Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 What is the basic numeral for 4.5378×10^{-4}, correct to 4 significant figures?

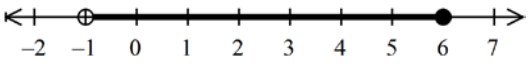
(A) 0.00045378

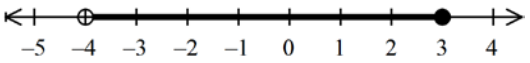
(B) 0.0004538

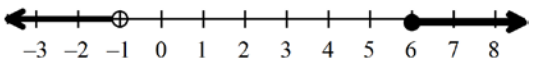
(C) 0.4538

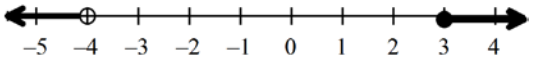
(D) 45380

2 What is the solution on a number line for $-5 < 2x-3 \le 9$?

(A) 

(B) 

(C) 

(D) 

3 What is the solution of $|x-3|=2x+1$?

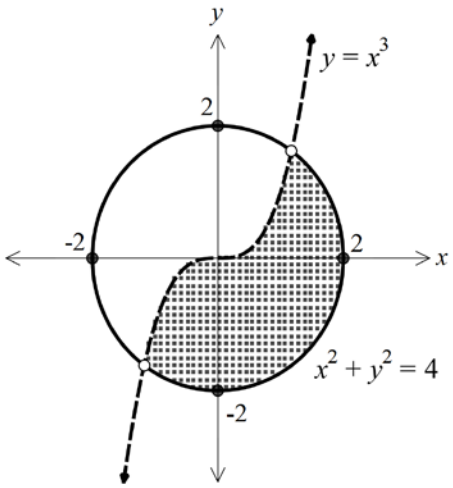
(A) $x=-4$

(B) $x=\frac{2}{3}$

(C) $x=\frac{4}{3}$

(D) $x=\frac{3}{2}$

4 Which inequalities define the shaded region shown in the diagram?



(A) $x^2 + y^2 \geq 4$ and $y < x^3$

(B) $x^2 + y^2 \geq 4$ and $y \geq x^3$

(C) $x^2 + y^2 \leq 4$ and $y > x^3$

(D) $x^2 + y^2 \leq 4$ and $y < x^3$

5 What is the domain and range of $f(x) = -\dfrac{1}{x^2 - 1}$?

(A) $x \neq 1, y \neq 0$

(B) $x \neq 1, y < 0$

(C) $x \neq -1, y \neq 0$

(D) $x \neq \pm 1, y \neq 0$

6 What is the gradient of the normal to the curve $y = \cos 2x$ at the point where $x = \frac{\pi}{8}$?

(A) $-\sqrt{2}$

(B) $-\frac{\sqrt{2}}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

7 A particle moves in a straight line with a displacement of x metres at time t seconds. When is the particle speeding up?

