Student No: ____________

ROSEVILLE COLLEGE

HSC MATHEMATICS
TRIAL EXAMINATION

2004

Time allowed: 3 hours + 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Board approved calculators may be used
- Total marks available - 120
- All questions are worth 12 marks
- The part marks for each section are shown on the right hand side of the page
- Please start each question on a new page.
- All necessary working should be shown. You may not be awarded the marks for an answer unsupported by working or badly arranged work.
- A table of standard integrals can be found at the back of this paper.

Question 1 (12 marks) **Marks**

(a) Evaluate $\log_e 1 \cdot 05$ correct to 3 significant figures. **2**

(b) Two fair coins are tossed. What is the probability that a head and a tail come up? **1**

(c) Differentiate $2x(x-1)$ with respect to x. **2**

(d) Integrate $\frac{2}{x} - 1$. **2**

(e) Solve $\frac{x-2}{2} - 5 = 2x$. **3**

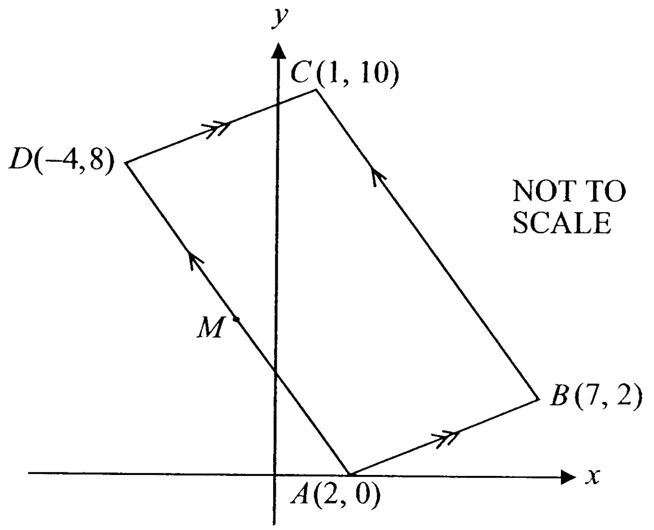
(f) Solve the simultaneous equations **2**

$$x - 3y = 5$$
$$2x + y = 3.$$

Question 2 (12 marks) ***(Begin a new page)*** **Marks**

(a) Find the equation of the normal to the curve $y = 3\sin x$ at the point $(\pi, 0)$. **3**

(b)



In the diagram above, *ABCD* is a parallelogram with the corner points $A(2, 0)$, $B(7, 2)$, $C(1, 10)$ and $D(-4,8)$. The point *M* is the midpoint of *AD*.

(i) Show that the gradient of *AD* is $\frac{-4}{3}$. **1**

(ii) Find the length of *AD*. **1**

(iii) Show that the coordinates of *M*, the midpoint of *AD*, are (-1, 4). **1**

(iv) Show that the equation of *BC* is $4x + 3y - 34 = 0$. **2**

(v) Find the perpendicular distance from *M* to *BC*. **2**

(vi) Find the area of the parallelogram *ABCD*. **2**

Question 3 (12 marks) ***(Begin a new page)*** **Marks**

(a) State the natural domain and range of the function $y = \sqrt{x-2}$. **2**

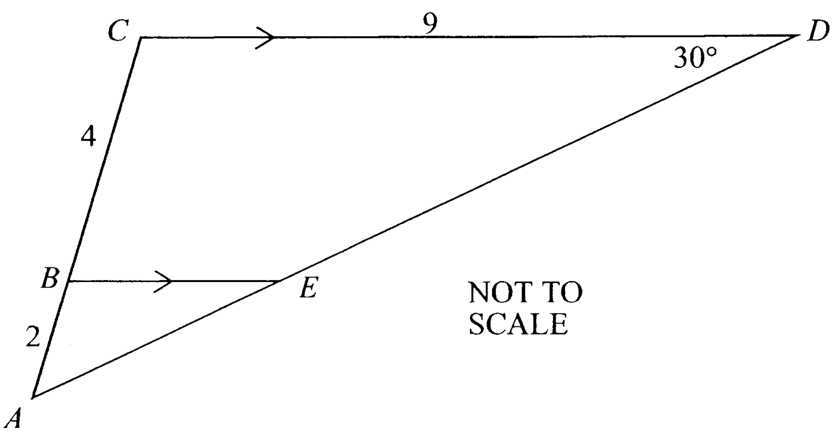
(b) Differentiate with respect to x:

(i) e^{5x} **1**

(ii) $\ln(2x^2 - 1)$ **1**

(iii) $x^3 \tan x$ **2**

(c)



In the diagram, ACD is a triangle where $AB = 2$ cm, $BC = 4$ cm, $CD = 9$ cm and $\angle CDE = 30^\circ$. Also, BE is parallel to CD.

