PQ}{10} = \frac{12}{9}$ $\frac{1}{2}$

$PQ = \frac{4\times10}{3}$

$= \frac{40}{3}$

$\therefore PQ = 13\frac{1}{3}$ cm 1

let $\angle PRQ = \angle PSR = \alpha$

Using sine rule (ΔPRQ)

$\frac{PQ}{\sin\alpha} = \frac{12}{\sin P}$

$PQ = \frac{12\sin\alpha}{\sin P}$ 1

$\frac{\sin P}{9} = \frac{\sin\alpha}{10}$ (ΔPSR)

$\sin P = \frac{9\sin\alpha}{10}$ 1

$\therefore PQ = \frac{12\sin\alpha}{\frac{9}{10}\sin\alpha}$

$= \frac{120}{9}$

$= \frac{40}{3}$

$= 13\frac{1}{3}$ cm 1

7(c) i) $x = 3t + e^{-2t}$

Sub $t = 0.5$ 1.

② MARKS

$x = 3\times0.5 + e^{-2\times0.5}$

$= 1.5 + e^{-1}$

$\doteqdot 1.9$ cm to nearest mm $\frac{1}{2}$.

The particle is 1.9 cm to the RIGHT of ORIGIN. $\frac{1}{2}$.

(ii) $V = \frac{dx}{dt} = 3 - 2e^{-2t}$ 1

② MARKS

Sub $t=0$

$v = 3-2e^{0}$ $\frac{1}{2}$

$= 3-2$

$= 1$

$\therefore$ Initial velocity is 1 cm/s $\frac{1}{2}$

(iii) $a = \frac{dV}{dt} = 4e^{-2t}$ 1.

① MARK

Sub $t=0$

$a = 4e^{0}$

$= 4\times1$

$\therefore$ Initial accel'n is 4 cm/s^2

(iv) As $t\to\infty$, $e^{-2t}\to0$ $\therefore V \to {}^{+}3$

① MARK

i.e. Particle starts by moving to the right and will continue moving to the right. Hence the particle never comes to rest

1 MARK: Correct Argument.

OR ESTABLISH THAT $V \neq 0$ for $t \geq 0$.

QUESTION 8

(a) $y = \log_e\left(\dfrac{\sqrt{x}}{3x-1}\right)$

3 MARKS

$= \log_e \sqrt{x} - \log_e(3x-1)$

$= \frac{1}{2}\log_e x - \log_e(3x-1)$ 1.

$\dfrac{dy}{dx} = \dfrac{1}{2x} - \dfrac{3}{3x-1}$ 1. + 1.

(b) $9x^2 - kx + 1 = 0$

$a = 9, \quad b = -k, \quad c = 1$

2 MARKS

Real Roots occur when $\Delta \geq 0$

$b^2 - 4ac \geq 0$

$(-k)^2 - 4 \times 9 \times 1 \geq 0$ 1.

$k^2 \geq 36$

$k \geq 6$ or $k \leq -6$ $\frac{1}{2}$. + $\frac{1}{2}$.

(c) (i) In ΔAPD and ΔCQB

2 MARKS

$\angle A = \angle C$ Opposite angles of parallelogram $ABCD$ $\frac{1}{2}$.

$AP = QC$ Given $\frac{1}{2}$.

$AD = BC$ Opposite sides of parallelogram $ABCD$ $\frac{1}{2}$.

$\therefore \Delta APD \equiv \Delta CQB$ (SAS) $\frac{1}{2}$.

(ii) $\angle APD = \angle PDQ$ Alternate angles, $AB \parallel CD$ $\frac{1}{2}$.

$\angle APD = \angle BQC$ Corresponding angles in congruent triangles APD and CQB 1.

2 MARKS

$\therefore \angle PDQ = \angle BQC$ (Both equal to $\angle APD$)

Hence $PD \parallel PQ$ as corresponding angles are equal. $\frac{1}{2}$.

ALTERNATIVE METHOD

8(c) (ii) $PD = BQ$ corresponding sides of congruent triangles in part (i) 1.

2 MARKS

Since

$PB = AB - AP$

$DQ = DC - QC$ 1.

