SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2010

MATHEMATICS

Directions to Students

• Reading Time : 5 minutes	• Total Marks **120**
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• Attempt Question 1 – 10
• Board approved calculators may be used	• All questions are of equal value
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

Question 1 (Start a new Booklet) Marks

(a) Calculate $e^{3 \cdot 1}$ correct to 2 significant figures — 2

(b) Solve for x: $x^2 + 5x = 24$ — 2

(c) Find the primitive of $(2x + 5)^4$ — 2

(d) Calculate the exact value of $\cos \dfrac{\pi}{6}$ — 2

(e) Expand and simplify $(3 + 2\sqrt{2})^2$ — 2

(f) Find the sum of the first 40 terms of the series $4 + 10 + 16 + \ldots$ — 2

Question 2 (Start a new Booklet) Marks

(a) Find the derivative of

(i) $x \sin 3x$ — 2

(ii) $\dfrac{e^x}{x}$ — 2

(b) Integrate the following

(i) $\dfrac{3x}{x^2 + 4}$ — 2

(ii) $3 \sec^2 \dfrac{x}{2}$ — 1

(c) Find the equation of the tangent to the curve $y = x^3 + 1$ at the point where $x = 1$. — 3

(d) Evaluate $\displaystyle\sum_{n=1}^{3} n^3 + 3$ — 2

Question 3 (Start a new Booklet) Marks

(a) $A(1,-4)$ is a point on the line J: $3x+2y+5=0$

(i) Show that the point $B(-3,2)$ lies on the line. 1

(ii) Find the equation of the line perpendicular to J passing through the point $C(3,1)$. 2

(iii) Calculate the distance AB. 2

(iv) Find the perpendicular distance from C to the line J. 2

(v) Calculate the area of ΔABC. 1

(b) The Pacific Star cruise ship travels 215 km on a bearing of 085° from Sydney. It then travels 112km on a bearing of 135°.

(i) How far is the ship from Sydney? 2

(ii) What is the final bearing of the ship from Sydney? 2
(give answer correct to the nearest degree)

Question 4 (Start a new Booklet) Marks

(a) For the parabola $x^2 + 4x - 12y + 40 = 0$:

(i) Use completing the square method to write the equation in the form $(x - h)^2 = 4a(y - k)$ 1

(ii) Find the focal length. 1

(iii) Write down the coordinates of the vertex. 1

(iii) Find the focus. 1

(b) Solve $\log x + \log (x + 4) = \log 12$ 2

(c) William drops a ball out of a window that is 25 m above the ground. On the first rebound, it rises to a height of 20 m. On subsequent rebounds, it rises to a height equal to $\frac{4}{5}$ of its previous height. If there is no interference with the ball, calculate the total distance through which the ball moves before coming to rest. 2

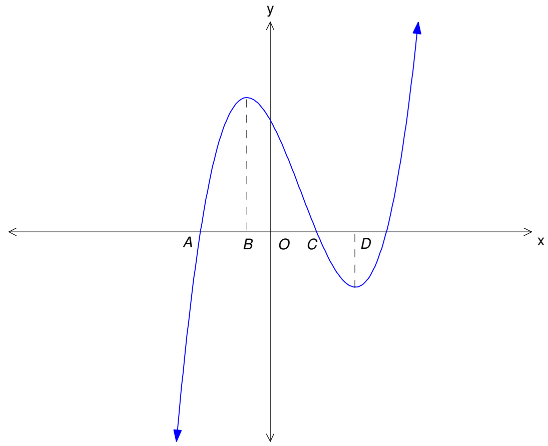
(d) At a Primary School Sports Carnival the combined year relay race has one participant from each year group from Kinder to Year 6. The Kinder child runs 15 m to a point and returns to the start, then the year 1 student runs 20 m and returns to the start. Each child runs in turn with each year group running 5 m further than the previous year group.

(i) How far does the year 6 child have to run? 2

(ii) How far is run by the students in one complete race? 2

Question 5 (Start a new Booklet) Marks

(a) Copy the following diagram into your answer booklet. 2



The curve represents the function $y = f(x)$. On the same set of axes draw the derivative function $y = f\,'(x)$.

(b) Consider the function $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the only x-intercept exists at the origin. 1

(ii) Find the stationary points and determine their nature. 3

(iii) Show that a point of inflexion exists at the point $(1.5, 4.5)$. 1

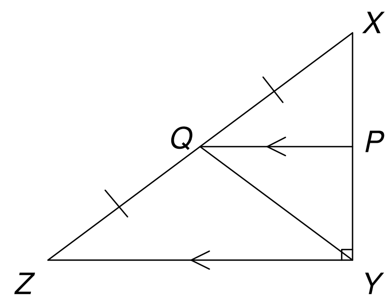
(iv) For what values of x is the curve monotonically decreasing? 1

(v) Sketch the curve of the function $y = 2x^3 - 9x^2 + 12x$ in the domain $0 \le x \le 3$. 2

(vi) Find the values of k for which $2x^3 - 9x^2 + 12x = k$ has only one solution. 2

Question 6 (Start a new Booklet) Marks

(a)



In the diagram above ΔXYZ is right angled. PQ is parallel to YZ and Q is the midpoint of XZ.

