SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2013

MATHEMATICS

Directions to Students

• Reading Time : 5 minutes	• Total Marks **100**
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• This paper contains two sections. Section 1 contains ten objective response questions. Section 2 contains six free response questions. All questions may be attempted.
• Board approved calculators may be used	• Section 1-all questions 1 mark each • Section 2- Q11-16, 15 marks each
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

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Section 1 10 Marks Answer on sheet provided.

1. What is the exact value of $\tan 330^\circ$?

(A) $-\sqrt{3}$

(B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $-\frac{1}{\sqrt{3}}$

---

2. What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1, -3)$?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y - 5 = 0$

(D) $2x + y + 5 = 0$

---

3. What is the value of $\sum_{n=1}^{4} n^2$?

(A) 576 (B) 120 (C) 30 (D) 16

---

4. What is the size of each interior angle in a regular octagon?

(A) $22\frac{1}{2}^\circ$ (B) 80° (C) 135° (D) 180°

---

5. Which of the following is the point of intersection of the two lines $3x - 4y + 6 = 0$ and $x - y - 1 = 0$?

(A) (0,0)

(B) $(-2, -3)$

(C) (10,9)

(D) (11,10)

---

6. What are the solutions of the equation $4^x - 5 \times 2^x + 4 = 0$?

(A) $x = 0, 2$ (B) $x = 1,2$ (C) $x = 1,4$ (D) $x = 4, 5$

---

7. Consider the series $\sqrt{5} + \sqrt{45} + \sqrt{125} + \ldots + z = 225\sqrt{5}$.
How many terms are there in this series?

(A) 15 (B) 16 (C) 225 (D) 226

---

8. Which of the following is equal to $\sin\theta$?

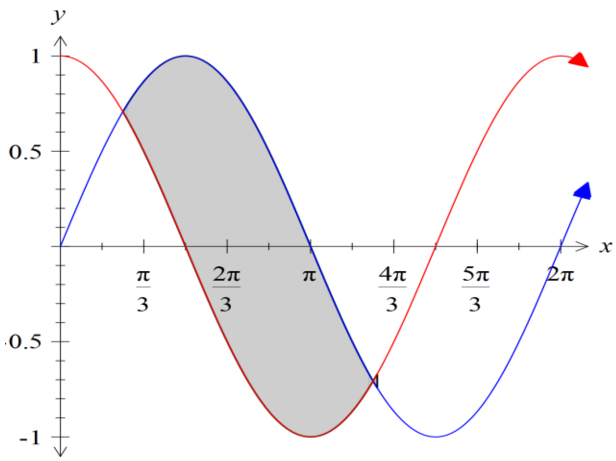
(A) $\tan\left(90^\circ - \theta\right)$

(B) $\cos\left(\theta - 90^\circ\right)$

(C) $\sin\left(180^\circ - \theta\right)$

(D) $\sin\left(360^\circ - \theta\right)$

9.



Which of the following describes the area given in the graph above?

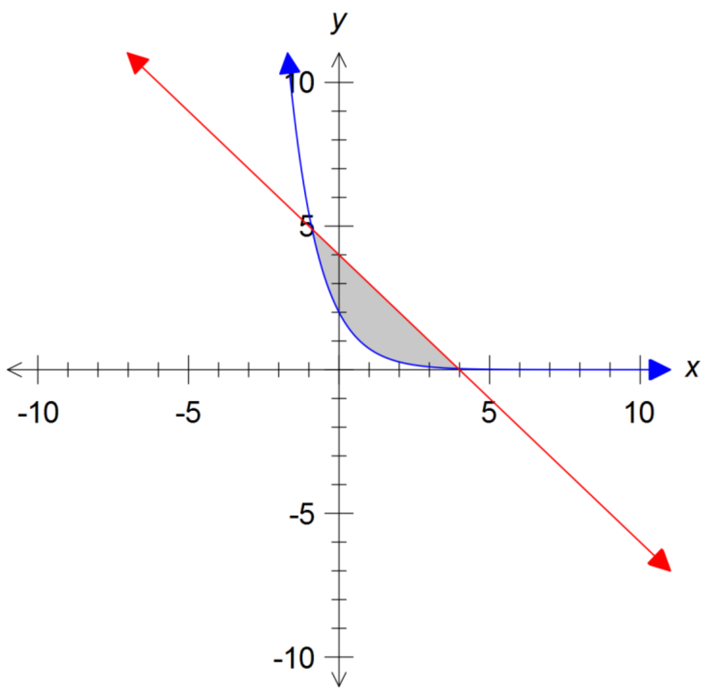
(A) $$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \sin x - \cos x \, dx$$

(B) $$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x - \cos x \, dx$$

(C) $$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \cos x - \sin x \, dx$$

(D) $$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos x - \sin x \, dx$$

10.



Which of the following describes the region given in the graph above?

(A) $y \geq e^{-2x}, x + y \geq 4$

(B) $y \geq 2e^{-x}, x + y \leq 4$

(C) $y \geq e^{-2x}, x + y \leq 4$

(D) $y \geq 2e^{-x}, x + y \geq 4$

Section 2

Question 11 (Start a new Booklet) Marks

(a) Factorise completely $4x^3 - 32$. 2

(b) Solve $|3x + 6| = 12$ 2

(c) Solve $10^x = 178$, correct to 4 decimal places 2

(d) Draw the graph of $x^2 + 4x - 21 + y^2 = 0$ 3

(e) $A(-2,4)$ and $B(6,-2)$ are points on the number line.

