SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2005

MATHEMATICS

8:50am – 11:55am
Monday 22nd August 2005

Directions to Students

• Reading Time : 5 minutes	• Total Marks 120
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• Attempt Question 1 – 10
• Board approved calculators may be used	• All questions are of equal value
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) The circumference of a circle is 9.29 metres. Find the radius of the circle to 3 significant figures, using the formula $C = 2\pi r$. **2**

(b) Differentiate $3x^2 - 4\sqrt{x}$. **2**

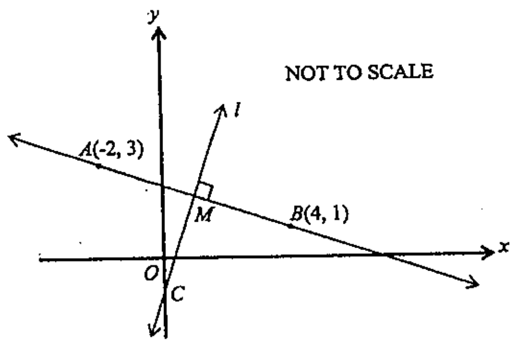
(c) Expand and simplify $(\sqrt{6}+\sqrt{2})^2$. **2**

(d) Solve the equation $a - \dfrac{a-2}{4} = 4$. **2**

(e) Find the values of c for which $|2c+3| > 1$. **2**

(f) The roots of an equation are $\dfrac{2}{3}$ and -4. Write down its equation in the form $ax^2 + bx + c = 0$ where a, b and c are integers. **2**

QUESTION 2 (12 MARKS) Use a SEPARATE writing Booklet **Marks**



The diagram shows the points $A(-2, 3)$ and $B(4, 1)$.
M is the midpoint of the interval AB. l is the line through M perpendicular to AB.

(i) Find the gradient of the line AB. 1

(ii) Find the coordinates of M. 1

(iii) Show that the equation of AB is $x+3y-7=0$. 1

(iv) Show that the equation of l is $3x-y-1=0$. 1

(v) Find the coordinates of C, the point of intersection of l and the y-axis. 1

(vi) Show that C is equidistant from A and B 2

(vii) Find the perpendicular distance from C to the line AB. 2

(viii) Hence find the area of the triangle ABC. 2

(ix) State the inequality that defines the region of the half-plane under the line AB. 1

QUESTION 3 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) Differentiate with respect to x:

(i) $x^3 \cos x$ 2

(ii) $(3+e^{2x})^4$ 2

(b) (i) Find $\int \frac{6x}{x^2+2}\,dx$. 2

(ii) Find $\int_0^{\frac{\pi}{6}} \sec^2 2x\,dx$. 2

(c) The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 - 4x + 1$.

The curve passes through the point (1, 6).

(i) Find the equation of the curve. 2

(ii) What is the equation of the tangent to the curve at the point (1, 6)? 2

QUESTION 4 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) Consider the function $f(x) = x^3 - 6x^2 + 9x$.

(i) Find the coordinates of the stationary points and determine their nature. 3

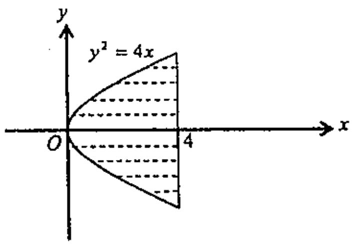
(ii) Find the x-coordinate of the point of inflexion. 2

(iii) Find where the curve meets the x-axis. 1

(iv) Sketch the graph of $y = f(x)$. 1

(v) Find the values of x for which the **gradient** is decreasing. 1

(b)



The diagram shows the region bounded by the curve $y^2 = 4x$ and the line $x = 4$.

(i) Find the exact area of the shaded region. 2

(ii) The region is rotated about the x-axis. 2
Calculate the exact volume of the solid formed.

QUESTION 5 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) Jim decides to start a daily exercise program, in which he increases the time spent on exercise each day by 2 minutes from the previous day. On the first day he does 10 minutes of exercise.

(i) For how long will Jim exercise on the 30th day? 1

(ii) What will be the total length of time spent on exercise in the first 30 days? 2

(iii) On which day would Jim have achieved 40 hours of exercise in total? 3

(b) Jill commences her working career, earning \$30 000 per year in her first year. If she stays with the same company, her annual salary will increase by 5% each year.