(i) Copy the diagram into your answer booklet.

(ii) Give a reason why $\angle XPQ = 90^\circ$. 1

(iii) Prove that $\Delta XPQ \equiv \Delta YPQ$. 2

(iv) Prove $QZ = QY$. 1

(b) A cylindrical tank is filled with water. The volume of water in the tank is determined by the function $V = 7t^3 + 15t^2 - 3t$, where t is time in seconds and the volume in litres. What is the rate of change of the volume of the tank after 12 seconds have elapsed? 2

(c) Find the **exact** length of the radius of a circle in which an arc length of 10 *cm* subtends an angle of 50° at the centre of the circle. 2

(d) Given that α and β are the roots of the quadratic equation $2x^2 - 7x + 5 = 0$, find ;

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

Question 7 (Start a new Booklet) Marks

(a) Prove that $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$ 3

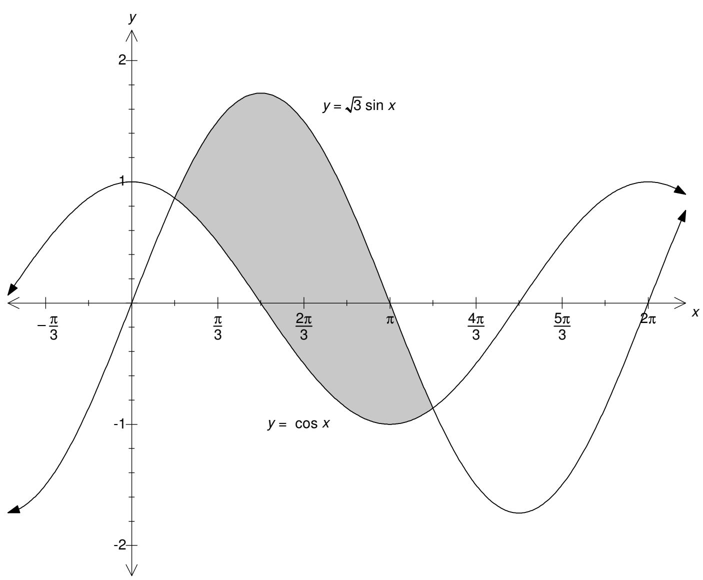
(b) The area bound by the curve $y = \sqrt{\sin x}$, $x = 0$, $x = \dfrac{\pi}{3}$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed. 3

(c) A function $f(x)$ has a table of values 2

x	0	0.5	1	1.5	2
f(x)	3.24	4.16	2.25	1.15	0

Use Simpson's Rule to calculate $\int_0^2 f(x)\,dx$ (correct to 2 decimal places)

(d)



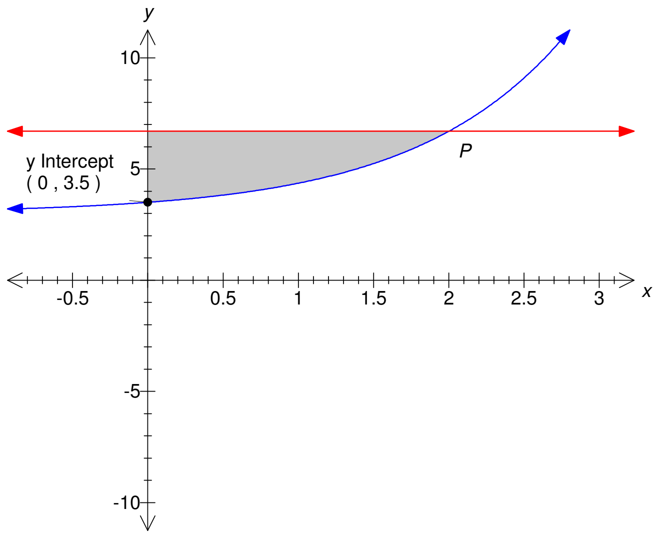
The graph above shows the sketch of the curves $y = \sqrt{3}\sin x$ and $y = \cos x$.

(i) Solve the curves simultaneously to show that the x-values of the points of intersection are $\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$. 1

(ii) Find the shaded area in the diagram. 3

Question 8 (Start a new Booklet) Marks

(a)



(i) The curve $f(x)$ displayed above is in the form $f(x) = ae^{x} + 3$. Given that the curve passes through the point $(0, 3.5)$, show that $a = 0.5$. 1

(ii) The x-value of the point P on the curve is 2. What is the y-value of the point P? 1

(iii) Find the area of the shaded region. 2

(b) A particle moves such that its position, x metres, from a fixed point O is given by the function $x = t^3 - 7\frac{1}{2}t^2 + 18t + 2$, where t is measured in seconds.

(i) Find the particle's initial position and velocity. 3

(ii) When is the particle at rest? 1

(iii) What is the acceleration of the particle when it is first at rest? 2

(iv) Find the distance travelled by the particle in the first three seconds. 2

Question 9 (Start a new Booklet) Marks

(a) Jenni invests $30000 into an account on the 1st of March. She receives 9% p.a. interest compounded monthly. On the first day of each month after that she withdraws $250 immediately after the interest is paid.

