Mr Antonio
Mrs Collett
Mr Dudley
Mrs Kerr
Ms Lau
Mrs Sear
Miss Single
Mrs Soutar

Name:..

Teacher:..

2011
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Start each question in a new booklet
- Marks may be deducted for careless or untidy work

Total Marks – 120

- Attempt Questions 1-10
- All questions are of equal value

Mark	/120
Rank	/
Highest Mark	/120

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Question 1 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Evaluate, correct to two decimal places $\dfrac{2.35}{\sqrt{43.7}+3.19}$. **2**

(b) Simplify $\dfrac{2}{x(x-3)} - \dfrac{1}{x}$. **2**

(c) Evaluate $\sum_{n=3}^{5}(2n-1)^2$. **1**

(d) Solve the simultaneous equations. **2**

$$2x+y=13$$
$$x-y=-7$$

(e) Solve $|2x+1|=5$. **2**

(f) Factorise $8x^3+27$. **1**

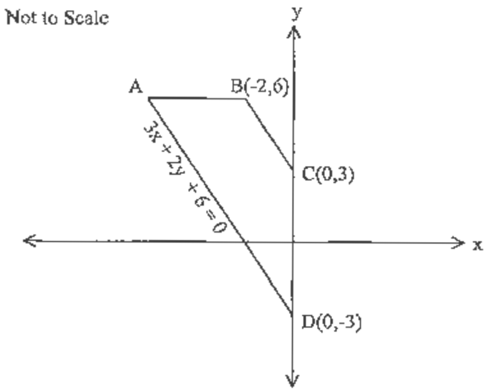
(g) Rationalise the denominator of the following fraction. **2**

$$\frac{4}{\sqrt{7}-2}$$

Question 2 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) In the diagram below, $ABCD$ is a quadrilateral.

Not to Scale



The equation of the line AD is $3x+2y+6=0$.

(i) Show that $ABCD$ is a trapezium by showing that BC is parallel to AD. **2**

(ii) The line AB is parallel to the x-axis. Find the coordinates of A. **1**

(iii) Find the equation of BC. **1**

(iv) Find the length of BC. **1**

(v) Show that the perpendicular distance from B to AD is $\dfrac{12}{\sqrt{13}}$. **2**

(vi) Hence, or otherwise, find the area of the trapezium $ABCD$. **2**

(b) Find the equation of the normal to the curve $y=x^2-5x+1$ when $x=-1$. **3**

Question 3 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Differentiate the following with respect to x.

(i) $e^x \ln 2x$. **1**

(ii) $4(x^2+2)^3$. **1**

(iii) $\dfrac{\tan x}{x^3}$. **2**

(b) Find:

(i) $\int \dfrac{6}{x^2} - \dfrac{2}{x^3}\,dx$. **2**

(ii) $\int 2\cos 6x \quad dx$. **2**

(c) Evaluate:

(i) $\int_2^5 \dfrac{3x}{x^2+1}\,dx$. **2**

(ii) $\int_0^\pi \sin 2x\,dx$. **2**

Question 4 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) The gradient of a curve is given by $g'(x) = 3x^2 - 4 + \dfrac{1}{x^2}$. **3**

The curve passes through the point (1, 4). What is the equation of the curve?

(b) Sketch the curve $y = \cos^2 x + 1$ for $0 \le x \le \pi$, clearly showing all relevant features. **3**

(c) The region enclosed by the curve $y = \dfrac{1}{2}\left(\sqrt{x-1}\right)$, the x and y -axes and the line $y = 2$ is rotated about the y –axis. **3**

Find the volume of this solid of revolution.

(d) Shade the region in the plane defined by $x^2 + y^2 \le 4$ and $y > x - 1$. **3**
Do not find the coordinates of the points of intersection.

Question 5 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Let $f(x) = 9x(x-2)^2$

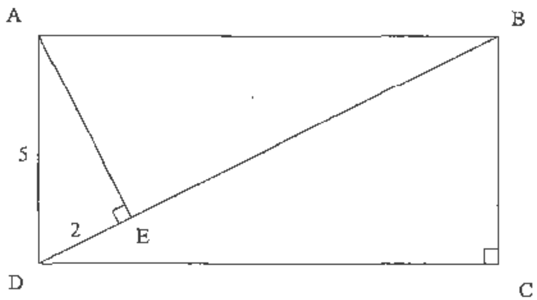
(i) Find the coordinates of the stationary points and determine their nature. **4**

(ii) Find the coordinates of the point(s) where the curve crosses the axes. **1**

(iii) Find the coordinates of the point(s) of inflexion. **2**

(iv) Sketch the graph of $y = f(x)$, clearly indicating all relevant features. **2**

(b) ABCD is a rectangle and E is a point on the diagonal BD such that AE is perpendicular to BD.



(i) Prove that ΔADE is similar to ΔDBC. **2**

(ii) If AD = 5cm and DE = 2cm, find the length of the diagonal BD. **1**

Question 6 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find the values of m for which the equation $x^2 + (m-2)x + 4 = 0$ has no real roots. **2**

(b) Consider the parabola $4y = x^2 - 6x + 1$

(i) Express in the form $(x-h)^2 = 4a(y-k)$. **1**

(ii) Write down the coordinates of the vertex. **1**

(iii) Find the coordinates of the focus. **2**

(c) Use Simpson's rule with five function values **3**

to estimate $\int_0^4 \frac{1}{\sqrt{x^3+1}}\,dx$.

Give your answer correct to 2 decimal places.