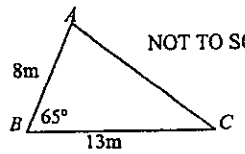
(i) What will be Jill's annual salary in her 20th year (to nearest dollar)? 2

(ii) In what year would her annual salary first exceed \$50 000? 2

(iii) What will be her total earnings in the first 20 years of employment? 2

QUESTION 6 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a)



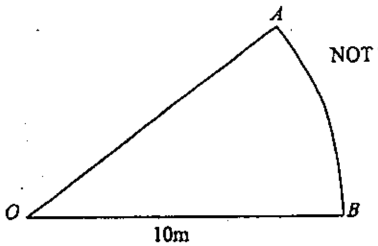
The diagram shows the triangular cross-section of the roofline of a modern building.
$AB = 8$m, $BC = 13$m, and $\angle ABC = 65°$. BC is horizontal.

(i) Find the length of the beam AC. **2**

(ii) Find the angle of elevation of the beam AC. **2**

(iii) Building regulations require a cross-sectional area of at least 48m² for adequate ventilation. Does this design satisfy this regulation? **1**

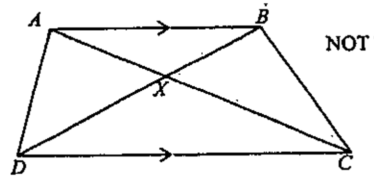
(b)



AOB is the sector of a circle, centre O and radius 10m. **3**
The area of the sector is 60m². Calculate the exact perimeter of the sector AOB.

Question 6 continues on page 7

(c)



$ABCD$ is a trapezium, and AB is parallel to DC.
The diagonals AC and BD intersect at X. $AX = 4$cm, $CX = 10$cm, $BD = 12$cm.

Copy the diagram neatly into your answer booklet.

(i) Prove ΔAXB is similar to ΔCXD. **2**

(ii) Find the length of BX. **2**

End of Question 6

QUESTION 7 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) Find the coordinates of the points of intersection of the line $2x - y - 2 = 0$ and the hyperbola $xy = 4$. **3**

(b) Solve the equation for x:

$$3 \times 3^{2x} + 26 \times 3^{x} - 9 = 0.$$ **3**

(c) For what values of c does the equation $x^2 + 5x + c = 0$ have two real and different roots? **2**

(d) α, β are the roots of the quadratic equation $2x^2 - 5x - 8 = 0$.
Without finding the roots, find the value of:

(i) $\alpha^2 + \beta^2$. **2**

(ii) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$. **2**

QUESTION 8 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) Evaluate $\displaystyle\sum_{r=2}^{4} \frac{1}{r(r-1)}$. **2**

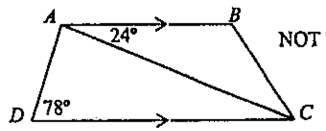
(b) The equation of a parabola is $8y = x^2 - 4x - 4$.
By expressing the equation in the form $(x-h)^2 = 4a(y-k)$, find:

(i) the coordinates of the vertex. **2**

(ii) the coordinates of the focus. **1**

(iii) the equation of the directrix. **1**

(c)



NOT TO SCALE

$ABCD$ is a trapezium and AB is parallel to DC. $\angle BAC = 24°, \angle ADC = 78°$. **2**

Explain why ΔACD is isosceles.

(d) Consider the function $f(x) = \begin{cases} 3 + 2^{-x} & \text{for } x \geq 0 \\ x + 4 & \text{for } x < 0 \end{cases}$.

(i) Evaluate $f(-1) + f(1)$. **2**

(ii) Find $\lim\limits_{x \to \infty} f(x)$. **1**

(iii) Is the function continuous at $x = 0$? Give reasons. **1**

QUESTION 9 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a) The value of a car depreciates exponentially according to the formula $V = Ae^{-kt}$ where V dollars is the value of the car after t years. A car's new price is \$32 000, and after 6 years its value is \$14 000.

(i) Show that $V = Ae^{-kt}$ satisfies the equation $\frac{dV}{dt} = -kV$. 1

(ii) What is the value of A? 1

(iii) Find the value of k correct to 3 significant figures. 2

(iv) What will be the value of the car after 10 years? 1

(v) At what rate will the value of the car be decreasing when it is 10 years old? 1

(b) A particle moves along a straight line. Its position x metres from O, t seconds after starting, is given by $x = 3 + \cos 2t$.

