Roseville College

YEAR 12

TRIAL EXAMINATION 1995

MATHEMATICS

2/3 UNIT

Time Allowed - 3 hours

Directions to Candidates

* All questions may be attempted
* All questions are of equal value
* All necessary working must be shown
* Each question must begin on a new page
* Note that the marks for parts of questions are shown in parentheses

Question 1 **(Start each question on a new page)**

a) Evaluate correct to 3 significant figures
(2)

$$\sqrt{\frac{16.4 + 93.7}{45.6 - 29.4}}$$

b) Solve: $8 - (x - 7) = 5 + x$
(2)

c) What is the exact value of $\sin 300°$?
(2)

d) Factorise $ax^2 - 25a$
(2)

e) The volume of a sphere is $\frac{4}{3}\pi r^3$.
(2)
Find correct to two decimal places the radius of a ball whose volume is 100cm^3

f) Solve $|x - 3| < 7$
(2)

Question 2 (Start a new page)

a) (2) Simplify $\dfrac{\cos(180° - A)}{\sin(90° - A)}$

b) (2) Find the arc length of a sector of 55° cut from a circle of radius 10cm (answer correct to 1 decimal place).

c) (2) What is the equation of the circle whose centre is at the origin and which passes through the point (2, -5)?

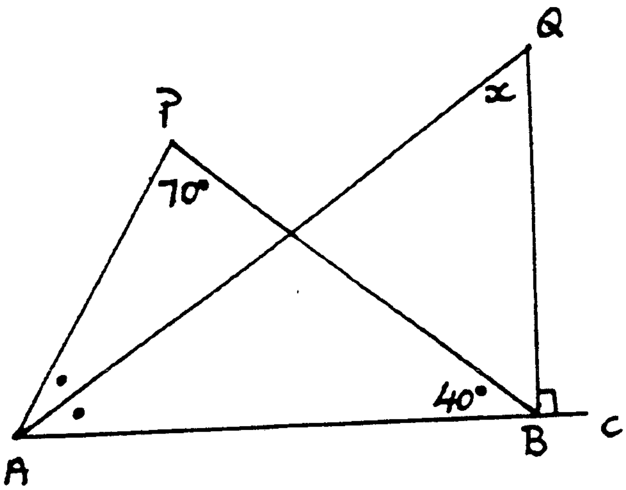
d) (2) Comment **briefly** on the validity of the following statement.
"As there are 26 letters in the alphabet, if I choose any letter at random on a page of print, the probability that it is a "k" is $\frac{1}{26}$."

e) (2) Find the largest angle in the triangle with sides 5cm, 6cm and 9cm. Answer to the nearest degree.

f) (2) The Mothers' Club sewed special school library bags. By recruiting more volunteers, production increased steadily each week.
The number of bags made each week were: 15, 20, 25, 30 ......
Using your knowledge of arithmetic series, how many bags were made altogether in the first 12 weeks?

Question 3 **(Start a new page)**

a) (3) In the figure (not drawn to scale),



AQ bisects ∠ PAB.

QB is perpendicular to AC

∠ ABP = 40°

∠ APB = 70°

∠ AQB = x°

i) Draw a neat sketch of the diagram

ii) Calculate x giving reasons for your answer

b) (2) Differentiate $(x^2 - 5)^8$

c) (3) If $y = \dfrac{\sin x}{1 + \cos x}$ show that $\dfrac{dy}{dx} = \dfrac{1}{1 + \cos x}$.

d) (2) Find the primitive of $5\sqrt{x}$.

e) (2) A number is selected at random from the integers 20 to 30.

Find the probability that it is either a prime or a perfect square.

Question 4 (Start a new page)

a) (4) If α and β are the roots of the equation $3x^2 + 2x - 4 = 0$, find the value of:

i) $\alpha + \beta$

ii) $\alpha\beta$

iii) $(\alpha + 1)(\beta + 1)$

b) (4) The coordinates of three points are A (-2, -1), B (-2, 2) and C (3,7)

i) Find the area of the triangle

ii) Find the midpoint of AC. Call this point M.