(i) Calculate the gradient of the line *AB*. 1

(ii) Hence show that the equation of the line *AB* is $3x + 4y = 10$ 1

(iii) Find the distance between the x and y intercepts of the line *AB*. 2

(iv) On the same graph show the region described by 2

$3x + 4y > 10, x \geq 0, y \geq 0$

Question 12 (Start a new Booklet) Marks

(a) Differentiate the following:

(i) $3x\,e^{2x^2}$ 2

(ii) $\left(3+\sin\left(x^2\right)\right)^4$ 2

(b) (i) Evaluate $\displaystyle\int_1^e \frac{5}{x}\,dx$ 2

(ii) Evaluate $\displaystyle\int_0^3 2\sqrt{x}+x^3\,dx$ 2

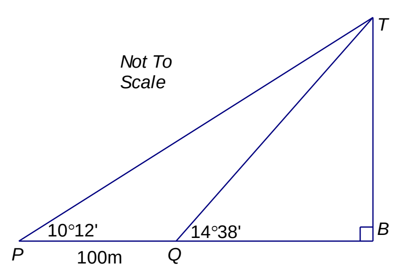
(c) An AP has a first term of 2 and a last term of 126. If there are 32 terms in the series, find the sum of the series. 2

(d) 3

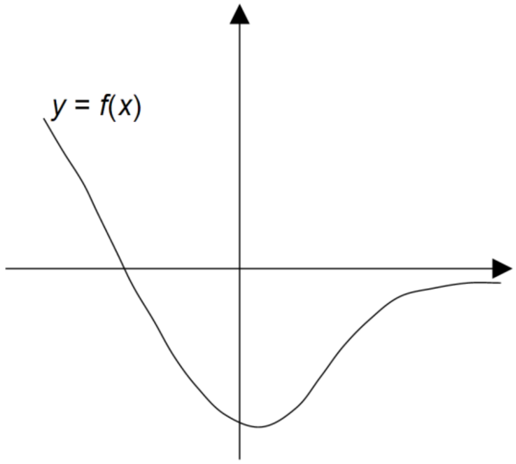
The angle of elevation of the top of tree *BT* when viewed from point *P* is 10°12'.

After walking 100m directly towards the tree one arrives at *Q* where the angle of elevation is 14°38'.

Find the height of the tree to the nearest centimetre.



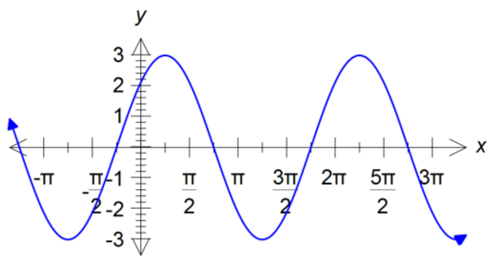
(e) Copy the following graph into your answer booklet and on the same graph draw the function $y = f'(x)$ 2



Question 13 (Start a new Booklet) Marks

(a) Solve $\cos 2\theta = \dfrac{1}{\sqrt{2}}$ in the domain $0 \le \theta \le 2\pi$ 3

(b) 2



The graph given is in the form $y = A\sin(x + \alpha)$.

Find the values of A and α .

(c) Given the parabola $x^2 + (m-2)x + 4 = 0$, find the values of m for which the parabola has no real roots. 3

(d) If α and β are the roots of the quadratic equation $x^2 + 4x - 8 = 0$, calculate:

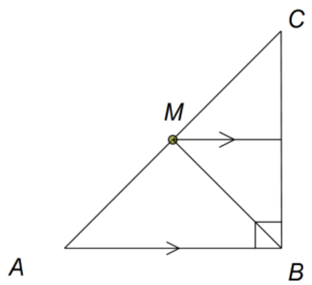
(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2\beta + \alpha\beta^2$ 1

(iv) $\alpha^2 + \beta^2$ 1

(e) In the triangle ABC, M is the midpoint of AC. Prove that M is equidistant from all three vertices of the right angle triangle. 3



Question 14 (Start a new Booklet) Marks

(a) Find the equation of the tangent to the curve $y = 2x\,e^{x}$ at the point $(1, e)$. 2

(b) Consider the parabola $y = x^{2} + 12$

(i) Find the coordinates of the vertex and focus of the parabola. 2

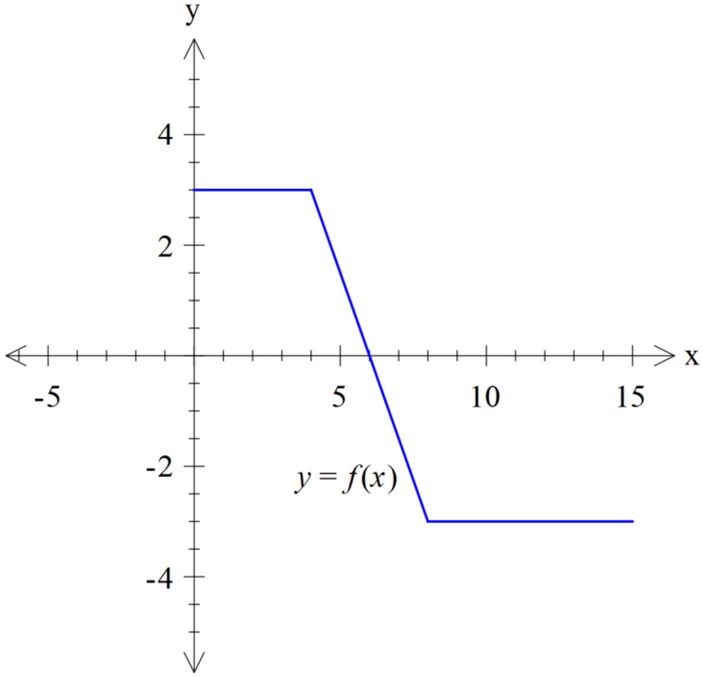
(ii) The area between the parabola and the line $y = 16$ is rotated about the y-axis. Calculate the volume of the solid formed by this rotation leaving your answer in terms of π. 3

(c) Calculate the approximate area (to two decimal places) between the curve $y = \ln 2x$, the x-axis and the line $x = 2$, using the Trapezoidal Rule with four function values. 4

Question 14 continues on page 12

Question 14 continued

(d)



(i) Calculate $\int_0^4 f(x)\ dx$ 1

(ii) Explain why $\int_4^8 f(x)\,dx = 0$ 1

(iii) What is the value of a if $\int_1^a f(x)\,dx = -6$ 2

Question 15 (Start a new Booklet) Marks

(a) Radioactive material is decaying according the function $R = R_0 e^{-kt}$.
There is initially 1 kg of the material and after 20 years there is 0.95 kg of the material remaining.

