QUESTION 1.

Marks

a) Factorise fully *i)* $9x^2 - 49$ — 2

ii) $m^3 - 9m^2 + 3m - 27$

b) Solve $3n - 7 \leq 5 - n$ — 2

c) Differentiate with respect to t: $y = (5t^3 - 3)^7$ — 2

d) Evaluate exactly: $\sin 135° + \tan 120°$ — 2

e) Find a primitive for $3x^2 + \cos 5x$ — 2

f) Evaluate $\log_5 8$ correct to 4 significant figures — 1

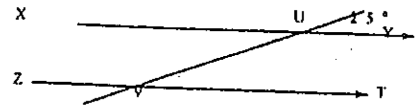
g) Evaluate: $|-7 - 8| - |3 - 11|$ — 1

135

QUESTION 2 (Start a new page)

Marks

a) — 2



In the diagram above, XY || ZT.

Find the size of UVZ and give a reason for your answer.

b) On a number plane, sketch the region which is described by — 3

$$x^2 + y^2 \leq 9 \text{ and } y > x^2 + 1$$

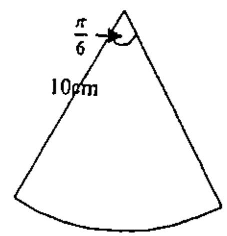
c) Differentiate the following functions with respect to x: — 5

i. $7 \tan x$;

ii. $\dfrac{x-1}{3x-4}$;

iii. $x e^{\sin x}$

d) — 2



NOT TO SCALE

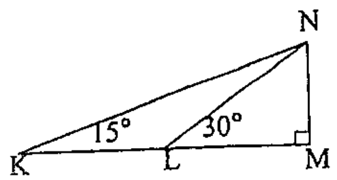
A sector drawn in a circle of radius 10cm has a sector angle of $\frac{\pi}{6}$ radians. Find the length of the arc correct to 3 decimal places.

Marks

QUESTION 3 (Start a new Page)

a) Fifty identical cards numbered from 1 to 50 are placed in a bag, and one card is drawn at random. What is the probability that this card will be either less than 20 or divisible by 3? 2

b)



i. Find LNM, KNL in degrees, and show that KL = LN. 2

ii. Given that the length of NM is 1 unit, find the *exact* lengths of LM and LN. 2

iii. Deduce that $\tan 15° = 2 - \sqrt{3}$ 1

b) Find the following indefinite integrals: 5

i. $\int 3\cos 2x\,dx$;

ii. $\int \frac{4x+5}{2x^2+5x}dx$;

iii. $\int \frac{x+1}{\sqrt{x}}dx$.

136

Marks

QUESTION 4 (Start a new page)

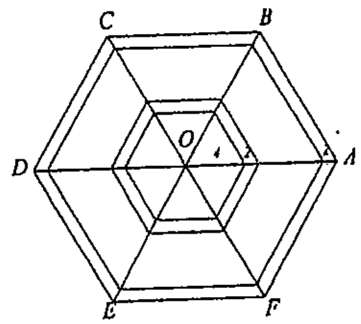
(a) Write down the formula for:

(i) the nth term of an arithmetic series with first term a and common difference d; 1

(ii) the sum of the first n terms of this series. 1

A particular spider's web consists of a series of regular hexagons with a common centre O, held together by rays through O, as in the figure, where only some of the hexagons are shown.

NOT TO SCALE, LENGTHS IN CM.



The vertices of the smallest hexagon are *4* cm from O, the vertices of the next are *2* cm further away and they continue at *2* cm intervals along the rays until the vertices of the last hexagon ABCDEF are 60 cm from O (i.e. OA = 60cm).

(iii) Using parts (i) and/or (ii), show that there are 29 hexagons in the spider's web. 1

(iv) What is the length, in cm, of the perimeter of the smallest hexagon? 2

(v) What is the total length of thread used by the spider in making this web (including the six rays from O)? 3

b) A function $y = f(x)$ has a stationary point at (2, -6) and $f''(x) = 6x - 4$

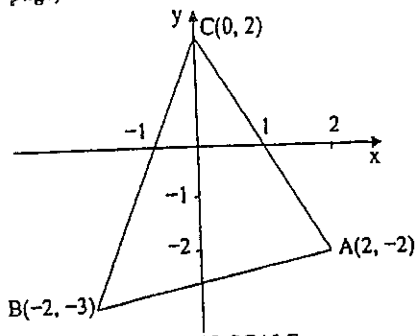
Find

i) the nature of the stationary point at (2, -6) 1

ii) the equation of the function 3

Marks

QUESTION 5 (Start a new page)



NOT TO SCALE

A(2, −2), B(−2, −3) and C(0, 2) are the vertices of a triangle ABC.

i. Find the length of the interval AC

ii. Find the gradient of AC.