(i) Find the position of the particle after $\frac{3\pi}{4}$ seconds. 1

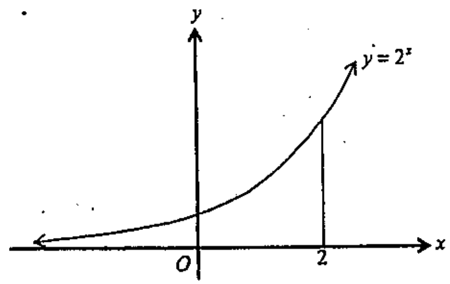
(ii) In what direction does the particle start to move? 2

(iii) Sketch the graph of v as a function of t for $0 \le t \le 2\pi$. 2

(iv) What is the maximum acceleration of the particle? 1

QUESTION 10 (12 MARKS) Use a SEPARATE writing Booklet **Marks**

(a)



The area between the curve $y = 2^x$ and the x-axis for $0 \le x \le 2$, is rotated about the x-axis to form a solid.

(i) Write down the definite integral which represents the volume of the solid. 1

(ii) Use Simpson's Rule with three function values to find the approximate volume of the solid giving your answer to 2 decimal places. 2

(iii) Using integration and the result $\int a^x dx = \frac{1}{\ln a} a^x$, evaluate the volume of the solid, giving your answer to 2 decimal places. 2

(b) Consider the function $f(x) = xe^{-x}$.

(i) Show that the graph of $y = f(x)$ has one stationary point at $x = 1$. 2

(ii) Show that the stationary point is a maximum. 2

(iii) Show that there is one point of inflexion and it occurs at $x = 2$. 1

(iv) By considering the signs of x and e^{-x} for large positive and large negative values of x, sketch the graph of $y = f(x)$. 2

End of paper

Mathematics: Question 1

Suggested Solutions | Marks Awarded | Marker's Comments

(a) $C = 2\pi r \therefore r = \frac{C}{2\pi}$

$r = \frac{9.29}{2\pi} = 1.4785\ldots$

$= 1.48$ (3 sig. fig.) (2)

(b) $\frac{d}{dx}(3x^2 - 4\sqrt{x}) = 6x - 4 \times \frac{1}{2} x^{-\frac{1}{2}}$

$= 6x - \frac{2}{\sqrt{x}}$ (2)

(c) $(\sqrt{6} + \sqrt{2})^2 = 6 + 2\sqrt{12} + 2$

$= 8 + 4\sqrt{3}$ (2)

(d) $a - \frac{a-2}{4} = 4$

$4a - (a-2) = 16$

$3a + 2 = 16$

$a = 4\frac{2}{3}$ (2)

(e) $|2c+3| > 1$

$2c+3 < -1$ or $2c+3 > 1$

$2c < -4$ or $2c > -2$

$c < -2$ or $c > -1$ (2)

(f) $(x - \frac{2}{3})(x+4) = 0$

$x^2 + 4x - \frac{2}{3}x - \frac{8}{3} = 0$

$3x^2 + 10x - 8 = 0$ (2)

OR $(3x-2)(x+4) = 0$

$3x^2 + 10x - 8 = 0$

OR $x^2 - (\frac{2}{3} + -4)x + (\frac{2}{3})(-4) = 0$

i.e. $x^2 + 3\frac{1}{3}x - \frac{8}{3} = 0$

i.e. $3x^2 + 10x - 8 = 0$

Mathematics: Question 2.

Suggested Solutions | Marks Awarded | Marker's Comments

(i) Grad. $AB = \frac{3-1}{-2-4} = -\frac{1}{3}$ (1)

(ii) $M: \left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = (1, 2)$ (1)

(iii) $AB: y - 3 = -\frac{1}{3}(x+2)$ or $y - 1 = -\frac{1}{3}(x-4)$

$3y - 9 = -x - 2$ $\qquad 3y - 3 = -x + 4$

$x + 3y - 7 = 0$ $\qquad x + 3y - 7 = 0$ (1)

(iv) $l: y - 2 = 3(x-1)$

$y - 2 = 3x - 3$

$3x - y - 1 = 0$ (1)

(v) Subst. $x = 0$ into l: $0 - y - 1 = 0$

$y = -1 \therefore C$ is $(0, -1)$ (1)

(vi) $AC = \sqrt{(-2-0)^2 + (3 - -1)^2} = \sqrt{4+16} = \sqrt{20}$

$BC = \sqrt{(4-0)^2 + (1 - -1)^2} = \sqrt{16+4} = \sqrt{20}$

$\therefore C$ is equidistant from A and B. (2)