(i) Calculate the value of R_0 and k in exact form 2

(ii) Determine the half-life of the material 2

(b) A particle is traveling with the acceleration in terms of time given by the expression $\ddot{x} = 4e^{-2t}$. The particle is initially at rest.

(i) Explain why the particle moves in a positive direction for $t > 0$ 1

(ii) Find an expression for the velocity of the particle. 2

(iii) Find the value of the velocity as the acceleration approaches zero. 2

(c) A couple is wishing to buy a home for \$650 000. They take out a loan at 12% p.a. interest compounded monthly. The term of the loan is 25 years, with repayments paid monthly.

(i) Show that the after the second repayment has been made, the amount outstanding is given by the expression. 1

$$A_2 = 650\,000(1.01)^2 - M(1.01) - M$$

where M is the amount of the monthly repayment.

(ii) Calculate the value of M. 2

(iii) Instead of paying the amount in (ii) for the loan repayment, the couple pays \$250 more on their loan so that they will pay the amount in less time. By paying this extra money per month, how many months does the couple save on their home loan? 3

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Question 16 (Start a new Booklet) Marks

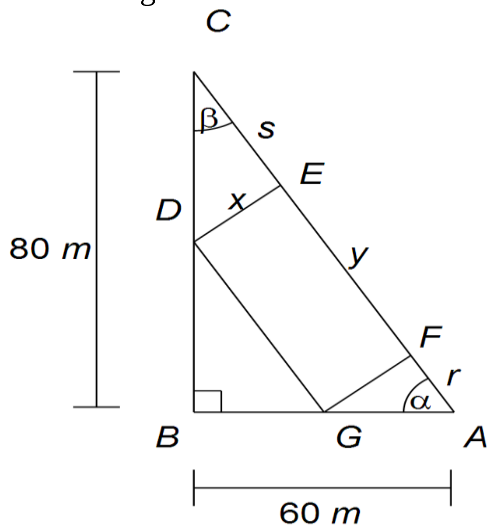
(a) Consider the curve $y = \frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x + 2$.

(i) Find the stationary points of the curve and determine their nature. 3

(ii) Show that there is an inflexion point at $x = 1\frac{1}{4}$ 1

(iii) Sketch a graph of the function for the domain $-2 \le x \le 6$. 2

(b) A lot of land has the form of a right triangle, with perpendicular sides 60 and 80 metres long.



(i) Show that $r = \frac{3}{4}x$ and $s = \frac{4}{3}x$ 2

(ii) Show that $y = 100 - \frac{25}{12}x$ 1

(iii) Find the length and width of the largest rectangular building that can be erected, facing the hypotenuse of the triangle. 3

(c) The centres of two circles are 7 cm apart, with one circle having a radius of 5 cm and the other a radius of 3 cm. Find the area of their intersection. 3

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Saint Ignatius' College, Riverview

Year 12 Mathematics

Trial HSC 2013

Suggested Solutions

Section 1 (10 marks)

1. D
2. B
3. C
4. C
5. C
6. A
7. A
8. C
9. B
10. B

Markers Comments

* well done on the whole . . .

* Qu 9 tripped up many students who didn't look at graph accurately POI was $\frac{\pi}{4}$ not $\frac{\pi}{3}$!

QUESTION 11 (JPN) | MARKER COMMENT

a. $4x^3 - 32$
$= 4(x^3 - 8)$ ✓
$\boxed{= 4(x-2)(x^2+2x+4)}$ ✓

factorising
1 mark for common factor
1 mark for diff. of two cubes

b. $|3x+6| = 12$

$3x+6 = 12$
$3x = 6$
$x = 2$
Check: LHS $= |3\times2+6|$
$= |12|$
$= 12$
$=$ RHS

$-3x-6 = 12$
$-3x = 18$
$x = -6$
Check LHS $= |3\times-6+6|$
$= |-12|$
$= 12$
$=$ RHS

$\therefore$ solution $\boxed{x = 2, x = -6}$ ✓✓

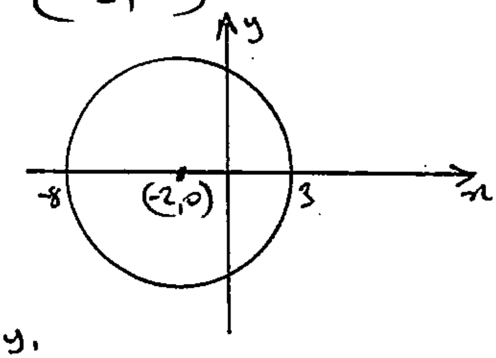
1 mark for each correct solution. ie (2 marks)

c. $10^x = 176$
$\log_{10} 10^x = \log_{10} 178$ ✓
$x = 2.250420002$
$\boxed{x = 2.2504}$ (4 d.p) ✓

1 mark for log both sides
1 answer to 4 d.p.

d. $x^2 + 4x - 21 + y^2 = 0$
$x^2 + 4x + y^2 = 21$
$x^2 + 4x + 4 + y^2 = 25$
$(x+2)^2 + y^2 = 25$ ✓
$\therefore$ centre $(-2, 0)$ radius $= 5$ units ✓

1 mark for expression correctly
1 identifying centre & radius.



✓

1 correct sketch

USE a TEMPLATE OR COMPASS

e. (i) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-2-4}{6+2}$
$= \frac{-6}{8}$
$= \boxed{-\frac{3}{4}}$

1 correct answer.

