Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

---

1 Write 0.00000350793 in scientific notation correct to three significant figures.

(A) 0.351×10^{-5}

(B) 3.508×10^{-6}

(C) 35.1×10^{-7}

(D) 3.51×10^{-6}

2 Solve $|2x+1|\leq 5$.

(A) $-3\leq x\leq 2$

(B) $-2\leq x\leq 3$

(C) $x\geq 2,\ x\leq -3$

(D) $x\geq 3,\ x\leq -2$

3 A parabola has equation $(y-3)=8(x+2)^2$. The coordinates of its vertex are

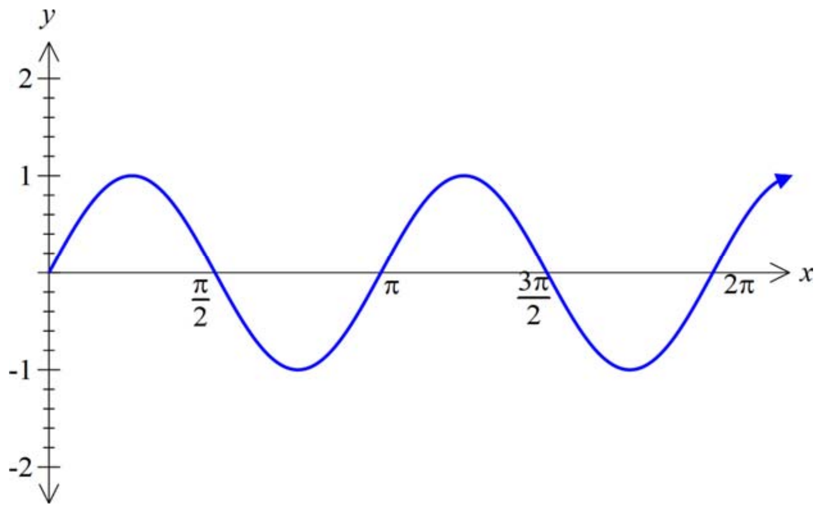
(A) $(3,-2)$

(B) $(-3,2)$

(C) $(-2,3)$

(D) $(2,-3)$

4 Choose the equation of the graph below.



(A) $y = 2 + \sin x$

(B) $y = 2\sin x$

(C) $y = \sin 2x$

(D) $y = \sin(x + 2)$

5 Find $\frac{1}{2}\int e^{\frac{t}{2}}\,dt$.

(A) $e^{-\frac{t}{2}} + C$

(B) $e^{\frac{t}{2}} + C$

(C) $2e^{\frac{t}{2}} + C$

(D) $e^{t} + C$

Multiple Choice (continued).

---

6 If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b \left(f(x)+5\right)dx$ is given by

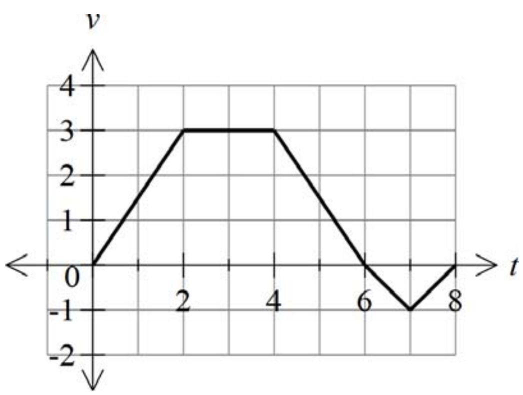
(A) $a+2b+5$

(B) $a+2b-5$

(C) $7b-5a$

(D) $7b-4a$

Questions 7 and 8 refer to the following graph:



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v cm/s of the bug at time t seconds, $0 \le t \le 8$, is given by the function whose graph is shown above.

7 After how many seconds does the bug change direction?

(A) 2

(B) 4

(C) 6

(D) 7

8 What is the total distance the bug travelled from $t = 0$ to $t = 8$?

(A) 13 cm

(B) 11 cm

(C) 8 cm

(D) 4 cm

---

9 The quadratic function $y = 2x^2 - 5x + 1$ has

(A) No real roots.

(B) Two real, distinct, rational roots.

(C) Two real, distinct, irrational roots.

(D) Two equal roots.

10 If $e^y = \sin x$, $0 \le x \le \pi$, find $\frac{dy}{dx}$.

(A) $\tan x$

(B) $\cot x$

(C) $-\cot x$

(D) $-\tan x$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a **separate** writing booklet.

(a) Find a and b such that $\dfrac{1}{2\sqrt{3}-1} = a\sqrt{3}+b$. **2**

(b) Solve $\dfrac{x-1}{2} - \dfrac{2x-3}{3} < 1$. **2**

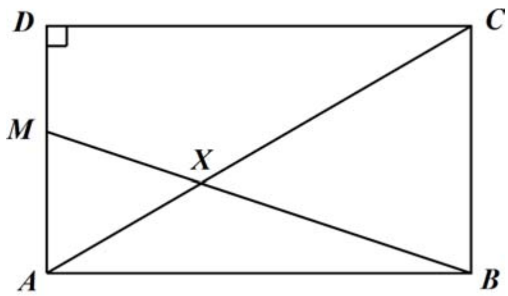
(c) Find the equation, in general form, of the normal to the curve $y = \dfrac{1-x}{1+x}$ at the point where $x = 0$. **3**

(d) Find a primitive function of $\dfrac{x^2}{x^3-2}$. **2**

(e) Find the exact value of x for which $5^{x+2} = 7$. **2**

(f) The area of a sector of a circle is 28.86 cm^2 and contains an angle at the centre of $\dfrac{\pi}{6}$ radians. **2**
Find the radius of the circle, correct to one decimal place.