(A) $\frac{dx}{dt} < 0$ and $\frac{d^2x}{dt^2} > 0$

(B) $\frac{dx}{dt} > 0$ and $\frac{d^2x}{dt^2} < 0$

(C) $\frac{dx}{dt} > 0$ and $\frac{d^2x}{dt^2} = 0$

(D) $\frac{dx}{dt} < 0$ and $\frac{d^2x}{dt^2} < 0$

8 What is the derivative of $\frac{x}{\sqrt{x}}$?

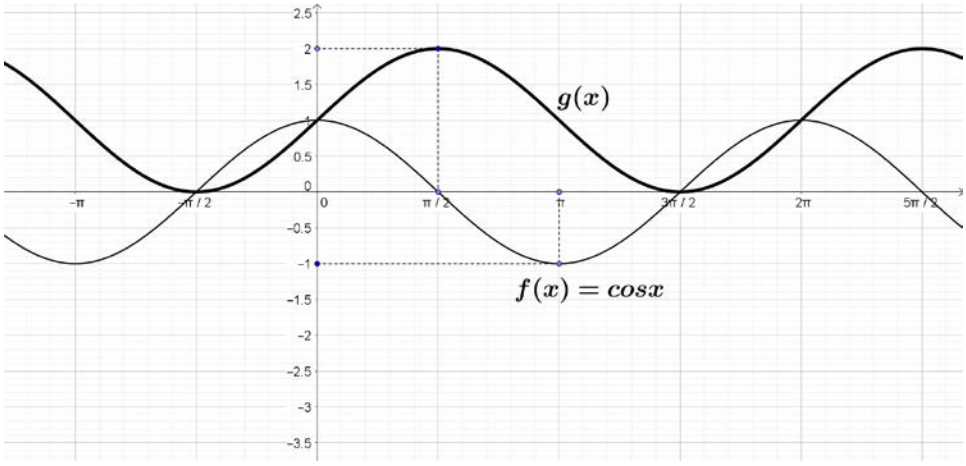
(A) $\frac{1}{2\sqrt{x}}$

(B) $\frac{1}{\sqrt{2x}}$

(C) $\frac{\sqrt{x}}{2}$

(D) $2\sqrt{x}$

9 The diagram shows the graphs of $f(x)=\cos x$ and $g(x)$.
What is the equation of $g(x)$?



(A) $g(x)=f\left(x-\frac{\pi}{2}\right)+2$

(B) $g(x)=f\left(x+\frac{\pi}{2}\right)+1$

(C) $g(x)=f\left(x-\frac{\pi}{2}\right)+1$

(D) $g(x)=f\left(x+\frac{\pi}{2}\right)+2$

10 Which of the following statements is true if $(2,-5)$ is a minimum turning point of $f(x)$ and $f(x)=-f(-x)$?

(A) $(-2,-5)$ is a maximum turning point of $f(x)$

(B) $(-2,5)$ is a minimum turning point of $f(x)$

(C) $(-2,-5)$ is a minimum turning point of $f(x)$

(D) $(-2,5)$ is a maximum turning point of $f(x)$

END OF SECTION I

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a Writing Booklet.

(a) Find the primitive function of $(2x-1)^3$. **1**

(b) Find $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{x+1}$. **2**

(c) Find the value of A given that $\sqrt{28} + 3\sqrt{7} = \sqrt{A}$, where A is a positive integer. **2**

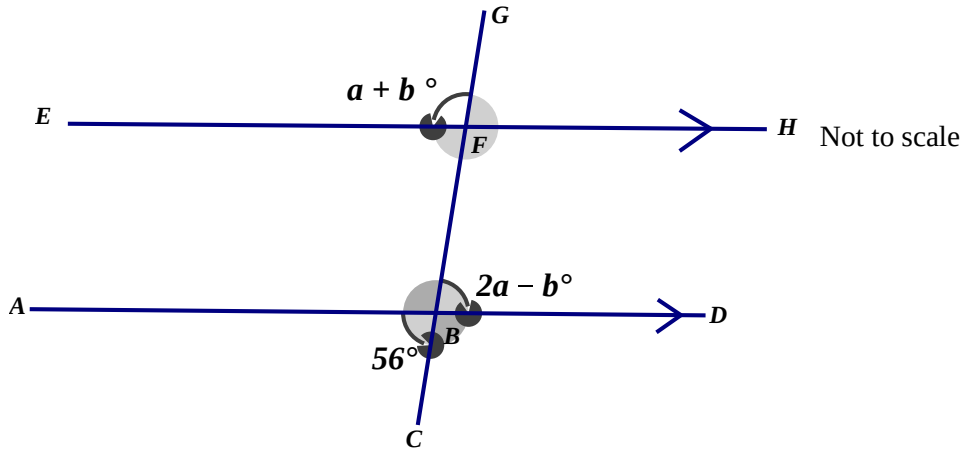
(d) Factorise completely the expression $m^3 - 9m^2 - 4m + 36$. **3**

(e) Find the period of the curve $y = 3\cos\left(\dfrac{x}{2}\right)$. **1**

Question 11 continues on next page

(f) If $\frac{d^2y}{dx^2} = 4x - 2$, find an expression for y, given that $\frac{dy}{dx} = 8$ when $x = -2$ and $y = -4$ when $x = 3$. **3**

(g)



In the diagram above, $AD \parallel EH$, $\angle ABC = 56°$,
$\angle DBG = (2a - b)°$, and $\angle EFG = (a + b)°$.