(d) (i) Find the limiting sum of the geometric series **2**

$3 + \frac{3}{1+\sqrt{3}} + \frac{3}{\left(1+\sqrt{3}\right)^2} + \ldots\ldots$

(ii) Explain why the geometric series **1**

$3 + \frac{3}{1-\sqrt{3}} + \frac{3}{\left(1-\sqrt{3}\right)^2} + \ldots\ldots$

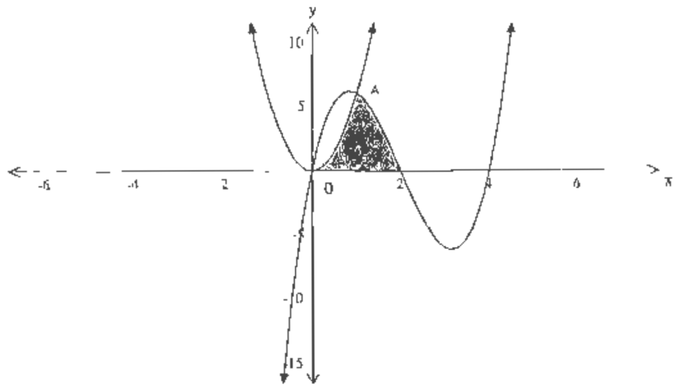
does NOT have a limiting sum.

Question 7 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) (i) Solve $e^{x-2}-1=0$. **1**

(ii) Sketch $y=e^{x-2}-1$, clearly showing all the relevant features. **2**

(b)



In the diagram, the shaded region is bounded by $y=6x^2$, $y=2x(x-2)(x-4)$ and the x-axis. The curves intersect in three places, two of which (0, 0) and A, are shown on the diagram.

(i) Find the x coordinate of A. **2**

(ii) Find the area of the shaded region bounded by $y=6x^2$, $y=2x(x-2)(x-4)$ and the x-axis. **3**

(c) A vessel initially contains 100 litres of liquid. It is being emptied and the rate of change of volume is $\frac{dV}{dt}=-\left(2+\frac{20}{t+1}\right)$ where V is the volume in litres after t minutes.

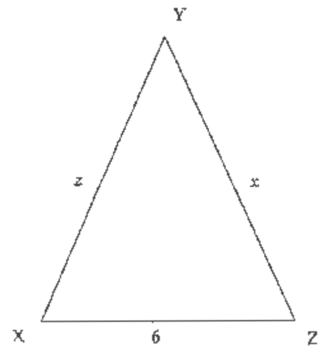
(i) At what rate is the vessel emptying initially? **1**

(ii) Find how many litres of liquid remain in the vessel after five minutes. Answer correct to 3 significant figures. **3**

Question 8 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find the maximum value of the function $y=-16x^2+160x-256$. **2**

(b) The triangle XYZ has $XZ=6$, $YZ=x$, $XY=z$ as shown below. The perimeter of triangle XYZ is 16. All measurements are in centimetres.



(i) Express z in terms of x. **1**

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$. **1**

(iii) Hence show that $\cos Z=\frac{5x-16}{3x}$. **2**

(iv) Let the area of triangle XYZ be A. **1**

Show that $A^2=9x^2\sin^2 Z$.

(v) Hence, show that $A^2=-16x^2+160x-256$. **2**

(vi) What is the maximum area for triangle XYZ ? **1**

(c) Let $f(x)=1+10^x$. **2**

Show that $f(x)\times f(-x)=f(x)+f(-x)$.

Question 9 (12 marks) Use a **SEPARATE** Writing Booklet **Marks**

(a) Assume the population, P, of Tasmanian devils has been decreasing at a rate proportional to P due to the infection of individuals by Devil facial tumour disease. That is $\frac{dP}{dt} = -kP$ where k is a positive constant. **4**

It was estimated that 130 000 individuals were in Tasmania in 1985. Twenty three years later, in 2008, the estimated number stood at just 10 000.

If the population continues to decrease at this rate, how many Tasmanian devils will there be in Tasmania in 2020?

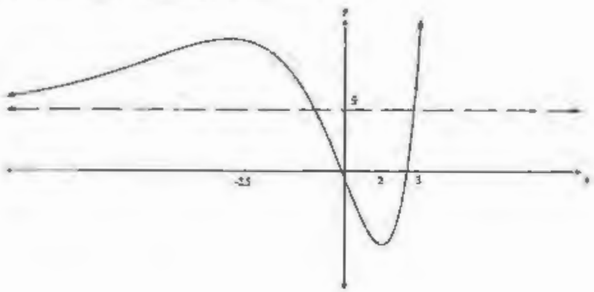
(b) A particle moves in a straight line such that after t seconds its acceleration function is $\ddot{x} = (6t - 2)$ ms^{-2}. Initially the velocity of the particle is -1ms^{-1}.

(i) Find the particle's velocity after 2 seconds. **2**

(ii) Find the time at which the particle is stationary. **1**

(iii) Find the distance travelled by the particle in the third second of motion. **2**

(c) The diagram shows the graph of $y = f(x)$.



(i) For which values of x is the derivative, $f'(x)$ negative? **1**

(ii) Sketch the graph $y = f'(x)$. **2**

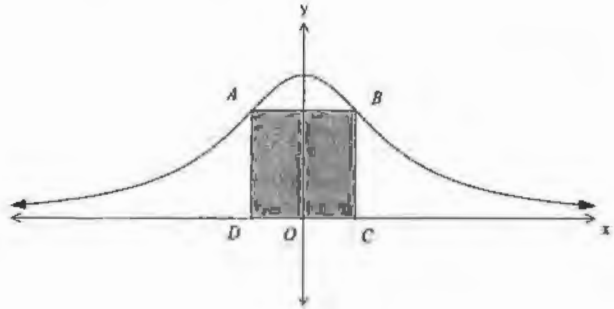
Question 10 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Herculè decides to save for a deposit on a new igloo. The Greater Antartic Savings Bank is offering an interest rate of 7.2% per annum, compounded monthly. Herculè receives his salary on the first day of the month and at this time puts \$500 into his savings account.

(i) What will be the value of the first \$500 invested after 2 years? **1**

(ii) Herculè's dream igloo requires a deposit of \$35000. If he continues to invest \$500 each month from his salary, how long will it take him to be able to afford the deposit? **3**

(b)



In the diagram above the function $y = \frac{b}{x^2 + a}$ has a maximum turning point at (0,4) and passes through (3,1).

