SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2002

MATHEMATICS

9:00am – 12:05 pm
Thursday 29th August 2002

General Instructions

- Reading time : 5 minutes
- Working time: 3 hours
- Write using blue or black pen
- ***Write your name and teacher's name on each answer booklet***
- Board approved calculators may be used
- A table of standard integrals is provided

- Total Marks **120**
- Attempt Questions 1 – 10
- All questions are of equal value

Students are reminded that this is a trial examination only and cannot in any way guarantee the content or the format of the 2002 Mathematics Higher School Certificate examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $(5-2a)-(3-7a)$. **2**

(b) Rationalise the denominator and simplify $\frac{8}{3\sqrt{2}}$. **2**

(c) Solve $|x-1| \geq 5$, and graph your solution on a number line. **2**

(d) Find the point of intersection of the lines $x+2y=1$ and $2x+y=8$. **2**

(e) Find the value of $x \sin x$ if $x = 0.8$ radians, giving your answer correct to three significant figures. **2**

(f) A landscape gardener quotes the cost of a job at \$4763, which includes 10% GST. What is the cost of the job before GST is added? **2**

QUESTION 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate the following functions:

(i) $\dfrac{1}{x^2-3}$ **2**

(ii) $x^2 \sin 2x$ **2**

(iii) $\dfrac{x}{\log_e x}$ **2**

(b) Find the following indefinite integrals:

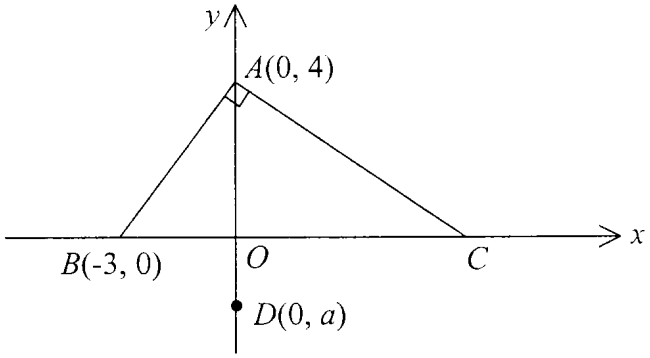
(i) $\int (2x+5)^3 \, dx$ **2**

(ii) $\int 4e^{3-2x} \, dx$ **2**

(c) $\int_4^8 \dfrac{dx}{x-2} = \log_e N$. Find the value of N. **2**

QUESTION 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



$A(0, 4)$ and $B(-3, 0)$ are points on a number plane. The line through A perpendicular to AB meets the x axis at C.

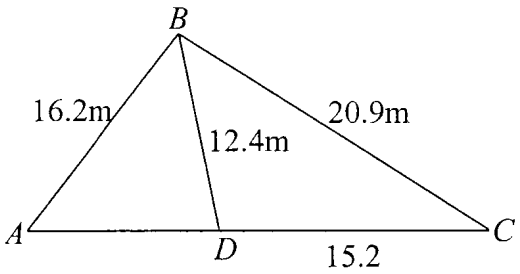
(i) Show that the equation of AC is $3x + 4y - 16 = 0$. **2**

(ii) Find the coordinates of C. **1**

(iii) Find the area of the triangle ABC. **1**

(iv) The point $D(0, a)$ lies on the y axis below the point A. Find the coordinates of D if it is 4 units from AC. **3**

(b)



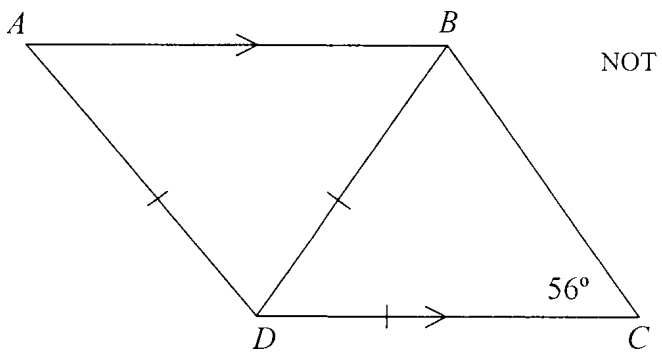
A mast BD on a yacht is inclined to the vertical and is held in position by two wires AB and CB, where A, B, C, D are in the same vertical plane. AB is 16.2 metres and CB is 20.9 metres, as shown in the diagram.
The wire CB is attached to the deck of the yacht at C, 15.2 metres from D.
A, D, and C are all in the same horizontal line.