Q11 continued

MARKER COMMENT

e (ii)
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\tfrac{3}{4}(x + 2)$$
$$4y - 16 = -3x - 6$$
$$3x + 4y = 10$$
(as required) ✓

1 mark showing this line or equivalent. (ie could have used $(6, -2)$)

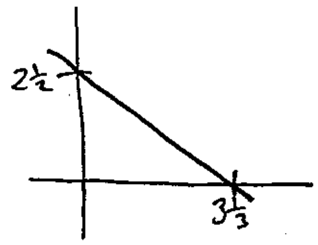
e (iii) for x intercept let $y = 0$

$$3x + 4\times 0 = 10$$
$$3x = 10$$
$$x = \tfrac{10}{3}$$

for y-intercept let $x = 0$

$$3\times 0 + 4y = 10$$
$$4y = 10$$
$$y = \tfrac{5}{2}$$

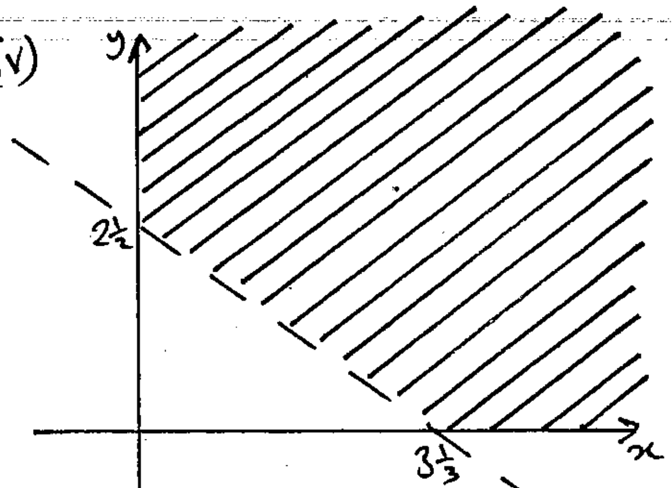
1 mark finding intercepts



$$d^2 = 2\tfrac{1}{2}^2 + 3\tfrac{1}{3}^2$$
$$\boxed{d = \tfrac{25}{6}} \text{ units}$$

1 calculating distance (some used distance formula)

(iv)



$$3x + 4y > 10,\ x \geq 0,\ y \geq 0$$

1 broken line

1 shading correct region.

Question 12

(a) (i) $\frac{d}{dx}\left(3xe^{2x^2}\right)$

$= 3x \times 4xe^{2x^2} + e^{2x^2} \times 3$

$= 12x^2e^{2x^2} + 3e^{2x^2}$

(ii) $\frac{d}{dx}\left(3 + \sin(x^2)\right)^4$

$= 4 \times 2x\cos(x^2)\left(3 + \sin(x^2)\right)^3$

$= 8x\cos(x^2)\left(3+\sin(x^2)\right)^3$

Question 12

Markers Comments

(b) (i) $\int_1^e \frac{5}{x}\,dx = 5\ln x + c$

$= 5(\ln e - \ln 1)$

$= 5$

* be careful, quite a few thought it was $\frac{1}{5}\ln x$! No!

(ii) $\int_0^3 \left(2x^{\frac{1}{2}} + x^3\right) dx$

$= \left[2 \times \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{4}x^4\right]_0^3$

$= \left[\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^4\right]_0^3$

$= \frac{4}{3}\times 3\sqrt{3} + \frac{1}{4}\times 81 - 0$

$= 4\sqrt{3} + \frac{81}{4} = \frac{16\sqrt{3}+81}{4} = 27.1782$

* answers here many & varied accepted most . . .

* more care needed when $\int 2\sqrt{x}$.

(c) $T_1 = 2$, $T_{32} = 126$

$S_n = \frac{n}{2}(a + l)$

$= \frac{32}{2}(2+126)$

$= 2048$

* well done
* some need to relearn formula!

(d) $\angle PTQ = 14°38' - 10°12' = 4°26'$

in $\triangle PQT$ using sine rule

* again many varied ways

Q 12

(d) $\angle PTQ = 14°38' - 10°12' = 4°26'$

in $\triangle PQT$ using sine rule

$$\frac{QT}{\sin 10°12'} = \frac{100}{\sin 4°26'}$$

$$QT = \frac{100 \sin 10°12'}{\sin 4°26'}$$

$$= 229.09037$$

in $\triangle QBT$ $\quad \sin 14°38' = \frac{BT}{QT}$

$$\therefore BT = QT \sin 14°38'$$

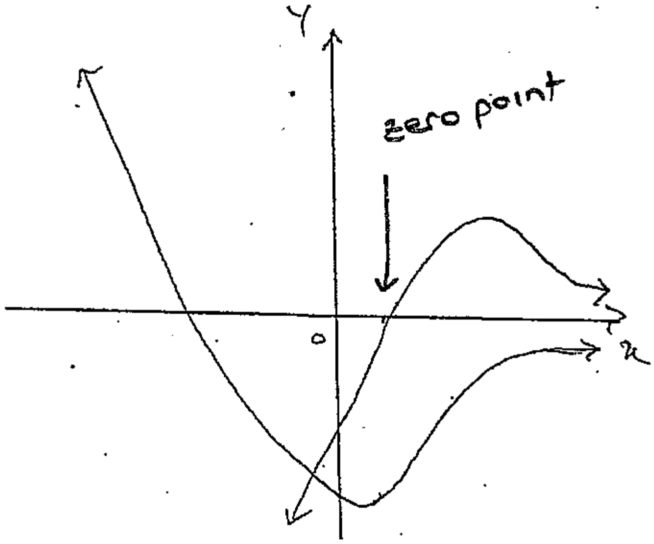
$$= 57.8756\ldots$$

$$= \cancel{58}\ \underline{57.88\ m} \text{ (nearest cm)}$$

* again many varied ways appeared...
* c. through marks awarded...
* errors in transferring angles in workings
* Be careful – asked to nearest cm! $\underline{\underline{57.88m}}$!