A rectangle $ABCD$ is inscribed within the curve as shown.
OY is the axis of symmetry.

(i) Find the value of a and b. **2**

(ii) If C has coordinates $(p, 0)$, find the coordinates of B in terms of p. **1**

(iii) Show that the area of $ABCD = \frac{24p}{p^2 + 3}$. **1**

(vi) Hence find the exact maximum area of $ABCD$. **4**

End of Paper

2011 TRIAL HSC SOLUTIONS

QUESTION ONE

a) $\dfrac{2.35}{\sqrt{43.7}+3.19}$

$= 0.24$ to 2 dec. pl. ②

b) $\dfrac{2}{x(x-3)} - \dfrac{1}{x} = \dfrac{2}{x(x-3)} - \dfrac{(x-3)}{x(x-3)}$ ① correct numerator

$= \dfrac{2-x+3}{x(x-3)}$

$= \dfrac{5-x}{x(x-3)}$ ① correct answer

c) $\sum_{n=3}^{5}(2n-1)^2 = (2\times3-1)^2+(2\times4-1)^2+(2\times5-1)^2$

$= 5^2+7^2+9^2$

$= 25+49+81$

$= 155$ ① r/w

d) $2x+y=13$ —— ①

$x-y=-7$ —— ②

① + ②

$3x=6$

$x=2$ ①

Sub $x=2$ in ②

$2-y=-7$

$-y=-9$

$y=9$ ①

$\therefore x=2,\ y=9$

e) $|2x+1|=5$

$2x+1=5$ $\quad 2x+1=-5$

$2x=4$ $\quad 2x=-6$

$x=2$ or $x=-3$

① ①

f) $8x^3+27=(2x+3)(4x^2-6x+9)$ ① r/w

g) $\dfrac{4}{\sqrt{7}-2} = \dfrac{4}{\sqrt{7}-2}\times\dfrac{\sqrt{7}+2}{\sqrt{7}+2}$ ① correct multiple

$= \dfrac{4\sqrt{7}+8}{7-4}$

$= \dfrac{4\sqrt{7}+8}{3}$ ① Answer simplified

QUESTION TWO

a) i. Find gradient AD

$3x+2y+6=0$

$2y=-3x-6$

$y=-\frac{3}{2}x-3$

$m_{AD}=-\frac{3}{2}$

Find gradient BC:

$m_{BC}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{6-3}{-2-0}$ ① Gradients

① statement justifying

$=\frac{3}{-2}$ why $BC \parallel AD$

Since $m_{AD}=m_{BC}$, $AD \parallel BC$ and ABCD is a trapezium.

ii. Since AB is parallel to the x-axis, it has the same y-coordinate as B, that is $y=6$

sub $y=6$ into $3x+2y+6=0$

$3x+2(6)+6=0$

$3x+18=0$

$3x=-18$

$x=-6$

$\therefore$ A is $(-6,6)$ ① r/w

iii. Equation of BC:

$y-y_1=m(x-x_1)$

$y-3=-\frac{3}{2}(x-0)$

$2y-6=-3x$

$3x+2y-6=0$ ① r/w

iv. length BC:

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-2-0)^2+(6-3)^2}$

$=\sqrt{(-2)^2+(3)^2}$

$=\sqrt{4+9}$

$=\sqrt{13}$ units ① r/w

v. perpendicular distance

$p=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\frac{|3(-2)+2(6)+6|}{\sqrt{3^2+2^2}}$ ① correct substitution

$=\frac{|-6+12+6|}{\sqrt{9+4}}$

$=\frac{|12|}{\sqrt{13}}$

$=\frac{12}{\sqrt{13}}$ units ① Evaluation

vi. length AD:

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-6-0)^2+(6--3)^2}$

$=\sqrt{(-6)^2+(9)^2}$

$=\sqrt{36+81}$

$=\sqrt{117}$ units ① length

Area trapezium ABCD:

$A=\frac{1}{2}\times h\times(a+b)$

$=\frac{1}{2}\times\frac{12}{\sqrt{13}}\times(\sqrt{117}+\sqrt{13})$

$=\frac{6\sqrt{117}}{\sqrt{13}}+6$

$=24$ units^2 ① Answer

QUESTION THREE

a) i. Differentiate $e^x \ln 2x$

$u = e^x \quad v = \ln 2x$

$u' = e^x \quad v' = \frac{2}{2x} = \frac{1}{x}$

$y' = vu' + uv'$

$= \ln 2x \times e^x + e^x \times \frac{1}{x}$

$= e^x(\ln 2x + \frac{1}{x})$ ① r/w

ii. Differentiate $4(x^2+2)^3$

$y' = 4 \times 3 \times 2x \times (x^2+2)^2$

$= 24x(x^2+2)^2$ ① r/w

iii. Differentiate $\frac{\tan x}{x^3}$

$u = \tan x \quad v = x^3$

$u' = \sec^2 x \quad v' = 3x^2$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{x^3 \times \sec^2 x - \tan x \times 3x^2}{(x^3)^2}$ ① correct subs into quotient rule

$= \frac{x^3 \sec^2 x - 3x^2 \tan x}{x^6}$

$= \frac{x^2(x\sec^2 x - 3\tan x)}{x^6}$

$= \frac{x\sec^2 x - 3\tan x}{x^4}$ ① Correct simplification

b) $y = x^2 - 5x + 1$

$\frac{dy}{dx} = 2x - 5$

when $x = -1$,

$\frac{dy}{dx} = 2(-1) - 5$

$= -2 - 5$

$= -7$ ① Correct calc of gradient at point

so the tangent at $x = -1$ has gradient -7.