(i) Find the size of $\angle BED$. Give a reason for your answer. **1**

(ii) Find the length of BE. **2**

(d) (i) Find $\int \left(4 - e^{3x}\right)dx$. **1**

(ii) Evaluate $\int_0^\pi \cos\frac{x}{2}\,dx$. **2**

Question 4 (12 marks) ***(Begin a new page)*** **Marks**

(a) For the quadratic equation

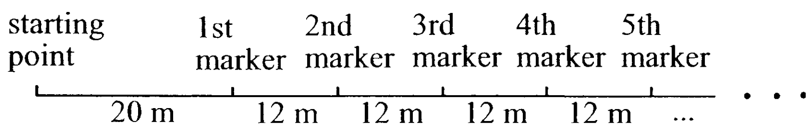
$$2x^2 + kx + 8 = 0,$$

find the values of k for which the equation has

(i) one real root. **1**

(ii) no real roots. **2**

(b) In a training drill at an athletics club an athlete must run from a starting point to the first marker and back to the starting point then out to the second marker and back to the starting point, then out to the third marker and back to the starting point and so on. The markers and the starting point are in a straight line with the first marker 20 m from the starting point and each marker 12 m apart thereafter as indicated in the diagram below.



NOT TO SCALE

Tonight the coach has had 18 markers laid out.

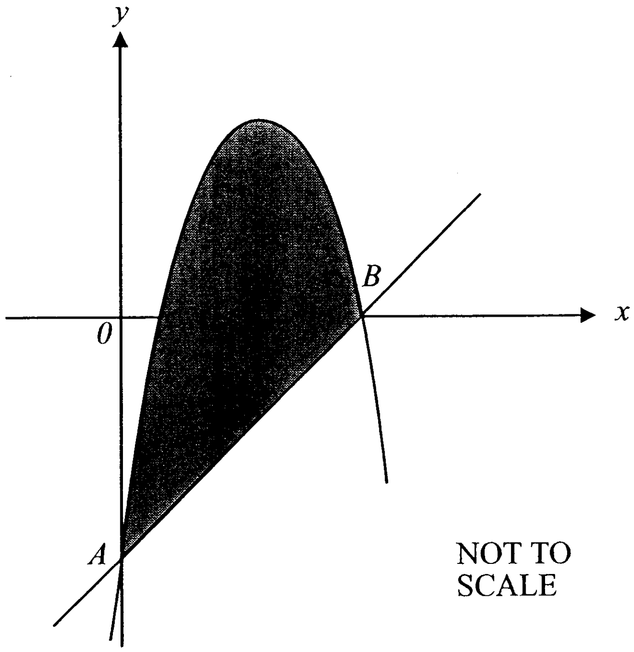
(i) How far from the starting point would the 18th marker have been placed? **2**

(ii) How far in total would an athlete run in completing the drill tonight? **2**

Question 4 continues on the next page.

Question 4 (cont'd) **Marks**

(c)



The graphs of $y = x - 5$ and $y = -x^2 + 6x - 5$ intersect at the points A and B as shown on the diagram.

(i) Find the x-coordinates of point A and point B. **2**

(ii) Find the area of the shaded region bounded by $y = -x^2 + 6x - 5$ and $y = x - 5$. **3**

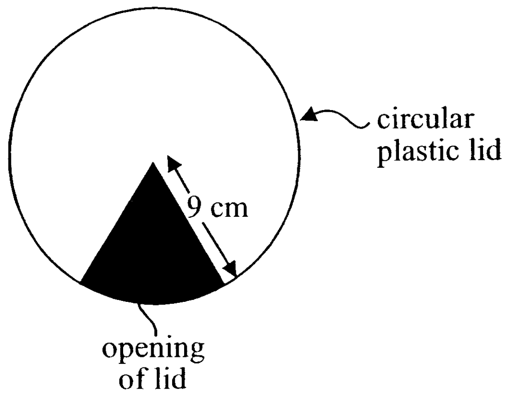
Question 5 (12 marks) ***(Begin a new page)*** **Marks**

(a) Solve the equation **2**

$$\tan x - 3 = 0 \quad \text{for} \quad 0 < x < 2\pi \, .$$

Give your answer(s) in radians correct to two decimal places.

(b) An opening on a circular plastic lid is in the shape of a sector indicated by the shaded region on the diagram below. **2**

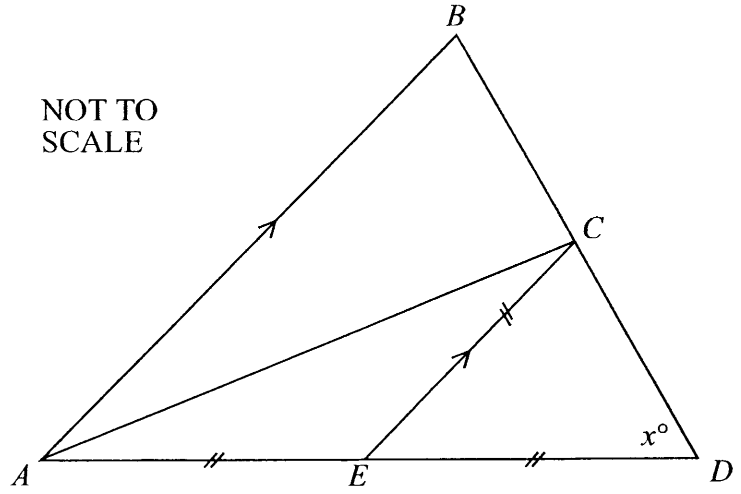


Find the exact value of the area of the opening of the plastic lid.

