Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\dfrac{x+\frac{1}{x}}{x-\frac{1}{x}}$ when $x = 2.7$, correct to three significant figures. **2**

(b) If $f(x) = 3x^2 - 5x$, evaluate f(-2). **1**

(c) Completely factorise $x^3 + 3x^2 - 4x - 12$. **2**

(d) Rationalise the denominator and simplify: $\dfrac{6}{3\sqrt{2} - 4}$. **2**

(e) If $\tan\theta = 2.8$ and θ is an acute angle measured in radians, find the value of θ correct to 2 decimal places. **1**

(f) Sketch the graph of $y = \sqrt{9 - x^2}$. **2**

(g) Find $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$. **2**

QUESTION 2 (12 Marks) Use a SEPARATE writing booklet. **Marks**

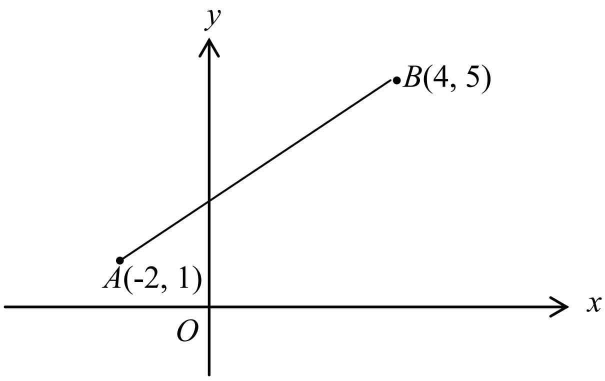
(a) The centre of a circle is (2, -3) and one end-point of a diameter is (4, 1).

(i) Find the coordinates of the other endpoint of the diameter. **2**

(ii) Find the radius of the circle. **1**

(iii) Write down the equation of the circle. **1**

(b)



On the number plane, $A(-2, 1)$ and $B(4, 5)$ are points and M is the midpoint of AB.

(i) Show that the equation of the line AB is $2x - 3y + 7 = 0$. **2**

(ii) Find the equation of the line through B which is perpendicular to AB, writing your answer in general form. **2**

(iii) Find the angle which the line AB makes with the positive direction of the x axis, to the nearest degree. **1**

(iv) The line $2x - 3y - 19 = 0$ is parallel to AB.

(α) Find the y coordinate of the point on $2x - 3y - 19 = 0$ which has an x coordinate of 5. **1**

(β) Hence find the perpendicular distance between the lines $2x - 3y + 7 = 0$ and $2x - 3y - 19 = 0$. **2**

QUESTION 3 (12 Marks) Use a SEPARATE writing booklet. **Marks**

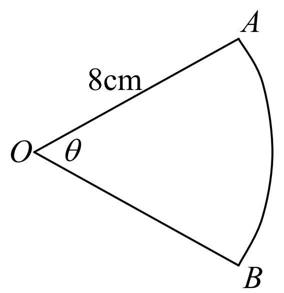
(a) Solve $(\tan x + 1)(2\sin x - 1) = 0$, for $0 \le x \le 2\pi$. **3**

(b) Differentiate the following functions:

(i) $\cos(3x + 2)$ **1**

(ii) $\dfrac{x}{\tan x}$ **2**

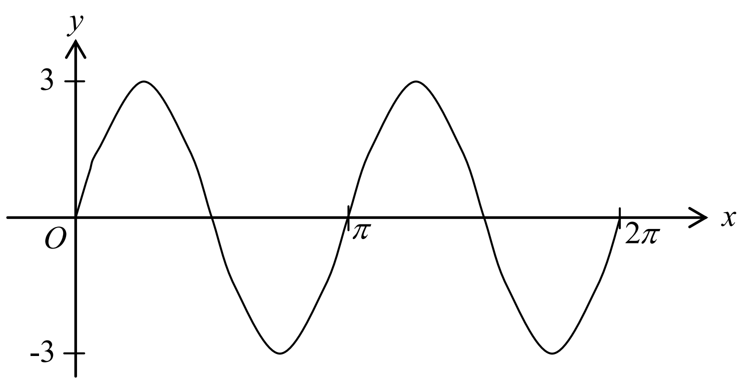
(c) **2**



The area of the sector OAB is 19.2cm², and its radius is 8cm.