(i) How much money did she have in the account immediately after making the first withdrawal? 1

(ii) Show that after making the n *th* withdrawal the balance of the account is given by $\left(33\,333\frac{1}{3} - 3\,333\frac{1}{3} \times 1.0075^n\right)$ 2

(iii) Find the number of withdrawals that Jenni can make before there is no money left in the account. 2

(b) An ant colony has a population that is described by the function $P = P_0\, e^{kt}$. If the ant conlony initially had 300 ants and after 100 days the population had increased to 550 ants, find:

(i) the value of P_0 and k . 3

(ii) the time taken for the population of the colony to reach 1000 ants (write your answer correct to the nearest day). 2

(c) It was found that on the 1st of June, 30 students in a school had the flu. Over the next month the number of cases of students being sick with the flu increased at decreasing rate. Draw a graph that would describe this situation. 2

Question 10 (Start a new Booklet) Marks

(a) Find the derivative of the function $y = x \log_e x$ and hence find $\int \log_e x \, dx$. 2

(b) A sector of a circle has an area of $\frac{3\pi}{4} cm^2$, while its arc length is $\frac{\pi}{4} cm$. Find the radius and angle of the sector. 4

(c) A cylindrical can is to hold $20\pi m^3$. The material for the top and bottom costs $\$10/m^2$ and material for the side costs $\$8/m^2$.

(i) Show that the total cost of the material for the can be expressed by the formula $C = 20\pi r^2 + 16\pi r h$ 1

(ii) Show that $h = \frac{20}{r^2}$ 1

(iii) Find an expression for the cost in terms of r and hence find the values of r and h such that the cost of the materials is a minimum. 4

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2010 Riverview Mathematics Trials – Solutions

Question 1:

(a) 22 (2 significant figures)

(b) $x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$x = -8$ or $x = 3$

(c) $y = \frac{(2x+5)^3}{10} + C$

(d) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(e) $9 + 12\sqrt{2} + 8 = 17 + 12\sqrt{2}$

(f) $S_n = \frac{n}{2}\left(2a + 6(n-1)\right)$

$\therefore S_{40} = 20(8 + 6 \times 39) = 4840$

Question 2:

(a) (i)

Let $u = x$ therefore $\frac{du}{dx} = 1$

Let $v = \sin 3x$ therefore $\frac{dv}{dx} = 3\cos 3x$

Since $\frac{dy}{dx} = u'v + v'u$ we have:

$$\frac{dy}{dx} = \sin 3x + 3x\cos 3x$$

(ii)

Let $u = e^x$ therefore $\frac{du}{dx} = e^x$

Let $v = \frac{1}{x}$ therefore $\frac{dv}{dx} = -\frac{1}{x^2}$

Since $\frac{dy}{dx} = u'v + v'u$ we have:

$$\frac{dy}{dx} = \frac{e^x}{x} - \frac{e^x}{x^2}$$

$$= \frac{e^x}{x^2}(x - 1)$$

(b) (i)

$$\int \frac{3x}{x^2 + 4}\,dx$$

$$= \frac{3}{2}\ln|x^2 + 4| + C$$

(ii)

$$\int 3\sec^2\frac{x}{2}\,dx$$

$$= 6\tan\frac{x}{2} + C$$

(c) $\frac{dy}{dx} = 3x^2$

Therefore at $x = 1$, $m_T = 3$

$y - 2 = 3(x - 1)$

$y = 3x - 1$ (or in general form: $3x - y - 1 = 0$)

(d) $(1 + 3) + (8 + 3) + (27 + 3) = 4 + 12 + 30$

Q3 a)

$A(1,-4)$ $\quad 3x+2y+5=0 \leftarrow J$

i) sub $x=-3,\ y=2$ into J:

LHS: $3(-3)+2(2)+5 \Rightarrow$

$-9+4+5=0=RHS$

$\therefore$ $P_B(-3,2)$ lies on the line

ii) let new line be R

$m_R \cdot m_J = -1$ $\quad \therefore m_J = -\frac{3}{2}$

$C(3,1)$

$m_R = \frac{2}{3}$

$y-1=\frac{2}{3}(x-3)$

$3y-3=2x-6$

$\therefore 2x-3y-3=0$

iii) $d_{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(d_{AB})^2=(1+3)^2+(-4-2)^2$

$=\sqrt{52}$

$=2\sqrt{13}\ u$

iv) $C(3,1)$

$a=3$ $\quad x=3$

$b=2$ $\quad y=1$

$c=5$

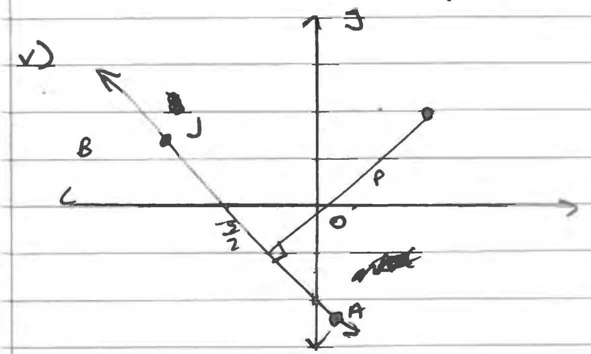
$P=\frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|3(3)+(2)(1)+5|}{\sqrt{3^2+2^2}}$