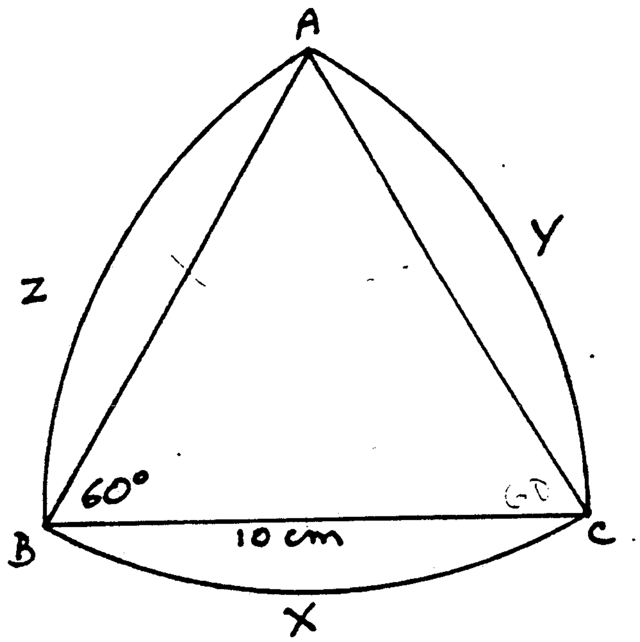
iii) Hence or otherwise find the point D so that ABCD is a parallelogram.

c) (4) In the figure (not drawn to scale), ABC is an equilateral triangle of side 10cm. The circular arcs AZB, BXC, CYA are constructed with centres at C, A, B respectively.

Find:

i) The area of the sector CAZB

ii) The total area enclosed by BZAYCX.



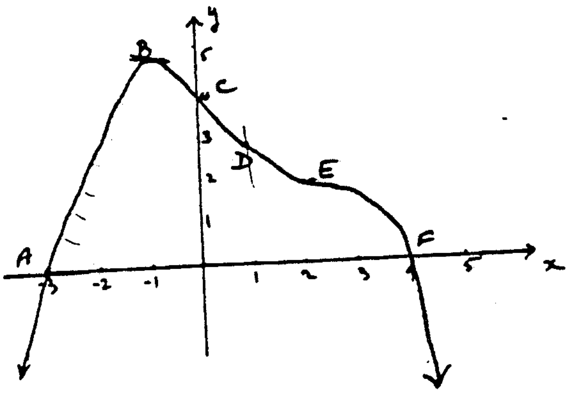
Question 5 (Start a new page)

a) (3) $\int (e^{2x+3} + 5)\,dx$

b) (3) Evaluate $\int_0^2 \frac{1}{2x+1}\,dx$ correct to 1 decimal place.

c) (3) Evaluate exactly $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\,dx$. Leave your answer as a surd.

d) (3)



$y = f(x)$ is the function illustrated.

A is the point (-3,0)

B is the point (-1,5)

C is the point (0,4)

D is the point (1,3)

E is the point (2,2)

F is the point (4,0)

Given B is a maximum turning point, D is a point of inflexion and E is a horizontal point of inflexion. State the values of x for which:

i) $y < 0$

ii) $y' < 0$

iii) The polynomial has zero value.

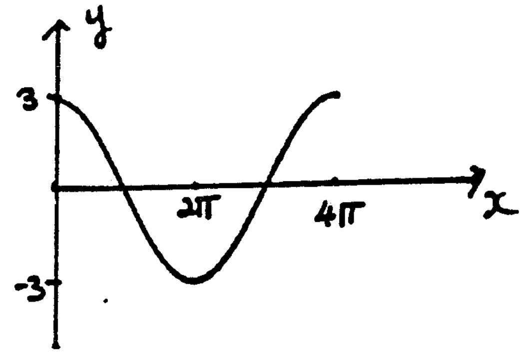
Question 6 (Start a new page)

a) (3) The diagram below represents a possible sine or cosine curve.

i) Give the amplitude.

ii) Give the period.

iii) Write down a possible equation for the curve.



b) (3) *Moss, Kuswadi & Ryan Pty. Ltd.* went into business on the first of January. The company profits increased each month until September ($500, 000). Due to industrial strikes, their supply of goods was depleted and there was a sudden decline in profitability until December when they just managed to break even.

i) Draw a possible graph showing profit against time (in months).

ii) Comment on the nature of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ over time.