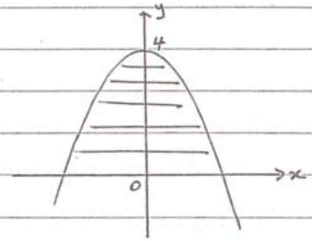
$\therefore PB = DQ$ since $AB = DC$ Opposite sides of parallelogram and $AP = QC$ given

$PBQD$ is a parallelogram as two pairs of opposite sides are equal.

$\therefore PD \parallel QB$ Opposite sides of a parallelogram are parallel.

(d)

3 MARKS



$y = 4 - 3x^2$

$3x^2 = 4 - y$

$x^2 = \frac{1}{3}(4 - y)$ $\frac{1}{2}$

$\text{Volume} = \pi \int_a^b x^2 \, dy$

$= \pi \int_0^4 \dfrac{4-y}{3} \, dy$ 1.

$= \dfrac{\pi}{3} \int_0^4 (4-y) \, dy$

$= \dfrac{\pi}{3}\left[4y - \dfrac{y^2}{2}\right]_0^4$ 1.

$= \dfrac{\pi}{3}\left\{4 \times 4 - \dfrac{4^2}{2} - (0)\right\}$

$= \dfrac{8\pi}{3}$ units^3 $\frac{1}{2}$.

Q9

(a)

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$\sin^2 x$	0	0.14645	$\frac{1}{\sqrt{2}}=\frac{1}{2}$

$x = \sin^2 x$ ← $\frac{1}{2}$

$A = \int_0^{\frac{\pi}{4}} \sin^2 x$

$= \frac{1}{2}\cdot\frac{\pi}{8}\left[0 + 2\times 0.14645 + \frac{1}{2}\right]$ 1.

$= 0.155685\ldots$

$= 0.156$ // (3 d.p.) $\frac{1}{2}$ (2)

(b) $\frac{dv}{dt} = -kv$

(i) $v = v_0 e^{-kt}$ (1) mark: correct method.

$\frac{dv}{dt} = -k\cdot v_0 e^{-kt} = -kv$

$\therefore$ $v = v_0 e^{-kt}$ is a soln. to ---

(ii) (α) Given $v_0 = 500$, $t = 1$ & $v = 400$, find k.

$\cancel{400} = \cancel{500}\, e^{-k}$ $\frac{1}{2}$

$\frac{4}{5} = e^{-k}$

$-k \cancel{\ln e} = \ln\frac{4}{5}$ $\frac{1}{2}$

$k = -\ln\frac{4}{5} = \ln\frac{5}{4} = \ln 1.25$ // $\frac{1}{2}$ (2)

(β) Find t when $v = 150$ $\frac{1}{2}$

$15\cancel{0} = 50\cancel{0}\, e^{\ln\frac{4}{5}\times t}$ $\frac{1}{2}$

$\ln\left(\frac{15}{50}\right) = \ln\frac{4}{5}\, t$ $\frac{1}{2}$

$t = 5.39\ldots \text{ min} = 5 \text{ min } 24 \text{ s}$ //

$\frac{1}{2}$ $\frac{1}{2}$ (2)

(c) $P = \$1500$, $r = 3\%$ compounded half yearly.

(i) $A = P\left(1 + \frac{r}{100}\right)^n$ (1)

$A = \$1500\,(1.03)^{40}$

$= \$4893.06$ //.

(ii) 1st Inv. becomes $\$1500\times 1.03^{40}$.

2nd Inv $\$1500\times 1.03^{38}$.

3rd $\$1500\times 1.03^{36}$.

⋮

19th Inv $\$1500\times 1.03^{4}$

20th Inv $\$1500\times 1.03^{2}$. $\frac{1}{2}$

Total $= \$1500\left[1.03^2 + 1.03^4 + \cdots + 1.03^{40}\right]$ 1
G.P.

$= \$1500\times 1.03^2\left[1 + 1.03^2 + 1.03^4 + \cdots + 1.03^{38}\right]$

$= \$1500\times 1.03^2 \cdot \frac{1\left[(1.03^2)^{20} - 1\right]}{1.03^2 - 1}$ $\frac{1}{2}$ 1.