Find the values of a and b, giving reasons for your answer. **3**

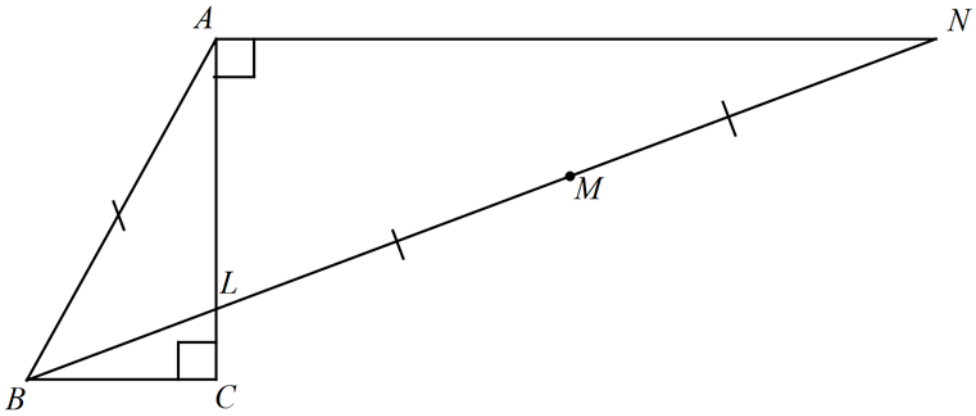
End of Question 11

Question 12 (15 marks) Use a New Writing Booklet.

(a) Solve $9^x + 4\left(3^x\right) - 21 = 0$. **2**

(b) Show that the equation $x + 3 + \dfrac{2}{x+3} = 5$ has 2 distinct irrational roots. **2**

(c) The triangles *ABC* and *ALN* are both right angled as shown.
AB = *LM* = *MN*.



Copy or trace the diagram into your writing booklet.

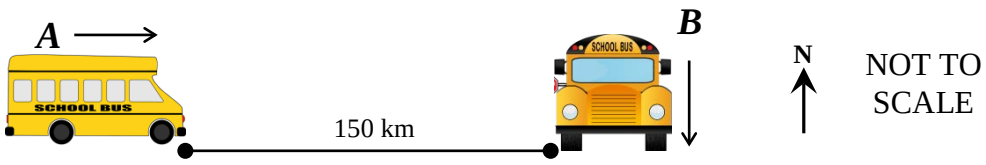
(i) Explain why $\angle LBC = \angle LNA$. **2**

(ii) The point *O* is the fourth vertex of the rectangle *ALON*.
Explain why *M* is the midpoint of *AO* **and** why *LM* = *AM*. **1**

(iii) Hence prove that $\angle LBA = 2\angle LBC$. **3**

Question 12 continues on next page

(d) School bus ***A*** is 150 km due west of school bus ***B***. At the same time, school bus ***A*** drives due east at 50 km/h while school bus ***B*** drives due south at 40 km/h.



(i) After x hours of driving, show that **2**

$$d = \sqrt{4100x^2 - 15000x + 22500},$$

where d is the distance between the buses and $0 < x < 3$.

(ii) Find the value of x that gives the minimum distance between the buses.

