Mrs Israel	**Mrs Kerr**	**Name:**…………………………………..………..
Ms Lau	**Mr Morrison**	
Ms Prosser	**Mrs Semler**	**Teacher:**………………………………………..
Ms Stott		

Pymble Ladies' College

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
2016

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Use pencil for Questions 1-10.
- Write using a black or blue pen for Questions 11-16. Black pen is preferred.
- Board approved calculators may be used.
- A reference sheet is provided.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations.

Total Marks – 100

Section I Pages 1-5

10 marks
- Attempt Questions 1-10
- Allow about 15 mins for this section

Section II Pages 7-16

90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

Mark	**/100**
Highest Mark	**/100**
Rank	

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Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 The line with equation $2y = 3x + 5$ is perpendicular to the line with equation $y = kx$. What is the value of *k*?

(A) $-\frac{3}{2}$

(B) $-\frac{2}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

2 A function *f*, defined on a suitable domain, is given by $f(x) = \dfrac{6x}{x^2 + 6x - 16}$.

What restrictions are there on the domain of *f*?

(A) $x \neq -8$ or $x \neq 2$

(B) $x \neq -4$ or $x \neq 4$

(C) $x \neq 0$

(D) $x \neq 10$ or $x \neq 16$

3 The functions *f* and *g* are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers.

Which expression is equivalent to $g(f(x))$?

(A) $3x^2 - 1$

(B) $9x^2 - 15$

(C) $9x^2 + 17$

(D) $3x^3 - 4x^2 + 3x - 4$

4 Given that $f(x) = 4\sin 3x$, what is $f'(0)$?

(A) 0

(B) 1

(C) 12

(D) 36

5 What is $\int x(3x+2)\,dx$?

(A) $x^3 + c$

(B) $x^3 + x^2 + c$

(C) $\frac{1}{2}x^2\left(\frac{3}{2}x^2 + 2x\right) + c$

(D) $3x^2 + 2x + c$

6 If $e^{4t} = 6$, which of the following is an expression for t?

(A) $t = \log_e \frac{3}{2}$

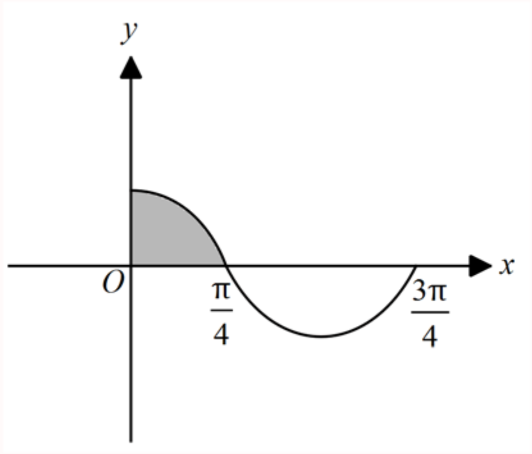
(B) $t = \frac{\log_e 6}{4}$

(C) $t = \frac{6}{\log_e 4}$

(D) $t = \frac{\log_e 6}{\log_e 4}$

7 The diagram shows part of the graph of $y = a\cos bx$. The shaded area is $\frac{1}{2}$ unit ^{2}.

What is the value of $\int_0^{\frac{3\pi}{4}} \left(a\cos bx\right)\, dx$?



(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $1\frac{1}{2}$

8 The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

What is the rate of change of V with respect to r, at $r = 2$?

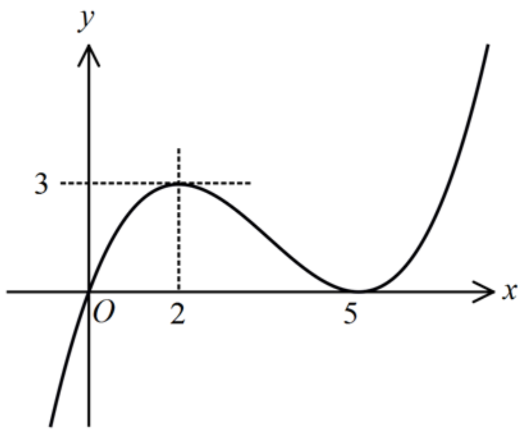
(A) $\frac{16\pi}{3}$

(B) $\frac{32\pi}{3}$

(C) 16π

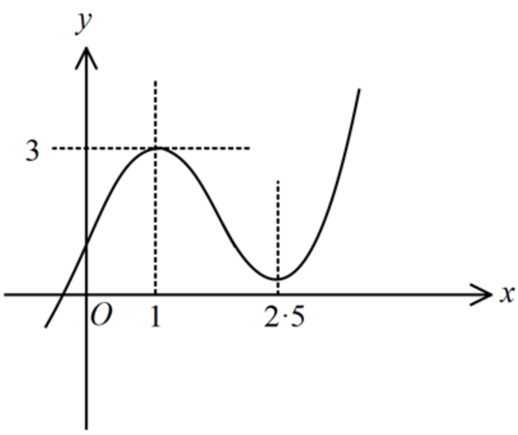
(D) 32π

9 The diagram shows part of the graph of $y = f(x)$.

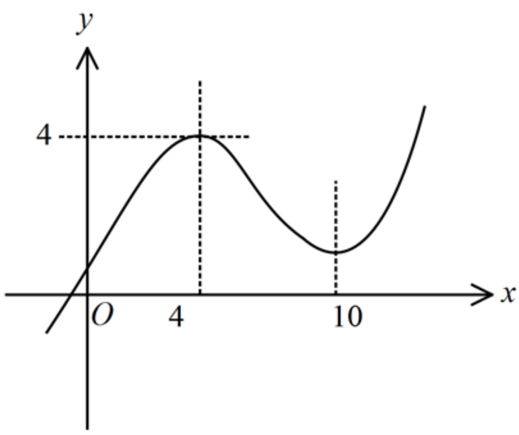


Which of the following diagrams could be the graph of $y = 2f(x) + 1$?