$=\frac{9+2+5}{\sqrt{13}}$

$P=\frac{16}{\sqrt{13}}$ units

$\therefore P=\frac{16}{\sqrt{13}}\ u$

v)



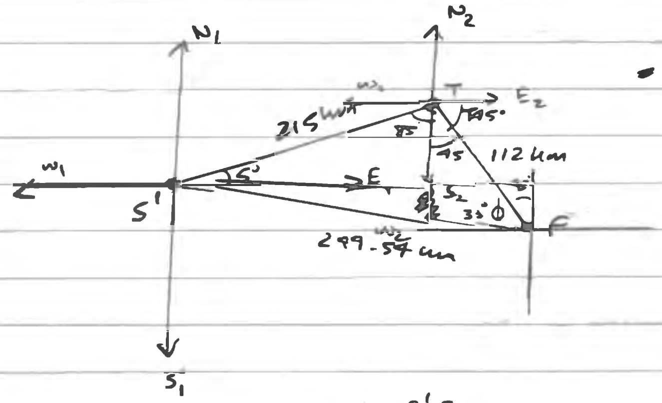
$d_{AB}=2\sqrt{13}$

$d_P=\frac{16}{\sqrt{13}}$

$A=\frac{1}{2}ab$

$=\frac{1}{2}\cdot\frac{16}{\sqrt{13}}\cdot 2\sqrt{13}=16$

b)



- let S' = Sydney
  T = Turn
  F = Stop

Join $S'F$.

$\angle FTS = 45°$ (adjacent angles reduced)

$\angle S'TS_2 = 85°$ (adjacent angles of parallel lines are eq. c.l)

$\angle S'TF = 85+45 = 130°$

$S'F = ?$

$a^2=b^2+c^2-2bc\cos\varphi$

$(S'F)^2=(S'T)^2+(TF)^2-2(TF)(S'T)\cos\varphi$

$(S'F)^2=(215)^2+(112)^2-2(112)(215)\cos 130°$

$=58769-48160\cos 130°$

$S'F=\sqrt{89725.65}=299.5424031$ km

$S'F=299.54$ km (2 d.p.)

ii) let $\angle TFS'=\phi$

$\frac{\sin\phi}{215}=\frac{\sin 130}{299.54}$

$\sin\phi=\frac{215}{299.54}\sin 130 \Rightarrow$

$\phi=33°21'$

$\angle S'FW_2 = 45°-33°21'$

$=11°39'$

$\therefore$ Bearing is $270°+11°39'$

$=282°$ (nearest degree)

Q 4c)