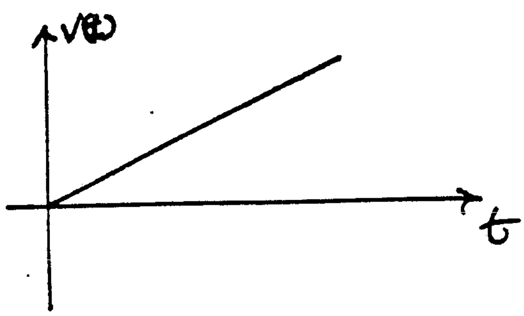
c) (5) A particle moves in a straight line and after t seconds, its velocity v m/s is given by $v = 12t - 3t^2$.

i) When is the particle at rest?

ii) Find the acceleration at $t = 1$.

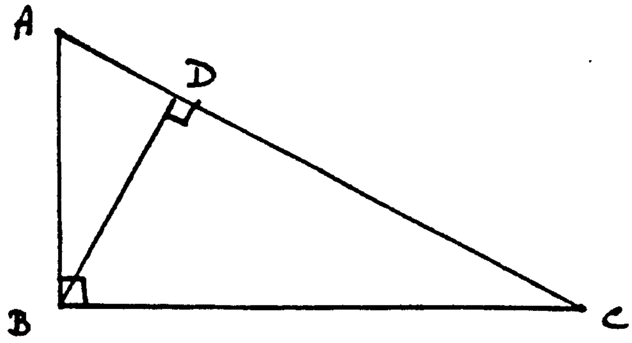
iii) Find the distance travelled in the fourth second.

d) (1) Copy this velocity graph onto your paper and then draw a neat sketch of a possible acceleration graph.



Question 7 (Start a new page)

a) (4) Diagram not to scale.



$\triangle$ ABC has a right angle at B.

BD is perpendicular to AC.

i) Show that $\triangle$ ABC ||| $\triangle$ADB.

ii) If BC = 9cm, DB = 4.5cm, AB = 5cm, find AD correct to one decimal place.

b) (3) A water tank had 2000 litres of water in it. Water is flowing into the tank at the rate R litres per minute where $R = 1.6t$ and t is time in minutes.

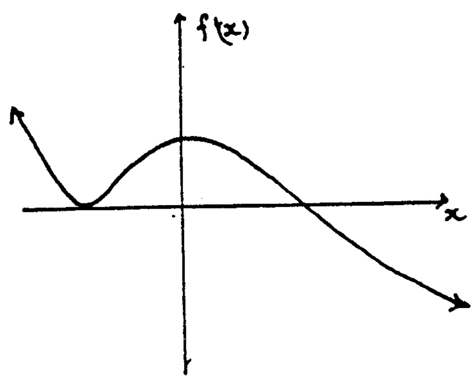
i) Find the formula for the volume, V litres of water in the tank at any time t.

ii) How much water is in the tank after a quarter of an hour?

c) (3) David's four wheel drive broke down in the Simpson Desert. He knew that he would have to walk 180km to get help and that he could not do it in one day. He planned to walk 80km on the first day then half of the previous day's distance each day thereafter. Will he be able to reach help? Justify your answer.

d) (2) The sketch below represents $f'(x)$ of a function, $f(x)$.

Copy the diagram and on the same axes sketch clearly a possible function, $f(x)$.



Question 8 (Start a new page)

a) (5) $y = 2 + 9x - 3x^2 - x^3$.

i) Find any stationary points and indicate clearly whether it is a maximum or minimum turning point.

ii) Locate any point of inflexion.

b) (3) Find the area bounded by the curves $y = 4 - x^2$ and $y = x^2 - 4$.

c) (4) The speed of a cyclist in the Olympic Games was recorded every 15 minutes.
A table was drawn up of the time in minutes and the corresponding speeds, S in km/h.

Time (min)	0	15	30	45	60
Speed (km/h)	0	52	49	53	55

Use Simpson's Rule to find the approximate value of $\int_0^1 S\,dt$ (where t is in hours).

Question 9 **(Start a new page)**

a) (5) The population of Australia can be estimated by using the formula $P = P_0e^{kt}$ where t is the time in years, and P_0 and k are constants. If the population of Australia at the beginning of 1980 was 14.741 million and at the beginning of 1991 was 16.849 million.

i) Find the annual growth rate, k correct to four significant figures.

ii) What will the population be at the beginning of the year 2000?

iii) During which year will the population be expected to reach 25 million?

b) (7) When Elizabeth was born, her grandparents set up a trust fund for her to receive on her 18th birthday. The conditions agreed upon for this trust fund involved investing \$100 every four months. Interest is calculated on the balance at the end of each year and is credited at the rate of 8% p.a.

i) Show that after two years the trust fund will amount to $\$(300 \times 1.08^2 + 300 \times 1.08)$

ii) How much will Elizabeth receive on her 18th birthday from this trust fund?

iii) Calculate the simple interest rate that would produce the same profit.