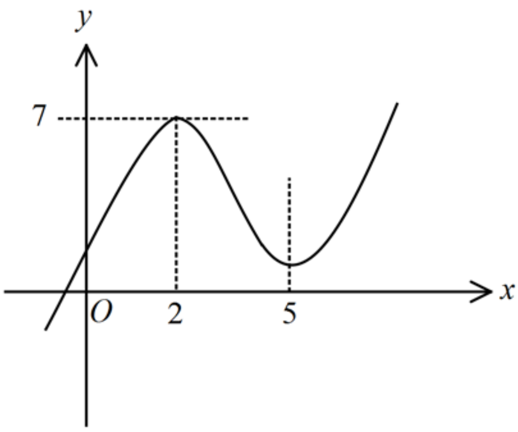
(A)



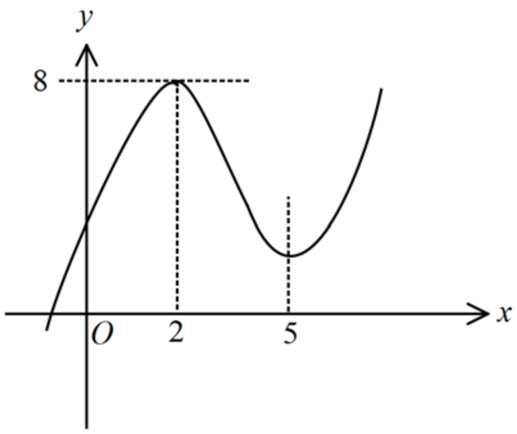
(B)



(C)



(D)

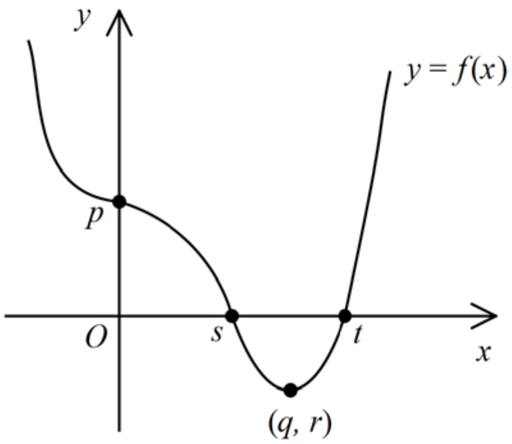


10 The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r).

Here are two statements about $f(x)$

(i) $f(x) < 0$ for $s < x < t$.

(ii) $f'(x) < 0$ for $x < q$.



Which of the following is true?

(A) Neither statement is correct.

(B) Only statement (i) is correct.

(C) Only statement (ii) is correct.

(D) Both statements are correct.

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Question 11 (15 marks) Use a separate writing booklet.

(a) Express $\dfrac{2}{\sqrt{6}-2}$ with a rational denominator in its simplest form. **2**

(b) Solve $|x-4| \le 2$. **2**

(c) Solve the equation $\dfrac{2-x}{3} - \dfrac{3-x}{2} = \dfrac{1}{5}$. **2**

(d) Find the gradient of the tangent to the curve $y=(3x+1)^4$ when $x=\dfrac{1}{3}$. **2**

(e) Simplify $\dfrac{\log_b a^m}{\log_m a}$ and express it in terms of base *b*. **2**

(f) Differentiate $\sqrt[3]{x}$. **1**

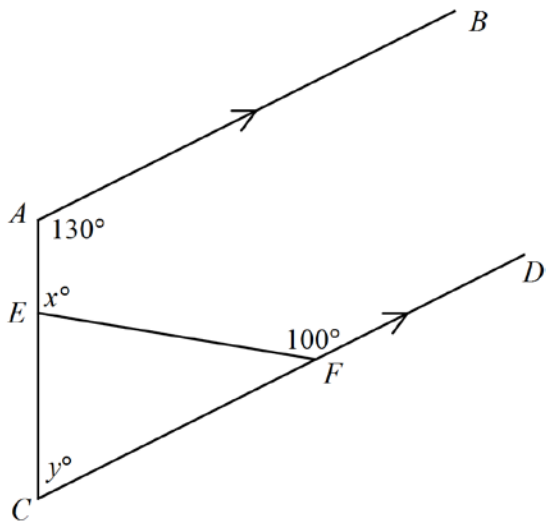
(g) Find $\displaystyle\int \frac{x+2}{x^2+4x}\,dx$. **2**

(h) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{x}{2}\right) dx$. **2**

End of Question 11

Question 12 (15 marks) Use a separate writing booklet.

(a)



In the diagram, $AB \parallel CD$, $\angle BAE = 130°$, $\angle EFD = 100°$.

(i) Find the value of y, giving reasons. **1**

(ii) Find the value of x, giving reasons. **2**

(b) A function *f* is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$.

(i) Find where the graph of $y = f(x)$ cuts

(1) the *x*-axis. **1**

(2) the *y*-axis. **1**

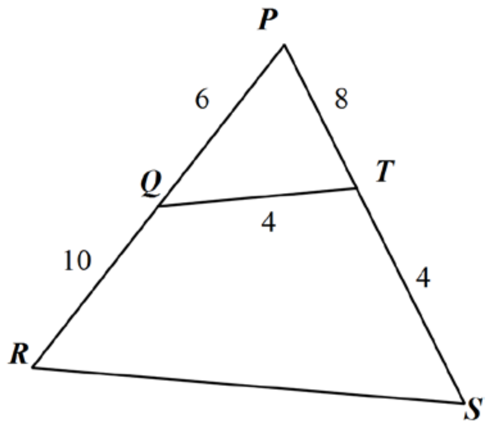
(ii) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **4**

(iii) Sketch the graph of $y = f(x)$, showing all important features. **3**

Question 12 continues on page 9.