(i) Find the angle at which the mast BD is inclined to the vertical, to the nearest degree. **3**

(ii) Use the Sine Rule to find the angle at which the wire AB is inclined to the horizontal. **2**

QUESTION 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram, $AB \parallel DC$ and $AD = BD = DC$. $\angle BCD = 56°$.

(i) Find the size of $\angle ADB$, giving reasons. **3**

(ii) Determine if $AD \parallel BC$. Give reasons. **1**

(b) Solve $4\sin^2 x = 3$ for $0 \leq x \leq 2\pi$. **3**

(c) $f(x) = x\sin x - \cos x$. Determine if $f(x)$ is odd, even, or neither. **2**

(d) Prove that the roots of the quadratic equation $2x^2 + mx + (m-2) = 0$ are rational for all rational values of m. **3**

QUESTION 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A parabola has equation $(x-1)^2 = 12(y+2)$.

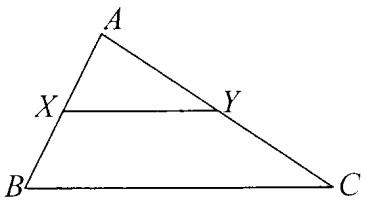
(i) What is the vertex? **1**

(ii) What is the focal length? **1**

(iii) Find the coordinates of the focus. **1**

(iv) Find the equation of the directrix. **1**

(b)

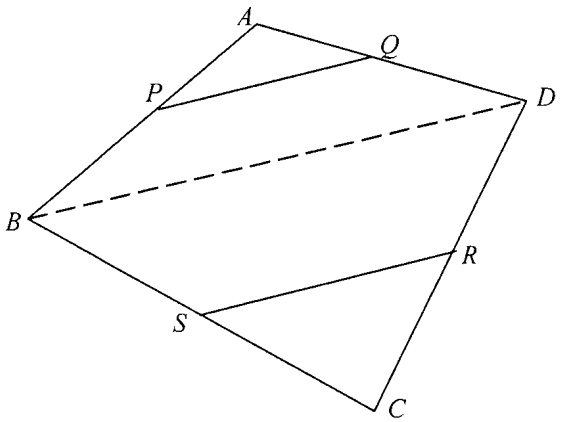


In ΔABC, X is the midpoint of AB and Y is the midpoint of AC.

(i) Prove ΔAXY is similar to ΔABC. **3**

(ii) State why XY is parallel to BC. **1**

(c)



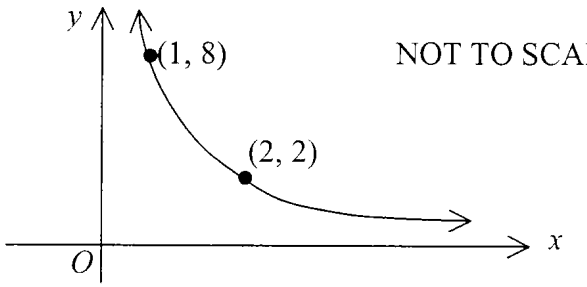
P, Q, R, S are the midpoints of the sides of the quadrilateral $ABCD$ as shown in the diagram.

(i) Using the result of part (b), state why PQ is parallel to SR. **2**

(ii) Hence prove that $PQRS$ is a parallelogram. **2**

QUESTION 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

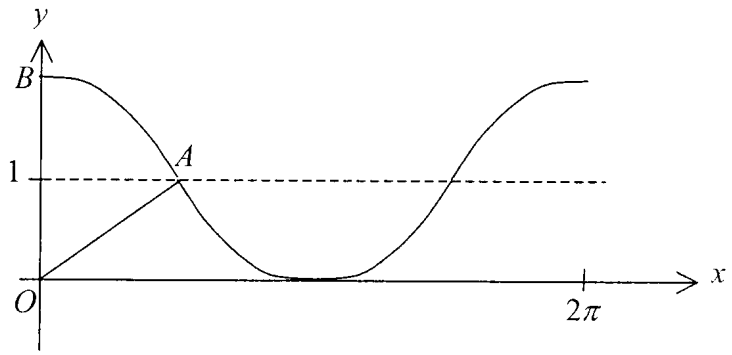
(a)



The diagram shows the graph of $y = \frac{8}{x^2}$ (i.e. $x^2 = \frac{8}{y}$) for $x > 0$. **3**

The arc of the graph between (1, 8) and (2, 2) is rotated about the y-axis.
Find the volume of the solid formed (in exact form).