Question 5 continues on the next page.

Question 5 (cont'd)

(c)



In the diagram, ABD is a triangle. The point C lies on BD and E lies on AD. Also $AE = CE = DE$, $AB \parallel EC$ and $\angle EDC = x°$.

Copy or trace the diagram onto your working page.

(i) Show that $\angle ACB = 90°$. **3**

(ii) Express $\angle ABD$ in terms of $x°$. **1**

(d) For the parabola $(y-1)^2 = -12x - 24$, find

(i) the coordinates of the vertex **1**

(ii) the focal length **1**

(iii) the equation of the directrix. **1**

(iv) the coordinates of the focus. **1**

Question 6 (12 marks) ***(Begin a new page)*** **Marks**

Consider the function defined by $f(x) = x^3 - 6x^2 + 9x + 2$.

(a) Find $f'(x)$. **1**

(b) Find the coordinates of the two stationary points. **2**

(c) Determine the nature of the stationary points. **2**

(d) Sketch the curve $y = f(x)$ for $0 \le x \le 5$ showing the stationary points. **2**

(e) Apply Simpson's Rule with 5 function values to find an approximation to an area between $f(x) = x^3 - 6x^2 + 9x + 2$ and the x-axis between $x = 0$ and $x = 4$ **3**

(f) Find the exact size of the area in part (e). **2**

Question 7 (12 marks) ***(Begin a new page)*** **Marks**

(a) Evaluate $\sum_{n=1}^{10} 5 \times 2^{n-1}$ **3**

(b) In a laboratory, a food product is heated in an oven set at 210°C in an effort to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After ten minutes the number of bacteria recorded is 550. It is found that the number N, of bacteria remaining t minutes after being in the oven is given by

$$N = N_0 e^{-kt}$$

where N_0 and k are constants.

(i) Find the values of N_0 and k. **3**

(ii) If an acceptable number of bacteria present is 100 or less, For how long should the food product be in the oven? Express your answer to the nearest second. **3**

(c) At a tennis tournament run during the school holidays, two Year 6 boys and four Year 5 boys have entered the Under 11 section. The names of the six boys are randomly selected one after the other with the first two names selected to play each other, the next two to play each other and so on.

(i) What is the probability that the first two names drawn are those of Grade 5 boys? **1**

(ii) What is the probability that the second pair drawn are Grade 6 boys? **2**

Question 8 (12 marks) ***(Begin a new page)*** **Marks**

(a) (i) Use one application (two function values) of the trapezoidal rule to find an approximation to **2**

$$\int_0^2 \sqrt{16-x^2}\, dx .$$

(ii) Explain whether this approximation is greater than or less than the exact value. **1**

(b) The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle is at $x = 5$m and the initial velocity of the particle is -36 m/s . Find:

(i) An expression for velocity v. **2**

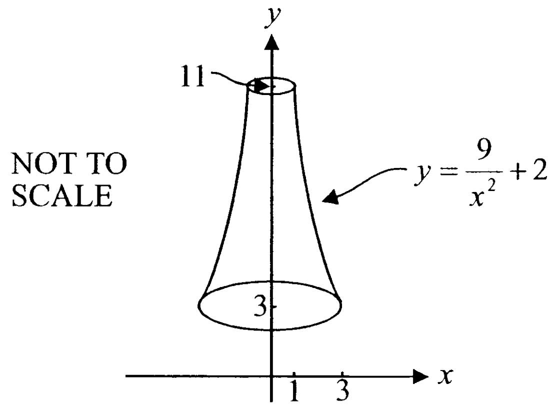
(ii) When is the particle at rest. **2**

