QUESTION ONE

(a) If $\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{z^2}$ find the value of z correct to one decimal place, when x = 6.53 and y = 7.48.

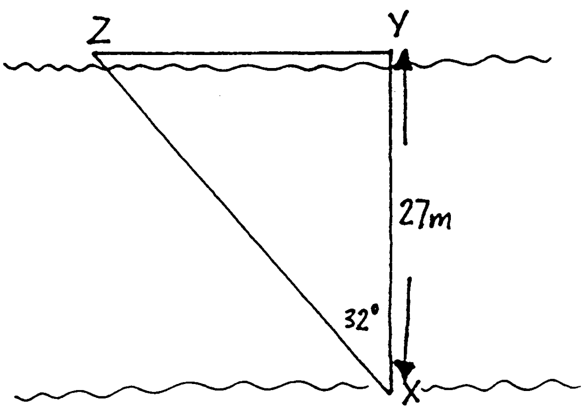
(b) Solve (i) $\sqrt{6a - 3} = 8$

(ii) $\frac{3}{4}(x+3) - \frac{1}{3}(2x-3) = \frac{5}{6}$

(c) Find the value of $\frac{926.1 - 34.72}{\sqrt{16.84} + 2.97}$

correct to 4 significant figures

(d)



X and Y are on opposite banks of a river 27 metres wide. A boy swims from X across the river but because of the current, ends up at point Z. If $\widehat{YXZ}$ is 32°, find the distance the boy actually swam. (answer correct to 3 sig. figures)

(e) A computer was sold for \$3621 including 27½% sales tax. Find the cost of the computer before the sales tax was added.

QUESTION TWO

(a) Determine $\lim_{x \to 3} \frac{x^2-9}{x-3}$

(b) Differentiate the following with respect to x.

(i) $3x^3 - \frac{5}{\sqrt{x}}$

(ii) $x^2 e^x$

(iii) $(2 - \cos x)^3$

(c) If α and β are the roots of the equation

$2x^2 - 4x + 1 = 0$

find:

(i) $\alpha + \beta$

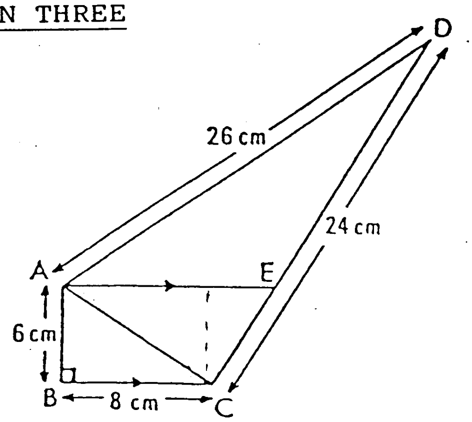
(ii) $\alpha\beta$

(iii) $\alpha^2 + \alpha\beta + \beta^2$

(d) The point P(0, -4) is the midpoint of A(g,-2) and B(-5,h). Find the values of g and h.

QUESTION THREE

(a)



In the diagram (not to scale) angle ABC = 90°, AB = 6 cm, BC = 8 cm, CD = 24 cm and AD = 26 cm, AE is parallel to BC.

(i) Calculate the length of AC and then prove that the angle ACD = 90°.

(ii) Calculate the size of the angle CAE. (to the nearest minute)

(iii) What is the length of DE? (to one decimal place)

(b) Find the area of the sector of a circle in which the arc length is 15 cm and the radius is 7 cm.

(c) Find the primitive function of each of the following:

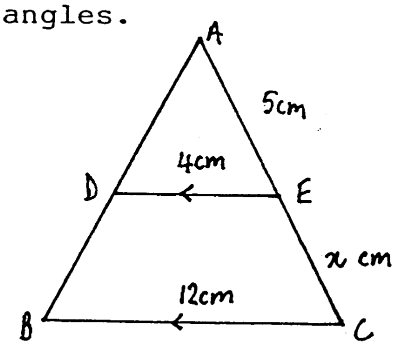
(i) $\cos 2x - e^{7x}$

(ii) $\frac{x}{2x^2+1}$

QUESTION FOUR

(a) Find the size of each internal angle of a regular pentagon. Hence, or otherwise, find the size of each of the external angles.