Give the answer correct to 2 decimal places. **3**

End of Question 12

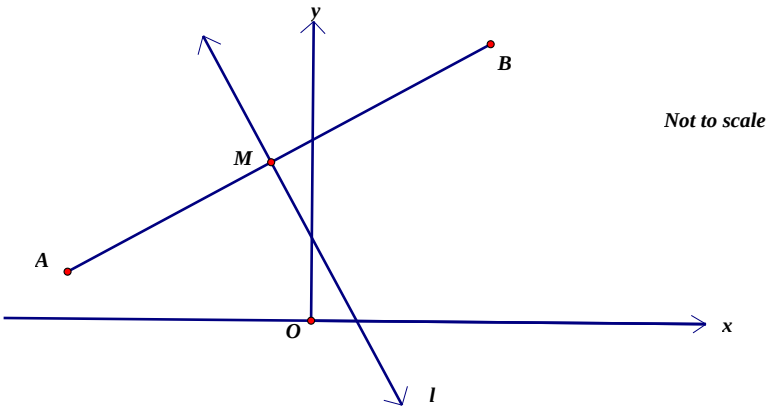
Question 13 (15 marks) Use a New Writing Booklet.

(a) (i) Differentiate $e^x - \cos 3x$. **1**

(ii) Hence, or otherwise, find $\int \frac{3e^x + 9\sin 3x}{e^x - \cos 3x}\, dx$. **2**

(b) Evaluate $\int_0^{\ln 5} \frac{3}{e^x}\, dx$, giving the answer as a single fraction. **2**

(c)



In the above diagram, Point M is the midpoint of the interval AB.
Point B has coordinates (4, 8) and Point M has coordinates (−1, 5).

(i) Find the coordinates of Point A. **1**

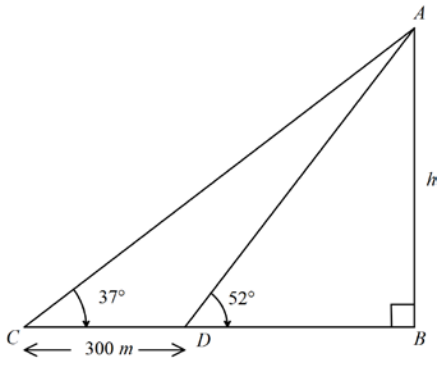
(ii) Line l is the perpendicular bisector of AB. Find the equation of line l. Give answer in general form. **2**

(iii) Find the equation of the circle with diameter AB. **2**

(iv) Find the angle of inclination of Line l to the positive direction of the x axis. Answer to nearest degree. **1**

Question 13 continues on next page

(d) In the diagram, $\angle ADB = 52°$, $\angle ACD = 37°$ and DC = 300 metres.
Let $AB = h$ metres.



(i) Show that $AD = \dfrac{300 \sin 37°}{\sin 15°}$. **2**

(ii) Hence find the value of h, correct to 2 decimal places. **1**

(e) Differentiate $y = \dfrac{1+3x}{2-x}$. **1**

End of Question 13

Question 14 (15 marks) Use a New Writing Booklet.

(a) Given that $\tan\theta = -\frac{5}{12}$ and $\cos\theta > 0$, find the exact value of $\sin\theta$. **1**

(b) Find the equation of the locus of a point *P* which moves so that the ratio of *PA*: *PB* is 3:2, given $A = (4, 5)$ and $B = (6, 0)$ **2**

(c) (i) Let $f(x) = 2x^2 e^{-x}$
Show that $f'(x) = 2xe^{-x}(2 - x)$ **1**

(ii) Find the two stationary points on $y = f(x)$ and determine their nature. **3**

(iii) Sketch the graph of $y = f(x)$, for $-\frac{1}{2} \le x \le 4$ **2**

(d) A particle moves in a straight line. Its displacement *x* in metres is given by

$$x = 70e^{-\frac{t}{10}} - 20t,$$

where *t* is the time in seconds.

(i) Find the initial displacement of the particle. **1**

(ii) Will the particle ever come to rest? **2**
Justify your answer using appropriate calculations.

(iii) Find the distance travelled by the particle in the first 3 seconds. **2**
Give the answer correct to 2 decimal places.