Question 12 (continued).

(c)



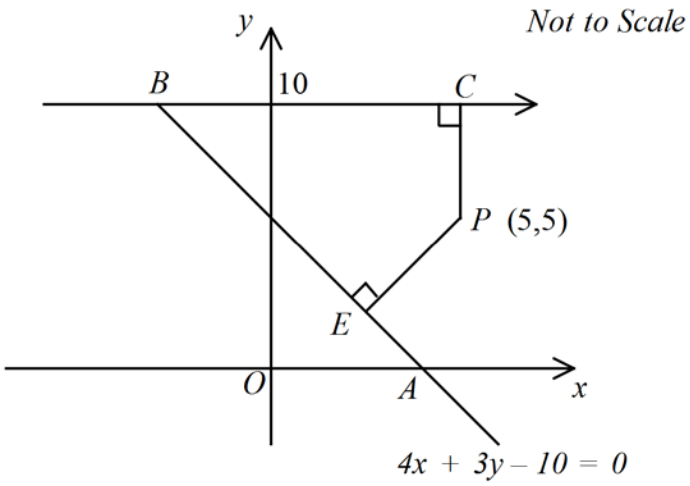
(i) Prove that ΔPQT and ΔPSR are similar. **2**

(ii) Hence, find the length of RS. **1**

End of Question 12

Question 13 (15 marks) Use a separate writing booklet.

(a)



In the diagram above, the equations of the lines *BE* and *BC* are $4x+3y-10=0$ and $y=10$ respectively.

P is the point $(5,5)$.
$PE \perp BE$, and $BC \perp PC$.

(i) Show that the perpendicular distance from *P* to *BE* is 5 units. **1**

(ii) Hence prove that $\Delta BCP \equiv \Delta BEP$. **3**

(iii) Show that the coordinates of *B* are $(-5, 10)$. **1**

(iv) Show that the locus of points which are equidistant from the lines *BC* and *BE* is given by the equation $x+2y-15=0$. **2**

Question 13 continues on page 11

(b) (i) Sketch the parabola *P*, whose focus is $(-2,\ 2)$ and whose directrix is the line $x = -\ 6$. **2**

Indicate on your diagram the coordinates of the focus, the vertex and the equation of the directrix.

(ii) Determine the equation of the parabola, *P*. **1**

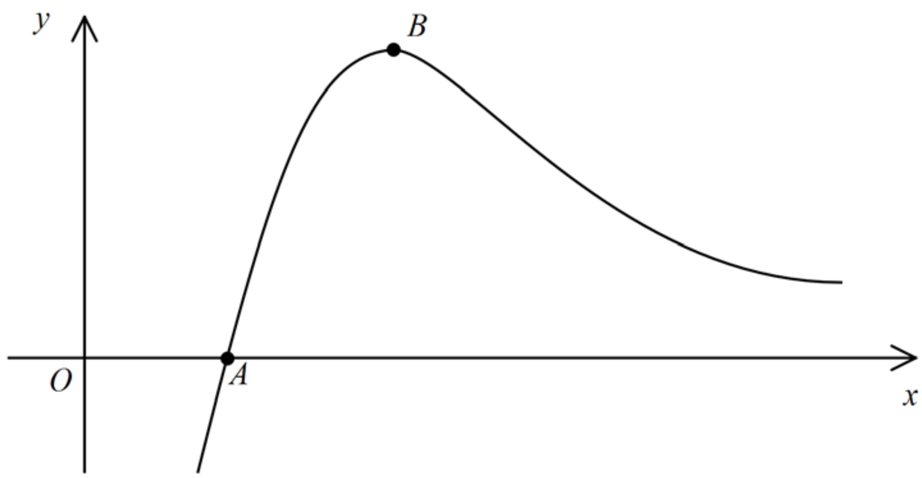
(c) For what values of *a* will the equation $ax^2 + 5x + a$ be positive definite? **3**

(d) A ball is dropped from a height of 10 metres and each time it bounces, it reaches $\frac{4}{5}$ of it's previous height. What is the total distance travelled by the ball? **2**

End of Question 13

Question 14 (15 marks) Use a separate writing booklet.

(a) The diagram show the curve $y = \frac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(i) State the coordinates of A. **1**

(ii) Find the coordinates of the stationary point B, of the curve, giving your answer in an exact form **2**

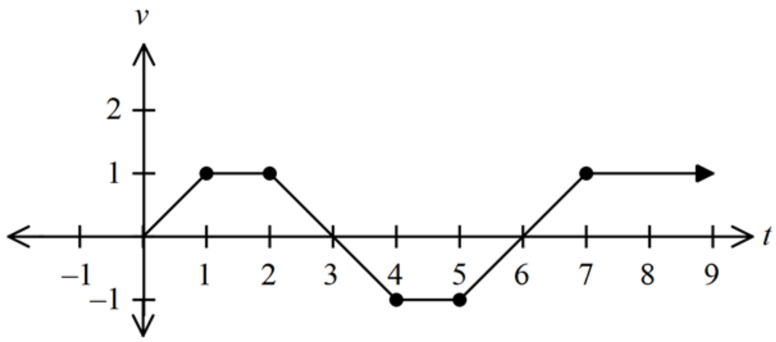
(iii) Find the exact value of the equation of the normal to the curve at the point where $x = e^3$. **2**

(b) Sketch the graph $y = 2 - \cos 2x$ for $-\pi \le x \le \pi$. **2**

Question 14 continues on page 13

Question 14 (continued).