Find the size of the angle θ at the centre of the sector.

(d)



The diagram shows the graph of a trigonometric function $y = f(x)$.

(i) State the period of the function. **1**

(ii) Write down the equation of the graph. **1**

(iii) Copy the graph of $y = f(x)$ into your writing booklet. **2**
On the same set of axes, draw the graph of $y = 3 - 2x$.
How many solutions does the equation $f(x) = 3 - 2x$ have?

QUESTION 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

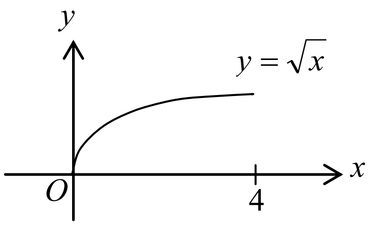
(a) Find the indefinite integrals:

(i) $\int (2x-1)^5 \, dx$ **2**

(ii) $\int e^{3x-1} \, dx$ **1**

(iii) $\int \frac{1}{5x} \, dx$ **1**

(b)



The diagram shows the graph of $y = \sqrt{x}$, for $0 \le x \le 4$.

(i) Find the area of the region bounded by the graph of $y = f(x)$, the x axis, and the line $x = 4$. **3**

(ii) If this region is rotated around the x axis, find the volume of the solid formed. **3**

(c) **2**

x	0	2	4	6
$f(x)$	3	7	9	5

Using the table of values, find an estimate of $\int_0^6 f(x)\, dx$ using the trapezoidal rule.

QUESTION 5 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) The limiting sum (i.e. sum to infinity) of the series $1+(x+1)+(x+1)^2+(x+1)^3+.......$ is 4. Find the value of x. **2**

(b) In an arithmetic sequence, the sixth term is 27 and the sum of the first and fifth terms is 30. **4**
Find the first term and common difference of the sequence.

(c) The profits of a company increased by 5% per annum each year.
The profit in the first year of operation was \$35 000.

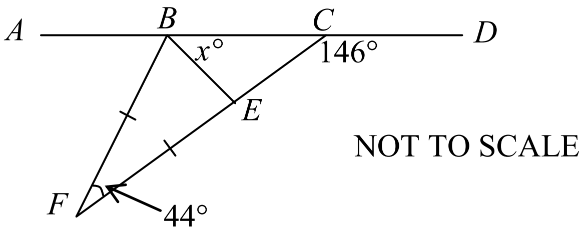
(i) Show that the profit in the tenth year of operation was \$54 296. **1**

(ii) What was the total profit in the first ten years of operation? **2**

(iii) If the profit continues to increase at the same rate, in what year would the total profits exceed \$600 000? **3**

QUESTION 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)

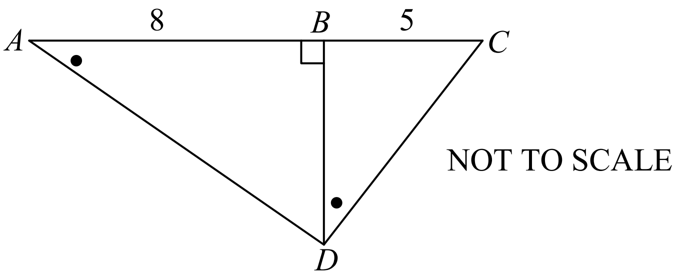


In the diagram, $ABCD$ is a straight line, and E lies on CF.
$BF = EF$, $\angle BFE = 44°$, $\angle DCE = 146°$, $\angle CBE = x°$.

(i) Find the value of x giving reasons. **3**

(ii) State why $BE = EC$. **1**

(b)

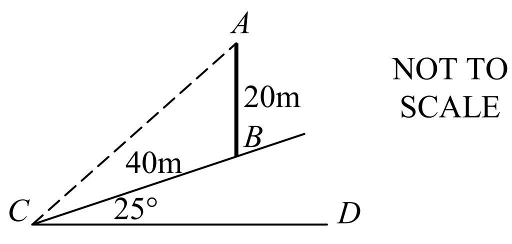


In the diagram, $BD \perp AC$, $\angle BAD = \angle BDC$.

(i) State why ΔABD is similar to ΔBCD. **1**

(ii) If AB = 8cm and BC = 5cm, find the length of BD. **2**

(c)



A 20 metre high vertical mast AB is placed 40 metres from the base C of a slope inclined at 25° to the horizontal. A wire support AC is used to keep it in position.