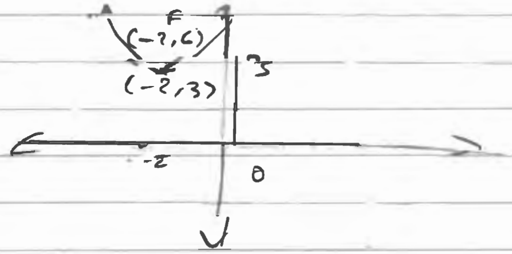
$x^2 + 4x - 12y + 40 = 0$

$(x+2)^2 - 4 - 12y + 40 = 0$

$(x+2)^2 - 12y = -36$

$(x+2)^2 = -36 + 12y$

$(x+2)^2 = 12(y-3)$



$4a = 12$

$a = 3$

i) $4a = 12$

$a = 3$

iii) $(-2, 3)$

iv) $(-2, 6)$

b) $\ln|x| + \ln|x+4| = \ln(12)$

$\ln|x(x+4)| = \ln|12|$

$x^2 + 4x - 12 = 0$

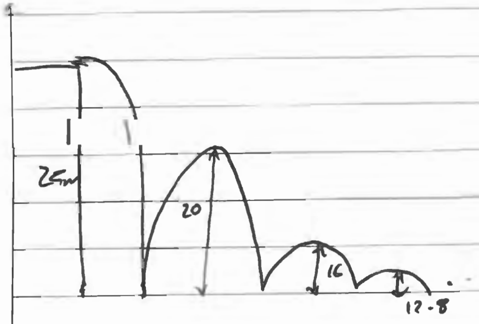
$(x+6)(x-2) = 0$

$x = 2, x = -6$

But $\ln|x|$, $x > 0$

$\therefore x \neq -6$

$\{x = 2\}$



$T_n = \{ 40, 32, 25.6, \ldots \} \longrightarrow \{40, 32, 25.6, \ldots\}$

let us define as $T_n \longrightarrow 0$, its at rest

T_n is a GP $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = 0.8$

$T_n = ar^{n-1}$

$T_n = 40(0.8)^{n-1}$

But window is 25m let defined set R_n

$R_n = 25 + T_n$

Since $|r| < 0.8$

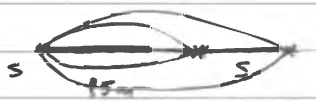
S_∞ exist

$S_\infty = \frac{a}{1-r}$

$= \frac{40}{1-0.8} = 200\text{ m}$

$R_n = 200 + 25 = 225\text{m}$

$\therefore$ The ball travelled 225m



i) $T_n = \{30, 40, 50, 60, 70, 80, 90\}$

$T_n = a + (n-1)d$

let $a = 30$

$d = 10$

$T_n = 30 + (10)(n-1)$

$= 30 - 10 + 10n$

$= 20 + 10n$

let $n = 7$

$T_7 = 20 + 10(7) = 90m$

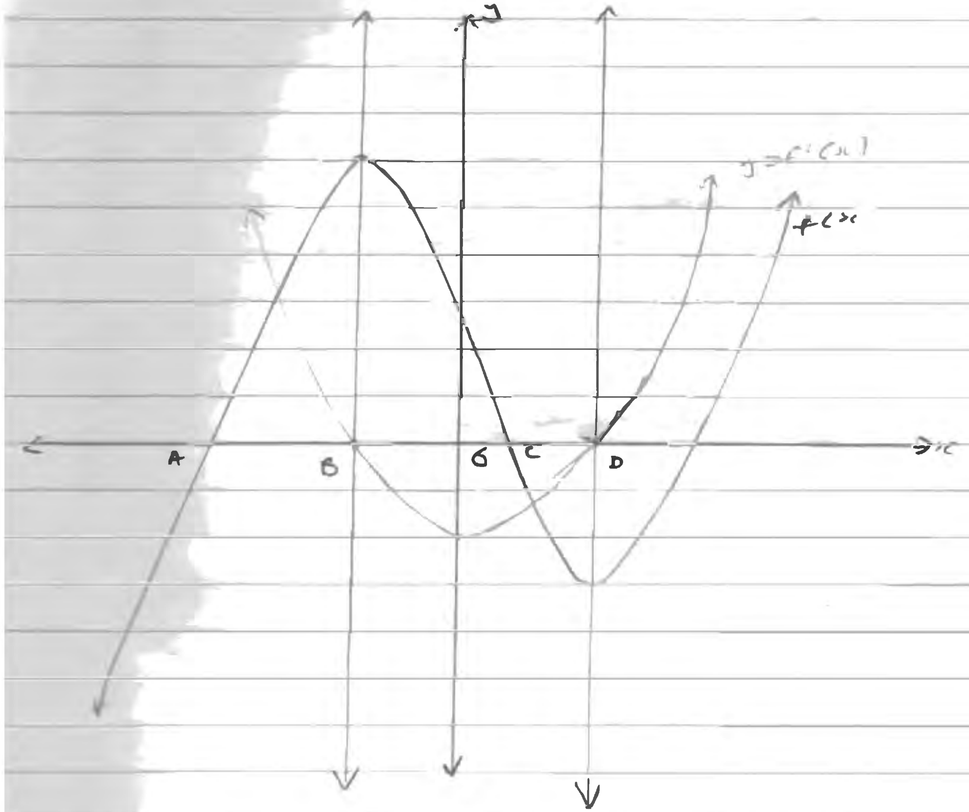
ii) $S_n = T_1 + T_2 + \dots + T_7$

$S_n = \frac{n}{2}(a + l)$

$= \frac{7}{2}(30 + 90) = 420m$

∴ The students ... 420m altogether

Question 5 (a)



b) ii) $y = 2x^3 - 9x^2 + 12x$

$y' = 6x^2 - 18x + 12$

let $y' = 0$ (TP)

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x = 2, \ x = 1$

$y = 4, \ y = 5$

i) $y = x(2x^2 - 9x + 12)$

let $y = 0$, $x = 0$, $2x^2 - 9x + 12 = 0$

But $\Delta = -15$ (<0) ∴ no Real roots

∴ $x = 0$

continued

ii) $y'' = 12x - 18$

sub $x = 1$ in y''

$y'' = 12 - 18 = -6$ (<0) Maxima

$\therefore P_1 (1, 5)$ TP(Max)

sub $x = 2$ in y''

$y'' = 12(2) - 18 = 6$ (>0) Minima

$P_2 (2, 4)$ (TP) (Min)

$\therefore P_1 (1, 5)$ (TP)(Max)

$P_2 (2, 4)$ (TP)(Min)

iii) sub $y'' = 0$ (IP)

$0 = 12x - 18$

$x = \frac{18}{12} = 1.5$

$y = 4.5$

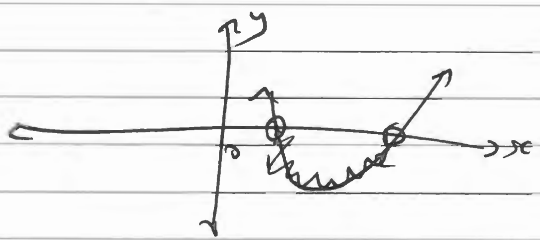
IP (1.5, 4.5)