(iii) An expression for the position of the particle. **2**

(iv) The distance travelled by the particle in the first 4 seconds. **3**

Question 9 (12 marks) ***(Begin a new page)*** **Marks**

(a) **4**



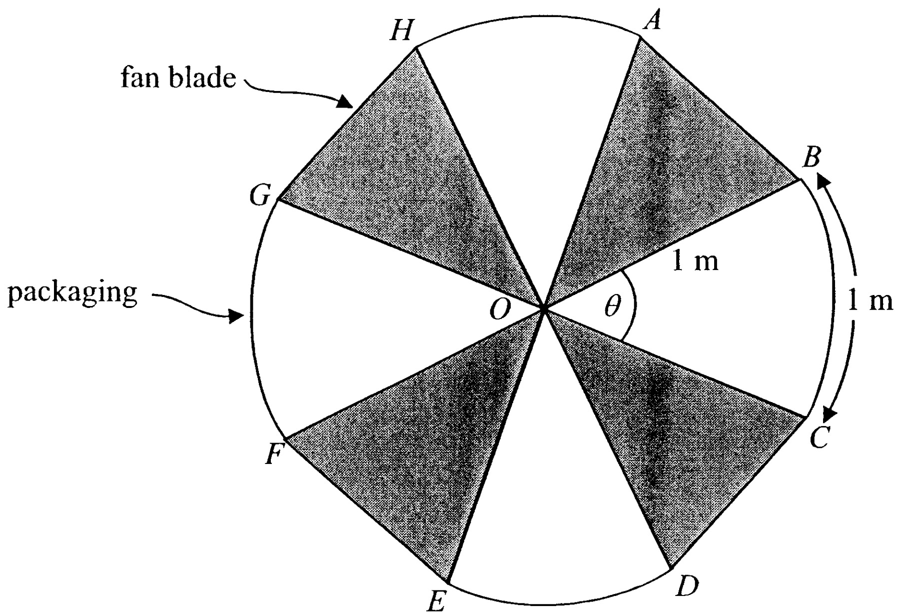
The part of the curve $y = \frac{9}{x^2} + 2$ between $x = 1$ and $x = 3$ is rotated around the y-axis.

Find the volume of the solid of revolution.

Question 9 continues on the next page.

Question 9 (cont'd) **Marks**

(b)



A large industrial fan blade is made up of four identical sectors *BOC, DOE, FOG* and *HOA* and *AE, BF, CG* and *DH* are straight lines with *O* as their midpoint. The fan blade rests in packaging so that the congruent triangles of packaging material *AOB, COD, EOF* and *GOH* are visible and are shaded in the diagram above. The arc length of the sector *BOC* is 1 metre and the length of *BO* is 1 metre.

(i) Show that $\angle AOB = \frac{\pi}{2} - \theta$. **2**

(ii) Use the cosine rule to show that $AB = \sqrt{2(1-\sin\theta)}$. **2**

(iii) Find the angle θ in radians. **1**

(iv) Find the total area covered by the fan blade and the visible packaging material. Express your answer correct to 2 significant figures. **3**

Question 10 (12 marks) ***(Begin a new page)*** **Marks**

(a) Ethel pays a single sum of \$40 000 to a financial institution to provide an annual payment of \$5 000 which is to be divided between her grandchildren. The account attracts interest of 5% per annum compounding yearly. The first payment is made to the grandchildren one year after Ethel sets up the account and just after the interest has been calculated.

(i) How much is left in the account after the first payment has been made? **1**

(ii) Let the amount in the account after n payments have been made, be $\$A_n$. **2**

Show that

$$A_n = 100\,000 - 60\,000 \times (1 \cdot 05)^n$$

(iii) Another type of account offers identical features except that the interest paid is 5% per annum compounding six monthly. Payments of \$5 000 are still to be made annually. Let the amount in this account after n years be $\$B_n$. **2**

Show that

$$B_n = 98765 \cdot 43 - 58765 \cdot 43 \times 1 \cdot 025^{2n}.$$

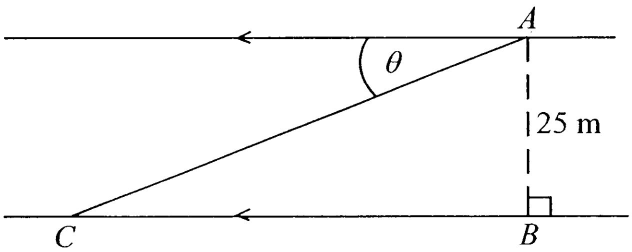
Question10 continues on the next page.

Question 10 (cont'd) **Marks**

(b) George is walking along a straight section of a river which is 25m wide and which has parallel riverbanks.
When George is at point A, he spots Mildred directly opposite on the other side of the river at point B.

George loves Mildred and so immediately dives into the river and swims in a straight line at an angle of θ to the riverbank. George swims at $1\,\text{ms}^{-1}$.

Meanwhile Mildred hasn't seen George and has continued walking along her side of the riverbank towards point C at $2\,\text{ms}^{-1}$.
George reaches Mildred's side of the riverbank at point C.