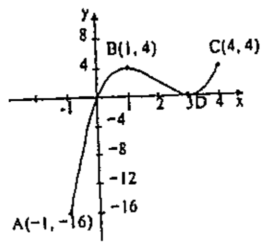
iii. Show that the equation of the line AC is $y = -2x + 2$.

iv Calculate the perpendicular distance of B from the side AC

v. Hence find the area of ΔABC.

vi. Find the co-ordinates of D such that ABCD is a parallelogram 7

b)



i) Using the words positive, negative or zero, complete the following sentences:
At B and D, f'(x) is ______________. (answer on your answer paper). 1

Between D and C, f'(x) is ______________. (answer on your answer paper). 1

ii) On Page 12, the above diagram is reproduced. On the axes underneath it, sketch the graph of $y = f'(x)$ 3

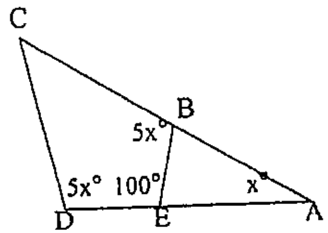
137

Marks

QUESTION 6 (Start a new page)

a) Evaluate $\lim_{x \to 3} \dfrac{x^3 - 27}{x - 3}$. 2

b)

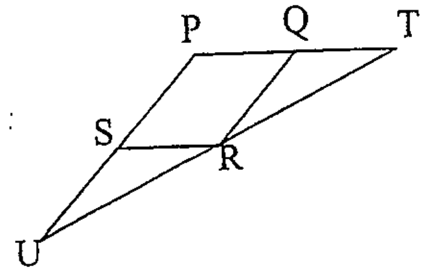


As shown in the figure (which is not to scale), B and E lie on the sides AC and DA respectively of ΔACD.
Use the information shown on the figure to

(i) find the value of x 2

(ii) and hence give ACD in degrees. 1
Give reasons for your answers.

c)



PQRS is a parallelogram. PQ is produced beyond Q to T so that QT = QR and PS is produced beyond S to U so that SU = PS. T, R and U are collinear.
Prove that PQRS is a rhombus. 3

d) If α and β are the roots of $2x^2 + 3x - 4 = 0$, find the value of:

i) $\alpha + \beta$; 2

ii) $\alpha^2 + \beta^2$.

2

e) Solve $2x^2 + 3x - 2 < 0$

QUESTION 7 **(Start a new page)** **Marks**

a) A parabola has equation $x^2 + 6x - 33 = 12y$. Find:

i) the axis of symmetry

ii) the coordinates of the vertex;

iii) the coordinates of the focus;

iv) the equation of its directrix. 4

b) A ship sails from port A on a course of 075° for 20 nautical miles then changes its course to 130° and continues sailing for 30 nautical miles.

i) Draw a neat sketch of the ship's course. 1

ii) How far is it from its starting point? (Answer to the nearest nautical mile). 1

iii) What is the ship's bearing from it's starting point? (Answer to the nearest degree). 2

c) Use

(i) the Trapezoidal rule and

(ii) Simpson's Rule to estimate $\int_4^8 \log_e(x^2)\,dx$, (one application of each)

rounding your answer to one decimal place. 4

138

QUESTION 8 (Start a new page) **Marks**

a) For $\frac{\pi}{2} < A < \frac{3\pi}{2}$, find A if $\sin^2 A = \frac{1}{4}$. 2

b) Two ordinary dice are tossed. What is the probability that 3

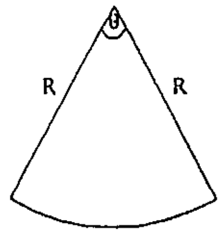
i) the uppermost faces will show a pair of "5's"

ii) the uppermost faces will show exactly one "3" and one "5"

iii) the uppermost faces of at least one die will show a "5"

c)

d)