iv) $\frac{dy}{dx} < 0$

$y' = 6x^2 - 18x + 12$

$6x^2 - 18x + 12 < 0$

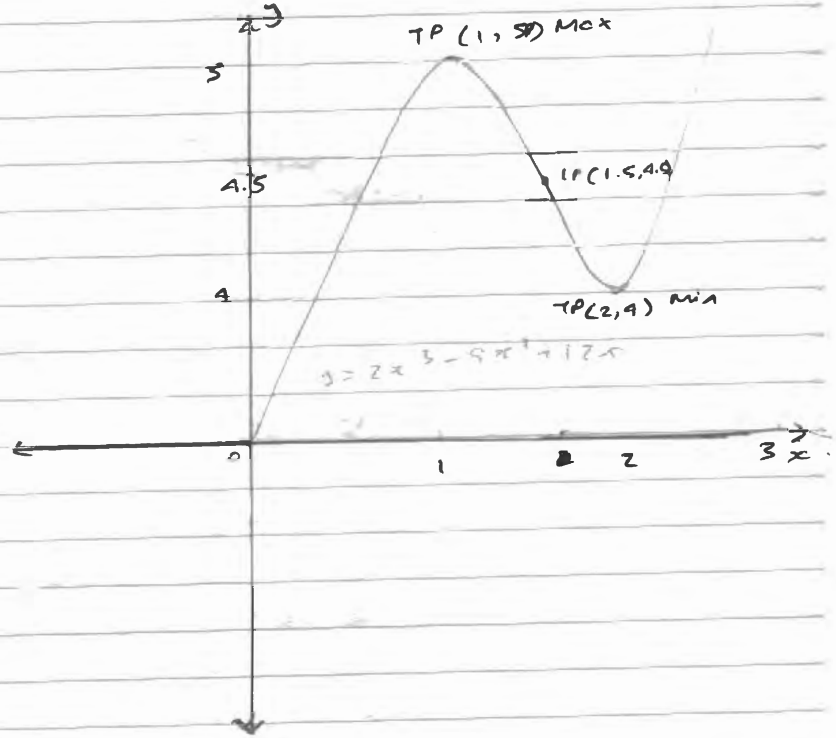
$(x-2)(x-1) < 0$



$\{1 < x < 2\}$

$\therefore$ graph is decreasing at $1 < x < 2$

v)



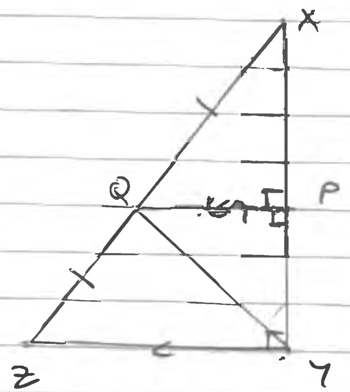
vi) $2x^3 - 9x^2 + 12x - k = 0$

$k = 0$ is a possible soln

$\{k < 4, \; k > 5\}$

Question 6

a)



ii) $\angle XPQ = 90° = \angle PYZ$ (corresponding angles of parallel lines are equal)

iii) in ΔQXP and ΔXZY:

$\angle QXP$ is common angle (included angle)

$\angle XPQ = \angle XYZ = 90°$ (angle)

$\therefore \Delta QXP \parallel \Delta XZY$ (AAA)

$\frac{XQ}{QZ} = \frac{XP}{PY} = 1$ (common ratio of similar triangles)

$\therefore XP = PY$

in ΔXPQ and ΔYPQ:

QP is common side (SIDE)

$PY = XP$ (SIDE)

$\angle QPY = \angle QPX = 90°$ (INCLUDED ANGLE)

$\therefore \Delta XPQ \equiv \Delta YPQ$ (SAS)

iv) $QX = QY = QZ$ (given ? equal sides of congruent triangles)

$\therefore QZ = QY$

b) $V = 7t^3 + 15t^2 - 3t$

$\frac{dV}{dt} = 21t^2 + 30t - 3$

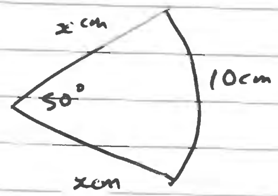
let $t = 12$

$\frac{dV}{dt} = 21(12)^2 + 30(12) - 3$

$= 3381$ l/s

$\therefore \frac{dV}{dt} = 3381$ l/s at $t = 12$ seconds

c)



$l = r\theta^c$

$10 = x\left(\frac{50}{180}\pi\right)$

$10 = x\left(\frac{5}{18}\pi\right)$

$\frac{180}{5\pi} = x$

$x = \frac{36}{\pi}$ cm

d) i) $\alpha + \beta = \frac{-b}{a} = \frac{7}{2}$

ii) $\alpha\beta = \frac{5}{2}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{7}{2}\right)^2 - 2\left(\frac{5}{2}\right)$

$= \frac{29}{4}$

Question 7

a) $\frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

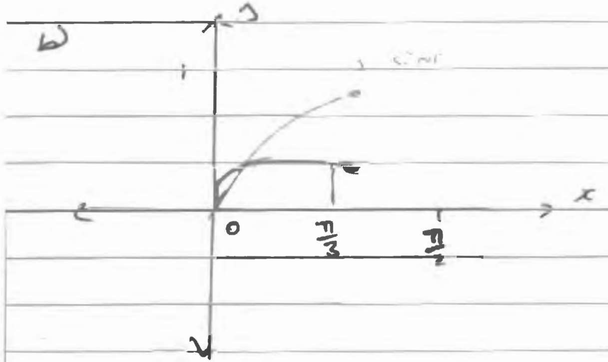
LHS:

$\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(\sin\theta)(1-\cos\theta)}{(1-\cos^2\theta)}$ $\quad 1-\cos^2\theta = \sin^2\theta$

$= \frac{\sin\theta}{\sin^2\theta}(1-\cos\theta) = \frac{1-\cos\theta}{\sin\theta}$ = RHS QED

b)



$V_y = \pi\int y^2\,dx$

$= \pi\int_0^{\pi/3} \sin x\,dx$

$= -\pi\left[\cos x\right]_0^{\pi/3}$

$= -\pi\left[\cos\frac{\pi}{3} - \cos 0\right]$

$= -\pi\left[\frac{1}{2} - 1\right]$

$= \frac{\pi}{2}\,u^3$

c) $\int_0^2 f(x)\,dx$

$A = \frac{h}{3}(f + l + 2\,\text{odd} + 4\,\text{even})$

$= \frac{0.5}{3}(0 + 3.24 + 2(2.25) + 4(4.16 + 1.15))$

$= \frac{0.5}{3}(28.98) = 4.83$ (2dp)

d) i) $y = \sqrt{3}\sin x$ —— (1)

$y = \cos x$ —— (2)

$\frac{(1)}{(2)} =$

$1 = \sqrt{3}\tan x$

$\tan x = \frac{1}{\sqrt{3}}$ $\quad$ S (A) / T C

$x^c = \frac{\pi}{6}$

1st:

$x^c = \frac{\pi}{2}$

3rd:

$x^c = \pi + \frac{\pi}{6}$

$x^c = \frac{7\pi}{6}$

$\therefore x^c = \frac{\pi}{6}, \frac{7\pi}{6}$ QED

ii) $A = \int_{\pi/6}^{7\pi/6}(\sqrt{3}\sin x - \cos x)\,dx$

$= \left[-\sqrt{3}\cos x - \sin x\right]_{\pi/6}^{7\pi/6}$

$= -\left[\left(\sqrt{3}\cos\frac{7\pi}{6} + \sin\frac{7\pi}{6}\right) - \left(\sqrt{3}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right)\right]$

$= -\left[-\frac{\sqrt{3}}{2}\times\sqrt{3} + \left(-\frac{1}{2}\right) - \left(\sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\right] = -\left(\frac{3}{2} - \frac{1}{2}\right) - \left(\frac{3}{2} + \frac{1}{2}\right)$

$\quad -2 \qquad\qquad -2$

$= -[-2 - 2]$

$= 4\,u^2$

Question 8

a) i) $f(x) = ae^{x} + 3$

sub $(0, 3.5)$

$f(0) = ae^{0} + 3 = 3.5$

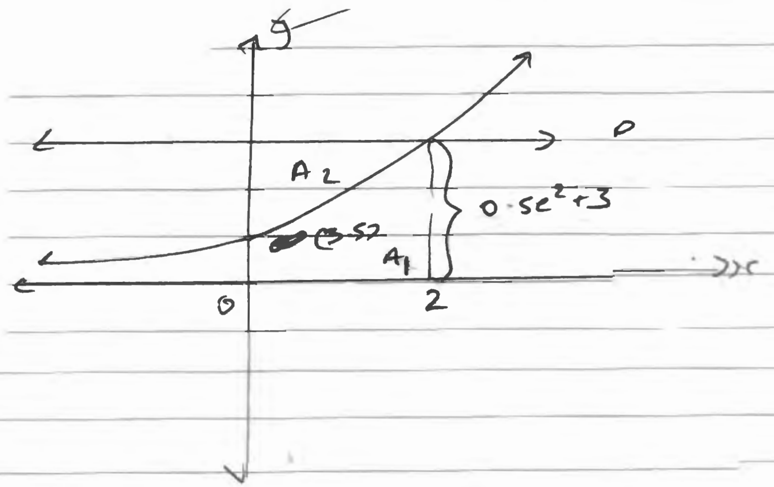
$\therefore a(1) = 0.5$ QED

ii) sub $x = 2$

$f(2) = 0.5e^{2} + 3$

$\therefore$ the y value $= 0.5e^{2} + 3$

iii)



$A_{Rectangle} = (e^{2}+6) u^{2}$

$A_2 = A_{Rect} - A_1$

$A_1 = \int_0^2 0.5e^{x} + 3 \, dx$

$= \left[0.5e^{x} + 3x\right]_0^2$

$= 0.5e^{2} + 6 - 0.5 = 0.5e^{2} + 5.5$

$A_2 = e^{2} + 6 - (0.5e^{2} + 5.5)$

$= 0.5e^{2} + 0.5$

$= 0.5(e^{2}+1) \; u^{2}$

$\therefore$ shaded region $= 0.5(e^{2}+1) \; u^{2}$

Q9) c) let $L = \$30000$

$K = 1 + \frac{9\%}{12} = 1.0075$

$n = n$

$k = \$250$

i) $A_1 = LK$

$= 30000(1.0075) = \$30225$

ii) $A_2 = Lk^{2} - P$

$A_3 = Lk^{3} - Pk - P$

$A_4 = Lk^{4} - P(1 + k + k^{2})$ GP

$S_n = \frac{a(k^{n}-1)}{k-1}$

$A_n = Lk^{n} - \frac{P(k^{n}-1)}{k-1}$

$= Lk^{n} - \frac{Pk^{n}}{k-1} + \frac{P}{k-1}$

$A_n = k^{n}\left(L - \frac{P}{k-1}\right) + \frac{P}{k-1} = 3333\frac{1}{3} - 3333\frac{1}{3}(1.0075)^{n}$