(i) Find an expression in terms of θ for the time taken by Mildred to walk from point B to point C. **1**

(ii) Find an expression in terms of θ for the time taken by George to swim from point A to point C. **1**

(iii) Show that George doesn't arrive at point C at the same time as Mildred. **1**

(iv) Find the least time by which George can miss Mildred at point C.
Justify your answer and express it to the nearest second. **4**

END OF TRIAL EXAM

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question (1)

(a) $0.048790164 \rightarrow 1$

$\doteqdot 0.0488$ (3 sig figs $\rightarrow 1$ 2

(b) $P(T,H)$ or (H,T)

$= \left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)$

$= \frac{1}{2}$ 1

(c) $\frac{d}{dx}\underbrace{2x^2-2x}$ $\rightarrow 1$

$= 4x-2 \rightarrow 1$ 2

(d) $\int \frac{2}{x} - 1\,dx \rightarrow 1$

$= 2\ln x - x + c \rightarrow 1$ 2

(e) $\frac{x-2}{2} - 5 = 2x$

$x-2-10 = 4x \rightarrow 1$

$x-12 = 4x$

$-12 = 3x \rightarrow 1$

$x = -4 \rightarrow 1$ 3

(f) $x-3y=5 \;...\; (1)$

$2x+y=3 \;...\; (2)$

$2x-6y=10 \;...\; (3)$

$(2)-(3)$

$7y=-7$

$y=-1 \rightarrow 1$

sub into (1)

$x+3=5$

$x=2 \rightarrow 1$

$\therefore (2,-1)$ 2

Question (2)

(a) $y=3\sin x$

$y'=3\cos x$

at $x=\pi$, $y=0$

$y'=-3 \rightarrow 1$

$m_T=-3$

$m_N=\frac{1}{3} \rightarrow 1$

$\therefore y=\frac{1}{3}(x-\pi) \rightarrow 1$ 3

$3y=x-\pi$

(b) $A(2,0)$ $D(-4,8)$

(i) $m_{AD}=\frac{8-0}{-4-2}$

$=-\frac{4}{3}$ 1

(ii) $d=\sqrt{(8)^2+(6)^2}$

$=10$ 1

(iii) $M=\left(\frac{2-4}{2},\frac{0+8}{2}\right)$

$=(-1,4)$ 1

(iv) $m_{BC}=-\frac{4}{3} \rightarrow 1$

$\therefore y-10=-\frac{4}{3}(x-1)$

$3y-30=-4x+4$

$4x+3y-34=0 \rightarrow 1$ 2

as required

(v) $d=\frac{|-4+12-34|}{\sqrt{16+9}} \rightarrow 1$

$=\frac{26}{5} \rightarrow 1$ 2

(vi) $A=b.h$

$=10\times\frac{26}{5} \rightarrow 1$

$=52u^2 \rightarrow 1$ 2

Question (3)

(a) $y = \sqrt{x-2}$

domain $x - 2 \geqslant 0$

$x \geqslant 2 \rightarrow$ (1)

range $y \geqslant 0 \rightarrow$ (1) 2

($x > 2$, $y < 0$ 1mark).

(b)(i) $\frac{d}{dx} e^{5x}$

$= 5e^{5x}$ 1

(ii) $\frac{d}{dx} \ln(2x^2 - 1)$

$= \frac{4x}{2x^2 - 1}$ 1

(iii) $\frac{d}{dx} x^3 \tan x$

(1)

$= \underbrace{3x^2 . \tan x + \sec^2 x . x^3}_{\text{prod. rule (1)}}$

$= 3x^2 \tan x + x^3 \sec^2 x$ 2

(c)(i) $\angle BED = 150°$ (cointerior $\angle$'s CD // BE) 1

(ii) $\frac{BE}{9} = \frac{2}{6}$ (similar Δs) $\rightarrow$ (1)

$BE = 3 \rightarrow$ (1) 2

(d) (i) $\int (4 - e^{3x})\,dx$

$= 4x - \frac{e^{3x}}{3} + c$

(ii) $\int_0^{\pi} \cos \frac{x}{2}\,dx$

$= 2\left[\sin \frac{x}{2}\right]_0^{\pi} \rightarrow$ (1)

$= 2[1-0] = 2 \rightarrow$ (1) 2

Question (4)

(a) $2x^2 + kx + 8 = 0$

(i) $\Delta = 0$.

$k^2 - 64 = 0$

$k = \pm 8$ 1

(ii) $\Delta < 0$

$k^2 - 64 < 0 \rightarrow$ (1)

$-8 < k < 8 \rightarrow$ (1) 2

(b)(i) 20, 12, 12, 12

(i) A.P. $a = 20$, $n = 18$, $d = 12$

$T_{18} = a + (n-1)d$

$= 20 + (17).12 \rightarrow$ (1)

$= 224$ m $\rightarrow$ (1) 2

(ii) $S_{18} = \frac{n}{2}(a + L) \times 2$.