(b)



The diagram shows the graph of $y = 1 + \cos x$ for $0 \le x \le 2\pi$.
The graph crosses the line $y = 1$ at A, and the y-axis at B.

(i) Show that the y coordinate of A is $\frac{\pi}{2}$. **1**

(ii) Show that the equation of OA is $y = \frac{2}{\pi}x$. **1**

(iii) Hence find the area bounded by the arc AB, the y-axis, and the line OA. **3**

(c) (i) A box of 12 golf balls contains 8 white and 4 yellow balls. **2**
A boy selects 2 golf balls at random.
What is the probability they are different colours?

(ii) Another box of 12 golf balls contains 6 white, 4 yellow and 2 pink balls. A boy selects 2 golf balls at random. **2**
What is the probability they are different colours?

QUESTION 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

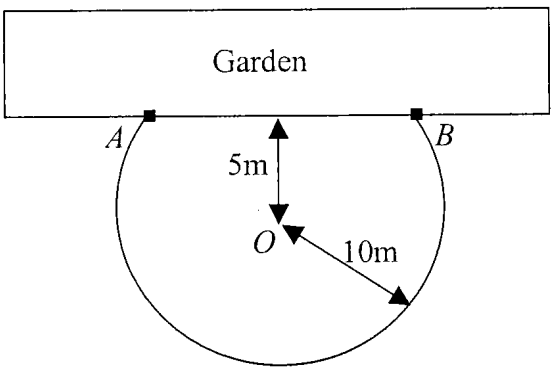
(a) The College gardener knows that the probability of a seedling growing to maturity is 0.95.

(i) If the gardener plants 2 seedlings, what is the probability that neither will survive to maturity? **1**

(ii) If the gardener plants 5 seedlings, what is the probability that at least one seedling will die before reaching maturity? **2**

(iii) If the gardener plants n seedlings, what is the maximum value of n if the probability that at least one seedling will die before reaching maturity is less than 0.5? **3**

(b)



A water sprinkler covers a circular lawn area of radius 10 metres.
The sprinkler (O) is placed 5 metres from a rectangular garden bed.

(i) Garden stakes are placed at A and B.
Show that $\angle AOB = \dfrac{2\pi}{3}$ radians. **1**

(ii) What area of lawn will the sprinkler cover? **2**

(i) What will be the total perimeter of the lawn area covered by the sprinkler? **3**

QUESTION 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The rate of growth of a certain population of rodents is given by the equation $\frac{dP}{dt} = 200(0.4 - 0.08t)$ where P is the population after t months.
Initially there are 200 rodents.

(i) Show that the population after t months is $P = 200(1 + 0.4t - 0.04t^2)$. **2**

(ii) In how many months will the initial population double itself? **2**

(b) Consider the function $y = 5xe^{-x}$.

(i) Copy and complete the following table, giving the value to 2 decimal places. **1**

x	0	1	2	3	4
y	0	1.84	1.35		0.37

(ii) Find $\int_0^4 5xe^{-x}dx$ using Simpson's Rule with 5 function values. **2**

(iii) Find $\frac{dy}{dx}$. **1**

(iv) Find the value of x for which $\frac{dy}{dx} = 0$. **1**

(v) Show that $\frac{d^2y}{dx^2} = 5e^{-x}(x-2)$. **2**

(vi) What is the x coordinate of the point of inflection on the graph of $y = 5xe^{-x}$? **1**

QUESTION 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Two particles A and B travelling in the same straight line, pass through the point O at the same time.
The positions x metres of each particle t seconds after passing through O are given by:
$x_A = \frac{1}{2}t(t-4)$ and $x_B = 4t$.

(i) Find when the particles are next at the same point. **2**

(ii) Find the velocity of each particle when they meet at the point determined in part (i). **2**

(iii) Find when the velocity of A is the same as the velocity of B. **1**

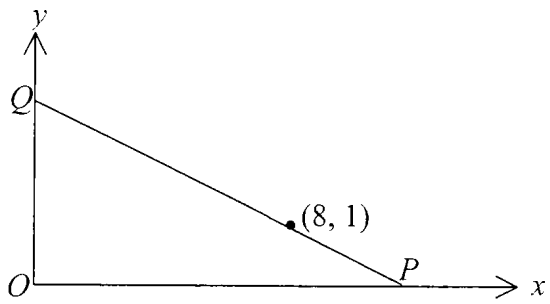
(iv) Are the accelerations of A and B ever equal? Give reasons. **1**

(b) A high-rise building consisting of 20 floors is to be constructed.