NOT TO SCALE

If the area of this sector is 625 m^2,

i) show that $\theta = \frac{1250}{R^2}$, and 1

ii) find an expression for P (the perimeter) in terms of R and θ, and hence show that

$$P = 2R + \frac{1250}{R}.$$ 2

iii) Find the values of R and θ (to the nearest degree) so that the sector has minimum perimeter. 2

QUESTION 9 (Start a new page) Marks

a) Evaluate $\sum_{k=2}^{8} 2^k$ 2

b) The rate at which water runs out of a tank is proportional to the volume of water in the tank, ie. $\frac{dV}{dt} = kV$. The tank is full to start with and has a capacity of 36 000 litres.

i) Show that $V = V_0e^{kt}$ satisfies this equation where V_0 is the volume of water in the tank initially. 1

ii) If $\frac{1}{4}$ of the water in the tank runs out in 30 minutes, find the volume of water remaining in the tank after 60 minutes. 3

c) The area under the curve $y = \sqrt{9 - x^2}$, $-3 \le x \le 3$, is rotated about the x axis.

i) Find the volume of the solid of revolution thus obtained. 3

ii) Describe the shape of this solid. 1

d) The sum of the first n terms of a series is given by $S_n = 3^n + 2n^2$. Find the 13th term. 2

QUESTION 10 (Start a new page) Marks

a) Solve for x: $2\ln x - \ln(2 - x) = \ln 2 - \ln 3$ 3

b) i) Expand $e^{-x}(1 - e^{-x})$. 1

ii) For the curve $y = e^{-x} - e^{-2x}$

a) Show that it cuts the axes at (0, 0) only. 1

b) Show that there is only one stationary point at $(\ln 2, \frac{1}{4})$, and determine the nature of that stationary point. 3

c) Hence determine the values of x for which the curve has a negative gradient, and so discuss the behaviour of the curve for large values of x. 1

d) Sketch the curve. 1

e) Calculate the area bounded by this curve, the x-axis and the ordinates x = 0 and x = ln 2. 2

End of Paper

Q1

a) (i) $(3x-7)(3x+7)$ (1)

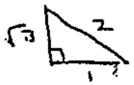
(ii) $m^2(m-9) + 3(m-9)$
$= (m-9)(m^2+3)$ (1)

b)
$3n - 7 \le 5 - n$
$4n \le 12$ (1)
$n \le 3$ (1)

c)
$y = (5t^3 - 3)^7$
$y' = 7(5t^3-3)^6 \times 15t^2$ (1)
$= 105t^2(5t^3-3)^6$ (1)

d) $\sin 135° + \tan 120°$
$= \sin 45° - \tan 60°$ (1)
$= \frac{1}{\sqrt{2}} - \sqrt{3}$.
(1)

S | A
T | C



e) $\int (3x^2 + \cos 5x)\,dx = x^3 + \frac{1}{5}\sin 5x + c$
(1) (1)

(f) $\log_5 8 = \frac{\log 8}{\log 5} = 1.292$ (1)

(g) $|-7-8| - |3-11| = 15 - 8$
$= 7$ (1)

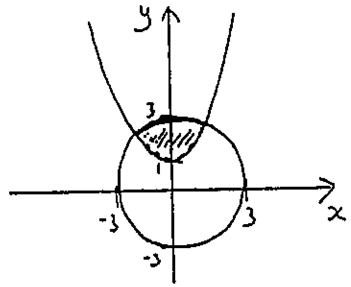
140

Marking Scheme Qn 2

a) (1) $\boxed{\angle UVZ = 155°}$

(1) $\boxed{\text{pair of reasons}}$ (e.g. vertically opp. angles equal
co-interior angles XY ∥ ZT)

b)



(1) $\boxed{\text{circle, centre origin, radius 3}}$

(1) $\boxed{\text{parabola, concave up, vertex at } y=1}$

(½) correct region is indicated

(½) circle edge included,
parabola edge is dashed

c) i) $y' = \boxed{7\sec^2 x}$

$\sec^2 x$ scores (½)

ii) $y' = \boxed{\frac{-1}{(3x-4)^2}}$

numerator × (using incorrect
denominator ✓ (½)

numerator ✓, denom. × (1)

numerator × (using correct f
but denom ✓ (1)

numerator × (using incorrect
but denom ✓ (½)

iii) $y' = \boxed{e^{\sin x} + x\cos x\, e^{\sin x}}$;
$= \boxed{e^{\sin x}(1 + x\cos x)}$

$y' = x\cos x\, e^{\sin x}$, (½

no product rule (1)

product rule and cos x appears somewhere
and $e^{\sin x}$ appears somewhere (1)

product rule and no [illegible]
mistake (1)

d) $l = 10 \times \frac{\pi}{6} = 5.23598$
$= \boxed{5.236 \text{ cm}}$ (3 d.pl.)
rounded correctly to 3 d.p.