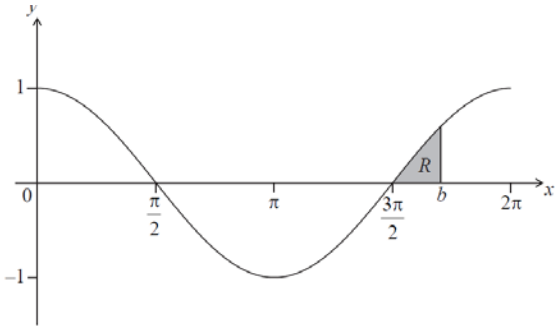
(iv) Find the speed that the particle is approaching as $t \to \infty$. **1**

End of Question 14

Question 15 (15 marks) Use a New Writing Booklet.

(a) Let $f(x) = \cos x$ for $0 \le x \le 2\pi$.

The shaded region *R* is enclosed by the graph of $f(x)$, the line $x = b$, where $b > \frac{3\pi}{2}$ and the *x*- axis.



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The area of R is $\left(1-\frac{\sqrt{3}}{2}\right)$ *units*^{2}. Find the value of *b*. **3**

(b) The amount of caffeine, $C(t)$, in milligrams in your system after drinking a cappuccino is given by,

$$C(t) = 105e^{-kt},$$

where *k* is a constant and *t* is the time in hours that have passed since drinking the cappuccino.

(i) After one hour the caffeine in your system has decreased by 40%. Find the exact value of *k*. **2**

(ii) When will there be 10 milligrams of caffeine remaining in your system? Give the answer correct to 2 significant figures. **1**

Question 15 continues on next page

(c) For the parabola $x^2 - 2x = 3 - y$, find:

(i) the coordinates of the vertex. **1**

(ii) the coordinates of the focus. **1**

(d) The rate at which a Jacaranda tree grows is given by

$$\frac{dh}{dt} = \frac{110}{(t+4)^2} \text{ metres per year},$$

where h is the height of the tree in metres and t is the number of years that have passed since the tree was planted from an established seedling, 0.5 m in height.

(i) Find the rate at which the tree is growing when $t = 5$. **1**

(ii) Find the height of the tree when $t = 5$, correct to 1 decimal place. **3**

(e) Find the volume of the solid formed when the semi-circle

$y = \sqrt{r^2 - x^2}$ is rotated about the y-axis.

(Where r is the radius of the semi-circle.)

Give your answer in terms of r. **3**

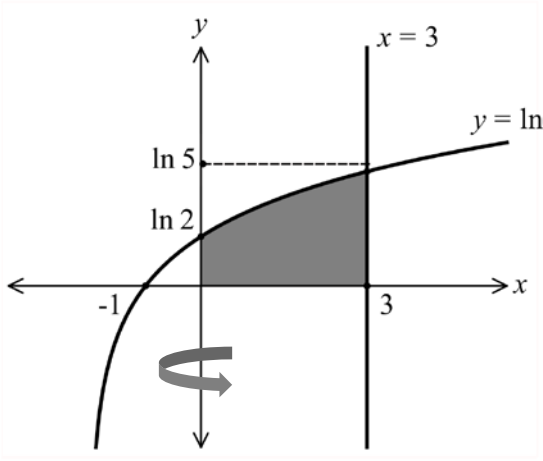
End of Question 15

Question 16 (15 marks) Use a New Writing Booklet.

(a) Solve for x. $\log_2(x+1) - \log_4(x+1) = 2$ **2**

(b) The diagram shows the graph of $y = \ln(x+2)$.

The shaded region is bounded by $y = \ln(x+2)$, $x = 3$ and the coordinate axes.