(b)



Find x, giving reasons in full. (Diagram not drawn to scale).

(c) JKLM is a quadrilateral.
Its diagonals are perpendicular, meeting at T, which is the midpoint of KM, but not of JL.

(i) Draw a diagram showing this information.

(ii) Prove that $J\hat{K}L = J\hat{M}L$

(d) If $\cos\theta^{\circ} = -\frac{3}{7}$ and $180^{\circ} < \theta < 360^{\circ}$ give the exact value of $\cot\theta^{\circ}$.

QUESTION FIVE

(a) P is the point $(-2,7)$

d is the line $2x + 5y + 1 = 0$
k is the line through P, perpendicular to d.

(i) Find the equation of k

(ii) If d meets the y-axis at D, and k meets the y-axis at K, find the area of the triangle PKD

(b) Find the values of x for which

$$|2x - 1| \geq 5$$

(c) A curve has the equation $y = ax^2 + b$,

where a and b are constants.

At the point $(3,4)$ on the curve, the gradient of the tangent is 6.

Find a and b

QUESTION SIX

(a) The first term of an arithmetic sequence is 18 and the fourth term is 9.
Show that the sum of the first four terms is equal to the sum of the first 9 terms.

(b) From a lighthouse, L, a ship S, bears 053° and is at a distance of 8 nautical miles. From L a boat B bears 293° and is a distance of 6 nautical miles.

(i) Draw a diagram, marking on it the information supplied.

(ii) Find the distance of ship S from boat B.
Give your answer as a surd.

(iii) Find the bearing of ship S from boat B.
Give your answer to the nearest degree.

(c) Sketch the curve $y = 3 \sin 2x$

over the interval $0 \leqslant x \leqslant 2\pi$

State the period and amplitude of the curve.

QUESTION SEVEN

(a) The velocity $\frac{dx}{dt}$ of a particle moving along the x-axis

is given by $\frac{dx}{dt} = 3t^2$.

If $t = 1$ when $x = -$ [illegible], find x in terms of t

Determine (i) the initial value of x

(ii) the acceleration of the particle after 4 seconds

(b) Express $0.\dot{3}\dot{5}$ as an infinite series and hence (or otherwise) calculate its value as a rational number.

(c) Solve $m^4 - 6m^2 - 40 = 0$

(d) Show that $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$

QUESTION EIGHT

(a) Use Simpson's Rule, with four sub divisions, to find an approximate value for

$$\int_1^2 \frac{1}{x}\,dx$$

Give your answer to two decimal places.

(b) (i) Show, by writing the equation in standard form, that the equation $x^2 + 6y - 8x + 4 = 0$ is a parabola

(ii) Find the focal length, the coordinates of the focus and the vertex.

(iii) Find the equation of the directrix.

(iv) Sketch the parabola

(c) If $\sqrt{45} + \sqrt{80} = \sqrt{x}$, find the value of x.

QUESTION NINE

(a) Find the exact value of volume generated by rotating about the x-axis, the area in the first quadrant bounded by the curve $xy^2 = 1$, the x-axis, the line $x = 1$ and the line $x = 4$.

(b) For the curve $y = x^3 - 3x^2 - 9x$

i) Find the stationary points and determine their nature.

ii) Find any points of inflexion.

iii) Using the above information and any other relevant information, sketch the curve of $y = x^3 + 3x^2 - 9x$.

iv) Find the set of values for which the curve is monotonically increasing.

QUESTION TEN

(a) Find the derivative of $\log_e(1 - \tan x)$

and hence evaluate

$$\int_0^{\pi/4} \frac{\sec^2 x}{1 + \tan x}\, dx$$

(b) The number (N) of bacteria in a culture at time t seconds is given by the equation

$$N = 5000e^{0.002t}$$

(i) What is the number of bacteria initially?

(ii) Determine the number of bacteria present after 20 seconds.

(iii) After what period of time will the number of bacteria double itself?

(iv) At what rate is the number of bacteria increasing when $t = 20$?