(g)



ABCD is a rectangle. **2**

The line *BM* meets *AC* at *X*.

Prove that the triangles *AXM* and *CXB* are similar.

Question 12 (15 marks) Use a **separate** writing booklet.

(a) Differentiate with respect to x.

(i) $\tan x$ **1**

(ii) $\dfrac{\ln x}{x}$ **1**

(iii) $\dfrac{1}{7x+4}$ **2**

(b) Find $\int \sqrt{5x-2}\ dx$. **2**

(c) Zoe is saving for a holiday. In the first month she saves \$20 and in the second month she saves \$25. In each subsequent month her savings are \$5 more than the month before.

(i) How much will she save in the 18^{th} month? **2**

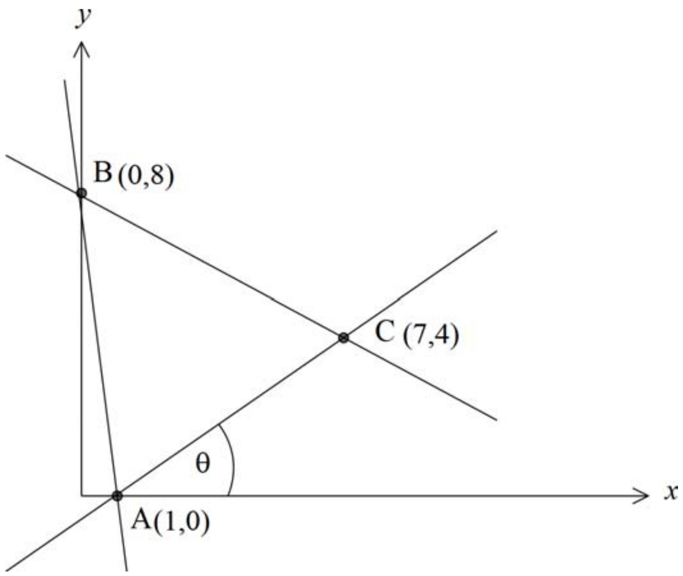
(ii) How much money will she have saved in total by the end of the 18^{th} month? **1**

(iii) Zoe needs \$1900 to pay for her plane ticket. How many whole months will it take for her to save at least this amount? **3**

(d) Use Simpson's rule with 5 function values to find an approximation to $\int_1^{17} \sqrt{x}\ dx$, giving your answer correct to 2 decimal places. **3**

Question 13 (15 marks) Use a **separate** writing booklet.

(a)



The points A, B and C have coordinates (1,0), (0,8) and (7,4) respectively.
The angle between the line AC and the positive direction of the x-axis is θ.

(i) Find the gradient of the line AC. **1**

(ii) Calculate the size of angle θ, correct to the nearest minute. **1**

(iii) Find the coordinates of D, the midpoint of AC. **1**

(iv) Show that AC is perpendicular to BD. **1**

(v) Find the area of ΔABC. **3**

(b) Consider the curve $y = x^3 - 6x^2 + 9x$.

(i) Find the coordinates of the stationary points and determine their nature. **4**

(ii) Find the coordinates of the point(s) of inflexion. **2**

(iii) Draw a neat sketch of the curve, showing all important features. **2**

Question 14 (15 marks) Use a **separate** writing booklet.

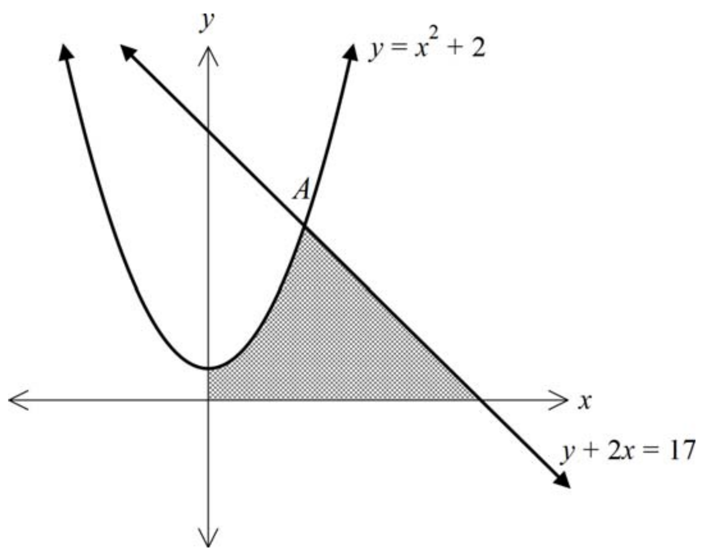
(a) Evaluate $\int_0^{\frac{3\pi}{2}} \cos 2\theta \; d\theta$. **2**

(b) Sophie planted a seed in the ground. During the first year it grew 50 cm. Each year after that the plant grew $\frac{2}{3}$ of the previous year's growth. **2**

What is the maximum height of the plant?