$= 18(20 + 224) \rightarrow$ (1)

$= 4392$m $\rightarrow$ (1) 2

(c)(i) $y = x - 5$ and $y = -x^2 + 6x - 5$

Equate

$x - 5 = -x^2 + 6x - 5$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0, 5$

A(0, −5) (1) B(5, 0) $\rightarrow$ (1) 2

(ii) $A = \int_0^5 -x^2 + 6x - 5 - x + 5\,dx$

$= \int_0^5 \underbrace{-x^2 + 5x}\,dx \rightarrow$ (1) or equivalent area

$= \left[-\frac{x^3}{3} + \frac{5x^2}{2}\right]_0^5 \rightarrow$ (1) integration.

$= -\frac{125}{3} + \frac{125}{2} - 0$

$= 20\frac{5}{6}\,u^2 \rightarrow$ (1) 3

Question (5)

(a) $\tan x - 3 = 0 \qquad 0 < x < 2\pi$

$\tan x = 3$

$x = 1{\cdot}25,\ 4{\cdot}39$

$\downarrow \qquad \downarrow$

(1) (1)

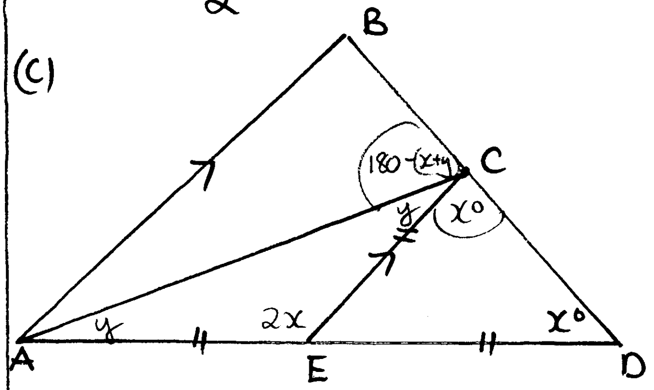
Subtract 1 for correct answer in o. 2

(b) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 9 \times 9 \times \frac{\pi}{3} \longrightarrow (1)$

$= \frac{27\pi}{2}\ \text{cm}^2 \longrightarrow (1)$ 2

(c)



(i) $\angle ECD = x$ (equal $\angle$ of isos Δ ECD) $\longrightarrow (1)$

$\angle AEC = 2x$ (external $\angle$ of Δ ECD)

ΔAEC is isosceles ($AE = EC$)

$\therefore$ let $\angle EAC = \angle ACE = y$

$\therefore \angle ACB = 180 - (x+y)$ ($\angle$ sum of st. line is 180°) $\to (1)$

In ΔAEC

$2x + 2y = 180^\circ$ ($\angle$ sum Δ is 180°)

$\left.\begin{array}{l} \therefore\ x + y = 90 \\ \therefore \angle ACB = 180 - 90 \\ \quad = 90^\circ \end{array}\right\} \to (1)$

as required. 3

(ii) $\angle ABD = x$ (corresponding $\angle$'s AB // EC) 1

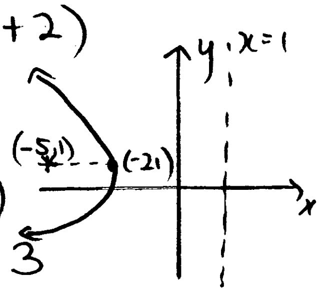
(d) $(y-1)^2 = -12x - 24$

(i) $(y-1)^2 = -12(x+2)$

in form

$y^2 = -4ax$

$\therefore$ vertex $(-2, 1)$ 1



(ii) focal length 3 1

(iii) directrix is $x = 1$ 1

(iv) focus $(-5, 1)$ 1

Question (6)

$f(x) = x^3 - 6x^2 + 9x + 2$

(a) $f'(x) = 3x^2 - 12x + 9$ 1

(b) $f'(x) = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

$\therefore$ when $\left.\begin{array}{l} x = 1 \\ y = 6 \end{array}\right\}$ $(1, 6) \longrightarrow (1)$

when $\left.\begin{array}{l} x = 3 \\ y = 2 \end{array}\right\}$ $(3, 2) \to (1)$ 2

(c) $f''(x) = 6x - 12$

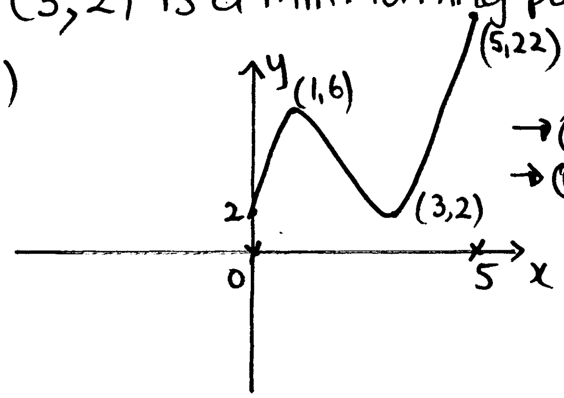
$\therefore f''(1) = -6 < 0\ \cap \longrightarrow (1)$