(vii) $C(0, -1)$ $AB: x + 3y - 7 = 0$

$d = \left|\frac{0 + 3(-1) - 7}{\sqrt{1^2 + 3^2}}\right| = \frac{10}{\sqrt{10}} = \sqrt{10}$ (2)

or Dist. $CM = \sqrt{(1-0)^2 + (2 - -1)^2} = \sqrt{10}$

(viii) $AB = \sqrt{6^2 + 2^2} = \sqrt{40}$ or $2\sqrt{10}$

Area $\triangle ABC = \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$

$= 10$ unit² (2)

(ix) Subst. $x = 0, y = 0$ in $x + 3y - 7 = 0$

$LHS = -7 < 0$

$\therefore$ Region is $x + 3y - 7 < 0$ (1)

Mathematics: Question 3

Suggested Solutions	Marks Awarded	Marker's Comments

(a) (i) $\frac{d}{dx} x^3 \cos x = \cos x\,(3x^2) + x^3(-\sin x)$

$= 3x^2 \cos x - x^3 \sin x$ ②

(ii) $\frac{d}{dx}(3+e^{2x})^4 = 4(3+e^{2x})^3 \times 2e^{2x}$

$= 8e^{2x}(3+e^{2x})^3$ ②

(b) (i) $\int \frac{6x}{x^2+2}\,dx = 3\int \frac{2x}{x^2+2}\,dx$

$= 3\log_e(x^2+2) + C$ ②

(ii) $\int_0^{\frac{\pi}{6}} \sec^2 2x\,dx = \left[\frac{1}{2}\tan 2x\right]_0^{\pi/6}$

$= \frac{1}{2}\tan\frac{\pi}{3} - \frac{1}{2}\tan 0$

$= \frac{\sqrt{3}}{2}$ ②

(c) $\frac{dy}{dx} = 6x^2 - 4x + 1$

(i) $y = 2x^3 - 2x^2 + x + C$

$x=1, y=6$: $6 = 2 - 2 + 1 + C$

$C = 5$

Curve is $y = 2x^3 - 2x^2 + x + 5$ ②

(ii) When $x=1$, $\frac{dy}{dx} = 6 - 4 + 1$

$= 3$.

Eqn. of tangent is

$y - 6 = 3(x - 1)$ ②

i.e. $3x - y + 3 = 0$ or $y = 3x + 3$.

Mathematics: Question 4

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $f(x) = x^3 - 6x^2 + 9x$

(i) $f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-1)(x-3) = 0$ when $x = 1, 3$

$f''(x) = 6x - 12$.

$f(1) = 1 - 6 + 9 = 4$; $f''(1) = 6 - 12 < 0$

∴ Maximum at $(1, 4)$

$f(3) = 27 - 54 + 27 = 0$; $f''(3) = 18 - 12 > 0$

∴ Minimum at $(3, 0)$ ③

(ii) Pt. of inflexion: $6x - 12 = 0$

$x = 2$.

If $x < 2$, $f''(x) < 0$; If $x > 2$, $f''(x) > 0$

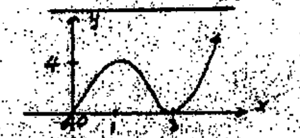
∴ change in concavity ∴ pt of inflexion at $x = 2$ ②

(iii) When $y = 0$, $x^3 - 6x^2 + 9x = 0$

$x(x-3)^2 = 0$

Meets x-axis at $0, 3$. ①

(iv)



①

(v) Grad. decreasing for $x < 2$. ①

(b) (i) $A = 2\int_0^4 2x^{\frac{1}{2}}\,dx = 4\left[\frac{2}{3}x^{3/2}\right]_0^4$

$= \frac{8}{3}\left[4^{3/2} - 0\right]$

Area $= 21\frac{1}{3}$ unit^2 ②

(ii) $V = \pi\int_0^4 4x\,dx = \pi\left[2x^2\right]_0^4$

$= 2\pi\left[4^2 - 0\right]$

Volume $= 32\pi$ unit^3 ②

Mathematics: Question 5

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $10, 12, 14, \ldots$

(i) $T_{30} = a + (n-1)d = 10 + 29 \times 2$

Time $= 68$ minutes (1)

(ii) $S_{30} = \frac{n}{2}(a + l)$ or $\frac{n}{2}[2a + (n-1)d]$

$= \frac{30}{2}(10 + 68)$

Total time $= 1170$ minutes (2)

(iii) 40 hours $= 2400$ minutes.