Question 12

(e)



Markers Comments

* Badly Drawn Graphs!
* Generous marking here
* Students must label axis, curves etc properly
* smoother lines needed!

(1mk) – awarded for showing curve going upwards through correct zero point

(1mk) – showing curve come back

QUESTION 13 (JPN) | MARKER COMMENT

a. $\cos 2\theta = \frac{1}{\sqrt{2}}$ $\quad 0 \leqslant \theta \leqslant 2\pi$

$\therefore 0 \leqslant 2\theta \leqslant 4\pi$

many forgot this and so didn't get all 4 solution.

$\therefore 2\theta = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$ ✓✓

2 marks needed angle in 1st Q & 4th Q for 2 revolutions.

$\boxed{\theta = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}}$ ✓

finding solution

b. $\boxed{A = 3}$ ✓ $\boxed{\alpha = \frac{\pi}{4}}$ ✓

A = amplitude

α = units shifted to left.

1 mark for each.

c. $x^2 + (m-2)x + 4 = 0$

$\Delta = b^2 - 4ac$

$= (m-2)^2 - 4(1)(4)$

$= m^2 - 4m + 4 - 16$

$\Delta = m^2 - 4m - 12$ ✓

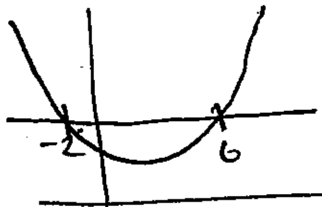
1 mark calculating Δ.

$\Delta < 0$ since no real roots.

ie $m^2 - 4m - 12 < 0$ ✓

1 mark quadratic inequality.

$(m-6)(m+2) < 0$



$\boxed{-2 < m < 6}$ ✓

1 mark correct solution

d. (i) $\alpha + \beta = -\frac{b}{a} = \boxed{-4}$ ✓

answer only

(ii) $\alpha\beta = \frac{c}{a} = \boxed{-8}$ ✓

" "

(iii) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= -8 \times -4$

$= \boxed{32}$ ✓

answer only

(iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

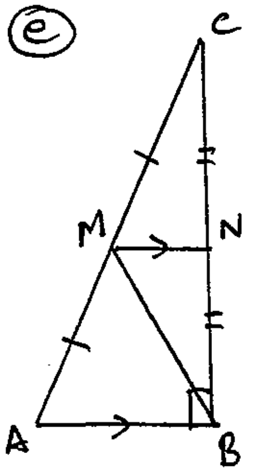
$= (-4)^2 - 2 \times -8$

$= 16 + 16 = \boxed{32}$ ✓

answer only.

Q 13 continued.

(e)



$AM = MC$ ... (M is midpoint of AC) ✓

In $\triangle CMN$ + $\triangle BMN$

$CN = NB$ ... (equal intercepts ie ratio of intercepts on parallel lines are equal since $AM = MC$ and $MN \parallel AB$)

$\angle CNM = \angle CBA = 90°$ (corresponding $\angle$s are equal as $MN \parallel AB$)

$\angle BNM = 180° - \angle CNM$ ($\angle CNB$ is a straight $\angle$)

$= 180° - 90°$

$= 90°$

$\therefore \angle BNM = \angle CNM$

MN is common

$\therefore \triangle CMN \equiv \triangle BMN$ (SAS)

$\therefore MC = MB$ ... (corresponding sides of congruent $\triangle$s)

and since $AM = MC$ and $MC = MB$

$\therefore AM = BM = CM$

$\therefore$ M is equidistant from all 3 vertices.

*Draw Diagram!! it is not given!!

1 mark to prove $\triangle CMN \equiv \triangle BMN$

1 mark logical argument to prove that vertices are equidistant to M.

Most attempts were quite poor.

Q14 (a) $y = 2xe^x$

$$y' = 2x.e^x + 2.e^x$$
$$= 2(x+1)e^x$$
$$\therefore m = 2(1+1)e^1 \quad \text{at } (1,e)$$
$$= 4e$$ ✓

Generally well done

$\therefore$ Eq^n of tangent: $y - e = 4e(x-1)$
$$y - e = 4ex - 4e$$
$$4ex - y - 3e = 0 ✓$$
$$(y = 4ex - 3e)$$

Common error: "$-4e^1 + e^1$" not combined to $-3e$

/2

(b)(i) $y = x^2 + 12$

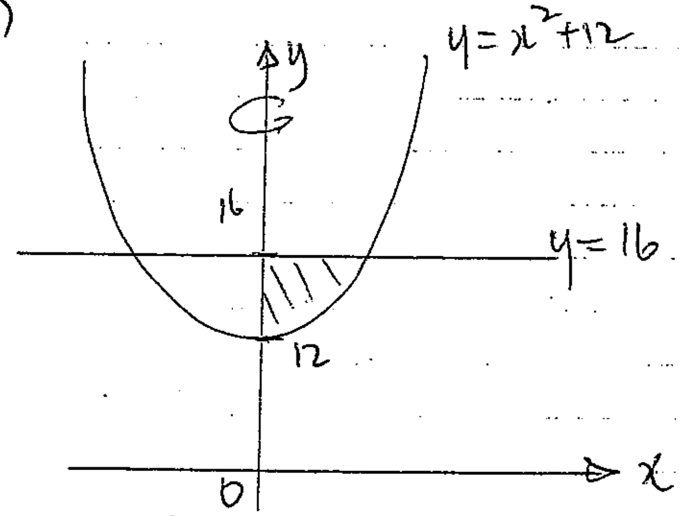
$$\therefore x^2 = 4a(y-12)$$
$$\therefore 4a = 1$$
$$a = \frac{1}{4}$$