(c)



The diagram above describes the velocity, *v*, of an electrical pulse along a wire in metres/second. Refer to the diagram and answer the questions below.

(i) When is the pulse travelling in a positive direction? **1**

(ii) When is the pulse stopped? **1**

(iii) Describe the motion of the particle for $0 < t < 3$. **2**

(iv) Find the area between the curve and the *t* axis for $3 \le t \le 6$. **1**

(d) If α and β are the roots of the quadratic equation $3x^2 - 4x - 1 = 0$, find

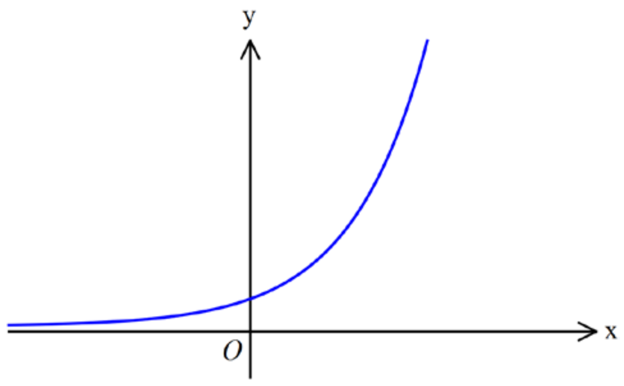
(i) $\alpha + \beta$ and $\alpha\beta$. **1**

(ii) $(\alpha + 2)(\beta + 2)$ **2**

End of Question 14

Question 15 (15 marks) Use a separate writing booklet.

(a) The diagram shows a sketch of the curve $y = 2^{4x}$.



(i) Use the trapezoidal rule with three function values to find an approximate value for $\int_0^1 2^{4x}\,dx$. **2**

(ii) Is the approximate for $\int_0^1 2^{4x}\,dx$, an under approximation or an over approximation? Explain your choice. **1**

(b) The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where

- P_0 is the initial concentration
- P_t is the concentration at time t
- t is the time, in days, after the application of the pesticide.

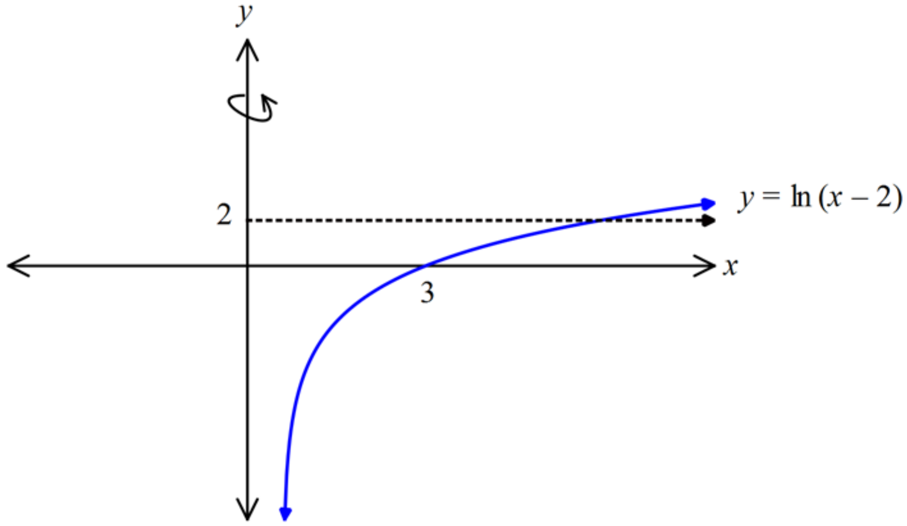
(i) Once in the soil, the half-life of a pesticide is the time taken for it's concentration to be reduced to one half of its initial value. If the half-life of *Xpesto* is 25 days, find the value of k correct to 2 significant figures. **2**

(ii) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

Question 15 continues on page 15

Question 15 (continued).

(c)



The region bounded by the curve $y = \ln(x-2)$ and the y-axis between $y = 0$ and $y = 2$ is rotated about the y-axis to form a solid. **4**

Find the exact volume of the solid.

(d) Given that $\int_0^a 5\sin 3x \, dx = \frac{10}{3}$, $0 \le a < \pi$, calculate the value of a. **3**

End of Question 15

Question 16 (15 marks) Use a separate writing booklet.

(a) Solve $2\tan x \sin^2 x = \tan x$ for $0 \le x \le 2\pi$. **3**

(b) An open cylindrical can is to have a surface area of $20\pi \text{ cm}^2$. The can has no lid.

(i) Let r centimetres be the radius of the can and h centimetres be its height. **1**
Show that $h = \dfrac{20 - r^2}{2r}$.

(ii) Hence, show that the total volume of the can is given **1**
by $V = 10\pi r - \dfrac{1}{2}\pi r^3$.

(iii) Show that the maximum volume is obtained when the height of the can is equal to it's radius. **4**

(c) (i) Show that the equation $4\operatorname{cosec}^2\theta - \cot^2\theta = k$, where $k \ne 4$, can be written in the form **2**

$$\sec^2\theta = \frac{k-1}{k-4}.$$

(ii) Hence, or otherwise, solve the equation **4**

$$4\operatorname{cosec}^2(2x + 75°) - \cot^2(2x + 75°) = 5,$$

giving all values of x in the interval $0° < x < 180°$.