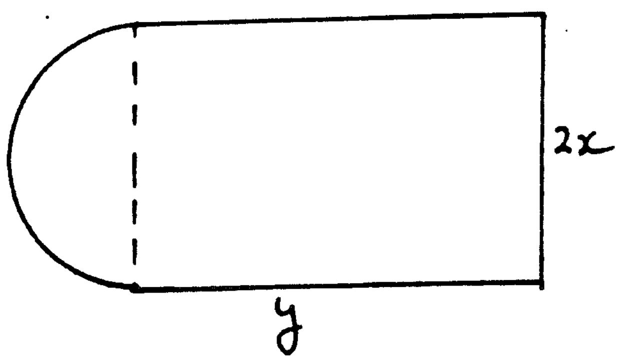
Question 10 (Start a new page)

a) (3) Solve for x: $e^{2\ln x} = x + 20$.

b) (3) Find the exact volume of the solid formed when the curve $y = \log_e x$ is rotated about the y axis from $y = 1$ to $y = 3$.

c) (6) A surfboard is in the shape of a rectangle and a semi circle as shown. The perimeter is to be 4m.

i) Show that $y = 2 - x - \frac{\pi x}{2}$



ii) Show that the area of the board can be expressed as

$$A = 4x - 2x^2 - \frac{\pi x^2}{2}$$

iii) Find the maximum area of the surfboard.

Question 1 (12)

2.4069707 ✓

2.41 to 3 sig figs. ✓ (2)

$8 - x + 7 = 5 + x$ ✓

$2x = 10$ (2)

$x = 5$ ✓

$\sin 300^\circ$

$-\sin 60^\circ$ ✓ (2)

$-\frac{\sqrt{3}}{2}$ ✓

S	A
T	C ✓

$ax^2 - 25a$

$a(x^2-25)$ ✓ (2)

$a(x+5)(x-5)$ ✓

Volume $= \frac{4}{3}\pi r^3$

$100 = \frac{4}{3} \times \pi \times r^3$

$\frac{300}{4\pi} = r^3$ ✓ (2)

$r^3 \doteqdot 23.87\ldots$

$r \doteqdot 2.8794119\ldots$ ✓

$\therefore r = 2.88$ cm to 2 dec. pl.

$x - 3 < 7$ or $-(x-3) < 7$

$x < 10$ ✓ $\quad -x + 3 < 7$

$-x < 4$

$x > -4$ ✓

$\therefore -4 < x < 10$ (2)

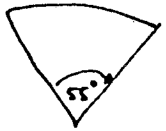
Question 2 (12)

a) $\frac{\cos(180 - A)}{\sin(90 - A)}$

$= \frac{-\cos A}{\cos A}$ ✓✓ (2)

$= -1$

b)



$l = r\theta$

$l = 10 \times \frac{55}{360} \times 2\pi$ ✓

$\doteqdot 9.599\ldots$

$l = 9.6$ cm ✓ (2)

c) $x^2 + y^2 = r^2$ through $(2,-5)$

$4 + 25 = r^2$ ✓

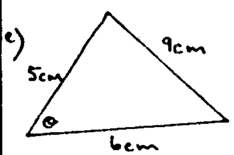
$r^2 = 29.$ (2)

$\therefore$ equation is $x^2 + y^2 = 29$. ✓

d) As k is not evenly distributed throughout the words of the language and occurs much less than many other letters, especially the vowels, then this probability would not hold. (2)

0, 1, 2

e)



$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos\theta = \frac{5^2+6^2-9^2}{2 \times 5 \times 6}$ ✓

$\theta^\circ \doteqdot 109.47\ldots$ (2)

$= 109^\circ$ to the nearest degree ✓

Arithmetic series in which $a = 15$ $d = 5$

$n = 12$.