Since $m_1 = -7$ and $m_1 \times m_2 = -1$ when they are perpendicular,

$-7 \times m_2 = -1$

$m_2 = \frac{1}{7}$ ① correct calculation of normal gradient

so the normal has gradient $\frac{1}{7}$

when $x = -1$

$y = (-1)^2 - 5(-1) + 1$

$= 1 + 5 + 1$

$= 7$

using point gradient formula,

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{1}{7}(x - (-1))$ ① Correct equation from pt gradient formula

$7y - 49 = x + 1$

$x - 7y + 50 = 0$ — eqn of normal

b) i. $\int \frac{6}{x^2} - \frac{2}{x^3}\, dx = \int (6x^{-2} - 2x^{-3})\, dx$

$= \frac{6x^{-1}}{-1} - \frac{2x^{-2}}{-2} + c$ ① integration

$= -6x^{-1} + x^{-2} + c$

$= \frac{-6}{x} + \frac{1}{x^2} + c$ ① returning to original format

ii. $\int 2\cos 6x\, dx = 2\int \cos 6x\, dx$

$= 2\left[\frac{1}{6}\sin 6x\right] + c$ ① "$\frac{1}{3}$"

$= \frac{\sin 6x}{3} + c$ ① $\sin 6x$ and $+ c$

c) i. $\int_2^5 \frac{3x}{x^2+1}\, dx = \frac{3}{2}\int_2^5 \frac{2x}{x^2+1}\, dx$

$= \frac{3}{2}\left[\log_e (x^2+1)\right]_2^5$ ①

$= \frac{3}{2}\left[\log_e (5^2+1) - \log_e (2^2+1)\right]$

$= \frac{3}{2}(\log_e 26 - \log_e 5)$ ①

ii) $\int_0^{\pi} \sin 2x\, dx$

$= \left[\frac{-\cos 2x}{2}\right]_0^{\pi}$ ① integral

$= \frac{-\cos 2\pi}{2} - \left(\frac{-\cos 0}{2}\right)$

$= -\frac{1}{2} + \frac{1}{2}$

① Answer

QUESTION FOUR

a) $g'(x) = 3x^2 - 4 + \frac{1}{x^2}$

$g(x) = \frac{3x^3}{3} - 4x + \frac{x^{-1}}{-1} + c$

$= x^3 - 4x - \frac{1}{x} + c$ ① Integration

when $x = 1$, $g(x) = 4$

$4 = (1)^3 - 4(1) - \frac{1}{1} + c$

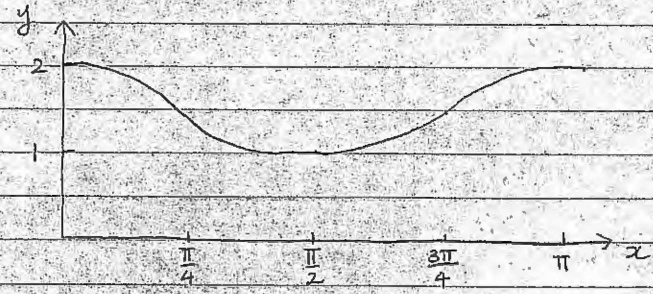
$4 = 1 - 4 - 1 + c$ ① correct c value.

$c = 8$

$\therefore g(x) = x^3 - 4x - \frac{1}{x} + 8$ ① correct eq^n

b) $y = \cos^2 x + 1$ for $0 \leq x \leq \pi$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	2	$1\frac{1}{2}$	1	$1\frac{1}{2}$	2



① correct domain

① correct range

① correct shape

c) $y = \frac{1}{2}(\sqrt{x-1})$

$2y = \sqrt{x-1}$

$4y^2 = x-1$

$x = 4y^2+1$

$V = \pi \int_a^b x^2 \, dy$

$= \pi \int_0^2 (4y^2+1)^2 dy$ ① Correct expression

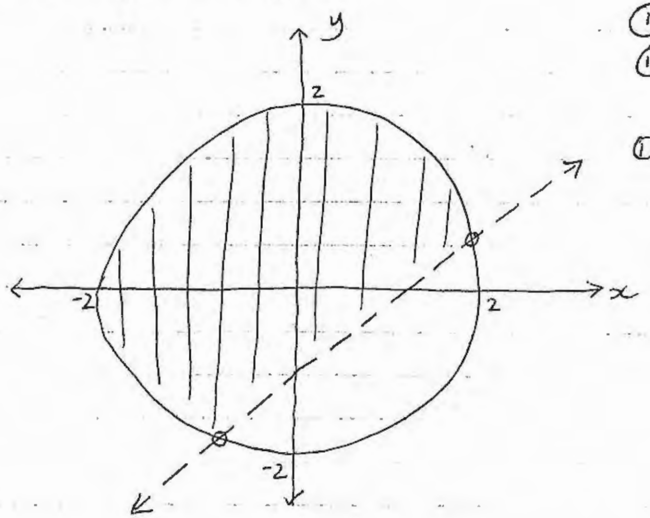
$= \pi \int_0^2 (16y^4 + 8y^2 + 1) \, dy$

$= \pi \left[\frac{16y^5}{5} + \frac{8y^3}{3} + y \right]_0^2$ ① Correct integral.