End of paper

Year 12 Mathematics Trial solutions

Multiple Choice

1. B
2. A
3. A
4. C
5. B
6. B
7. B
8. C
9. C
10. B

Question 11

a) $\frac{2}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2}$ ① correct multiplier

$= \frac{2\sqrt{6}+4}{6-4}$

$= \frac{2\sqrt{6}+4}{2}$

$= \sqrt{6}+2$ ① correct simplification

b) $|x-4| \leq 2$

$-2 \leq x-4 \leq 2$ ① correct boundaries

$2 \leq x \leq 6$ ① correctly expressing as a closed interval

c) $\frac{2-x}{3} - \frac{3-x}{2} = \frac{1}{5}$

$10(2-x) - 15(3-x) = 6$ — ① correctly removing fraction, or equivalen

$20 - 10x - 45 + 15x = 6$

d) $y = (3x+1)^4$

$\frac{dy}{dx} = 4(3x+1)^3 \times 3$

$= 12(3x+1)^3$ — ① correct derivative

When $x = \frac{1}{3}$ $\frac{dy}{dx} = 12\left(3 \times \frac{1}{3} + 1\right)^3$

$= 96$ — ① correct substitution and evaluation

e) $\frac{m \log_b a}{\log_m a}$

$= m \log_b a \div \frac{\log_b a}{\log_b m}$ — ① correct change of base and multiplying

$= m \log_b a \times \frac{\log_b m}{\log_b a}$

$= m \log_b m$

$= \log_b m^m$ ① for either answer

f) $y = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$ — ① r/w

$= \frac{1}{3\sqrt[3]{x^2}}$

g) $\int \frac{x+2}{x^2+4x}\, dx$ ① — for $\frac{1}{2}$

$= \frac{1}{2} \ln|x^2+4x| + c$ ① — correct log and constant

h) $\int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right) dx$

$= 2\left[\tan \frac{x}{2}\right]_0^{\pi/2}$ — (1)

$= 2\{\tan \pi/4 - \tan 0\}$

$= 2$ — (1)

Question 12

(a)(i) $y° = 180° - 130°$ (cointerior angles supplementary AB||CD)

$= 50°$

(1)

(ii) $\angle EFC = 180° - 100°$ (adjacent supplementary angles)

$= 80°$

$x° = y° + \angle EFC$ (exterior angle of ΔEFC equals

$= 50° + 80°$ sum of 2 opposite interior angles).

$= 130°$

(2) * some found $\angle CEF$ Δ sum

then angles on straight line

(b)(i) $f(x) = (x-2)(x^2+1)$

(1) cuts x axis $y=0$

$x = 2$ x^2+1 no solution

$(2, 0)$ (1)

(2) cuts y axis $x=0$

$y = (0-2)(0^2+1)$

$= -2$

$(0, -2)$ (1)

(ii) let $f(x) = y$ $y = (x-2)(x^2+1)$

$= x^3 - 2x^2 + x - 2$

$y' = 3x^2 - 4x + 1$

stat pts can occur $y' = 0$

$0 = 3x^2 - 4x + 1$

$0 = (x-1)(3x-1)$

$x = 1 \quad y = (1-2)(1^2+1)$
$\quad = -2 \qquad (1,-2)$

$x = \frac{1}{3} \quad y = \left(\frac{1}{3}-2\right)\left(\left(\frac{1}{3}\right)^2+1\right)$
$\quad = \frac{-50}{27} \qquad \left(\frac{1}{3}, \frac{-50}{27}\right)$

use y'' to determine nature

* if you used y' as test you must show substitutions and values.

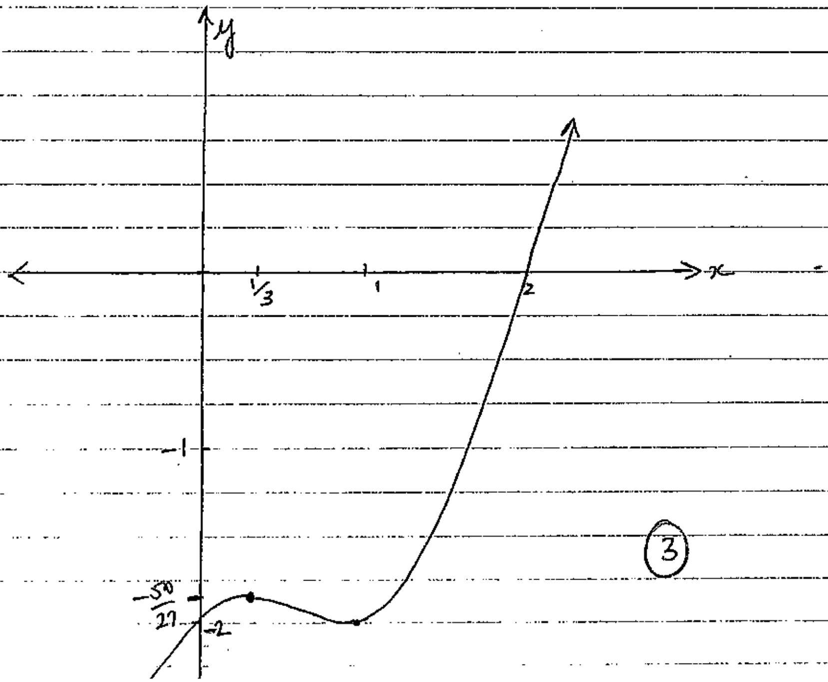
$y'' = 6x - 4$

(4)

$x = 1 \quad y'' = 6(1) - 4$
$\quad = 2 > 0 \quad \therefore (1,-2)$ minimum.