$\therefore (1, 6)$ is a max. turning point

$f''(3) = 6 > 0\ \cup \longrightarrow (1)$

$(3, 2)$ is a min. turning point. 2

(d)



$\to$ (1) curve

$\to$ (1) end points 2

(e)

	$n=1$				
x	0	1	2	3	4
$f(x)$	2	6	4	2	6

$A \doteq \frac{1}{3}\left[2 + 4(6) + 2(4) + 4(2) + 6\right]$ ①

$\doteq 16u^2$

(f) $A = \int_0^6 x^3 - 6x^2 + 9x + 2$

$= \left[\underbrace{\frac{x^4}{4} - 2x^3 + \frac{9x^2}{2} + 2x}_{①}\right]_0^4$

$= 16u^2$

Question (7)

(a) $\sum_{n=1}^{10} 5 \times 2^{n-1}$

$= 5 + 10 + 20 + 40 .. + 2560$ ①

G.P. $a = 5$, $r = 2$, $n = 10$

$S_{10} = \frac{5(2^{10} - 1)}{2 - 1}$ ①

$= 5115 \longrightarrow$ ① 3

(b) $N = N_0 e^{-kt}$

(i) $N_0 = 1800 \longrightarrow$ ①

when $N = 550$, $t = 10$

$550 = 1800 e^{-10k} \longrightarrow$ ①

$\frac{11}{36} = e^{-10k}$

$-10k = \ln\left(\frac{11}{36}\right)$

$k = 0.011 \longrightarrow$ ① 3

(ii) $N \leq 100$

$1800 e^{-kt} \leq 100 \longrightarrow$ ①

$e^{-kt} \leq \frac{1}{18}$

$-kt \leq \ln\left(\frac{1}{18}\right) \rightarrow$ ①

$t \geq 24.378$

$t \doteq 24$ [illegible] $\rightarrow$ ① 3

(c) 2 x Year 6

4 x Year 5

(i) $P(5,5) = \frac{4}{6} \times \frac{3}{5}$

$= \frac{2}{5}$ 1

(ii) $P(\bar{6}, \bar{6}), (6, 6)$.

$= \left(\frac{4}{6} \times \frac{3}{5}\right) \times \left(\frac{2}{4} \times \frac{1}{3}\right) \longrightarrow$ ①

$= \frac{1}{15} \longrightarrow$ ① 2

Question (8)

(a) (i) $\int_0^2 \sqrt{16 - x^2}\, dx$

x	0	2
y	4	$\sqrt{12}$

$\doteq \frac{2}{2}\left[4 + \sqrt{12}\right] \longrightarrow$ ①

$= 4 + 2\sqrt{3}$ (7.46) $\rightarrow$ ① 2

(ii)  less because the trapezium is below the curve. 1

(b) (i) $v = 6t^2 + 6t + C \rightarrow$ ①

when $t = 0$ $v = -36$ $\therefore \underbrace{C = -36}_{①}$ 2

$v = 6t^2 + 6t - 36$

(ii) at rest when $v = 0$ ① 2

$6t^2 + 6t - 36 = 0$

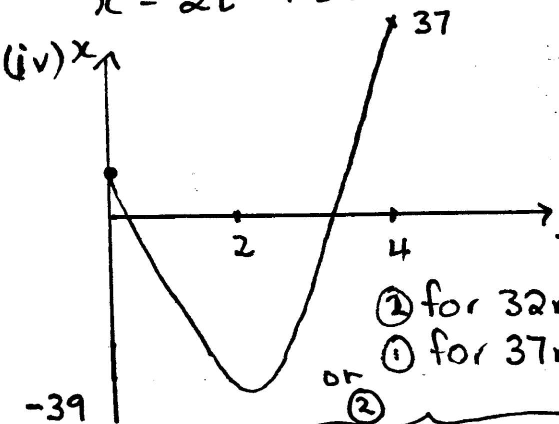
$(t+3)(t-2) = 0$

$t = 2$ since

(iii) $x = 2t^3 + 3t^2 - 36t + K \rightarrow$ ①

when $t = 0$, $x = 5$ $\therefore K = 5 \rightarrow$ ①

$x = 2t^3 + 3t^2 - 36t + 5$ 2

(iv) 

② for 32m

① for 37m

or

Distance $= \overbrace{5 + 39 + 39 + 37}^{②}$

$= 120\text{m} \rightarrow$ ① 3

Question (9)

(a) $V = \pi\int_3^{11} x^2\,dy$

$y = \frac{9}{x^2} + 2$

$x^2 y = 9 + 2x^2$

$x^2 y - 2x^2 = 9$

$x^2(y-2) = 9$

$x^2 = \frac{9}{y-2} \longrightarrow 1$

$\therefore V = \pi\int_3^{11} \frac{9}{y-2}\,dy \longrightarrow 1$

$= \pi\left[9.\ln(y-2)\right]_3^{11} \longrightarrow 1$

$= \pi\left[9\ln 9 - 9\ln 1\right]$

$= \pi.9\ln 9$

$= 9\pi\ln 9. \longrightarrow 1$ 4

(b)