$= \$59\,108.27$ //

OR $\$1500 \times \frac{(1.03^2)\left[(1.03^2)^{20} - 1\right]}{1.03^2 - 1}$ (3)

(a) $\sec 2\theta = -\frac{2\sqrt{3}}{3}$, $\quad 0 \le \theta \le 2\pi$

$\cos 2\theta = -\frac{3}{2\sqrt{3}}$ $\quad 0 \le 2\theta \le 4\pi$

$\cos 2\theta = -\frac{\sqrt{3}}{2}$ $\quad \frac{1}{2}$

2θ lies in 2^{nd} & 3^{rd} Quadrants

$2\theta = \frac{5\pi}{6}, \frac{7\pi}{6}, \; 2\pi + \frac{5\pi}{6}, \; 2\pi + \frac{7\pi}{6}$ $\quad 1$

$2\theta = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$

$\theta = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}$ and $\frac{19\pi}{12}$ // $\quad 2$

$\frac{1}{2}$

(b) $\frac{dV}{dt} = 0.4t - 40$. / V in cm^3, t in sec.

(i) Find $\frac{dV}{dt}$ when $t = 30$

$\frac{dV}{dt} = 0.4 \times 30 - 40 = -28$.

Volume is decreasing at the rate of <u>28</u> cm^3 per sec. // $\quad 1$

(ii) When $t=0$, $V = 2.5L = 2500\,mL = 2500\,cm^3$.

Integrating $\frac{dV}{dt} = 0.4t - 40$

$V = \frac{0.4t^2}{2} - 40t + C$

$V = 0.2t^2 - 40t + C$ $\quad 1$

When $t=0$, $V = 2500$, $\quad 2500 = C$ $\quad 1$

Hence $\boxed{V = 0.2t^2 - 40t + 2500}$ $\quad 2$

(iii) $V = 0.2t^2 - 40t + 2500$

Let $V = 0$, $\quad$ Show $V \ne 0$

$0.2t^2 - 40t + 2500 = 0$

$0.1t^2 - 20t + 1250 = 0$

$t^2 - 200t + 12500 = 0$ $\quad \frac{1}{2}$

$t = \frac{200 \pm \sqrt{40000 - 4 \times 12500}}{2}$

$= \frac{200 \pm \sqrt{40000 - 50000}}{2}$ $\quad \frac{1}{2}$

No sol^n.

1

Hence volume was never zero in the bucket

i.e. The hole is in the side of the bucket

OR From $\frac{dV}{dt} = 0.4t - 40$.

Water ceased to leak when $\frac{dV}{dt} = 0$

i.e. $0.4t - 40 = 0$. $\quad \frac{1}{2}$

$t = 100$ sec. $\quad \frac{1}{2}$

$\therefore$ water been leaking for 100 sec

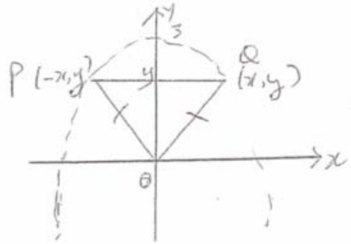
sub $t = 100$ into $V = 500L$ and established that water is still in the bucket.

1 mark: Correct Explanation.

(c)(i) Given $12y = 36 - x^2$

$$y = \frac{36}{12} - \frac{x^2}{12}$$

ie. $y = 3 - \frac{x^2}{12}$. 1.



Area of $\triangle OPQ = \frac{1}{2}$(base)(height)

$A = \frac{1}{2}\cdot(2x)\,y$ ½✓

$A = xy$ ↓sub ½✓

$A = x\left(3 - \frac{x^2}{12}\right) = 3x - \frac{x^3}{12}$ // . (2)

(ii) $\frac{dA}{dx} = 3 - \frac{3x^2}{12} = 3 - \frac{x^2}{4}$

Let $\frac{dA}{dx} = 0$, $\frac{x^2}{4} = 3$ ½✓

$x^2 = 12$

$x = 2\sqrt{3}$ ½✓

$\frac{d^2A}{dx^2} = -\frac{2x}{4} = -\frac{x}{2}$

When $x = 2\sqrt{3}$, $\frac{d^2A}{dx^2} = \frac{-2\sqrt{3}}{2} = -\sqrt{3} < 0$] 1 mark.

Hence $x = 2\sqrt{3}$ gives a Max. T.P. for A.

Max $A = 3(2\sqrt{3}) - \frac{(2\sqrt{3})^3}{12}$ ½✓

$= 6\sqrt{3} - \frac{8\cdot 3\sqrt{3}}{12}$

$= 4\sqrt{3}$ sq. units ½

(3)