$= \pi \left\{ \frac{16}{5} \times 2^5 + \frac{8}{3} \times 2^3 + 2 - \left(\frac{16}{5} \times 0^5 + \frac{8}{3} \times 0^3 + 0 \right) \right\}$

$= \frac{1886\pi}{15}$ units^3 ① Correct answer.

d) $x^2+y^2 \leq 4$, $y > x-1$



① circle correct

① line correct and broken

① correct shading

QUESTION FIVE

a) i. $f(x) = 9x(x-2)^2$

$u = 9x \quad v = (x-2)^2$

$u' = 9 \quad v' = 2(x-2)$

$f'(x) = vu' + uv'$

$= (x-2)^2 \times 9 + 9x \times 2(x-2)$

$= 9(x-2)^2 + 18x(x-2)$

$= 9(x-2)(x-2+2x)$

$= 9(x-2)(3x-2)$ ① correct $f'(x)$

Stat. points occur when $f'(x) = 0$

$9(x-2)(3x-2) = 0$

$(x-2)(3x-2) = 0$

$x-2 = 0 \quad 3x-2 = 0$

$x = 2 \quad 3x = 2$

$x = \frac{2}{3}$ ① correct x values of st. pts

when $x = 2$,

$f(2) = 9(2)(2-2)^2$

$= 0$

when $x = \frac{2}{3}$

$f(\frac{2}{3}) = 9(\frac{2}{3})(\frac{2}{3}-2)^2$

$= \frac{32}{3}$

$= 10\frac{2}{3}$ ① correct y values

$f(x) = 9x(x-2)^2$

$f'(x) = 9(x-2)(3x-2)$

$u = 9(x-2) \quad v = 3x-2$

$u' = 9 \quad v' = 3$

$f''(x) = vu' + uv'$

$= (3x-2) \times 9 + 9(x-2) \times 3$

$= 9(3x-2) + 27(x-2)$

$= 27x - 18 + 27x - 54$

$= 54x - 72$

when $x=2$,

$f''(2)=54(2)-72$

$=36$

>0

$\therefore$ concave up with minimum turning point

when $x=\frac{2}{3}$

$f''(\frac{2}{3})=54(\frac{2}{3})-72$

$=-36$

<0

$\therefore$ concave down with maximum turning point

$\therefore$ Stat point at $(2,0)$ is a minimum — ① correct testing of nature

Stat point at $(\frac{2}{3}, 10\frac{2}{3})$ is a maximum

ii. $f(x)=9x(x-2)^2$

when $x=0$,

$f(0)=9(0)(0-2)^2$

$=0$

when $f(x)=0$

$9x(x-2)^2=0$

$9x=0 \qquad (x-2)^2=0$

$x=0 \qquad x-2=0$

$x=2$

Curve crosses at $(0,0)$, $(2,0)$. ① r/w

iii. $f''(x)=54x-72$

when $f''(x)=0$,

$54x-72=0$

$54x=72$

$x=1\frac{1}{3}$

There is a possible point of inflexion at $x=1\frac{1}{3}$

Test concavity changes:

x	1	$1\frac{1}{3}$	$1\frac{2}{3}$
$f''(x)$	-18	0	18

Since concavity changes, there is a point of inflexion at $x=1\frac{1}{3}$.

when $x=1\frac{1}{3}$,

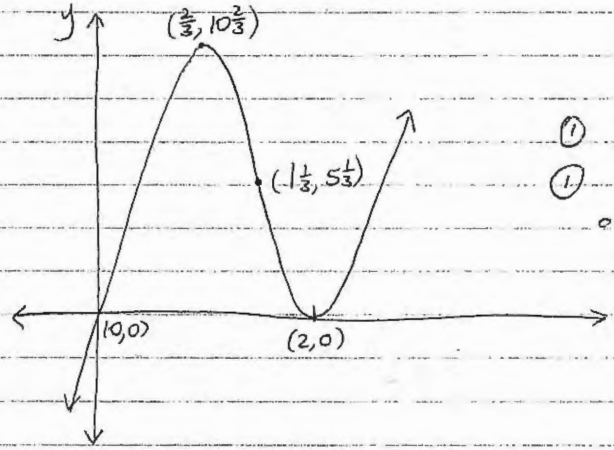
$f(1\frac{1}{3})=9(1\frac{1}{3})(1\frac{1}{3}-2)^2$

$=\frac{16}{3}$ — ① Testing

$=5\frac{1}{3}$ — ① correct point (x,y)

$\therefore$ Point of inflexion at $(1\frac{1}{3}, 5\frac{1}{3})$

iv.



① Correct shape

① correct labelling of relevant features

b) i. Since ABCD is a rectangle,

AD ∥ BC and AB ∥ DC (opposite sides of a rectangle are parallel) ①

In △ADE and △DBC

∠AED = ∠DCB (given)

∠ADE = ∠DBC (alternate angles on parallel lines are equal, AD ∥ BC) ①

$\therefore$ △ADE ||| △DBC (equiangular)

ii. $\frac{BD}{DA} = \frac{BC}{DE}$ (corresponding sides in similar triangles are in the same ratio)

$\frac{BD}{5} = \frac{5}{2}$

$BD = \frac{25}{2}$

$= 12.5$ ① r/w

QUESTION SIX

a) $x^2 + (m-2)x + 4 = 0$ has no real roots when $\Delta < 0$

$\Delta = b^2 - 4ac$

$= (m-2)^2 - 4 \times 1 \times 4$

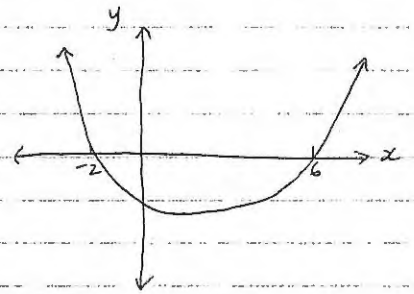
$= m^2 - 4m + 4 - 16$

$= m^2 - 4m - 12$ ① Evaluation of Δ

$\Delta < 0$

$m^2 - 4m - 12 < 0$

$(m-6)(m+2) < 0$



From the sketch,

$-2 < m < 6$ ① correct answer

b) i. $4y = x^2 - 6x + 1$

$4y - 1 = x^2 - 6x$

$4y - 1 + 9 = x^2 - 6x + 9$

$4y + 8 = (x-3)^2$

$4(y+2) = (x-3)^2$

$(x-3)^2 = 4(y+2)$ ① r/w

ii. Vertex : $(3, -2)$ ① / r/w)

iii. Focal length : $4a = 4$

$a = 1$ ① Focal length

Focus : $(3, -1)$ ① Answer

c) $\int_0^4 \frac{1}{\sqrt{x^3+1}}\,dx$

x	0	1	2	4	5
$f(x)$	1	$1/\sqrt{2}$	$1/3$	$1/\sqrt{28}$	$1/\sqrt{65}$

① calculation of values

$\int_a^b f(x)\,dx \doteqdot \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3)+2(y_2)\right]$ ① correct subs into a version of Simpsons rule