(c) Solve $\sin 2x + 1 = 0$ for $0 \le x \le 2\pi$. **2**

(d) The figure below shows part of the curve $y = x^2 + 2$ and the line $y + 2x = 17$.



(i) Show that the coordinates of A are (3,11). **1**

(ii) Find the area of the shaded region. **3**

Question 14 continues on next page.

Question 14 (continued).

(e) The rate of decay of a radioactive substance is proportional to the mass present at time t years and is given by $\frac{dM}{dt} = -kM$.

(i) Show that $M = M_o e^{-kt}$ satisfies the equation $\frac{dM}{dt} = -kM$. **1**

(ii) If it takes 24 000 years for half of the substance to decay, show that $k = \frac{\ln 2}{24\ 000}$. **2**

(iii) How long would it take for $\frac{2}{3}$ of this substance to decay? Give your answer correct to the nearest thousand years. **2**

Question 15 (15 marks) Use a **separate** writing booklet.

(a) Given that $\dfrac{d^2y}{dx^2} = 2 - \dfrac{4}{x^2}$ and $\dfrac{dy}{dx} = 3$ at the point (1,6), find y in terms of x. **3**

(b) A particle moving in a straight line on the x-axis has displacement x metres after t seconds given by the function

$$x = 2t^2 - 19t + 35.$$

(i) What is the initial position of the particle? **1**

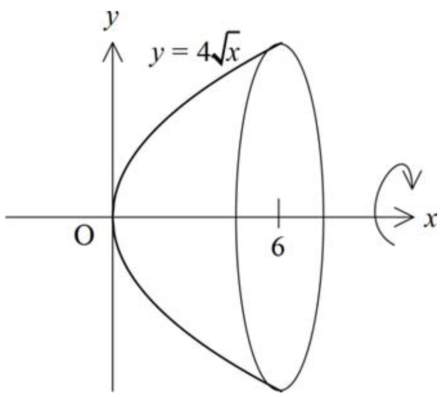
(ii) What is the initial velocity of the particle? **1**

(iii) At what times was the particle at the origin? **2**

(iv) At what time was the particle instantaneously at rest? **1**

(v) How far did the particle travel in between its visits to the origin? **2**

(c) **2**



The region enclosed by the curve $y = 4\sqrt{x}$ and the x axis between $x = 0$ and $x = 6$ is rotated about the x axis as shown.

Find the volume of the solid of rotation.

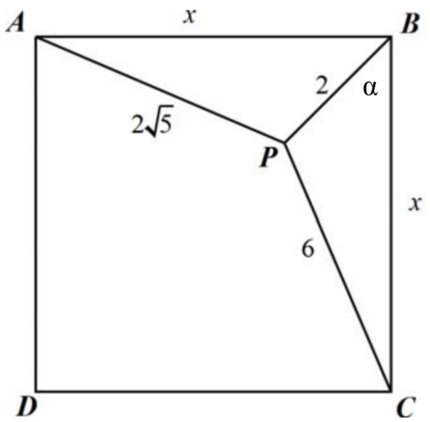
(d) Prove $\left(\cot\theta + \operatorname{cosec}\theta\right)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **3**

Question 16 (15 marks) Use a **separate** writing booklet.

(a) The current of I amperes flowing in an electrical circuit is given by $I = \frac{dQ}{dt}$, where Q is the charge on a capacitor and t is the time in seconds. For a particular electrical circuit, $Q = 4\sin \pi t$. **2**

Calculate the current flowing in the circuit after 2 seconds.

(b)



Not to Scale

The diagram shows a square $ABCD$ of side length x cm. P is a point inside the square, such that $PC = 6$ cm, $PB = 2$ cm and $AP = 2\sqrt{5}$ cm.

Let $\angle PBC = \alpha$.

(i) Using the cosine rule in triangle PBC, show that $\cos\alpha = \frac{x^2 - 32}{4x}$. **1**

(ii) By considering triangle PBA, show that $\sin\alpha = \frac{x^2 - 16}{4x}$. **2**

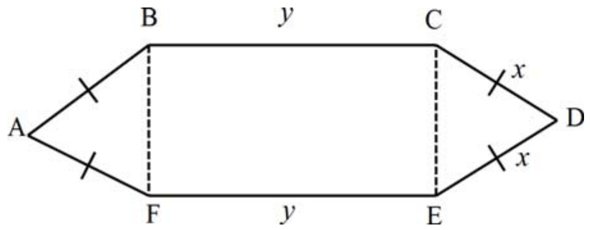
(iii) Hence, or otherwise, show $x^4 - 56x^2 + 640 = 0$. **2**

(iv) Find the only solution for x, giving reasons. **2**

Question 16 continues on next page.

Question 16 (continued).

(c) A wire of length 120 cm is bent to form the perimeter of a frame *ABCDEF,* where *ABF* and *CDE* are equilateral triangles of side x cm and $BC = EF = y$ cm.