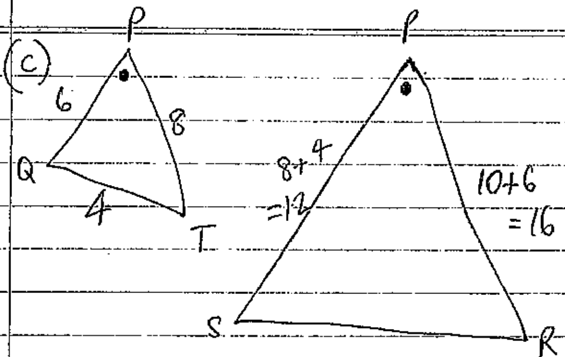
$x = \frac{1}{3} \quad y'' = 6\left(\frac{1}{3}\right) - 4$
$\quad = -2 < 0 \quad \therefore \left(\frac{1}{3}, \frac{-50}{27}\right)$ maximum.

(iii)



(3)

(c)



(i) In ΔPQT and ΔPSR

$\angle QPT = \angle SPR$ (common angle)

$\frac{PQ}{PS} = \frac{6}{12} = \frac{1}{2}$ given (1) ratios

$\frac{PT}{PR} = \frac{8}{16} = \frac{1}{2}$ given (1) reason

$\therefore \Delta PQT \,|||\, \Delta SPR$ (2 pairs of matching sides in proportion and included angle equal)

(ii) $\frac{QT}{SR} = \frac{1}{2} = \frac{4}{x}$

$\therefore x = 8$ (1) matching sides in similar Δs.

Question 13

a) i. $4x + 3y - 10 = 0$

Point $(5,5)$

$$\text{perp dist} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|4\times5 + 3\times5 - 10|}{\sqrt{4^2+3^2}}$$

$$= \frac{|20+15-10|}{\sqrt{25}}$$

$$= \frac{|25|}{5}$$

$= 5$ units (1)

ii. C is $(5,10)$, 5 units vertically above $P(5,5)$.

Construct BP.

(1) $CP = EP$

(1) structure and rest

(1) conclusion & test

In $\triangle BCP$ and $\triangle BEP$

$\angle BCP = \angle BEP$ (given)

BP is common

$CP = EP$ (part (i) and above, both 5 units)

$\therefore \triangle BCP \equiv \triangle BEP$ (RHS)

iii. B lies on the line $y=10$,

$\therefore$ y coordinate of B is 10.

$4x + 3y - 10 = 0$

when $y = 10$

$4x + 3(10) - 10 = 0$

$4x + 20 = 0$

$4x = -20$

$x = -5$ (1)

iv. As established in (ii), P is equidistant from line BA and line BC.

$\therefore$ The locus passes through B and P.

$$m_{BP} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-10}{5-(-5)}$$

$$= \frac{-5}{10}$$

$$= \frac{-1}{2}$$

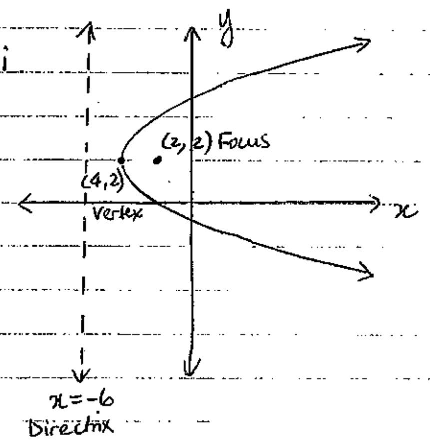
$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{2}(x-5)$ (1) progress

$2y - 10 = -x + 5$ (1) correct locus

$x + 2y - 15 = 0$ (several methods possible)

b) i.



(2) all correct

-1 each error

ii. Focal length $= 2$

$(y-2)^2 = 4a(x+4)$

$(y-2)^2 = 8(x+4)$ (1)

c) For $ax^2 + 5x + a$ to be positive definite,

$a > 0$, $\Delta < 0$.

$\Delta = b^2 - 4ac$

$= 5^2 - 4 \times a \times a$

$= 25 - 4a^2$

$25 - 4a^2 < 0$ ①

$25 < 4a^2$

$4a^2 > 25$

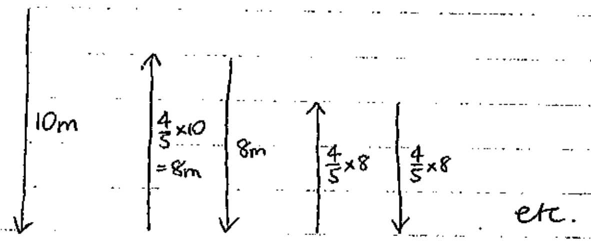
$a^2 > \frac{25}{4}$

$a < -\frac{5}{2}$, $a > \frac{5}{2}$ ①

Since $a > 0$,

$a > \frac{5}{2}$ is the only soln. ①

d)



$a = 8$

$r = \frac{4}{5}$

$(|r| \leq 1)$

Total distance $= 10 + 2 \times S_\infty$

$= 10 + 2 \times \frac{a}{1-r}$

$= 10 + 2 \times \frac{8}{1 - \frac{4}{5}}$

$= 90\text{m}$

① S_∞

① Total distance

Question 14

a) $y = \frac{\ln x}{x}$

(i) y-coordinate of A is 0

$\therefore \quad 0 = \frac{\ln x}{x}$

$\ln x = 0$

$x = e^0$

$x = 1$

$\therefore \quad A(1, 0)$ (1) R/W

(ii) $y = \frac{\ln x}{x}$ $\quad u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$ (1)

Find x when $\frac{dy}{dx} = 0$ for stationary point.