$\therefore$ Total $= 15 + 20 + 25 + \ldots\ldots$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$ ✓

$= \frac{12}{2}\{2 \times 15 + 11 \times 5\}$

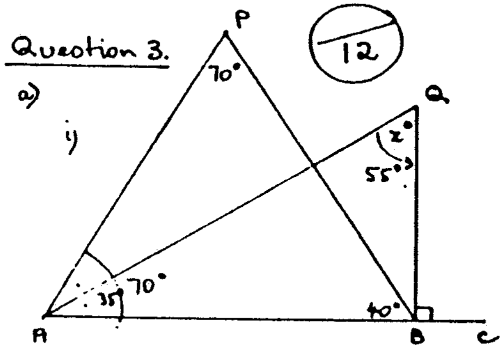
$= 510$ ✓ (2)

$\therefore$ 510 bags in 1st 12 weeks.

Question 3. (12)

a)

i)



ii) $\angle PAB = 70^\circ$ [angle sum of $\triangle PAB$]

given $\angle PBA = 40^\circ$

given $\angle A\hat{P}B = 70^\circ$ ✓

$P\hat{A}Q = Q\hat{A}B$ [given]

$\therefore 2\angle QAB = \angle PAB$

$\therefore 2\angle QAB = 70^\circ$ [from above]

$\angle QAB = 35^\circ$. ✓

$\angle QBA = 90^\circ$ [supplementary adjacent angles]

$\therefore A\hat{Q}B = 180 - 90^\circ - 35^\circ$ [angle sum of $\triangle QAB$]

$\therefore x^\circ = 55^\circ$. ✓ (3)

b) $y = (x^2-5)^8$

$y' = 8(x^2-5)^7 \times 2x$ ✓✓ (2)

$= 16x(x^2-5)^7$.

c) $y = \frac{\sin x}{1 + \cos x}$

$u = \sin x$

$u' = \cos x$.

$v = 1 + \cos x$

$v' = -\sin x$. ✓

$\therefore y' = \frac{(1+\cos x) \times \cos x - \sin x \times -\sin x}{(1+\cos x)^2}$

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ ✓

$= \frac{1 + \cos x}{(1+\cos x)^2}$ (3)

$= \frac{1}{1 + \cos x}$ ✓

d) $f'(x) = 5x^{1/2}$ ✓

$f(x) = 5\frac{x^{3/2}}{3/2} + c$ (2)

$= \frac{10}{3}x^{3/2} + c$ ✓

e) 20, 21, 22 (23) 24 (25) 26 27 28 ✓

Question 4 (12)

$3x^2 + 2x - 4 = 0$

i) $\alpha + \beta = -\frac{b}{a}$ 1

$= -\frac{2}{3}$

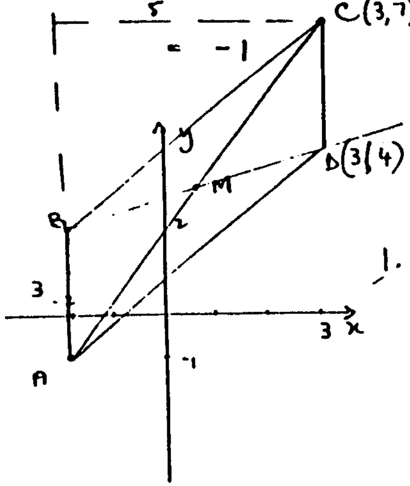
ii) $\alpha\beta = \frac{c}{a}$ 1

$= \frac{-4}{3}$ 2

iii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= -\frac{4}{3} + -\frac{2}{3} + 1$

$= -1$



1

Area $= \frac{1}{2}bh$ 1

$= \frac{1}{2} \times 3 \times 5$

$= 7\frac{1}{2}$ units^2

(4)

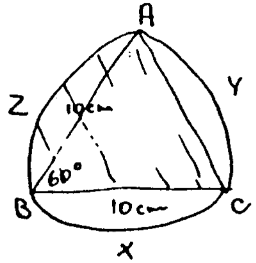
$M\left(\frac{-2+3}{2}, \frac{-1+7}{2}\right)$

$M\left(\frac{1}{2}, 3\right)$ 1

$D(3, 4)$ 1

(4)

c)



Area CAZB $= \frac{60}{360} \times \pi \times 10^2$

1 $\doteqdot 52.3598\ldots$ [2]

$= 52$ cm^2 to nearest cm^2

Area Δ ABC $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 10^2 \times \sin 60°$

1 $\doteqdot 43.30127\ldots$ [1]