(i) Express y in terms of x. **1**

(ii) Show that the total area of *ABCDEF,* $Z\,\text{cm}^2$, is given by $Z = \left(\frac{\sqrt{3}}{2} - 2\right)x^2 + 60x$. **2**

(iii) Find the value of x for which Z will be a maximum, leaving your answer in the form $a + b\sqrt{3}$, where a and b are constants. **3**

End of paper

2013 Mathematics Trial Solutions.

1. $3.50793 \times 10^{-6} = 3.51 \times 10^{-6}$ D

2. $|2x+1| \leq 5$
$-5 \leq 2x+1 \leq 5$ A.
$-6 \leq 2x \leq 4$
$-3 \leq x \leq 2$

3. $(y-3) = 8(x+2)^2$. C
$(-2, 3)$

4. $y = \sin 2x$ C

5. $\frac{1}{2}\int e^{t/2}\,dt$
$= e^{t/2} + c$ B

6. $\int_a^b f(x)\,dx = a + 2b.$
$\int_a^b (f(x)+5)\,dx$
$= [5x]_a^b + a + 2b$ D.
$= 5b - 5a + a + 2b$
$= 7b - 4a.$

7. 6 sec C.

8. 13 cm. A

9. $y = 2x^2 - 5x + 1$
$\Delta = (-5)^2 - 4 \times 2 \times 1$ C
$= 17$ $\therefore$ Two real, distinct irrational

10. $e^y = \sin x$ $\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x.$ B
$y = \ln(\sin x)$

Question 11

a) $\frac{1}{2\sqrt{3}-1} = a\sqrt{3} + b.$

$$\frac{1}{2\sqrt{3}-1} = \frac{1}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$$

$$= \frac{2\sqrt{3}+1}{12-1} \quad (1)$$

$$= \frac{2}{11}\sqrt{3} + \frac{1}{11}$$

$$a = \frac{2}{11} \qquad b = \frac{1}{11} \quad (1)$$

b) $\frac{x-1}{2} - \frac{2x-3}{3} < 1$

$$\frac{3(x-1) - 2(2x-3)}{6} < 1$$

$$3x - 3 - 4x + 6 < 6. \quad (1)$$

$$-x + 3 < 6$$

$$-x < 3$$

$$x > -3. \quad (1)$$

c) $y = \frac{1-x}{1+x}.$ $\qquad u = 1-x \qquad v = 1+x$

$\qquad u' = -1 \qquad v' = 1$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1+x)\times(-1) - (1-x)\times 1}{(1+x)^2}$$

$$= \frac{-1 - x - 1 + x}{(1+x)^2}$$

$$= \frac{-2}{(1+x)^2} \quad (1)$$

when $x = 0$

$$\frac{dy}{dx} = \frac{-2}{(1+0)^2}$$

$$= -2$$

ie gradient of tangent is $m_T = -2$

$\therefore$ gradient of normal $m_N = -\frac{1}{m_T}$

$$= \frac{1}{2}. \quad (1)$$

when $x = 0$

$$y = \frac{1-0}{1+0}$$

$$= 1 \quad \text{ie } (0, 1)$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 0)$$

$$2y - 2 = x$$

$$x - 2y + 2 = 0. \quad (1)$$

d) $$\int \frac{x^2}{x^3 - 2}\,dx$$

$$= \frac{1}{3}\int \frac{3x^2}{x^3 - 2}\,dx$$

$$= \frac{1}{3}\ln(x^3 - 2) + c.$$

(1) (1)

e) $$5^{x+2} = 7$$

$$(x+2)\ln 5 = \ln 7.$$

$$x + 2 = \frac{\ln 7}{\ln 5}$$

$$x = \frac{\ln 7}{\ln 5} - 2.$$

OR $$5^{x+2} = 7$$

$$x + 2 = \log_5 7$$

$$x = \log_5 7 - 2$$

(2)

f) $A = 28.86$ $\theta = \frac{\pi}{6}$

Area of sector $A = \frac{1}{2} r^2 \theta$.

$28.86 = \frac{1}{2} \times r^2 \times \frac{\pi}{6}$ (1)

$r^2 = \frac{28.86 \times 12}{\pi}$

$= 110.2370798$

$r = \sqrt{110.23707\ldots}$

$= 10.49938473$

$\doteqdot 10.5$ cm. (1)

(correct rounding required)

g) $AM \parallel CB$ (opposite sides of a rectangle are parallel) (1)

$\angle MAX = \angle BCX$ (alternate angles in parallel lines are equal; $AM \parallel CB$)

$\angle AXM = \angle CXB$ (vertically opposite angles are equal) (1)

$\therefore \triangle AXM \parallel\!| \triangle CXB$ (equiangular)

Question 12.

a) (i) $\frac{d}{dx} \tan x = \sec^2 x$ (1)

(ii) $\frac{d}{dx}\left(\frac{\ln x}{x}\right)$ $\quad u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$

$= \frac{vu' - uv'}{v^2}$

$= \frac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$ (1)

(iii) $\frac{d}{dx}\left(\frac{1}{7x+4}\right)$

$= \frac{d}{dx} (7x+4)^{-1}$

$= -(7x+4)^{-2} \times 7$

$= \frac{-7}{(7x+4)^2}$ (1) (1)

b) $\int \sqrt{5x-2}\, dx$

$= \int (5x-2)^{1/2}\, dx$

$= \frac{2}{3} \times \frac{(5x-2)^{3/2}}{5} + c$ (1) (1)

$= \frac{2(5x-2)^{3/2}}{15} + c$

$= \frac{2}{15}\sqrt{(5x-2)^3} + c.$

c) $T_1 = \$20$ $\quad a = 20$

$T_2 = \$25$ $\quad d = 5$

$T_3 = \$30$. $\quad$ ie arithmetic sequence (1)