$V = (0, 12)$ ✓

$S = (0, 12\frac{1}{4})$ ✓

Generally well done

(ii)

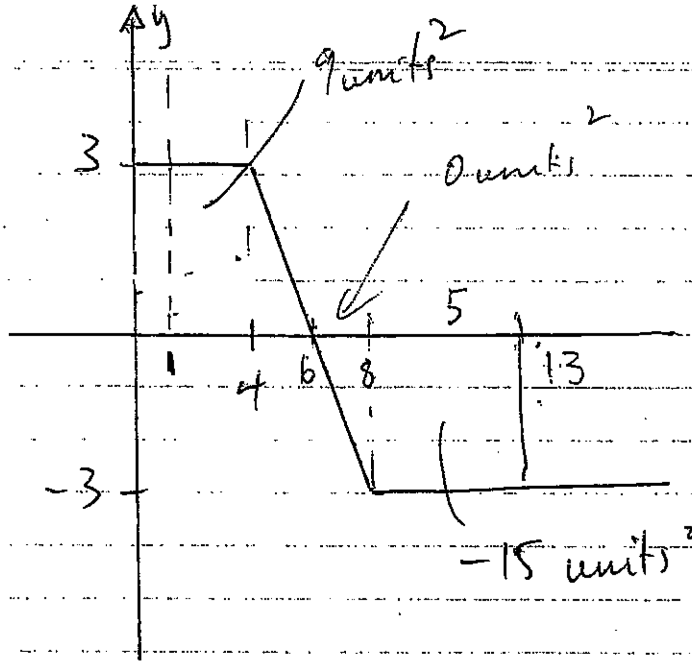


Many did not bother to sketch – it usually helps

$$Vol = \pi \int x^2 . dy$$
$$= \pi \int_{12}^{16} (y - 12) . dy$$
$$= \pi \left[\frac{1}{2}y^2 - 12y \right]_{12}^{16}$$

$y = x^2 + 12$
$\Rightarrow x^2 = y - 12$ ✓

Q14 (d)



Again, quickly copying the diagram helps. Many answers were just a jumble of numbers on the page – hard to follow

(i) $\int_0^4 f(x)\,dx = \int_0^4 3\,dx$ OR By inspection $= 4\times 3$
$= 12$

$= \left[3x\right]_0^4$

$= 12$ ✓

(ii) $\int_4^8 f(x)\,dx = 0$ as the area above the x axis is equal and opposite to the area below the x axis. ✓

N.B. $\int_4^6 f(x) = \frac{1}{2}\times 3\times 2$
$= 3$

$\int_6^8 f(x) = \frac{1}{2}\times(-3)\times 2$
$= -3$

(iii) $\int_1^a f(x)\,dx = -6$

Area ($x=1$ to $x=4$) $= 9$
Area $x=4$ to $x=8$ $= 0$
Area below axis from $x=8$ must be -15 units^2 $\therefore$ ✓
$a = 8+5$
$= 13$ ✓

Many missed that the lower limit of integration was 1.

4

Q14 (b)(ii) Continued.

$$Vol = \pi\left[\tfrac{1}{2}y^2 - 12y\right]_{12}^{16}$$ ✓

$$= \pi\left[\tfrac{1}{2}(16^2-12^2) - 12(16-12)\right]$$

$$= \pi\left[\tfrac{1}{2}(28)(4) - 12(4)\right]$$

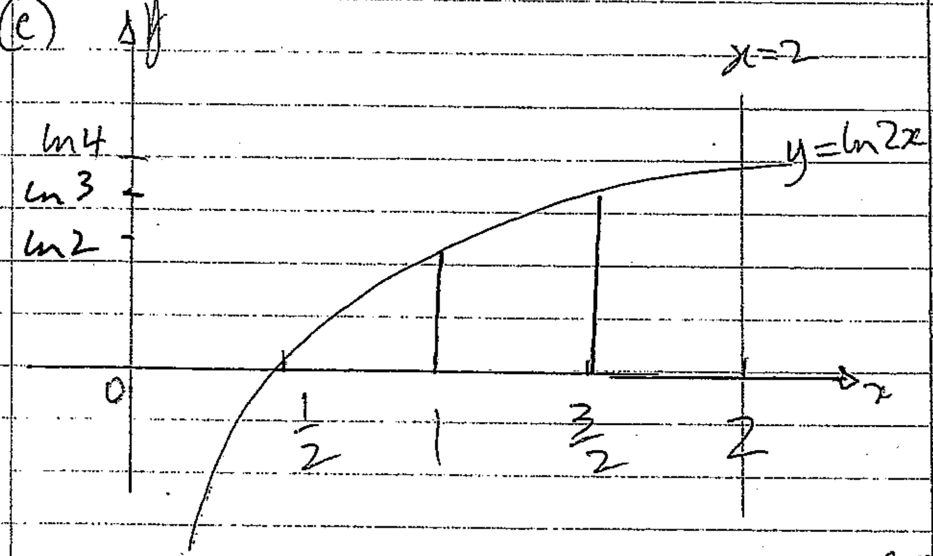
$$= 8\pi \text{ units}^3.$$ ✓

$\frac{1}{2}.16^2 - 12\times16 - (\frac{1}{2}12^2 - 12^2$

$128 - 192 - (-72$

/5

(c)



Large, clear sketches help!