NOT TO SCALE

(i) Rewrite $y = \ln(x+2)$ in the form $x = e^y + b$, where b is a constant. **1**

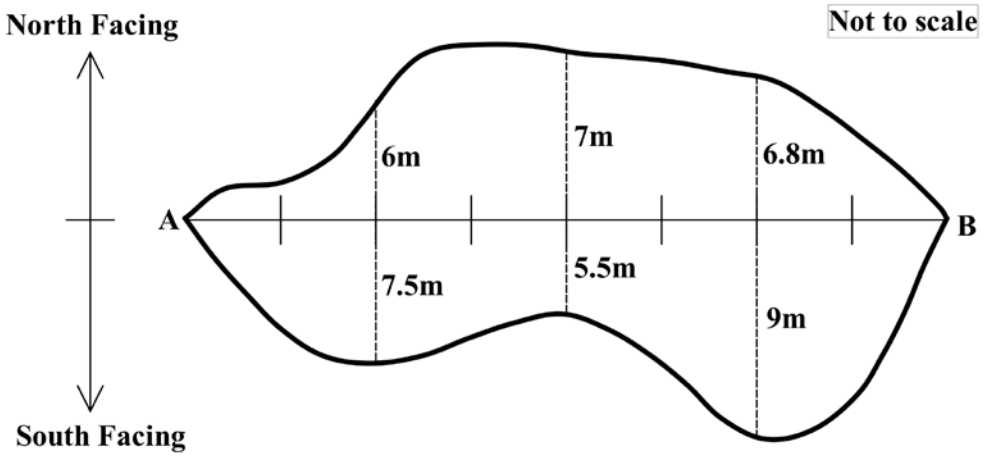
(ii) Find the volume of the solid formed when the shaded area is rotated about the y - axis. Give the answer correct to 1 decimal place. **3**

Question 16 continues on next page

Question 16 (continued)

(c) The diagram shows a sun shade that is to be installed in the playground of a pre-school. *AB* is 20 m long and is divided into 4 equal intervals. *AB* divides the sun shade into the north facing and south facing sections.

The installers of the sun shade have measured the vertical lengths from *AB* to the edges of the north facing and south facing sections.



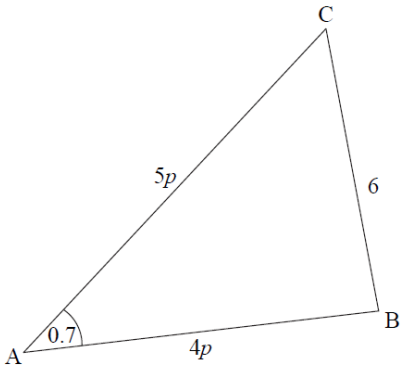
In order to calculate an approximation for the area of the sun shade the installers decided to use the Trapezoidal rule for the north facing section and the Simpson's rule for the south facing section.

(i) Give a reason as to why the Simpson's rule is a better option than the Trapezoidal rule for calculating the approximate area of the south facing section of the sun shade. **1**

(ii) Find the approximate area of the sun shade calculated by the installers giving the answer to the nearest square metre. **2**

Question 16 continues on next page

(d) In ΔABC, $BC = 6$ metres, $\angle CAB = 0.7$ radians, $AB = 4p$ and $AC = 5p$, where $p > 0$.



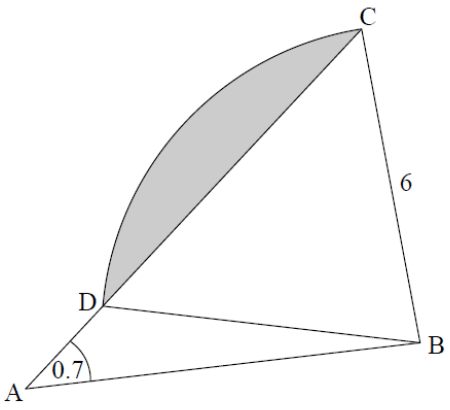
Not to scale

(i) Find the value of p, correct to 2 decimal places. **2**

(ii) A circle is constructed on ΔABC with centre B, passing through point C.

Part of this circle is shown in the diagram below.

The circle intersects the line CA at D and $\angle ADB$ is obtuse.



Not to scale

Find the size of $\angle ADB$ in radians correct to 2 decimal places. **2**

(iii) Find the shaded area correct to 2 decimal places. **2**

END OF PAPER