(i) $T_n = a + (n-1)d$

$T_{18} = 20 + (18-1)\times 5$

$= \$105$ (1)

(ii) $S_n = \frac{n}{2}(a+l)$ $\quad l = 105$

$S_{18} = \frac{18}{2}(20+105)$

$= \$1125$ (1)

(iii) $S_n \geqslant \$1900$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$\frac{n}{2}(2\times20+(n-1)\times5) \geqslant 1900$ (1)

$n(40+5n-5) \geqslant 3800$

$35n + 5n^2 \geqslant 3800$

$n^2 + 7n \geqslant 760$

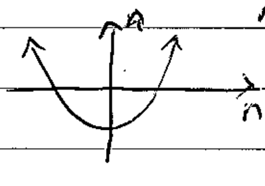
$n^2 + 7n - 760 \geqslant 0$.

Solve $n^2 + 7n - 760 = 0$.

$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-7 \pm \sqrt{7^2 - 4\times1\times760}}{2\times1}$

$= \frac{-7 \pm \sqrt{3089}}{2}$



$n = -31.289\ldots$ or $n = 24.289$. (1)

ie since n is a positive whole number,

$n \geqslant 25$

$\therefore$ 25 whole months. (1)

d) $f(x) = \sqrt{x}$ $\qquad \int_1^{17} \sqrt{x}\, dx$

x	1	5	9	13	17
$\sqrt{x}$	1	$\sqrt{5}$	3	$\sqrt{13}$	$\sqrt{17}$

1

$h = \frac{17-1}{4}$
$= 4$

$$\int_1^{17} \sqrt{x}\, dx \doteq \frac{4}{3}\left((1+\sqrt{17}) + 2 \times 3 + 4 \times (\sqrt{5}+\sqrt{13})\right) \quad 1$$

$$= 45.986\ldots$$

$$\doteq 45.99 \quad 1$$

Question 13.

a) i) $m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1}$ A (1, 0) C (7, 4)

(x_1, y_1) (x_2, y_2)

$= \dfrac{4-0}{7-1}$

$= \dfrac{4}{6}$

$= 2/3$. 1

(ii) $m = \tan\theta$

$\tan\theta = 2/3$

$\theta = \tan^{-1}(2/3)$

$= 33°41'24.24''$

$\doteqdot 33°41'$ 1

(iii) midpoint D.

$D = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{1+7}{2}, \dfrac{0+4}{2}\right)$

$= (4, 2)$ 1

(iv) gradient BD. B (0, 8) D (4, 2)

(x_1, y_1) (x_2, y_2)

$m_{BD} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2-8}{4-0}$

$= \dfrac{-6}{4}$

$= -3/2$

$m_{AC} \times m_{BD} = \dfrac{2}{3} \times -\dfrac{3}{2}$ 1

$= -1$

$\therefore$ AC is perpendicular to BD

(v) $A = \frac{1}{2}bh$

base is length AC
height is length BD.

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(7-1)^2 + (4-0)^2}$$
$$= \sqrt{6^2 + 4^2}$$
$$= \sqrt{52}. \quad (1)$$

$$BD = \sqrt{(4-0)^2 + (2-8)^2}$$
$$= \sqrt{4^2 + (-6)^2}$$
$$= \sqrt{52} \quad (1)$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$$
$$= 26 \text{ units}^2 \quad (1)$$

b) $y = x^3 - 6x^2 + 9x$.

(i) $\frac{dy}{dx} = 3x^2 - 12x + 9$.

Stationary points when $\frac{dy}{dx} = 0$.

$$3x^2 - 12x + 9 = 0 \quad (1)$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 3 \text{ or } x = 1 \quad (1)$$

When $x = 3$, $y = 3^3 - 6 \times 3^2 + 9 \times 3$
$= 27 - 54 + 27$
$= 0$ ie $(3, 0)$

When $x = 1$, $y = 1 - 6 + 9$
$= 4$ ie $(1, 4)$ (1)

$$\frac{d^2y}{dx^2} = 6x - 12$$

At $(3,0)$ $\quad \frac{d^2y}{dx^2} = 6\times 3 - 12$

$= 6 > 0$

ie concave up $\therefore$

$(3,0)$ is a minimum turning point.

At $(1,4)$ $\quad \frac{d^2y}{dx^2} = 6\times 1 - 12$

$= -6 < 0$

ie concave down.

$\therefore (1,4)$ is a maximum turning point

1

(ii) For points of inflexion

$$\frac{d^2y}{dx^2} = 0$$

$6x - 12 = 0$

$x = 2$.