5.235 s

use this

2 a)

minus one for each bit missing
- Factorising the top correctly
- Cancelling $(x-3)$ top and bottom
- Substituting $x=3$ (½), to get 27 (½)

$(x-3)(x^2+3x+9)$ or some such followed then correctly scores 1.5 out of 2

3 b)

QTR is isosceles → (½)

or

180 + (180 − 5x) = 180

angles on a straight line (½)

angle sum of a triangle (½)

x = 20 (½)

half each

① $\hat{BCD} + 5x + x = 180°$ (angle sum of $\Delta = 180°$)

① $\hat{BCD} + 5x + 5x + 100 = 360°$ (angle sum of quadrilateral = 360°)

① Solve the above simultaneously (or attempting)

① Finding $x = 20°$

① [Substituting $x = 20°$ to obtain $\alpha\hat{BCD} = 60°$

3 c)

① Explaining that QRT is an isosceles triangle

① Explaining that SUR is also isosceles

① Explaining that adjacent sides of PQRS are therefore the same

2 d)

① $\alpha + \beta = -3/2$

① $\alpha^2 + \beta^2 = \frac{25}{4}$ or $6\frac{1}{4}$

Correct formula and substitution but incorrect answer scores ½

2 e)

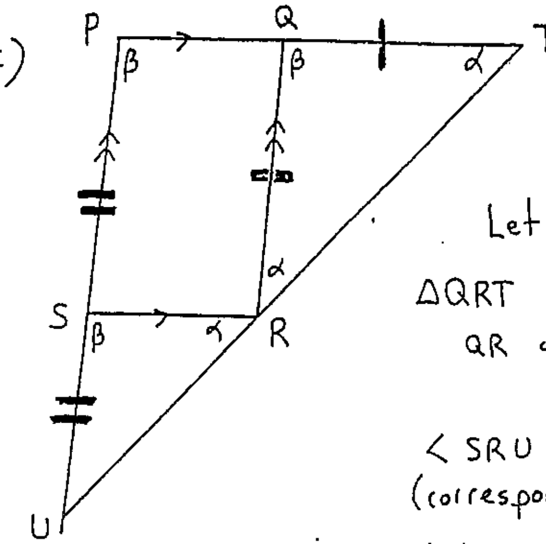
minus one for each bit missing
- Factorising correctly $(2x-1)(x+2)$
- Finding x values correctly (½ and −2)
- $-2 < x < ½$

$x > -2$ and $x < ½$ scores 2

$x < -2$ and $x < ½$ scores 1.5

141

c)



Let QTR = α

ΔQRT is isosceles (2 sides equal, QR and QT)

< SRU = α (corresponding angles, PT ∥ SR)

Let QPS = β

∴ ~~TR~~ TQR and ~~RSQ~~ RSU are both also equal to β (corresponding angles, parallel lines)

∴ SRU is similar to QTR (AAA)

∴ SRU is isosceles

∴ SU = SR (equal sides of isosceles triangle)

But PS = SU (given)

∴ PS = SR

∴ PQRS must be a rhombus (parallelogram with adjacent sides equal)

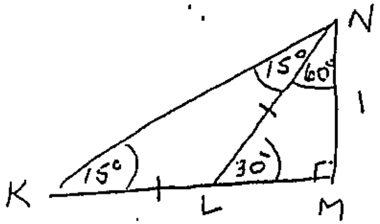
Q.E.D.