$h = \frac{b-a}{n}$

$= \frac{4-0}{4}$

$= 1$

$\int_0^4 \frac{1}{\sqrt{x^3+1}}\,dx \doteqdot \frac{1}{3}\left[\left(1+\frac{1}{\sqrt{65}}\right)+4\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{28}}\right)+2\left(\frac{1}{3}\right)\right]$

$\doteqdot 1.791685824$

$\doteqdot 1.79$ (to 2 dec. pl.) ① Answer

d) i. $3 + \frac{3}{1+\sqrt{3}} + \frac{3}{(1+\sqrt{3})^2} + \dots$

$a = 3$

$r = \frac{1}{1+\sqrt{3}}$

$S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-\frac{1}{1+\sqrt{3}}}$ ① correct S_∞ and substitution

$= \frac{3(1+\sqrt{3})}{1+\sqrt{3}-1}$

$= \sqrt{3}(1+\sqrt{3})$ ① correct answer
nothing for decimal approximation

ii. $r = \frac{1}{1-\sqrt{3}} \doteqdot -1.366025404$ ① r/w

Since we don't have $|r|<1$, we do not have a limiting su...

QUESTION SEVEN

a) i. $e^{x-2} - 1 = 0$

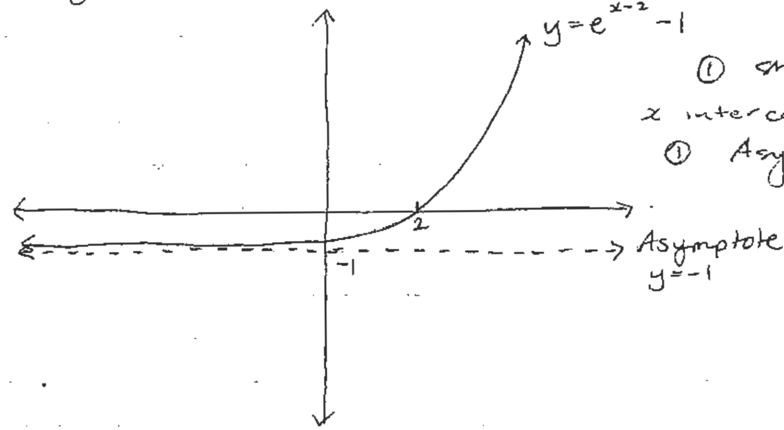
$e^{x-2} = 1$

$\log_e e^{x-2} = \log_e 1$

$x - 2 = 0$

$x = 2$ ① r/w

ii. $y = e^{x-2} - 1$



① shape, x intercept
① Asymptote

b) i. $y = 6x^2$ —— ①

$y = 2x(x-2)(x-4)$ —— ②

① = ②

$6x^2 = 2x(x-2)(x-4)$ ① solving simultaneously to achieve a correct eq^n.

$6x^2 = 2x(x^2-6x+8)$

$6x^2 = 2x^3 - 12x^2 + 16x$

$0 = 2x^3 - 18x^2 + 16x$

$0 = x^3 - 9x^2 + 8x$

$0 = x(x^2 - 9x + 8)$

$0 = x(x-8)(x-1)$

$x = 0 \quad x - 8 = 0 \quad x - 1 = 0$

$x = 8 \quad x = 1$

$\therefore$ x coordinate of A is $x = 1$ ① Answer

ii. $\int_0^1 6x^2\,dx + \int_1^2 2x(x-2)(x-4)\,dx$

$= \int_0^1 (6x^2)\,dx + \int_1^2 (2x^3 - 12x^2 + 16x)\,dx$ ① correct statement

$= \left[\frac{6x^3}{3}\right]_0^1 + \left[\frac{2x^4}{4} - \frac{12x^3}{3} + \frac{16x^2}{2}\right]_1^2$ ① integral

$= \left[2x^3\right]_0^1 + \left[\frac{x^4}{2} - 4x^3 + 8x^2\right]_1^2$

$= \left[2(1)^3 - 0\right] + \left[\left(\frac{2^4}{2} - 4(2)^3 + 8(2)^2\right) - \left(\frac{1^4}{2} - 4(1)^3 + 8(1)^2\right)\right]$

$= 2 + [(8 - 32 + 32) - (\frac{1}{2} - 4 + 8)]$

$= 2 + (8 - 4\frac{1}{2})$

$= 5\frac{1}{2}$ units^2 ① Substitution and evaluation

c) i. $\frac{dV}{dt} = -\left(2 + \frac{20}{t+1}\right)$

when $t = 0$

$\frac{dV}{dt} = -\left(2 + \frac{20}{0+1}\right)$

$= -22$

Emptying at 22L/min ① r/w

ii. $\frac{dV}{dt} = -2 - \frac{20}{t+1}$ $\quad V = \int -\left(2 + \frac{20}{t+1}\right) dt$ ①

$V = -2t - 20\log_e(t+1) + C$ correct integral and correct integration

When $t = 0$, $V = 100$

$100 = -2(0) - 20\log_e(0+1) + C$

$100 = C$

$V = -2t - 20\log_e(t+1) + 100$ ① calculation of "c"

When $t = 5$,

$V = -2(5) - 20\log_e(5+1) + 100$

$= 54.1648 1062$

$= 54.2$ (to 3 sig. fig) ① calculation of

QUESTION EIGHT

a) $y = -16x^2 + 160x - 256$

axis of symmetry

$x = \frac{-b}{2a}$

$= \frac{-160}{2 \times -16}$

$= 5$ ① For x value

when $x = 5$,

$y = -16(5)^2 + 160(5) - 256$

$= 144$

Maximum value is 144 ① Maximum value

b) i. Perimeter:

$P = 6 + x + z$

$16 = 6 + x + z$

$10 = x + z$

$z = 10 - x$ ① r/w

ii. $c^2 = a^2 + b^2 - 2ab\cos C$

$z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$

$z^2 = x^2 + 36 - 12x\cos Z$ ① r/w

iii. $(10-x)^2 = x^2 + 36 - 12x\cos Z$ ① substitution into cosine rule

$100 - 20x + \cancel{x^2} = \cancel{x^2} + 36 - 12x\cos Z$

$12x\cos Z = 20x - 64$

$\cos Z = \frac{20x - 64}{12x}$

$\cos Z = \frac{5x - 16}{3x}$ ① Correct rearrangement to achieve "show"

iv. $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 6 \times x \sin Z$

$= 3x \sin Z$

$A^2 = 9x^2 \sin^2 Z$ ① r/w

v. $A^2 = 9x^2 \sin^2 Z$

$= 9x^2(1-\cos^2 Z)$

$= 9x^2\left(1-\left(\frac{5x-16}{3x}\right)^2\right)$ ① Correct combination of earlier parts

$= 9x^2\left(1-\left(\frac{25x^2-160x+256}{9x^2}\right)\right)$

$= 9x^2\left(\frac{9x^2-25x^2+160x-256}{9x^2}\right)$

$= -16x^2+160x-256$ ① Correct expansion and simplification to achieve "show"

vi. from a) Maximum value for $A^2 = 144$

$\therefore$ Maximum area of $\triangle XYZ$ is $\sqrt{144}$

$= 12$ units^2

① r/w

c) $f(x) = 1+10^x$

$f(-x) = 1+10^{-x}$

$$\left.\begin{aligned} f(x)\times f(-x) &= (1+10^x)(1+10^{-x}) \\ &= 1+10^x+10^{-x}+10^x.10^{-x} \\ &= 1+10^x+10^{-x}+10^0 \\ &= 1+10^x+10^{-x}+1 \\ &= 10^x+10^{-x}+2 \end{aligned}\right\}$$ ① LHS

$$\left.\begin{aligned} f(x)+f(-x) &= 1+10^x+1+10^{-x} \\ &= 10^x+10^{-x}+2 \end{aligned}\right\}$$ ① RHS

$\therefore f(x)\times f(-x) = f(x)+f(-x)$

QUESTION NINE

a) $\frac{dP}{dt} = -kP$

$P = P_0 e^{kt}$ ① Correct diff Eq^n

when $t=0$ (1985), $P=130000$

$130000 = P_0 e^{k\times 0}$

$\therefore P_0 = 130000$ ① calc P_0

when $t=23$ (2008), $P=10000$

$10000 = 130000\, e^{k\times 23}$

$\frac{1}{13} = e^{23k}$

$\log_e\left(\frac{1}{13}\right) = \log_e e^{23k}$

$23k = \log_e\left(\frac{1}{13}\right)$

$k = \frac{1}{23}\log_e\left(\frac{1}{13}\right)$ ① Correct "k"

$= -0.1115195373$

$\therefore P = 130000e^{-0.1115195373t}$

when $t = 35$

$P = 130000\, e^{-0.1115195373\times 35}$

$= 2623.083792$ ① Answer.

$\doteqdot 2623$ (to nearest whole number)

b) i. $\ddot{x} = 6t-2$

$\dot{x} = \frac{6t^2}{2} - 2t + c$

$\dot{x} = 3t^2 - 2t + c$

when $t=0$, $\dot{x} = -1$

$-1 = 3(0)^2 - 2(0) + c$

$c = -1$

$\dot{x} = 3t^2 - 2t - 1$ ① Correct $\dot{x}$

when $t = 2$

$\dot{x} = 3(2)^2 - 2(2) - 1$

$= 12 - 4 - 1$

$= 7\ \mathrm{ms}^{-1}$ ① $\dot{x}$ when $t=2$

ii. The particle is stationary when $\dot{x} = 0$.

$\dot{x} = 3t^2 - 2t - 1$

$0 = 3t^2 - 2t - 1$

$0 = 3t^2 - 3t + t - 1$

$0 = 3t(t-1) + (t-1)$

$0 = (3t+1)(t-1)$

$3t+1 = 0 \qquad t-1 = 0$

$3t = -1 \qquad t = 1$

$t = \frac{-1}{3}$

since $t > 0$, $t = 1s$. ① r/w

iii. $\dot{x} = 3t^2 - 2t - 1$

$x = \frac{3t^3}{3} - \frac{2t^2}{2} - t + k$

$x = t^3 - t^2 - t + k$

when $t = 2$

$x = (2)^3 - (2)^2 - (2) + k$

$= 8 - 4 - 2 + k$

$= 2 + k$

when $t = 3$

$x = (3)^3 - (3)^2 - (3) + k$

$= 27 - 9 - 3 + k$

$= 15 + k$ ①

Distance travelled $= (15+k) - (2+k)$

$= 13$ m ①

or Distance $= \int_2^3 (3t^2 - 2t - 1)\,dt$

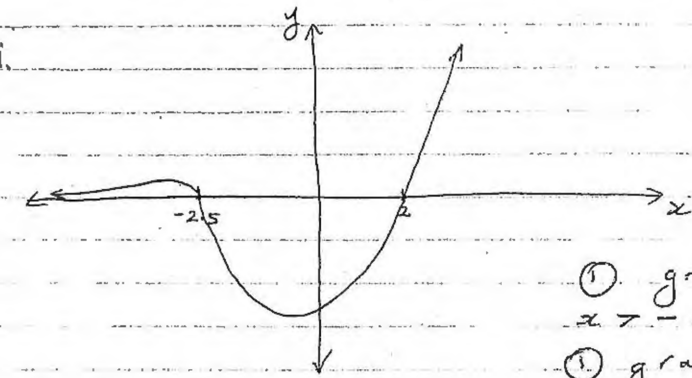
① $= \left[\frac{3t^3}{3} - \frac{2t^2}{2} - t\right]_2^3$

$= 3^3 - 3^2 - 3 - (2^3 - 2^2 - 2)$

$= 13$ m ①

c) i. $f'(x) < 0$ when $-2.5 < x < 2$ ① r/w

ii.