(i) The cost of constructing each floor is 10% more than the floor below it. If the first floor cost \$165 000, what is the total cost of the complete building? **3**

(ii) Each floor contains 4 square metres less glass than the floor below it. The total area of glass in the building is 1720 square metres. How much glass was used in the first floor? **3**

QUESTION 10 (12 marks) Use a SEPARATE writing booklet. **Marks**



A line is drawn through the point (8, 1) to cut the positive x axis at P and the positive y axis at Q. The gradient of PQ is m.

(a) Find the equation of PQ in terms of m. **1**

(b) Show that the coordinates of P are $\left(\frac{8m-1}{m}, 0\right)$. **1**

(c) Find the coordinates of Q. **2**

(d) Show that the area of ΔOPQ is $\frac{1}{2}\left(16-64m-\frac{1}{m}\right)$. **2**

(e) Find the value of m for which this area is a minimum, noting that m is negative. **4**

(f) Show that the minimum area is 16 unit^2. **2**

End of Examination

SAINT IGNATIUS' COLLEGE

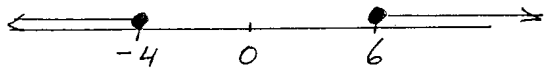
Trial Higher School Certificate

2002

MATHEMATICS

Suggested Solutions

Saint Ignatius' College Riverview 2002

Mathematics: Question 1		
Suggested Solutions	Marks Awarded	Marker's Comments
(a) $(5-2a)-(3-7a)$ $= 5-2a-3+7a$ $= 5a+2$ (2)		
(b) $\frac{8}{3\sqrt{2}} = \frac{8}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{6} = \frac{4\sqrt{2}}{3}$ (2)		
(c) $\lvert x-1 \rvert \geq 5$ $x-1 \leq -5$ or $x-1 \geq 5$ $x \leq -4$ or $x \geq 6$.  (2)		
(d) $x+2y=1 \Rightarrow 2x+4y=2$ (1) $2x+y=8 \Rightarrow 2x+y=8$ (2) (1) $-$ (2) $3y=-6$ $y=-2$ Subst. $y=-2$ into (1): $x-4=1$ $x=5$ Point of intersection is $(5,-2)$. (2)		
(e) $x \sin x = 0.8 \times \sin 0.8$ $= 0.574$ (3 sig. fig.) (2)		
(f) $1.1 \times \text{price} = 4763$ $\text{price} = \frac{4763}{1.1}$ Before GST, the price is \$4330 (2)		

Mathematics: Question 2

Suggested Solutions	Marks Awarded	Marker's Comments
(a) (i) $\frac{d}{dx}\frac{1}{x^2-3} = \frac{d}{dx}(x^2-3)^{-1}$ $= -1(x^2-3)^{-2} \times (2x)$ $= \frac{-2x}{(x^2-3)^2}$ (2)		
(ii) $\frac{d}{dx} x^2 \sin 2x = (\sin 2x)(2x) + (x^2)(2\cos 2x)$ $= 2x \sin 2x + 2x^2 \cos 2x$ (2)		
(iii) $\frac{d}{dx}\frac{x}{\log_e x} = \frac{\log_e x \times 1 - x \times \frac{1}{x}}{(\log_e x)^2}$ $= \frac{\log_e x - 1}{(\log_e x)^2}$ (2)		
(b)(i) $\int (2x+5)^3 dx = \frac{1}{4} \times \frac{1}{2}(2x+5)^4 + C$ $= \frac{1}{8}(2x+5)^4 + C$ (2)		
(ii) $\int 4e^{3-2x} dx = 4 \times \left(-\frac{1}{2}\right) e^{3-2x} + C$ $= -2e^{3-2x} + C$ (2)		
(c) $\int_4^8 \frac{dx}{x-2} = \left[\log_e(x-2)\right]_4^8$ $= \log_e 6 - \log_e 2$ $= \log_e 3$ $\therefore N = 3.$ (2)		

Mathematics: Question 3.