$\frac{n}{2}[2a + (n-1)d] = 2400$

$\frac{n}{2}[20 + (n-1)2] = 2400$

$n(10 + n - 1) = 2400$

$9n + n^2 - 2400 = 0$

$n = \frac{-9 \pm \sqrt{81 - 4(-2400)}}{2}$

$= 44.7$ (since $n > 0$).

40 hours reached on 45^{th} day (3)

(b)(i) $T_{20} = ar^{n-1} = 30\,000 \times (1.05)^{19}$

Salary $= \$75\,809$ (2)

(ii) $30\,000 \times 1.05^{n-1} > 50\,000$

$1.05^{n-1} > 1\frac{2}{3}$

$(n-1)\log 1.05 > \log 1\frac{2}{3}$

$n - 1 > \frac{\log 1\frac{2}{3}}{\log 1.05}$

$n > 11.47$

Exceeds \$50 000 in 12^{th} year (2)

(iii) $S_{20} = \frac{a(r^n - 1)}{r - 1} = \frac{30\,000(1.05^{20} - 1)}{1.05 - 1}$

Total salary $= \$991\,979$ (2)

Mathematics: Question 6

Suggested Solutions	Marks Awarded	Marker's Comments

(a)(i) $AC^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 65^\circ$

$AC = 12.05$ m. (2)

(ii) $\frac{\sin \angle ACB}{8} = \frac{\sin 65^\circ}{12.05}$ (2)

$\sin \angle ACB = \frac{8 \sin 65^\circ}{12.05} = 0.6017$

$\angle ACB = 37^\circ$ (nearest degree)

(iii) Area $= \frac{1}{2} \times 8 \times 13 \times \sin 65^\circ$

$= 47.12\ m^2$

Does not meet regulation. (1)

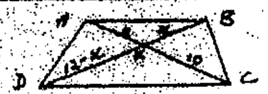
(b) $A = \frac{1}{2} r^2 \theta$ ∴ $60 = \frac{1}{2} \times 10^2 \times \theta$

$\theta = 1.2$

Perimeter $= 10 + 10 + 10 \times 1.2$

$= 32$ m. (3)

(c)



(i) In $\triangle AXB$, $\triangle CXD$,

Any 2 of.
- $\angle AXB = \angle CXD$ (vertically opposite)
- $\angle BAX = \angle DCX$ (alternate, $AB \parallel DC$)
- $\angle ABX = \angle CDX$ (alternate, $AB \parallel DC$)

∴ $\triangle AXB \parallel\!\mid \triangle CXD$ (2 angles equal) (2)

(ii) Let $BX = x$ cm, then $DX = (12 - x)$ cm

$\frac{4}{x} = \frac{10}{12 - x}$

$48 - 4x = 10x$

$48 = 14x$

$x = \frac{48}{14}$ (2)

$BX = \frac{24}{7}$ cm (or equivalent)

Mathematics: Question 7

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $2x - y - 2 = 0$ i.e. $y = 2x - 2$

$xy = 4$

$x(2x-2) = 4$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$ (3)

Points of intersection: $(2,2), (-1,-4)$.

(b) $3 \times 3^{2x} + 26 \times 3^{x} - 9 = 0$

Let $u = 3^x$: $3 \times u^2 + 26u - 9 = 0$

$(3u - 1)(u + 9) = 0$

$u = \frac{1}{3}$ or $u = -9$

$3^x = \frac{1}{3}$ or $3^x = -9$

$x = -1$ — no solution (3)

(c) $x^2 + 5x + c = 0$

For real and different roots, $\Delta > 0$

$25 - 4c > 0$

$c < 6\frac{1}{4}$ (2)

(d) $2x^2 - 5x - 8 = 0$

$\alpha + \beta = \frac{5}{2}$, $\alpha\beta = \frac{-8}{2} = -4$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{25}{4} - 2(-4)$

$= 14\frac{1}{4}$ (2)

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{14\frac{1}{4}}{-4}$

$= -\frac{57}{16}$ (2)

Mathematics: Question 8

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $\sum_{r=2}^{4} \frac{1}{r(r-1)} = \frac{1}{2\times1} + \frac{1}{3\times2} + \frac{1}{4\times3}$

$= \frac{3}{4}$ (2)

(b) $8y = x^2 - 4x - 4$

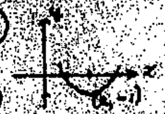
$x^2 - 4x + 4 = 8y + 4 + 4$

$(x-2)^2 = 8(y+1)$ NB $a = 2$.