Graph and/or correct table of values

x	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$2x$	1	2	3	4
$\ln 2x$	0	0.6931	1.0986	1.386

$$\text{Area} \doteq \frac{\frac{1}{2}}{2}\left[\ln 1 + 2(\ln 2 + \ln 3) + \ln 4\right]$$

$$= \frac{1}{4}\times 4.9698\ldots$$

$$= 1.2424\ldots$$

$$= 1.24 \text{ units}^2 \text{ (2 decimal pl.)}$$ ✓

Common mistakes.
1. FIVE function values used
2. $\ln 0 = 0$ ✗
$\ln 0$ is undefined (think $-\infty$).

x int: $\ln 2x = 0$
$2x = e^0$
$= 1$
$\therefore x = \frac{1}{2}$

Many assumed incorrectly that it was just a "standard" log graph $\therefore$ cuts at $x=1$

Question 14 (Cont)

d) (i) $\int_0^4 f(x)\,dx = 3\times4 = 12$

(ii) Since $\int_4^6 f(x)\,dx = -\int_6^8 f(x)\,dx$

Then $\int_4^8 f(x)\,dx = \int_4^6 f(x)\,dx + \int_6^8 f(x)\,dx$

$= 0$

(iii) $\int_1^4 f(x)\,dx = 9$

$\int_4^8 f(x)\,dx = 0$ (already shown)

Now $\int_8^{13} f(x)\,dx = -3\times5$

$= -15$

$\therefore \int_1^{13} f(x)\,dx = 9+0-15$

$= -6$

$\boxed{a=13}$

Question 15

a) (i) $R = R_0 e^{-kt}$

When $t=0$, $R = 1\text{kg}$ ← 1mk

$\therefore 1 = R_0 e^0 \therefore R_0 = 1$

$\underline{R = e^{-kt}}$

Well done.

When $t = 20$, $R = 0.95$

$\therefore 0.95 = e^{-20k}$

take log of both sides

$\ln(0.95) = \ln e^{-20k}$

$-20k = \ln(0.95)$ $\boxed{k = -\frac{\ln(0.95)}{20}}$ ← 1mk

Question 15 (cont)

Markers Comments

(i) For half life let $R = 0.5$

$\therefore 0.5 = e^{-kt}$ — 1mk

$\ln 0.5 = \ln e^{-kt}$

$\therefore -kt = \ln 0.5$

$t = \frac{\ln 0.5}{-k}$

$= \ln 0.5 \times \frac{20}{\ln 0.95}$

$= 270.3$ yrs — 1mk

Well done

---

b) (i) Since initially the particle is at rest but acceleration is acting in the positive direction.

Needed
* $t=0$ at rest
* $a>0$ for all t.
for 1mk.

(ii) $\ddot{x} = 4e^{-2t}$

$\therefore \dot{x} = 4\int e^{-2t}\,dt$

$\dot{x} = \frac{4}{-2}e^{-2t} + c$

$= -2e^{-2t} + c$ — 1mk

when $t=0$, $\dot{x}=0$

$\therefore 0 = -2e^{0} + c \qquad c = 2$ — 1mk

$\therefore \boxed{\dot{x} = -2e^{-2t} + 2}$

Too many neglected to consider "c".

(iii) $\lim_{t\to\infty} 4e^{-2t} = 0$ — 1mk

$\therefore \lim_{t\to\infty} -2e^{-2t} + 2 = 2$

$\therefore$ velocity is 2 in positive direction — 1mk

Too many showed no reasoning.

Question 15 (cont.)

(c) (i) let A_n be amount owed after nth repayment

$A_1 = 650000(1+0.01) - M$ ← 1mk

$A_2 = A_1(1.01) - M$

$= (650000(1.01) - M)(1.01) - M$

$= 650000(1.01)^2 - M(1.01) - M$

as req'd.

Generally Well done!

(ii) $A_3 = (650000(1.01)^2 - M(1.01) - M)(1.01) - M$

$= 650000(1.01)^3 - M(1.01)^2 - M(1.01) - M$

$= 650000(1.01)^3 - M(1 + 1.01 + 1.01^2)$

$A_{300} = 650000(1.01)^{300} - M(\underbrace{1 + 1.01 + 1.01^2 + \dots + 1.01^{299}}_{\text{This is a G.P}})$ ← 1mk

$\therefore A_{300} = 650000(1.01)^{300} - M\dfrac{(1.01^{300} - 1)}{0.01}$

But $A_{300} = 0$

$\therefore 650000(1.01)^{300} - M\dfrac{(1.01^{300} - 1)}{0.01} = 0$

$M\dfrac{(1.01^{300} - 1)}{0.01} = 650000(1.01)^{300}$

$M = \dfrac{650000(1.01)^{300}(0.01)}{(1.01^{300} - 1)}$

$= \underline{\$6845.96}$ ← 1mk

(cont)

Marker's Comments

(c) let $M = 6845.96 + 250$

$= \$7095.96$ ← 1mk

Most understood the question.

$$7095.96 = \frac{650000(1.01)^n(0.01)}{(1.01^{300}-1)}$$

$7095.96(1.01^{300}-1) = 650000(1.01)^n(0.01)$ ← 1mk

$7095.96(1.01)^n - 7095.96 = 6500(1.01)^n$

$595.96(1.01)^n = 7095.96$

$(1.01)^n = 11.907$

$$n = \frac{\ln(11.907)}{\ln(1.01)}$$

$= 248.95$ mths ← 1mk

Too many found $n = 249$ months but DID NOT then show a saving of 51 months

~~Difference~~ $= 300 - 248.95$

$\equiv 51$ months ← 1mk

Question 16

(a) (i) $y' = 2x^2 - 5x - 3$

$y'' = 4x - 5$

For S.P's let $y' = 0$

$\therefore 2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$\therefore x = -\frac{1}{2},\ x = 3$

When $x = -\frac{1}{2}$, $y = \frac{67}{24}$

$x = 3$, $y = -11\frac{1}{2}$

$\therefore$ SP's at $(-\frac{1}{2}, \frac{67}{24})$ and $(3, -11\frac{1}{2})$

need correct "y" values to get full marks.