When $x = 2$, $y = 2^3 - 6\times 2^2 + 9\times 2$

$= 8 - 24 + 18$

$= 2$.

ie $(2,2)$ 1

Check concavity.

x	1.5	2	2.5
$\frac{d^2y}{dx^2}$	-3	0	3

$\therefore$ since concavity changes at $(2,2)$ 1

it is a point of inflexion

(iii) x-intercepts when $y=0$

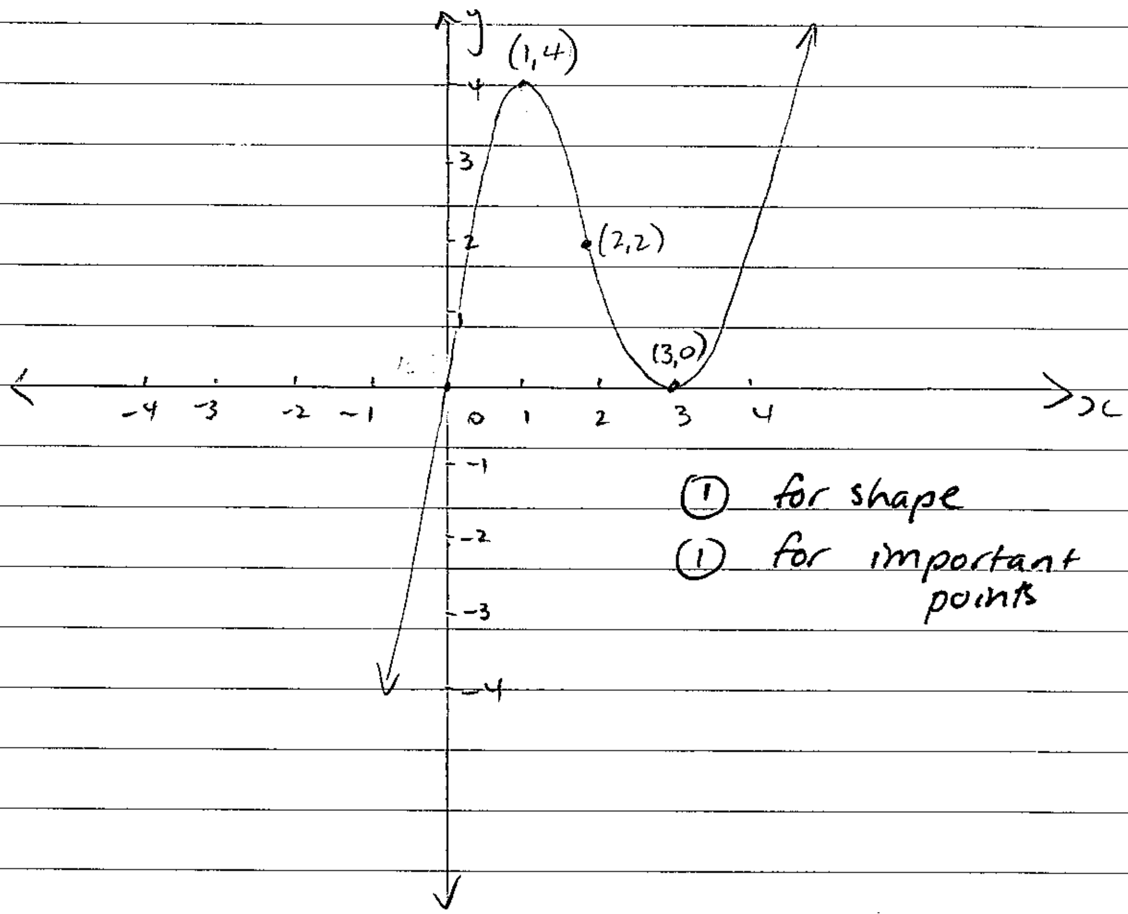
$$0 = x^3 - 6x^2 + 9x$$
$$= x(x^2 - 6x + 9)$$
$$= x(x-3)^2$$

$x = 0$ or $x = 3$

ie x intercepts at 0 or 3.

y-intercepts when $x = 0$

ie at 0.



① for shape

① for important points

Question 14.

a) $\int_0^{\frac{3\pi}{2}} \cos 2\theta \, d\theta$

$= \left[\frac{1}{2} \sin 2\theta \right]_0^{3\pi/2}$ ①

$= \frac{1}{2}\left(\sin 2\times\frac{3\pi}{2} - \sin 2\times 0 \right)$

$= \frac{1}{2}\left(\sin 3\pi - \sin 0 \right)$

$= 0$ ①

b) $a = 50 \qquad r = 2/3$

This is geometric series with $|r| < 1$

$S = \frac{a}{1-r}$

$= \frac{50}{1 - 2/3}$ ①

$= \frac{50}{1/3}$

$= 150$ ①

$\therefore$ maximum height of plant is 150 cm.

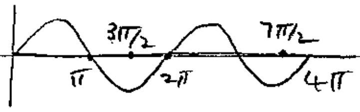
c) $\sin 2x + 1 = 0$

$\sin 2x = -1 \qquad 0 \le x \le 2\pi$

Let $\theta = 2x \qquad 0 \le \theta \le 4\pi$

$\sin \theta = -1$

$\therefore \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\therefore x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ ②

d) (i) Solve simultaneously

$$y + 2x = 17 \qquad -(1)$$
$$y = x^2 + 2 \qquad -(2)$$

Substitute (2) into (1)

$$x^2 + 2 + 2x = 17.$$
$$x^2 + 2x - 15 = 0.$$
$$(x-3)(x+5) = 0$$
$$x = 3 \quad \text{or} \quad x = -5$$

Substitute $x = 3$ into (2)

$$y = 3^2 + 2$$
$$= 11$$

(1)

since A is in first quadrant
(3, 11) is the point we require.