① graph $x > -2.5$

① graph $x < -2.5$

QUESTION TEN

a) i. $r = 0.072$ p.a

$= 0.006$ p.m

$P = \$500$

$n = 24$

$A = P(1+r)^n$

$= 500(1+0.006)^{24}$

$= \$577.1936461$

$= \$577.19$ (to 2 dec. pl) ① r/w

ii. Last amount:

$A = 500(1+0.006)^1$

Second last amount:

$A = 500(1+0.006)^2$

Third last amount:

$A = 500(1+0.006)^3$

↓

First amount:

$A = 500(1+0.006)^n$

Total amount:

$A = 500(1.006) + 500(1.006)^2 + 500(1.006)^3 + \ldots + 500(1.006)^n$ ① Correct Creation of Series

$= 500[1.006 + 1.006^2 + 1.006^3 + \ldots + 1.006^n]$

When is $A > \$35000$

$35000 < 500(1.006 + 1.006^2 + 1.006^3 + \ldots + 1.006^n)$

$1.006 + 1.006^2 + 1.006^3 + \ldots + 1.006^n$ is a GP

with $a = 1.006$, $r = 1.006$, $n = ?$

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$= \dfrac{1.006(1.006^n - 1)}{1.006 - 1}$

$35000 < \dfrac{500(1.006)(1.006^n - 1)}{1.006 - 1}$ ① Correct Series Sum

$210 < 503(1.006^n - 1)$

$0.4174950298 < 1.006^n - 1$

$1.4174950298 < 1.006^n$

$\log_e 1.4174950298 < \log_e 1.006^n$

$\log_e 1.4174950298 < n \log_e 1.006$

$\dfrac{\log_e 1.4174950298}{\log_e 1.006} < n$

$n > 58.32281342$

He will be able to afford the deposit after 59 months. ① Answer

b) i. $y = \dfrac{b}{x^2 + a}$

When $x = 0$, $y = 4$

$4 = \dfrac{b}{0^2 + a}$

$4 = \dfrac{b}{a}$ —— ①

When $x = 3$, $y = 1$

$1 = \dfrac{b}{3^2 + a}$

$1 = \dfrac{b}{a+9}$ —— ②

From ①, $b = 4a$

From ②, $b = a + 9$

$4a = a + 9$

$3a = 9$

$a = 3$ ① "a"

Sub in ① ① "b"

$$4 = \frac{b}{a}$$
$$4 = \frac{b}{3}$$
$$b = 12$$
$\therefore a = 3, b = 12$

ii. $C: (p, 0)$

sub $x = p$ into $y = \frac{b}{x^2+a}$

$$y = \frac{b}{p^2+a}$$
$$y = \frac{12}{p^2+3}$$
$$B: \left(p, \frac{12}{p^2+3}\right)$$ ① r/w

iii. Due to the symmetry:

D has coordinates $(-p, 0)$

A has coordinates $\left(-p, \frac{12}{p^2+3}\right)$

length $DC = p + p$
$= 2p$

$$\text{Area } ABCD = 2p \times \frac{12}{p^2+3}$$
$$= \frac{24p}{p^2+3}$$ ① r/w

iv. $A = \frac{24p}{p^2+3}$

$u = 24p \qquad v = p^2+3$

$u' = 24 \qquad v' = 2p$

$$A' = \frac{vu' - uv'}{v^2}$$
$$= \frac{(p^2+3)\times 24 - (24p)(2p)}{(p^2+3)^2}$$
$$= \frac{24p^2 + 72 - 48p^2}{(p^2+3)^2}$$
$$= \frac{72 - 24p^2}{(p^2+3)^2}$$ ① derivative

$A' = 0$ at stat. pts.

$$0 = \frac{72-24p^2}{(p^2+3)^2}$$
$$24p^2 = 72$$
$$p^2 = 3$$
$$p = \pm\sqrt{3}$$ ① "p" values.

$$A' = \frac{72-24p^2}{(p^2+3)^2}$$

$u = 72 - 24p^2 \qquad v = (p^2+3)^2$

$u' = -48p \qquad v' = 4p(p^2+3)$ ① testing

$$A'' = \frac{vu' - uv'}{v^2}$$
$$= \frac{(p^2+3)^2 \times -48p - (72-24p^2)(4p)(p^2+3)}{(p^2+3)^4}$$
$$= \frac{-48p(p^2+3)^2 - 4p(72-24p^2)(p^2+3)}{(p^2+3)^4}$$

or use of 1st derivative test

When $p = \sqrt{3}$

$$A'' = \frac{-48\sqrt{3}(\sqrt{3}^2+3)^2 - 4\sqrt{3}(72 - 24\sqrt{3}^2)(\sqrt{3}^2+3)}{(\sqrt{3}^2+3)^4}$$
$$= \frac{-48\sqrt{3}\times 36 - 4\sqrt{3}\times 0 \times 6}{6^4}$$
$$= \frac{-1728\sqrt{3}}{1296}$$
$$= -2.309401077$$

$\therefore$ maximum turning point

When $\rho = \sqrt{3}$

$$A = \frac{24\rho}{\rho^2+3}$$

$$= \frac{24\sqrt{3}}{(\sqrt{3})^2+3}$$

$$= \frac{24\sqrt{3}}{6}$$

$$= 4\sqrt{3} \text{ units}^2$$

1 calculation of Area