Suggested Solutions	Marks Awarded	Marker's Comments

(a) (i) Grad $AB = \frac{4}{3}$ $\therefore$ grad $AC = -\frac{3}{4}$.

Eqn. AC: $y = -\frac{3}{4}x + 4$ $[y = mx + b]$

$4y = -3x + 16$

$3x + 4y - 16 = 0$ (2)

(ii) When $y = 0$, $x = \frac{16}{3}$. C is $(\frac{16}{3}, 0)$ (1)

(iii) Area $\triangle ABC = \frac{1}{2} \times 8\frac{1}{3} \times 4$

$= 16\frac{2}{3}$ unit^2 (1)

(iv) Perp. dist.: $\left| \frac{0 + 4a - 16}{\sqrt{3^2 + 4^2}} \right| = 4$

$\frac{4a - 16}{5} = 4$ or $\frac{4a - 16}{5} = -4$

$a = 9$ or $a = -1$.

$\therefore$ D is $(0, -1)$. (3)

(b)(i) In $\triangle BDC$, by Cosine Rule.

$\cos \angle BDC = \frac{12 \cdot 4^2 + 15 \cdot 2^2 - 20 \cdot 9^2}{2 \times 12 \cdot 4 \times 15 \cdot 2}$

$= -0 \cdot 13797$

$\angle BDC = 98°$ (nearest degree)

$\therefore$ Mast is inclined at 8° to vertical. (3)

(ii) In $\triangle BAD$, by Sine Rule.

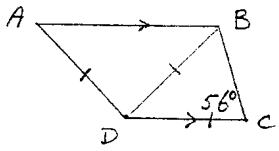
$\frac{\sin \angle BAD}{12 \cdot 4} = \frac{\sin 82°}{16 \cdot 2}$

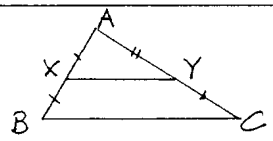
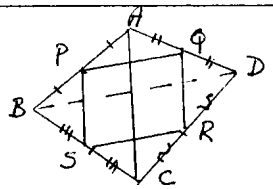
$\sin \angle BAD = \frac{12 \cdot 4 \sin 82°}{16 \cdot 2}$

$= 0 \cdot 75798$

$\angle BAD = 49°$ (nearest degree) (2)

Mathematics: Question 4

Suggested Solutions	Marks Awarded	Marker's Comments
(a).  (i) $\angle BDC = 180^\circ - 2\times 56^\circ$ (Angle sum of isosceles triangle) $= 68^\circ$ $\angle ABD = \angle BDC = 68^\circ$ (alternate angles, $AB \parallel DC$) $\angle ADB = 180^\circ - 2\times 68^\circ$ (Angle sum of isosceles triangle) $= 44^\circ$ (3) (ii) $\angle ADC = 44^\circ + 68^\circ = 112^\circ$. $\angle ADC + \angle BCD = 112^\circ + 56^\circ = 168^\circ$. Since co-interior angles are not supplementary, AD is not parallel to BC. (1)		
(b) $4\sin^2 x = 3$ $\sin x = \pm \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (3)		
(c) $f(x) = x\sin x - \cos x$. $f(-x) = (-x)\sin(-x) - \cos(-x)$ $= x\sin x - \cos x$ $= f(x)$ $\therefore f(x)$ is even. (2)		
(d) $2x^2 + mx + (m-2) = 0$ $\Delta = m^2 - 4\times 2\times(m-2)$ $= m^2 - 8m + 16$ $= (m-4)^2$. Since Δ is a perfect square, the roots are rational. (3)		

Mathematics: Question 5		
Suggested Solutions	Marks Awarded	Marker's Comments
(a) $(x-1)^2 = 12(y+2)$ (i) Vertex $(1,-2)$ (1) (ii) Focal length = 3 units (1) (iii) Focus $(1,1)$ (1) (iv) Directrix $y=-5$. (1)		
(b)(i)  (i) $\frac{AX}{AB} = \frac{1}{2}$, $\frac{AY}{AC} = \frac{1}{2}$, $\angle A$ is common $\therefore \triangle AXY \parallel\parallel \triangle ABC$ (2 sides in same ratio, included angles equal) (3) (ii) $\angle AXY = \angle ABC$ (corresponding angles in similar triangles) $\therefore XY \parallel BC$ (corresponding angles equal). (1)		
(c)  (i) By (b)(i) $PQ \parallel BD$ and $SR \parallel BD$ $\therefore PQ \parallel SR$ (2) (ii) In $\triangle ABC$, $PS \parallel AC$ by (b)(i) In $\triangle ADC$, $QR \parallel AC$ by (b)(i) $\therefore PS \parallel QR$. $\therefore$ Both pairs of opposite sides are parallel. $\therefore PQRS$ is a parallelogram. (2)		