Q3

(a) $P(x<20 \text{ or } x \div 3) = \frac{19+16-6}{50} = \frac{29}{50}$ (2)

$\frac{35}{50} \to 1$ $= \cancel{\frac{35}{50}}$

(b)



(i) $L\hat{N}M = 60°$

$K\hat{N}L = 180 - (90+60+15) = 15°$ } 1

Since $L\hat{K}N = K\hat{N}L$, $KL = LN$ (sides of isosceles ΔKLN) } 1

(ii) $\sin 30° = \frac{1}{2} = \frac{NM}{NL} = \frac{1}{NL}$

$\therefore NL = 2 = KL$ 1

$\sin 60 = \frac{\sqrt{3}}{2} = \frac{LM}{NL} = \frac{LM}{2}$

$\therefore LM = \sqrt{3}$ 1

(iii) $\tan K = \frac{NM}{KM} = \frac{1}{2+\sqrt{3}}$ } $\frac{1}{2}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$. } $\frac{1}{2}$

(b) (i) $\int 3\cos 2x \, dx = \frac{3}{2}\sin 2x + c$ (2)

(ii) $\int \frac{4x+5}{2x^2+5x} dx = \ln(2x^2+5x) + c$ (1)

(iii) $\int \frac{x+1}{\sqrt{x}} dx = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c.$ (2)

Question (4)

(i)(i) $T_n = a+(n-1)d$ (1)

(ii) $S_n = \frac{n}{2}(2a+(n-1)d)$ (1)

(iii) $a=4, d=2, T_n=60$

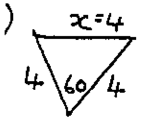
$60 = 4+(n-1).2$

$56 = 2n-2$

$2n = 58$

$n = 29$ (1)

(iv)



$4\times6 = 24cm$ (2)

(v) hex's = $24+36+\ldots+360$

Rays = $6\times60 = 360$.

Total = $\frac{29}{2}[24+360] + 360$

$= 5928cm$ (3)

) $f''(x) = 6x-4$

i) $f''(2) = 8 > 0$ (1)

$\therefore$ minimum Turning Pt

ii) $f'(x) = 3x^2-4x+c$

$f'(2) = 0$

$0 = 12-8+c$ 1 for integration no constants

$c = -4$

$\therefore f'(x) = 3x^2-4x-4$

$f(x) = x^3-2x^2-4x+k$

$f(2) = -6$

$-6 = \cancel{8}-\cancel{8}-8+k$

$k = 2$

$\therefore f(x) = x^3-2x^2-4x+2$

out of 12

Q5 a) i) $\boxed{AC = \sqrt{20}}$ or $2\sqrt{5}$

ii) $\boxed{m_{AC} = -2}$

iii) $\boxed{\text{substituting into } y = mx + b \text{ or } y - y_1 = m(x - x_1)}$

A(2, -2) or C(0, 2)

iv) $\boxed{d = \dfrac{|2\times -2 + 3\times -1 + -2|}{\sqrt{2^2 + 1^2}}}$

$\boxed{= \dfrac{9}{\sqrt{5}}}$ or $\dfrac{9\sqrt{5}}{5}$

v) $\boxed{A = 9\ u^2}$ or $\frac{1}{2} \times (i) \times (iv)$

vi) $\boxed{D \text{ is at } (4,3)}$

b) i) ~~positive~~ $\boxed{\text{zero}}$

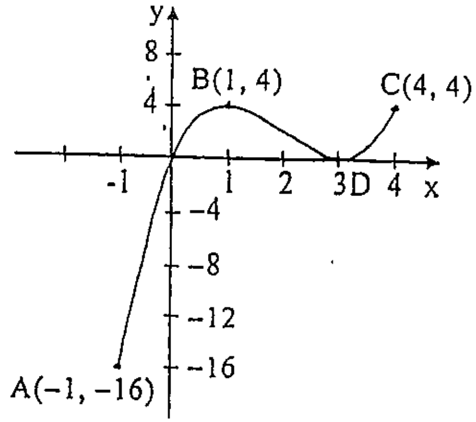
$\boxed{\text{positive}}$

ii) on sheet

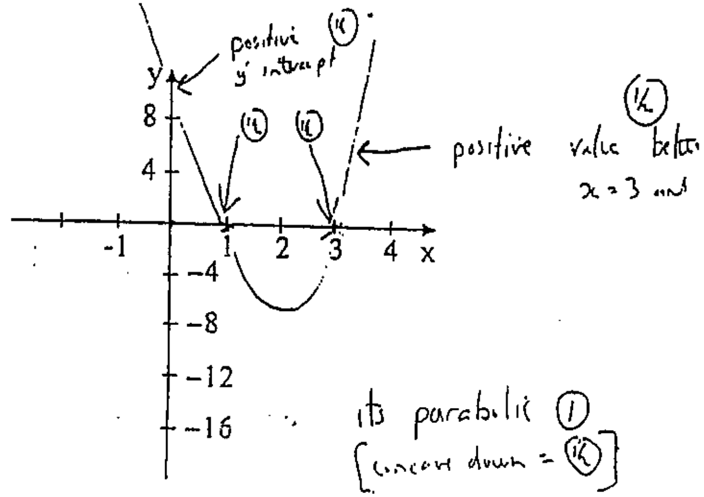
143

Question 5 (b)