(i) Vertex $(2,-1)$ (2)

(ii) Focus $(2,1)$ (1)

(iii) Directrix $y = -3$ (1)



(c)

$\angle ACD = \angle BAC$ (alternate, $AB \parallel DC$)

$= 24°$.

$\angle DAC = 180° - (78° + 24°)$ (angle sum of $\triangle ACD$)

$= 78°$

$\therefore \angle DAC = \angle ADC$

$\therefore \triangle ACD$ is isosceles (2)

(d) $f(x) = \begin{cases} 3 + 2^{-x} & \text{for } x \geq 0 \\ x + 4 & \text{for } x < 0 \end{cases}$

(i) $f(-1) + f(1) = (-1+4) + (3 + 2^{-1})$

$= 3 + 3\frac{1}{2}$

$= 6\frac{1}{2}$ (2)

(ii) $\lim_{x \to \infty} f(x) = 3 + 0 = 3$ (1)

(iii) As $x \to 0$ from below, $f(x) \to 0 + 4 = 4$

As $x \to 0$ from above, $f(x) \to 3 + 2^0 = 4$

$\therefore f(x)$ is continuous at $x = 0$. (1)

Mathematics: Question 9

Suggested Solutions	Marks Awarded	Marker's Comments

(a) $V = Ae^{-kt}$

(i) $\frac{dV}{dt} = -kAe^{-kt}$

$= -kV$ (1)

(ii) When $t=0$, $V = 32\,000$

$32\,000 = Ae^{0}$

$\therefore A = 32\,000$ (1)

(iii) $t=6$, $V = 14\,000$: $14\,000 = 32\,000\,e^{-6k}$

$e^{-6k} = \frac{14\,000}{32\,000}$

$-6k = \log_e\left(\frac{14}{32}\right)$

$k = 0.138$ (3SF) (2)

(iv) When $t=10$, $V = 32\,000 \times e^{-0.138 \times 10}$

Value $= \$8051$ (1)

(v) When $t=10$, $\frac{dV}{dt} = -0.138 \times 8051$

$= -1111$

Value decreasing at \$1111 per year (1)

(b) $x = 3 + \cos 2t$

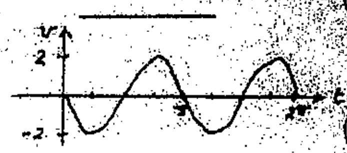
(i) When $t = \frac{3\pi}{4}$, $x = 3 + \cos\left(2 \times \frac{3\pi}{4}\right)$

$= 3$ (1)

(ii) $v = -2\sin 2t$.

It starts to move in negative direction (2)

(iii)



(2)

(iv) $a = -4\cos 2t$

Maximum acceleration is $4\,m/s^2$ (1)

Mathematics: Question 10

Suggested Solutions	Marks Awarded	Marker's Comments

(a) (i) $V = \pi\int_0^2 (2^x)^2\,dx$ (1)

(ii)

x	0	1	2
$f(x)$	1	4	16

$\int_0^2 (2^x)^2\,dx \doteq \frac{1}{3}[1 + 4 \times 4 + 16]$

$\doteq 11$ (2)

Volume $\doteq 11 \times \pi = 34.56$ unit^3 (2dp)

(iii) $V = \pi\int_0^2 4^x\,dx$

$= \pi\left[\frac{1}{\ln 4} 4^x\right]_0^2$

$= \frac{\pi}{\ln 4}[16 - 1]$

Volume $= 33.99$ unit^3 (2 d.p) (2)

(b) $f(x) = xe^{-x}$

(i) $f'(x) = e^{-x} \times 1 + x(-e^{-x})$

$= e^{-x}(1-x)$

$= 0$ only when $x = 1$

$\therefore$ only one stationary point (2)

(ii) $f''(x) = (1-x)(-e^{-x}) + e^{-x}(-1)$

$= xe^{-x} - 2e^{-x} = e^{-x}(x-2)$

$f''(1) = 1e^{-1} - 2e^{-1} < 0$ (2)

$\therefore$ Stat point is maximum at $x=1$

OR show derivative changes sign.

(iii) For $f''(x) = 0$, $x = 2$ (1)

$\therefore x = 2$ is the only point of inflexion.

(iv) For $x > 0$, $e^{-x} > 0 \therefore f(x) > 0$

For $x < 0$, $e^{-x} > 0 \therefore f(x) < 0$

(2)