QED

iii) from ii)

$A_n = Lk^n - \frac{P(k^n)}{k-1} + \frac{P}{k-1}$ let $A_n = 0$

$\Rightarrow k^n \left(L - \frac{P}{k-1} \right) = \frac{-P}{k-1}$

$k^n = \frac{-P}{Lk - L - P}$

$k^n = 10$

$n \ln|k| = \ln|10|$

$n = \frac{\ln(10)}{\ln|k|} = 309$

$\therefore n = 309$ months

$\therefore$ She will have no money on the 309^{th} month

) i) $P = P_0 e^{kt}$

let $t = 0$, $P = 300$

$\therefore P_0 = 300$

let $t = 100$, $P = 550$

$550 = 300 e^{k(100)}$

$\ln\left(\frac{11}{6}\right) = \ln(e) \, 100 k$

$k = \frac{\ln\left(\frac{11}{6}\right)}{100} = 0.006061358$

$\therefore P_0 = 300$

$k = 0.006061358$

ii) let $P = 1000$

$\frac{1000}{300} = \frac{300}{300} e^{kt}$

$\frac{10}{3} = e^{kt}$

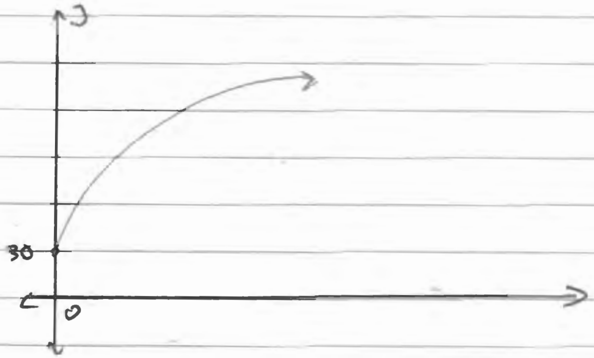
$\ln\left|\frac{10}{3}\right| = kt \ln(e)$

$t = \frac{\ln\left|\frac{10}{3}\right|}{k}$

$t = 198.6308$

$\therefore t = 199$ days

c)



Q10 a)

$y = x \ln|x|$

let $u = x$ $\quad u' = 1$

$v = \ln|x|$ $\quad v' = \frac{1}{x}$

$y' = \ln|x| + 1$

$\frac{dy}{dx} = \ln|x| + 1$

$\int dy = \int \ln|x| + 1 \, dx$

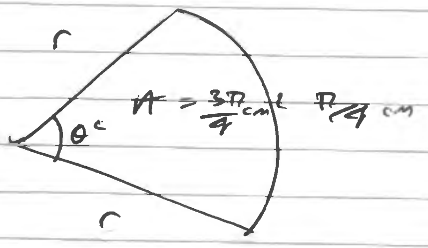
$y = \int \ln|x| \, dx + x$

$y - x = \int \ln|x| \, dx$

$x \ln|x| - x = \int \ln|x| \, dx$

$\therefore \int \ln|x| \, dx = x \ln|x| - x + C$

b)



$A = \frac{1}{2}r^2\theta$ —— (2) $\quad l = r\theta$

$\frac{\pi}{4} = r\theta$

$\theta = \frac{\pi}{4r}$ —— (1)

sub (1) into (2)

$\frac{3\pi}{4} = \frac{1}{2} r^2 \frac{\pi}{4r}$

$3\pi = \frac{1}{2} r \pi$

$6 = r$

$\therefore \begin{cases} r = 6 \\ \theta^c = \frac{\pi}{24} \end{cases}$

c) i) $V = 20\pi m^3$

$2\pi r^2 \longrightarrow \$10/m^2$

$2\pi r h \longrightarrow \$8/m^2$

i)

$C = 10(2\pi r^2) + 8(2\pi r h)$

$C = 20\pi r^2 + 16\pi r h$ QED

ii)

$V = \pi r^2 h$

$20\pi = \pi r^2 h$

$h = \frac{20\pi}{\pi r^2}$

$\therefore h = \frac{20}{r^2}$ QED

iii) sub h into C

$C = 20\pi r^2 + 16\pi r\left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + \frac{320\pi}{r}$

$C = 20\pi r^2 + 320\pi r^{-1}$

$\frac{dC}{dr} = 40\pi r - 320\pi r^{-2}$

let $\frac{dC}{dr} = 0$ (TP)

$0 = 40\pi r - \frac{320\pi}{r^2}$

$\overline{\quad 40\pi \quad}$

$0 = r - \frac{8}{r^2}$

$0 = \frac{r^3 - 8}{r^2}$

$r \neq 0$

$r = 2$

$h = 5$

1st D test:

1.9	2	2.1
\	—	/

$\therefore$ at $r = 2$, it is a minimum

$\therefore r = 2, h = 5$

$\therefore \begin{cases} r = 2m \\ h = 5m \end{cases}$