(ii) Find where $y + 2x = 17$ crosses x-axis.

$$0 + 2x = 17$$
$$x = 8.5$$

$$\text{Area} = \int_0^3 (x^2+2)\,dx + \int_3^{8.5} (-2x+17)\,dx \quad (1)$$

$$= \left[\frac{x^3}{3} + 2x\right]_0^3 + \left[\frac{-2x^2}{2} + 17x\right]_3^{8.5} \quad (1)$$

$$= \left(\frac{3^3}{3} + 2\times 3\right) - (0+0) + \left(-8.5^2 + 17\times 8.5\right) - \left(-3^2 + 17\times 3\right)$$

$$= (9+6) + 72.25 - 42$$

$$= 45.25 \text{ units}^2 \quad (1)$$

e) $\frac{dM}{dt} = -kM$

(i) $M = M_0 e^{-kt}$.

$\frac{dM}{dt} = M_0 \times -k e^{-kt}$

$= -k M_0 e^{-kt}$ 1

$= -k M$

(ii) When $t = 24000$ $M = \frac{M_0}{2}$.

ie $\frac{M_0}{2} = M_0 e^{-k \times 24000}$

$\frac{1}{2} = e^{-k \times 24000}$ 1 OR

$2 = e^{k \times 24000}$

$k \times 24000 = \ln 2$

$k = \frac{\ln 2}{24000}$.

$\ln \frac{1}{2} = -k \times 24000$

$\ln 2^{-1} = -k \times 24000$ 1

$-\ln 2 = -k \times 24000$

$k = \frac{\ln 2}{24000}$

(iii) $M = \frac{M_0}{3}$. find $t = ?$

$\frac{1}{3} M_0 = M_0 e^{-kt}$.

$1/3 = e^{-kt}$ 1

$3 = e^{kt}$

$\ln(3) = kt$

$t = \frac{\ln(3)}{\ln 2 / 24000}$

$k = \frac{\ln 2}{24000}$

$= 38039.10002$

ie 38000 years (nearest thousand) 1

Question 15.

a) $\frac{d^2y}{dx^2} = 2 - \frac{4}{x^2}$

$\frac{dy}{dx} = 3$ at $(1,6)$

$\frac{d^2y}{dx^2} = 2 - 4x^{-2}$

$\frac{dy}{dx} = 2x - \frac{4x^{-1}}{-1} + c$

$= 2x + \frac{4}{x} + c.$

At $(1,6)$ $\frac{dy}{dx} = 3$

$\therefore \quad 3 = 2\times 1 + \frac{4}{1} + c.$

$\underline{c = -3}.$

$\therefore \quad \frac{dy}{dx} = 2x + \frac{4}{x} - 3$

$= \underline{2x + 4x^{-1} - 3}$ (1)

$y = x^2 + 4\ln x - 3x + c.$

$(1,6)$ $\quad 6 = 1^2 + 4\times\ln 1 - 3\times 1 + c.$ (1)

$6 = 1 + 0 - 3 + c$

$6 = -2 + c$

$\underline{c = 8}$

$\therefore \quad y = x^2 + 4\ln x - 3x + 8.$ (1)

6) $x = 2t^2 - 19t + 35$

(i) when $t = 0$, $x = 35$

$\therefore$ initial position is 35 m. 1

(ii) $\dfrac{dx}{dt} = 4t - 19$

when $t = 0$, $\dfrac{dx}{dt} = -19$

$\therefore$ initial velocity is -19 m/s 1

(iii) $x = 0$

$2t^2 - 19t + 35 = 0$

$(2t - 5)(t - 7) = 0$ 1

$t = 2.5$ or $t = 7$

$2t \quad -5$

$t \quad -7$

ie particle is at origin when $t = 2.5$ seconds and $t = 7$ seconds. 1

(iv) instantaneously at rest when

$\dfrac{dx}{dt} = 0$

$4t - 19 = 0$

$4t = 19$

$t = 19/4$

$= 4\frac{3}{4}$ seconds. 1

(v) At $t = 4\frac{3}{4}$

$x = 2 \times (19/4)^2 - 19 \times (19/4) + 35$

$= -10.125$ 1

$\therefore$ distance travelled $= 2 \times 10.125$

$= 20.25$ m 1

c) $y = 4\sqrt{x}$

$$V = \pi \int_0^6 y^2 \, dx$$

$$= \pi \int_0^6 (4\sqrt{x})^2 \, dx \quad 1$$

$$= \pi \int_0^6 16x \, dx$$

$$= 16\pi \left[\frac{x^2}{2} \right]_0^6$$

$$= 8\pi (6^2 - 0^2)$$

$$= 8\pi \times 36$$

$$= 288\pi \text{ units}^3 \quad 1$$

d) $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$.

$$LHS = (\cot\theta + \operatorname{cosec}\theta)^2$$

$$= \cot^2\theta + 2\cot\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$$

$$= \frac{\cos^2\theta}{\sin^2\theta} + 2\frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} + \frac{1}{\sin^2\theta} \quad 1$$

$$= \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}$$

$$= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta} \quad 1$$

$$= \frac{(1 + \cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \quad 1$$

$$= \frac{1+\cos\theta}{1-\cos\theta}$$

$$= RHS.$$

Question 16

a) $Q = 4\sin \pi t$

$$I = \frac{dQ}{dt} = 4 \times \pi \cos \pi t$$

$$= 4\pi \cos \pi t \cdot \quad (1)$$

When $t = 2$ $\quad I = 4\pi \cos(\pi \times 2)$

$$= 4\pi \times 1$$

$$= 4\pi \text{ amperes.} \quad (1)$$

b) (i) In ΔPBC

$$\cos\alpha = \frac{x^2 + 2^2 - 6^2}{2 \times 2x} \quad (1)$$

$$= \frac{x^2 + 4 - 36}{4x}$$

$$= \frac{x^2 - 32}{4x}$$

(ii) In ΔPBA. $\angle ABP = 90 - \alpha$.