Mathematics: Question 6

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $V = \pi \int x^2 dy$ $= \pi \int_2^8 \frac{8}{y} dy$ $= \pi [8 \ln y]_2^8$ $= 8\pi [\ln 8 - \ln 2]$ Volume $= 8\pi \ln 4$ unit^3 (3)		
(b)(i) When $y=1$, $1+\cos x = 1$ $\cos x = 0 \therefore x = \frac{\pi}{2}$ (1) (ii) Grad. $OA = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$. $\therefore OA: y = \frac{2}{\pi} x$ (1) (iii) $A = \int_0^{\frac{\pi}{2}} (1 + \cos x - \frac{2}{\pi} x) dx$ $= [x + \sin x - \frac{1}{\pi} x^2]_0^{\pi/2}$ $= [\frac{\pi}{2} + 1 - \frac{1}{\pi} \times \frac{\pi^2}{4}] - [0 + 0 - 0]$ Area $= (1 + \frac{\pi}{4})$ unit^2 (3)		
(c)(i) P(diff. colours) $= P(WY) + P(YW)$ $= \frac{8}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{8}{11}$ $= \frac{64}{132}$ or $\frac{16}{33}$ (2) (ii) 6W, 4Y, 2P. P(diff. colours) $= P(W, \text{non } W) + P(Y, \text{non } Y) + P(P, \text{non } P)$ $= \frac{6}{12} \times \frac{6}{11} + \frac{4}{12} \times \frac{8}{11} + \frac{2}{12} \times \frac{10}{11}$ $= \frac{88}{132}$ or $\frac{22}{33}$. (2)		

Mathematics: Question 7

Suggested Solutions	Marks Awarded	Marker's Comments

(a) (i) $P(\text{neither survive}) = 0.05 \times 0.05$

$= 0.0025$ (1)

(ii) $P(\text{at least one dies}) = 1 - P(\text{none die})$

$= 1 - (0.95)^5$

$= 0.226$ (3 d.p.) (2)

(iii) $P(\text{at least one dies}) < 0.5$

$1 - P(\text{none die}) < 0.5$

$1 - (0.95)^n < 0.5$

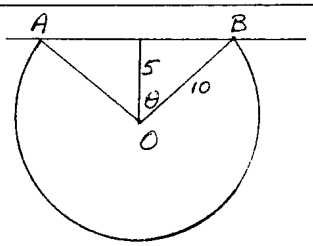
$(0.95)^n > 0.5$

$n < \dfrac{\log 0.5}{\log 0.95}$

$n < 13.5$

Maximum value of n is 13. (3)

(b)



(i) $\cos\theta = \frac{5}{10}$ $\therefore \theta = 60^\circ = \frac{\pi}{3}$.

$\therefore \angle AOB = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}$ radians. (1)

(ii) $\text{Area} = \frac{1}{2} \times 10^2 \times \frac{4\pi}{3} + \frac{1}{2} \times 10^2 \times \sin\frac{2\pi}{3}$

$= 50\left(\frac{4\pi}{3} + \frac{\sqrt{3}}{2}\right)$

$= 252.7\text{ m}$ (1 d.p.) (2)

(iii) $\text{Perimeter} = 10 \times \frac{4\pi}{3} + 2 \times \sqrt{75}$

$= \frac{40\pi}{3} + 10\sqrt{3}$

$= 59.2\text{ m}$ (1 d.p.) (3)

Mathematics: Question 8

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $\frac{dP}{dt} = 200(0.4 - 0.08t)$

(i) $P = 200(0.4t - 0.04t^2) + K.$

$t=0, P=200: \quad 200 = 200(0-0) + K$

$K = 200$

$\therefore P = 200(0.4t - 0.04t^2) + 200$

$= 200(1 + 0.4t - 0.04t^2)$ (2)