$\therefore$ Total area $=$ Area [1] $+ 3 \times$ [Area [2] $-$ Area [1]]

2 $\doteqdot 70.47\ldots$ cm^2

$= 70$ cm^2 to nearest cm

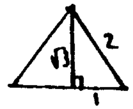
(4)

Question 5 (12)

a) $\int (e^{2x+3} + 5)\,dx = \frac{1}{2}e^{2x+3} + 5x + c$ ① ① ①

b) $\int_0^2 \frac{1}{2x+1}\,dx = \left[\frac{1}{2}\ln(2x+1)\right]_0^2 = \frac{1}{2}(\ln 5 - \ln 1) = 0.8$ ① ① ①

c)



$\int_{\pi/6}^{\pi/3} \sec^2 x\,dx = \left[\tan x\right]_{\pi/6}^{\pi/3}$ ①

$= \tan\frac{\pi}{3} - \tan\frac{\pi}{6} = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ ① ①

d) i) $x < -3,\ x > 4$ ①

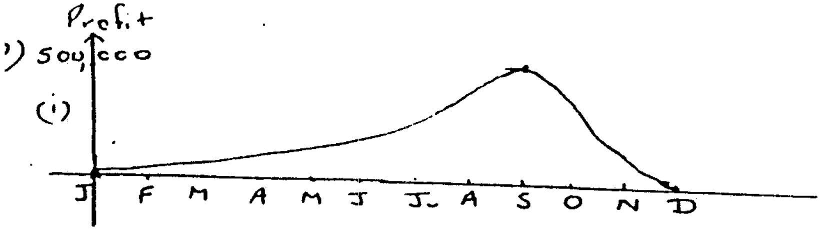
ii) $-1 < x < 2,\ x > 2$ ①

iii) $x = -3, 4.$ ①

Q6 (12)

(a) (i) 3 (1) (ii) 4π (1) (iii) $y = 3\cos\frac{x}{2}$ (1)

(b) (i)



(1)

(ii) $\frac{dP}{dt} > 0$ for time before September (1)
$\frac{dP}{dt} = 0$ " Sept. (and possibly December)
$\frac{dP}{dt} < 0$ from Sept. to Dec.

(iii) $\frac{d^2P}{dt^2} < 0$ for months close to Sept. (1)
It would be > 0 elsewhere.

(c) $v = 12t - 3t^2$

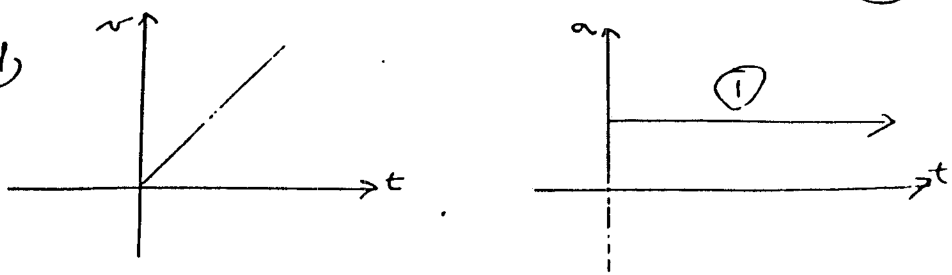
(i) $v = 0$ when $3t(4-t) = 0$ (1)
i.e. At rest when $t = 0$, 4 secs.

(ii) Acceln. $= 12 - 6t$ (1)
When $t = 1$, Acceln. $= 6\ ms^{-1}$ (1)

(iii) $d = \int (12t - 3t^2)\,dt$
$= 6t^2 - t^3 + c$. (1)

Distance in 4th second $= d(4) - d(3)$
$= (32 + c) - (27 + c) = 5\,m$. (1)

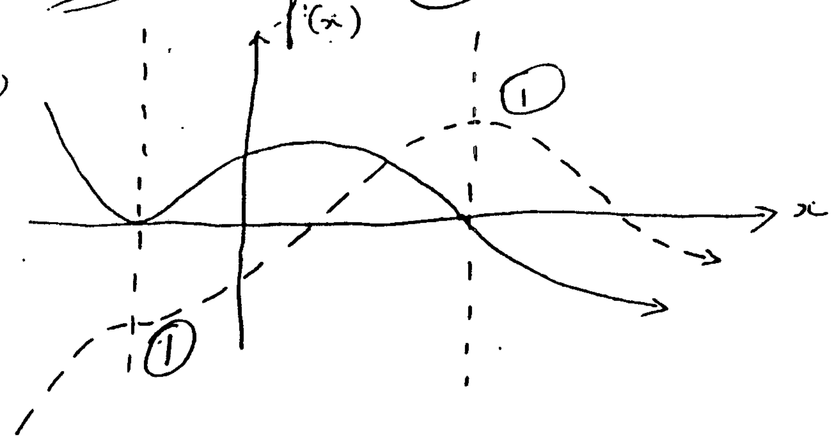
(d)