$0 = \frac{1 - \ln x}{x^2}$

$1 - \ln x = 0$

$\ln x = 1$

$x = e^1$

$x = e.$

$y = \frac{\ln e}{e}$

$= \frac{1}{e}$ $\quad \therefore \quad B(e, \frac{1}{e})$ (1)

(iii) When $x = e^3$

$\frac{dy}{dx} = \frac{1 - \ln e^3}{(e^3)^2}$

$= \frac{1 - 3}{e^6}$

$M_T = -2e^{-6}$

This is gradient of tangent.
So gradient of normal is

$M_N = \frac{-1}{M_T}$

$= \frac{-1}{-2e^{-6}}$

$= \frac{e^6}{2}$ (1)

When $x = e^3$ $\quad y = \frac{\ln e^3}{e^3}$

$= 3e^{-3}$

$\therefore$ equation of normal is

$y - 3e^{-3} = \frac{e^6}{2}(x - e^3)$ (1)

$2y - 6e^{-3} = e^6 x - e^9$

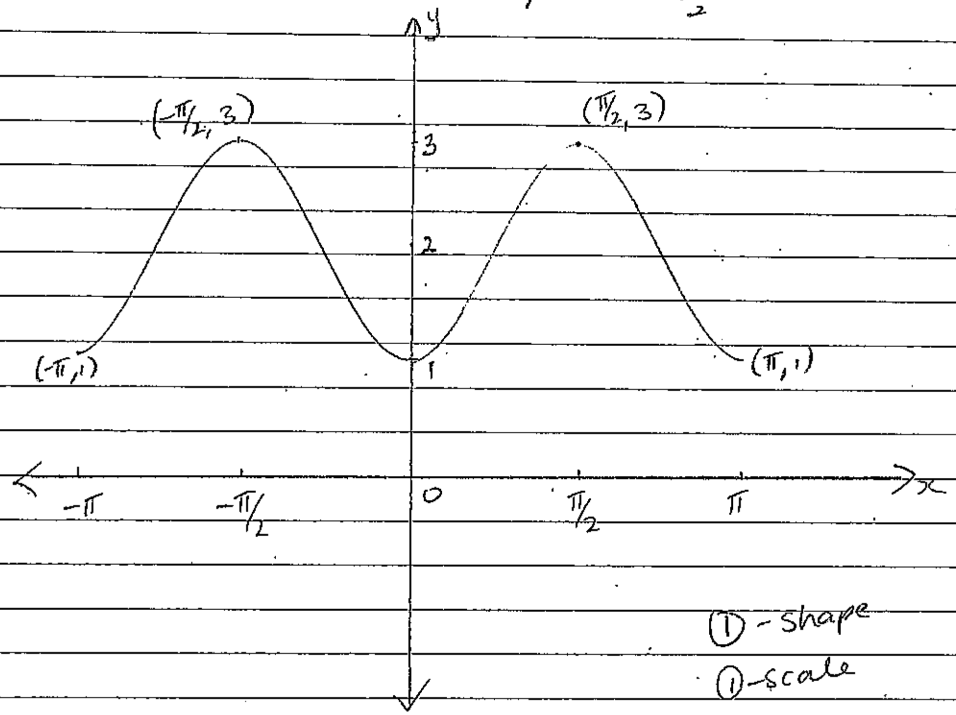
$e^6 x - 2y + 6e^{-3} - e^9 = 0 \quad (\times e^3)$

$e^9 x - 2e^3 y + 6 - e^{12} = 0.$

$y = \frac{e^6}{2}x - \frac{e^9}{2} + \frac{3}{e^3}$

b) $y = 2 - \cos 2x \qquad -\pi \le x \le \pi$

period $= \frac{2\pi}{2} = \pi$



(1) - shape

(1) - scale

c) (i) $0 < t < 3$ and $t > 6$ (1)

(ii) $t = 0, 3, 6$ (1)

(iii) Particle moves in positive direction for $0 < t < 3$

During $0 \le t < 1$ particle has constant acceleration so speed increases.

During $1 \le t \le 2$ particle has zero acceleration so speed is constant at 1m/s

During $2 < t < 3$ particle has constant deceleration so speed decreases

(1) - describe direction

(1) - describe speed.

(iv) $A = \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times 1 \times (1+3)$

$= 2$ units^2 (1)

d) $3x^2 - 4x - 1 = 0$

$a = 3 \quad b = -4 \quad c = -1$

(i) $\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$

$= -\frac{(-4)}{3} \qquad = -\frac{1}{3}$ (1)

$= \frac{4}{3}$

(ii) $(\alpha + 2)(\beta + 2)$

$= \alpha\beta + 2(\alpha + \beta) + 4$ (1)

$= -\frac{1}{3} + 2 \times \frac{4}{3} + 4$

$= \frac{19}{3}$ (1)

2 Unit Trial.

N.B. Do not use 2 columns or arrows in your solutions.

15 (a) (i)

x	0	$\frac{1}{2}$	1
y	1	4	16

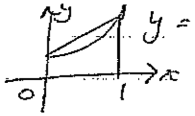
Using $A = \frac{h}{2}(a+b)$

$$\text{Area} = \frac{\frac{1}{2}}{2}(1+4) + \frac{\frac{1}{2}}{2}(4+16) \checkmark$$

$$= \frac{25}{4}\ u^2 \checkmark$$

(2)

(ii) Area under trapezium is greater than area under concave up curve as shown in diagram



$y = 2^{4x}$ ∴ Over estimation

(b)(i) $\frac{1}{2}P_0 = P_0 e^{-kt}$

$$\frac{1}{2} = e^{-k(25)} \quad (t = 25)$$

$$\ln\left(\frac{1}{2}\right) = -25k$$

$$k = -\frac{1}{25}\ln\left(\frac{1}{2}\right) \checkmark$$

$$= 0.027725\ldots$$

$$= 0.028 \quad (2 \text{ sig. figures}) \checkmark$$

(2)

(ii) $t = 80$

$$P = P_0 e^{-k(80)}$$

$$= 0.1088\ P_0$$

N.B. Students should use $0.027725\ldots$ for k

∴ 10.88% remaining

∴ 89.1181177… Used

89% (correct to nearest whole no.)