(ii) When $P = 400$, $400 = 200(1+0.4t-0.04t^2)$

$2 = 1 + 0.4t - 0.04t^2$

$0.04t^2 - 0.4t + 1 = 0$

$t = \frac{0.4 \pm \sqrt{(0.4)^2 - 4 \times 0.04 \times 1}}{0.08}$

$= 5$

Population doubles in 5 months. (2)

(b)(i)

x	0	1	2	3	4
y	0	1.84	1.35	0.75	0.37

(1)

(ii) $\int_0^4 f(x)\,dx \doteq \frac{1}{3}\left[0 + 4 \times 1.84 + 2 \times 1.35 + 4 \times 0.75 + 0.37\right]$

$\doteq 4.48$ (2 d.p.) (2)

(iii) $y = 5xe^{-x}$

$\frac{dy}{dx} = e^{-x} \times 5 + 5x \times (-e^{-x})$

$= e^{-x}(5 - 5x)$ (1)

(iv) $\frac{dy}{dx} = 0 : \quad x = 1$ (1)

(v) $\frac{d^2y}{dx^2} = (5 - 5x)(-e^{-x}) + e^{-5x} \times (-5)$

$= -e^{-x}(10 - 5x)$ (2)

(vi) At point of inflection, $\frac{d^2y}{dx^2} = 0$

i.e. at $x = 2$ (1)

Mathematics: Question 9

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $x_A = \frac{1}{2}t(t-4)$ $\quad x_B = 4t$ (i) $x_A = x_B$: $\frac{1}{2}t(t-4) = 4t$ $t^2 - 4t = 8t$ $t^2 - 12t = 0$ $t(t-12) = 0$. Particles next meet after 12 sec. (2) (ii) $v_A = t - 2$ $\quad v_B = 4$ Velocity of A = 10 m/s ; of B = 4 m/s. (2) (iii) $v_A = v_B$: $t - 2 = 4$ $t = 6$ Velocities are same after 6 sec. (1) (iv) $a_A = 1$, $\quad a_B = 0$ Accelerations are never the same. (1)		
(b) (i) $C = 165000 + 165000 \times 1{\cdot}1 + 165000 \times 1{\cdot}1^2 + \ldots$ $= 165000 \times \frac{1{\cdot}1^{20} - 1}{1{\cdot}1 - 1}$ Cost = \$ 9 450 375 (3) (ii) If A m² is area of glass on first floor $A + (A-4) + (A-8) + \ldots = 1720$ $\frac{20}{2}[2A + 19 \times (-4)] = 1720$ $2A - 76 = 172$ $A = 124$ First floor has 124 m² of glass. (3)		

Mathematics: Question 10

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $PQ: \quad y - 1 = m(x-8)$ (1)

(b) When $y=0, \quad -1 = m(x-8)$

$$mx = 8m - 1$$

$$x = \frac{8m-1}{m}$$

$\therefore P$ is $\left(\frac{8m-1}{m}, 0\right)$ (1)

(c) When $x=0, \quad y = -8m+1$

Q is $(0, 1-8m)$ (2)

(d) Area $\triangle POQ = \frac{1}{2} \times \frac{8m-1}{m} \times (1-8m)$

$$= \frac{1}{2}\left(8 - \frac{1}{m}\right)(1-8m)$$

$$= \frac{1}{2}\left(8 - 64m - \frac{1}{m} + 8\right)$$

$$= \frac{1}{2}\left(16 - 64m - \frac{1}{m}\right)$$ (2)

(e) $\frac{dA}{dm} = \frac{1}{2}\left(-64 + \frac{1}{m^2}\right)$

$= 0$ when $\frac{1}{m^2} = 64$

$$m = \pm \frac{1}{8}$$

But $m<0 \quad \therefore m = -\frac{1}{8}$.

$\frac{d^2A}{dm^2} = \frac{1}{2}\left(0 - \frac{2}{m^3}\right)$

When, $m = -\frac{1}{8}$, $\frac{d^2A}{dm^2} = \frac{1}{2} \times \frac{-2}{\left(-\frac{1}{8}\right)^3}$

$$> 0$$

$\therefore$ Area is minimum when $m = -\frac{1}{8}$ (4)

(f) When $m = -\frac{1}{8}$, $A = \frac{1}{2}\left[16 - 64 \times \left(-\frac{1}{8}\right) - \frac{1}{\left(-\frac{1}{8}\right)}\right]$

$$= \frac{1}{2}[16 + 8 + 8]$$

$$= 16$$

Minimum area is 16 unit^2 (2)