Question 16

Markers Comments

When $x = -\frac{1}{2}$ $y'' = -7$ (<0)

$\therefore \boxed{\left(-\frac{1}{2}, \frac{67}{24}\right) \text{ Max Turning point}}$

When $x = 3$, $y'' = 7$ (>0)

$\therefore \boxed{\left(3, -11\frac{1}{2}\right) \text{ min turning point}}$

(ii) Possible P.O.I. where $y'' = 0$

$\therefore 4x - 5 = 0$

$x = 1\frac{1}{4}$

* Must show change of concavity to get the mark.

test:

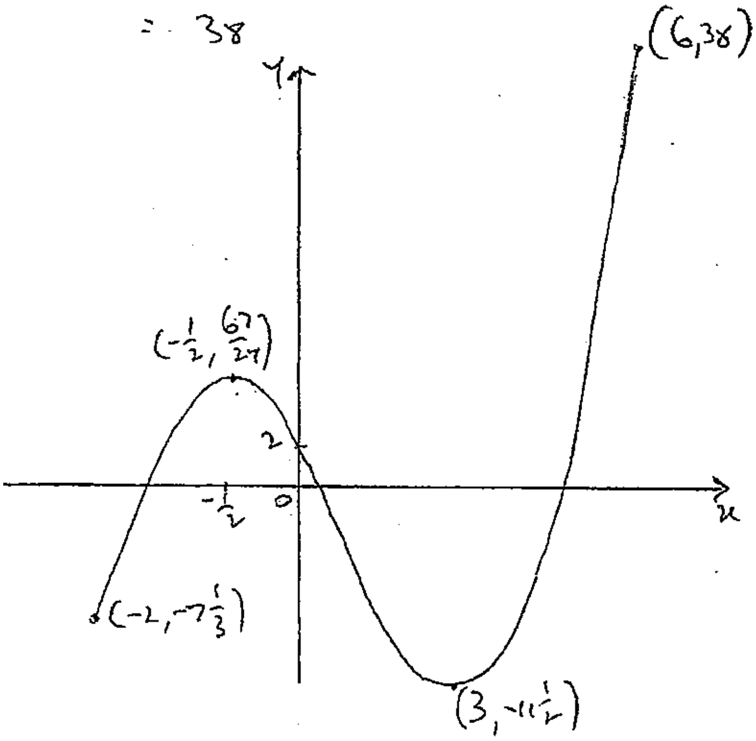
x	1	$1\frac{1}{4}$	1.5
y''	-1	0	1

Since change in concavity Inflexion at $x = 1\frac{1}{4}$

(iii) $f(-2) = -\frac{22}{3} = -7\frac{1}{3}$

$f(6) = \frac{2}{3}\times 6^3 - \frac{5}{2}\times 6^2 - 3\times 6 + 2$

$= 38$



Must show end points to get full marks. 1 Mark deducted for not having correct end poin

Question 16 (cont)

(b) (i) $\tan\alpha = \frac{80}{60}$ $\tan\beta = \frac{60}{80}$

$\therefore \tan\alpha = \frac{4}{3}$, $\tan\beta = \frac{3}{4}$

In $\triangle AFG$:

$\tan\alpha = \frac{x}{r}$

$\therefore r = \frac{x}{\tan\alpha}$

$\boxed{r = \frac{3}{4}x}$

Similarly in $\triangle CDE$:

$\tan\beta = \frac{x}{s}$

$\therefore s = x \div \frac{3}{4}$

$\boxed{s = \frac{4}{3}x}$

(ii) By Pythagoras $AC = 100$

now $AC = y + s + r$

$100 = y + \frac{4}{3}x + \frac{3}{4}x$

$y = 100 - \left(\frac{4}{3}x + \frac{3}{4}x\right)$

$y = 100 - \frac{25}{12}x$

(iii) $y = 100 - \frac{25}{12}x$

largest rectangular building is one with largest area

$A = xy$

$= x\left(100 - \frac{25}{12}x\right)$

$= 100x - \frac{25}{12}x^2$

$A' = 100 - \frac{25}{6}x$

$A'' = -\frac{25}{6}$ (<0)

$\therefore$ For max area $A' = 0$

Markers Comments

* Many students used similarity to do this question.

Question 16 (Cont)

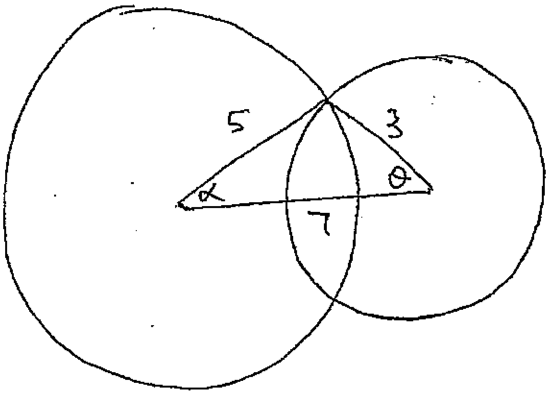
(iii) $100 - \frac{25}{6}x = 0$

$x = 24$

$y = 100 - 50 = 50$

$\therefore$ $\boxed{\begin{array}{l} \text{Breadth} = 24\text{m} \\ \text{length} = 50\text{m} \end{array}}$

(c)



$\cos\alpha = \dfrac{5^2 + 7^2 - 3^2}{2 \times 5 \times 7}$ $\quad$ $\cos\beta = \dfrac{3^2 + 7^2 - 5^2}{2 \times 3 \times 7}$

$\alpha = 21.8°$ $\quad$ $= 38.2°$

$$A = \frac{1}{2}(5)^2\left[\frac{2(21.8°) \times \pi}{180} - \sin(2 \times 21.8°)\right] + \frac{1}{2}(3)^2\left[\frac{2(38.2°) \times \pi}{180} - \sin(2(33.2))\right]$$

$= 2.518\text{cm}^2$

Markers Comments

* Need to show that this value is minimum ie using $A'' > 0$ or stating that it is a parabola with $a > 0$

* Very few students were successful with this question. Most students scored no marks.
Those student who attempted to do the question by integration usually do not get any marks with the occasional student recieving 1/3 if all their working was correct.