Graph of y = f(x)



Graph of y = f '(x)



Q7

(a) $12y = x^2 + 6x - 33$

$y = \frac{1}{12}x^2 + \frac{1}{2}x - 2\frac{3}{4}$

(i) Axis of symmetry $= -\frac{1}{2} / \frac{1}{6} = -3$

$\therefore x = -3$ (1)

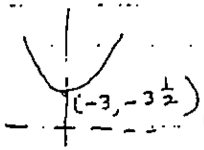
(ii) Vertex: $12y + 33 + 9 = (x+3)^2$

$12(y + 3\frac{1}{2}) = (x+3)^2$ (1)

Vertex $(-3, -3\frac{1}{2})$

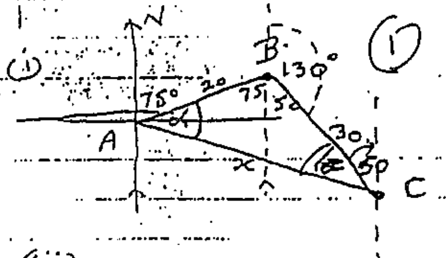
(iii) Focal length $= 3$ $(12 = 4a)$

$\therefore$ Focus $(-3, -\frac{1}{2})$ (1)



(v) $y = -6\frac{1}{2}$ is Directrix (1)

(b) (i)



(1)

(ii) $x^2 = 20^2 + 30^2 - 2 \cdot 20 \cdot 30 \cos 125$

$\therefore x = 44.2$ (1)

$= 44$ naut. miles

(iii) $\frac{\sin \alpha}{30} = \frac{\sin 125}{x}$

$\sin \alpha = \frac{30 \sin 125}{44} = 0.$ (2)

$\alpha = 33°57'$

$\therefore$ Bearing $=$ ~~$360° - (50 + 21°52')$~~

$=$ ~~$288°8'$~~ $108°57'$

$288°8'$ (1½)

(c) $\int_4^8 \ln x^2 \, dx \doteqdot$ (i) $\frac{8-4}{2}(\ln 16 + \ln 64) = 19.4$ (2)

OR (using 4,5,6,7,8) OR $\frac{8-4}{4}(\ln 16 + 2\ln 36 + \ln 64) = 14.1$

(ii) $\frac{8-4}{6}(\ln 16 + 4\ln 36 + \ln 64) = 14.2$ (2)

OR

144

Marking Scheme for ...

a) $\boxed{\sin A = \pm \frac{1}{2}}$ ½ for ± if they don't score for different answers below

$\boxed{A = \frac{5\pi}{6} \text{ or } \frac{7\pi}{6}}$ (minus half if answer in degrees 150°, 210°)

b) i) $\boxed{1/36}$

ii) $\boxed{1/18}$ $2/36$ is okay

iii) $\boxed{11/36}$

c) ignore

d) i) $\boxed{625 = \frac{1}{2}r^2\theta}$

ii) $\boxed{\text{Perimeter} = 2R + R\theta}$

$\boxed{= 2R + R \times \frac{1250}{R^2}}$ [substitution of expression for θ]

iii) $\frac{dP}{dR} = \boxed{2 - 1250R^{-2} = 0}$ [differentiating only scores ½]

$\boxed{R = 25 \text{ metres}}$ (½)

$\boxed{\theta = 2 \text{ radians}}$ (½) or 115 degrees

either is fine

Q9

(a) $\sum_{k=2}^{8} 2^k = 2^2 + 2^3 + 2^4 + \dots + 2^8$

$S_7 = \frac{4(2^7-1)}{2-1} = 4\times(128-1)$ (1)

$= 508$ (1)

(b) $\frac{dV}{dt} = kV$

$\therefore V = V_0 e^{+kt}, \quad V_0 = 36000$

$V = 36000\, e^{+kt},$

(i) differentiate: $\frac{dV}{dt} = +k V_0 e^{kt}$
$= kV,$ (1)