(i) Explain why $\angle ABC = 115°$. **1**

(ii) Show that the length of the wire AC is 51.7m (to 1 decimal place). **2**

(iii) Hence find the angle which the wire AC makes with the slope CB. **2**

QUESTION 7 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) A particle moves in a straight line. At time t seconds, its distance is x metres from a fixed point O and its velocity (v m/s) is given by the equation

$$v = 4t - 3t^2 .$$

Initially the particle is at $x = 3$.

(i) Find the position of the particle when $t = 2$. **3**

(ii) Find the velocity of the particle when the acceleration is zero. **2**

(iii) Find the acceleration of the particle when the particle becomes instantaneously at rest during the motion. **2**

(b) The rate of flow of water into a large container is given by:

$$\frac{dV}{dt} = \frac{30}{t+1}$$

where V is in litres and t is in minutes.
Initially, there is 40 litres of water in the container.

(i) Find the volume of water in the container after 4 minutes. **3**

(ii) How long does it take for the container to hold 160 litres? **2**

QUESTION 8 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\log(6x-1)-\log(x+2)=\log 4$. **2**

(b) Solve $3^{x-1}=7$, giving your answer to three significant figures. **2**

(c) Consider the function $y = x\log_e x$.

(i) Find $\dfrac{dy}{dx}$. **1**

(ii) Find the coordinates of the stationary point, and determine its nature. **3**

(iii) Copy and complete the table of values (to 2 decimal places). **2**

x	0.1	0.05	0.01
$x\log_e x$			

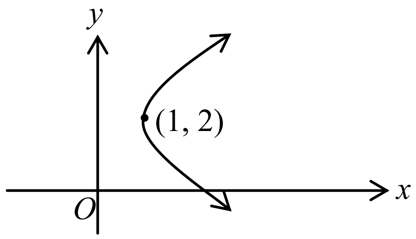
What is $\lim_{x\to 0}(x\log_e x)$?

(iv) Sketch the graph of the function $y = x\log_e x$. **2**

QUESTION 9 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the values of k if the roots of the equation $2x^2 - 5x + k = 0$ are real and different. **2**

(b)



The diagram shows the graph of the parabola $(y-2)^2 = 8(x-1)$, with vertex at (1, 2).

(i) What is the focal length of the parabola? **1**

(ii) Write down the coordinates of the focus. **1**

(iii) What is the equation of the directrix? **1**

(iv) Copy or trace this diagram onto your writing page. On the same set of axes, draw the graph of $(y-2)^2 = -8(x-1)$. **1**

(c) The probabilities that Alex, Bob and Colin will pass the next Probability test are 0.9, 0.8 and 0.7 respectively.

(i) Show that the probability that they all pass is greater than 50%. **1**

(ii) Show that the probability that Alex fails and Bob and Colin pass is 0.056. **1**

(iii) Find the probability that exactly two of the boys pass the test. **2**

(iv) Find the probability that at least one of the three boys pass the test. **2**

QUESTION 10 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Find the x coordinate of the point on the graph of $y = 5 - 14x - 2x^2$ where the tangent is parallel to the line $2x + y - 3 = 0$. **2**

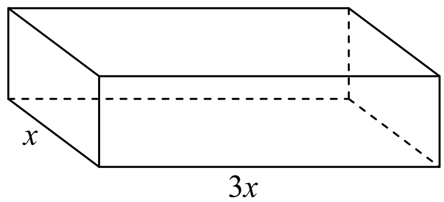
(ii) Hence find the equation of the tangent in general form. **2**

(b) For a particular function, $\frac{dy}{dx} = x^3 - x^2$.

(i) Show that it has stationary points at $x = 0$ and $x = 1$. **1**

(ii) Determine the nature of the stationary point at $x = 0$. **2**

(c)



An **open** rectangular container, is constructed. The edges where the surfaces meet have a special tape along them to strengthen the joins.
The length of the container is three times its width, and the volume of the container is 96 m³.

(i) If the width of the container is x metres, show that the total length of tape (L metres) that is needed is given by: **1**

$$L = 8x + \frac{128}{x^2}.$$

(ii) Find the width of the container if the length of the tape is to be a minimum. **4**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$