(1)

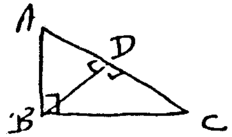
Q7 (12)

(d)



(1) (1)

(a)



(1) $\{ \hat{ADB} = \hat{ABC}$ (both $= 90°$, given)
$\hat{BAC} = \hat{BAD}$ (same angle)
$\therefore \hat{ABD} = \hat{ACD}$ ($\angle$ sum Δ's). (2)

$\therefore \Delta ABC \,|||\, \Delta ADB$ (equiangular)

(ii) Either $AD = \sqrt{5^2 - 4.5^2} = 2.2$ (by Pythag.)
(1) OR $\frac{AD}{5} = \frac{4.5}{9} \Rightarrow AD = 2.5$ (sim. Δ's).

(b) (i) $\frac{dV}{dt} = 1.6t \Rightarrow V = 0.8t^2 + c$ (1)
When $t = 0$, $V = 2000$ $\therefore V = 0.8t^2 + 2000$ (1)

(ii) When $t = 15$, $V = 2180$ litres (1)

(c) G.P with $a = 80$, $r = \frac{1}{2}$ (1)
$S_\infty = \frac{80}{1 - \frac{1}{2}} = 160$ (1)
$\therefore$ Even if walks forever only gets 160 km.
$\therefore$ does NOT get to help (180 km away)
(1)

(d) see above.

Question 8 (12)

a) $y = 2 + 9x - 3x^2 - x^3$

$y' = 9 - 6x - 3x^2 = 3(3 - 2x - x^2)$

at stationary pts $y' = 0 = 3(3+x)(1-x)$

$\therefore y' = 0$ at $x = -3$ or $x = 1$

$y'' = -6 - 6x$

at $x = -3$, y'' +ve $\therefore$ concave up $\therefore$ min

$y = 2 + 9(-3) - 3(-3)^2 - (-3)^3 = -25$

at $x = 1$; y'' = −ve $\therefore$ concave down $\therefore$ max

$y = 2 + 9.1 - 3.1^2 - 1^3 = 7$

$\therefore (-3, -25)$ minimum

$(1, 7)$ maximum

$y'' = -6 - 6x = -6(1+x)$

$y'' = 0$ for possible pt of inflexion

$-6(1+x) = 0$ at $x = -1$

x	-1^-	-1	-1^+
y	+ve	0	−ve

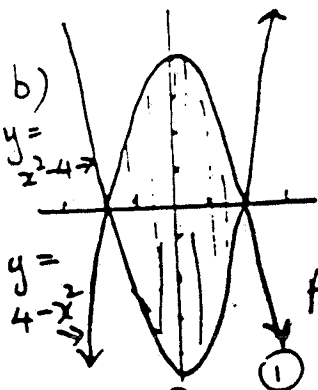
(1) $\therefore$ it is a pt of inflexion

$y = 2 + 9(-1) - 3(-1)^2 - (-1)^3 = -9$ (1)

$\therefore$ pt of inflexion at $(-1, -9)$ (1)

/5

b)



area bounded by $y = 4 - x^2$ and $y = x^2 - 4$

$A = 4\int_0^2 4 - x^2 \, dx$ (1)

$= 4\left[4x - \frac{x^3}{3}\right]_0^2 = 4\left(8 - \frac{8}{3}\right) - 0$ (1)

$= 4 \times 5\frac{1}{3} = 21\frac{1}{3}$ sq units (1)

/3

c) Simpson's Rule $= \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

using minutes

$= \frac{15}{3}\left[0 + 4\times52 + 2\times49 + 4\times53 + 55\right]$

$= 5[573] = 2865$

change to hours $= \frac{2865}{60} = 47.75$ km

change to hours first

$\frac{\frac{1}{4}}{3}\left[0 + 4(52) + 2(49) + 4(53) + 55\right]$

$= 47.75$ km

/4.