$\therefore$ satisfies,

(ii) $\frac{1}{4} \times 36000 = 9000$

$\therefore$ When $t = 30$, $V = 36000 - 9000 = 27000$

$27000 = 36000\, e^{k\times 30}$

$\frac{27}{36} = e^{30k}$

$\ln\left(\frac{27}{36}\right) = 30k$ (3)

$\therefore k = -0.009589402$

When $t=60$, $V = 36000 \times e^{-0.009589402\times 60}$

$= 20250$ L (17523 if k rounded)

(c) (i) $y = \sqrt{9-x^2}$ $\quad V = \pi\int_{-3}^{3} y^2\,dx = 2\pi\int_0^3 (9-x^2)\,dx$

$= 2\pi\left[9x - \frac{x^3}{3}\right]_0^3$

$= 2\pi\left[(27-9)-0\right] = 36\pi \doteq 113$ cu. units (3)

(ii) sphere (1)

(d) $S_n = 3^n + 2n^2$

$T_{13} = S_{13} - S_{12} = (3^{13} + 2\times 13^2) - (3^{12} + 2\times 12^2)$

$= 1062932$ (2)

145

Question 10

a) $2\ln x - \ln(2-x) = \ln 2 - \ln 3$

$\ln x^2 - \ln(2-x) = \ln\frac{2}{3}$

$\ln\frac{x^2}{2-x} = \ln\frac{2}{3}$

$\therefore \frac{x^2}{2-x} = \frac{2}{3}$

$3x^2 = 4-2x$

$3x^2 + 2x - 4 = 0$

$x = \frac{-2 \pm \sqrt{4+48}}{6}$

$x = \frac{-2 \pm \sqrt{52}}{6}$

but $x \neq \frac{-2-\sqrt{52}}{6}$

$\therefore x = \frac{-2+\sqrt{52}}{6} = \frac{-1+\sqrt{13}}{3}$

b) (i) $e^{-x}(1-e^{-x})$
$= e^{-x} - e^{-2x}$

(ii) (a) $y = e^{-x} - e^{-2x}$
$= e^{-x}(1-e^{-x})$

cuts at $y=0$

$e^{-x}(1-e^{-x}) = 0$

$e^{-x} \neq 0 \therefore e^{-x} = 1$

$x = 0$

$\therefore$ cuts at $(0,0)$ only

(b) $y' = -e^{-x} + 2e^{-2x}$

$y'' = e^{-x} - 4e^{-2x}$

s.p. at $y' = 0$

$-e^{-x} + 2e^{-2x} = 0$

$e^{-x}(2e^{-x} - 1) = 0$

$e^{-x} \neq 0, \quad e^{-x} = \frac{1}{2}$

$-x = \ln\frac{1}{2}$

$\therefore x = \ln 2$ only solution

when $x = \ln 2$

$y = e^{\ln\frac{1}{2}} - e^{\ln\frac{1}{4}}$

$= \frac{1}{2} - \frac{1}{4}$

$y = \frac{1}{4}$

at $x = \ln 2$

$y'' = e^{\ln\frac{1}{2}} - 4e^{\ln\frac{1}{4}}$

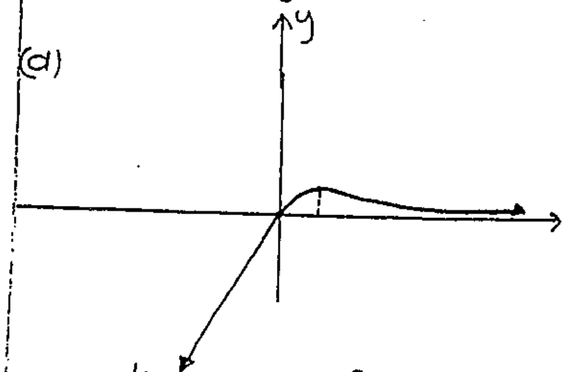
$= \frac{1}{2} - 1 < 0 \therefore$ max

$(\ln 2, \frac{1}{4})$ is a max

(c) $y' < 0$ when $x > \ln 2$

as $x \to \infty$ $y \to 0$

(d)



$A = \int_0^{\ln 2} e^{-x} - e^{-2x}\,dx$

$= \left[-e^{-x} + \frac{e^{-2x}}{2}\right]_0^{\ln 2}$