(c) $y = \ln(x-2)$

$$e^y = x - 2$$

$$x = 2 + e^y$$

$$x^2 = (2 + e^y)^2$$

$$= 4 + 4e^y + e^{2y}$$

(c) $V = \pi \int_0^2 (2 + e^y)^2\, dy$ ✓

$$= \pi \int_0^2 4 + 4e^y + e^{2y}\, dy \checkmark$$

$$= \pi \left[4y + 4e^y + \frac{e^{2y}}{2}\right]_0^2 \checkmark$$

$$= \pi \left[\left(4(2) + 4e^2 + \frac{e^{2(2)}}{2}\right) - \left(4(0) + 4e^0 + \frac{e^0}{2}\right)\right]$$

$$= \pi \left[\left(8 + 4e^2 + \frac{e^4}{2}\right) - \left(0 + 4 + \frac{1}{2}\right)\right]$$

$$= \pi \left[3\tfrac{1}{2} + 4e^2 + \frac{e^4}{2}\right] \checkmark$$

OR $\frac{\pi}{2}\left[e^4 + 8e^2 + 7\right]$

(4)

(d) $\int_0^a 5 \sin 3x\, dx = \frac{10}{3}$

$$-\frac{5}{3} \cos 3x \Big]_0^a = \frac{10}{3} \qquad 0 \le a \le \pi$$

$$-\frac{5}{3}\left[\cos 3a - \cos 0\right] = \frac{10}{3} \checkmark$$

$$\cos 3a - 1 = -2$$

$$\cos 3a = -1 \checkmark$$

$$3a = \pi \qquad 0 \le 3a < 3\pi\ ^*$$

$$a = \frac{\pi}{3} \checkmark$$

* Very important step

Question 16

a) $2\tan x \sin^2 x - \tan x = 0$

$\tan x (2\sin^2 x - 1) = 0$

$\therefore \tan x = 0$ or $\sin^2 x = \frac{1}{2}$ ① correct separation

$x = 0, \pi, 2\pi$ or $\sin x = \pm \frac{1}{\sqrt{2}}$

① correct values for $\tan x = 0$

$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

① correct values for $\sin x = \frac{\pm 1}{\sqrt{2}}$

b) i) $20\pi = 2\pi r h + \pi r^2$

$\therefore 20 = 2rh + r^2$

$2rh = 20 - r^2$

$h = \frac{20 - r^2}{2r}$ ① – correct demonstration

ii) $V = \pi r^2 h$

$= \pi r^2 \left(\frac{20 - r^2}{2r}\right)$

$= \frac{\pi r(20 - r^2)}{2}$

$= \frac{20\pi r}{2} - \frac{\pi r^3}{2}$ ① – correct demonstration

$V = 10\pi r - \frac{1}{2}\pi r^3$

iii) $\frac{dV}{dr} = 10\pi - \frac{3}{2}\pi r^2$

$\frac{d^2V}{dr^2} = -3\pi r$

When $\frac{dV}{dr} = 0$ ① differentiating and making

$\frac{3}{2}\pi r^2 = 10\pi$

$r^2 = \frac{20}{3}$

$r = \sqrt{\frac{20}{3}}$ – ① solving for r

When $r = \sqrt{\frac{20}{3}}$ $\frac{d^2V}{dr^2} = -3\pi \times \sqrt{\frac{20}{3}}$

< 0

$\therefore r = \sqrt{\frac{20}{3}}$ will produce maximum volume.

$r = \frac{\sqrt{20}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{5} \times \sqrt{3}}{3}$

$= \frac{2\sqrt{15}}{3}$

From i)

$h = \frac{20 - \left(\frac{2\sqrt{15}}{3}\right)^2}{2\left(\frac{2\sqrt{15}}{3}\right)}$

$= \frac{20 - \frac{60}{9}}{\frac{4\sqrt{15}}{3}}$

$= \frac{120}{9} \times \frac{3}{4\sqrt{15}}$

$= \frac{10}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$

$= \frac{10\sqrt{15}}{15}$

$= \frac{2\sqrt{15}}{3}$

$= r$

$\therefore$ Volume is maximum when $h = r$

c) i) $4\operatorname{cosec}^2\theta - \cot^2\theta = k$

$\frac{4}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} = k$

$4 - \cos^2\theta = k\sin^2\theta$ – ① to this point or equivalent

$\sin^2\theta = 1 - \cos^2\theta$

$4 - k = \cos^2\theta - k\cos^2\theta$

$4 - k = \cos^2\theta\,(1 - k)$

$\cos^2\theta = \dfrac{4-k}{1-k}$

$\dfrac{1}{\cos^2\theta} = \dfrac{-(k-1)}{-(k-4)}$

$\sec^2\theta = \dfrac{k-1}{k-4}$ — ① suitable demonstration

ii) $\sec^2(2x + 75) = \dfrac{5-1}{5-4}$

$\therefore \sec^2(2x + 75) = 4$ — ① linking to i)

$0^\circ < x < 180^\circ$

$75^\circ < 2x + 75 < 435^\circ$ — ① Adjustment of domain

$\therefore \dfrac{1}{\cos^2(2x + 75)} = 4$

$\cos^2(2x + 75) = \dfrac{1}{4}$

$\cos(2x + 75) = \pm\dfrac{1}{2}$ — ① Correctly square rooting and changing to cos or equivalent

$2x + 75 = 120^\circ, 240^\circ, 300^\circ, 420^\circ$

$x = 22.5^\circ, 82.5^\circ, 112.5^\circ, 172.5^\circ$ — ① correct solution