Question 9 (12)

a) $P = P_0 e^{kt}$

$16.849 = 14.741 e^{kt}$

(½) $\frac{16.849}{14.741} = e^{kt} = 1.143$

$11K \ln e = \ln 1.143$

(½) $K = \ln 1.143 \div 11$

$K = 0.01215078$

(1) $K = 0.01215$ (to 4 sig fig)

b) $P = 14.741 e^{20 \times 0.01215}$

$= 18.79578657$

$\doteqdot 18.796$ million. (1)

c) $25 = 14.741 e^{0.01215t}$ (½)

$\frac{25}{14.741} = \ln e \, 0.01215t$

$t = \ln \frac{25}{14.741} \div 0.01$ (½)

$= 43.476$ years (½)

$\therefore$ in year 2023 (½)

b) \$100 + \$100 + \$100 = \$300/year at 8% paid annually. (½)

$\therefore$ after 1 year $A_1 = 300 \times 1.08$

after 2 years $A_1 = (300 \times 1.08) \times 1.08$

$= 300 \times (1.08)^2$. (½)

In 2nd year \$300 invested at 8% p.a. (½)

$A_2 = 300 \times 1.08$

$\therefore$ at end of second year the two amounts will be

$A_1 + A_2 = 300 \times 1.08^2 + 300 \times$

Question 9. (continued)

(b)(ii) in 3rd year

$A_3 = 300\times 1.08^3 + 300\times 1.08^2 + 300\times 1.08$.

Continuing in this pattern

$A_{18} = 300\times 1.08^{18} + 300\times 1.08^{17} + \dots + 300\times 1.08^2 + 300\times 1.08$

This is the sum of a G.P.

$a = 300\times 1.08$, $r = 1.08$, $n = 18$

$S_n = \frac{a(r^n - 1)}{r-1} = \frac{300\times 1.08(1.08^{18}-1)}{1.08-1}$

$= \$12\,133.88$ — 3

iii) $P = \$300\times 18 = \$5400$

Interest $= \$12133.88 - \$5400 = \$6733.88$

$S.I = PRN$

$\$6733.88 = \$5400 \times 18 \times R$.

$R = \frac{6733.88}{5400\times 18} = 6.93\%$ — 2

Question 10 (12)

a) $e^{2\ln x} = x + 20$ $x > 0$

$2\ln x \ln e = \ln(x+20)$

$2\ln x = \ln(x+20)$

$\ln x^2 = \ln(x+20)$

$\therefore x^2 = x + 20$.

$x^2 - x - 20 = 0 = (x-5)(x+4)$

$\therefore x = 5$ or -4 (but x +ve)

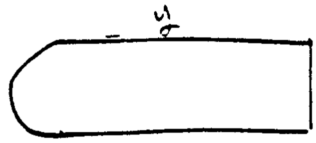
$\therefore$ solution is $x = 5$ — 3

b) $y = \log_e x \Rightarrow e^y = x$

$V = \pi\int x^2 dy = \pi\int_1^3 e^{2y} dy$

$= \pi\left[\frac{1}{2}e^{2y}\right]_1^3 = \frac{\pi}{2}\left[e^6 - e^2\right]$ — 3

c) i)



P = semicircle $+ 2y + 2x$

$4 = \pi x + 2y + 2x$

$4 - 2x - \pi x = 2y$

$y = 2 - x - \frac{\pi x}{2}$ — 2

ii) Area = semicircle + rectan.

$A = \frac{\pi x^2}{2} + \left(2 - x - \frac{\pi x}{2}\right)2x$

$= \frac{\pi x^2}{2} + 4x - 2x^2 - \pi x^2$

$= 4x - 2x^2 - \frac{\pi x^2}{2}$

iii) $\frac{dA}{dx} = 4 - 4x - \pi x$

$\frac{d^2A}{dx^2} = -4 - \pi$ which is -ve for all

If $\frac{dA}{dx} = 0$

$4 - 4x - \pi x = 0$

$4 - x(4+\pi) = 0$.

$x = \frac{4}{4+\pi}$

$= 0.56$ (2 d.p

Area $= 4(0.56) - 2(0.56)^2 - \frac{\pi}{2}[($

$\therefore$ Area $= 1.12\,m^2$ — 2