$$\cos(90 - \alpha) = \frac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times 2x}$$

$$= \frac{x^2 + 4 - 4 \times 5}{4x}$$

$$= \frac{x^2 - 16}{4x} \quad (1)$$

Also, $\cos(90 - \alpha) = \sin\alpha$ (1)

$$\therefore \sin\alpha = \frac{x^2 - 16}{4x}.$$

(iii) $\sin^2\alpha + \cos^2\alpha = 1$

$$\left(\frac{x^2-16}{4x}\right)^2 + \left(\frac{x^2-32}{4x}\right)^2 = 1 \quad (1)$$

$$\frac{(x^4-32x^2+256)+(x^4-64x^2+1024)}{16x^2} = 1$$

$$2x^4 - 96x^2 + 1280 = 16x^2 \quad (1)$$

$$\div 2 \qquad 2x^4 - 112x^2 + 1280 = 0$$

$$x^4 - 56x^2 + 640 = 0$$

(iv) let $u = x^2$

$$u^2 - 56u + 640 = 0$$

$$(u-40)(u-16) = 0$$

$$u = 40 \quad \text{or} \quad u = 16$$

$$x^2 = 40 \quad \text{or} \quad x^2 = 16$$

$$x = \pm\sqrt{40} \qquad x = \pm 4 \quad (1)$$

$$= \pm 2\sqrt{10}$$

x must be positive since it is a length.

The solution must be $2\sqrt{10}$ cm since for any triangle with sides a, b and c

$$c < a + b.$$

If $x = 4$

$6 < 2+4$ (false)

If $x = 2\sqrt{10}$

$6 < 2 + 2\sqrt{10}$ (true)

$2 < 6 + 2\sqrt{10}$ (true)

$2\sqrt{10} < 6 + 2$ (true)

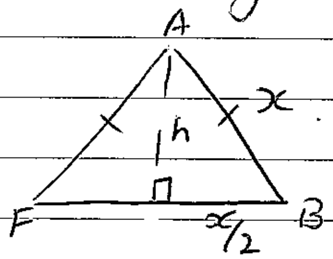
(1)

c) (i) $2y + 4x = 120$

$y + 2x = 60$

$y = 60 - 2x$. (1)

(ii)



Using Pythagoras'

$x^2 = h^2 + (x/2)^2$

$x^2 = h^2 + x^2/4$

$h^2 = x^2 - x^2/4$

$h^2 = \frac{3x^2}{4}$

$h = \frac{\sqrt{3}x}{2}$. (1)

$\therefore\ Z = xy + 2 \times \frac{1}{2} x \times \frac{\sqrt{3}}{2} x$

$= x(60-2x) + \frac{\sqrt{3}}{2} x^2$ (1)

$= 60x - 2x^2 + \frac{\sqrt{3}}{2} x^2$.

$= \left(\frac{\sqrt{3}}{2} - 2\right) x^2 + 60x$

(iii) $\frac{dZ}{dx} = \left(\frac{\sqrt{3}}{2} - 2\right) \times 2x + 60$. (1)

For maximum $\frac{dZ}{dx} = 0$

$2\left(\frac{\sqrt{3}}{2} - 2\right) x + 60 = 0$.

$2\left(\frac{\sqrt{3}}{2} - 2\right) x = -60$.

$\left(2 - \frac{\sqrt{3}}{2}\right) x = 30$

$\left(\frac{4 - \sqrt{3}}{2}\right) x = 30$

$$x = \frac{30 \times 2}{(4-\sqrt{3})} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$$

$$= \frac{240 + 60\sqrt{3}}{16 - 3}$$

$$= \frac{240 + 60\sqrt{3}}{13} \quad (1)$$

$$= \frac{240}{13} + \frac{60\sqrt{3}}{13}.$$

$$\frac{d^2Z}{dx^2} = 2\left(\frac{\sqrt{3}}{2} - 2\right)$$

$$= -2.2679\ldots$$

Since $\frac{d^2Z}{dx^2} < 0$, concave down (1)

so maximum Z for x value given above.

<u>OR</u> Using quadratic method.

$$Z = \left(\frac{\sqrt{3}}{2} - 2\right)x^2 + 60x.$$

Axis of symmetry $x = -b/2a$

$$= \frac{-60}{2\times\left(\frac{\sqrt{3}}{2} - 2\right)}$$

$$= \frac{60}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}}$$

$$= \frac{240 + 60\sqrt{3}}{16-3} \quad (1)$$

$$x = \frac{240}{13} - \frac{60\sqrt{3}}{13}.$$

Z is a maximum here since the (1)
coefficient of x^2 is $\frac{\sqrt{3}}{2} - 2 = -1.1339\ldots < 0$

$\therefore$ (parabola is concave down)