(i) $4\theta + 4\phi = 2\pi$

$\theta + \phi = \frac{\pi}{2}$

$\phi = \frac{\pi}{2} - \theta \longrightarrow 1$

$\angle AOB = \frac{\pi}{2} - \theta$ ↓ follow on $\longrightarrow 1$

(ii) $AB = 1^2 + 1^2 - 2.\cos(\frac{\pi}{2} - \theta)$

$= 2 - 2\cos(\frac{\pi}{2} - \theta) \longrightarrow 1$

$= 2 - 2\sin\theta$

$= 2(1 - \sin\theta) \longrightarrow 1$ 2

(iii) $L = r\theta$

$1 = 1.[\theta]$

$\theta = 1$

(iv) $A = 4(\frac{1}{2}r^2\theta) + 4(\frac{1}{2}1.1\sin(90-\theta)) \longrightarrow 1$

$= 4(\frac{1}{2}.1) + 4(\frac{1}{2}.\cos\theta)$

$= 2 + 2\cos(1)$ $\Big\}$ 1

$= 3.1\,m^2$ (1 d.p.) $\longrightarrow 1$ 3

Question (10)

(i) $40000(1.05) - 5000$

$= \$37000$ 1

(ii) $A_2 = 40000(1.05)^2 - 5000(1.05) - 5000$

$\therefore A_n = 40000(1.05)^n - 5000(1.05)^{n-1} \ldots - 5000$

$= 40000(1.05)^n - \underbrace{5000\left[1 + \ldots (1.05)^{n-1}\right]}_{\text{G.P. } a=1,\ r=1.05,\ n=n}$

$A_n = 40000(1.05)^n - 5000\frac{(1.05^n - 1)}{0.05} \longleftarrow 1$

$= 40000(1.05)^n - 100000(1.05^n - 1)$

$= 40000(1.05)^n - 100000(1.05)^n + 100000$

$= 100000 - 60000(1.05)^n \longrightarrow 1$ 2

as required.

(iii) $B_1 = 40000(1.025)^2 - 5000$

$B_2 = 40000(1.025)^4 - 5000(1.025)^2 - 5000$

$\vdots$

$B_n = 40000(1.025)^{2n} - \underbrace{5000\left[1 + \ldots + (1.025)^{2n}\right]}_{\text{G.P. } a=5000,\ r=(1.025)^2,\ n=n}$ 1

$B_n = 40000(1.025)^{2n} - \frac{5000(1.025^{2n} - 1)}{0.050625}$

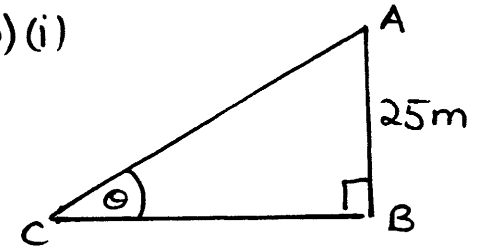
$= 40000(1.025)^{2n} - 98765.43[(1.025)^{2n} - 1]$

$= 400000(1.025)^{2n} - 98765.43(1.025)^{2n} + 98765.43$

$= 98765.43 - 58765.43 \times 1.025^{2n}$

as required. 1 2

(b)(i)



(I) $\tan\theta = \dfrac{25}{BC}$.

$S = \dfrac{D}{T}$

$\therefore \boxed{T = \dfrac{D}{S}}$

$BC = \dfrac{25}{\tan\theta}$

$T_{MILORED} = \dfrac{25}{2\tan\theta}$ s. 1

(ii) $\sin\theta = \dfrac{25}{AC}$

$AC = \dfrac{25}{\sin\theta}$

$\therefore T_{GEORGE} = \dfrac{25}{\sin\theta}$ s. 1

(iii) Equate T_M and T_G

$$\frac{\cancel{25}}{2\tan\theta} = \frac{\cancel{25}}{\sin\theta}$$

$$\frac{\cos\theta}{2\sin\theta} = \frac{1}{\sin\theta}$$

$$\therefore \cos\theta = 2$$

Not possible as $-1 \leq \cos\theta \leq 1$ 1

$\therefore$ they will not meet.

(iv) $T = T_G - T_M$

$$T = \frac{25}{\sin\theta} - \frac{25}{2\tan\theta}$$

$$= \frac{25}{\sin\theta} - \frac{25\cos\theta}{2\sin\theta}$$

$$\left.T = \frac{50 - 25\cos\theta}{2\sin\theta}\right\} - (1)$$

$$T' = \frac{25\sin\theta . 2\sin\theta - 2\cos\theta(50 - 25\cos\theta)}{4\sin^2\theta}$$

$$= \frac{50\sin^2\theta - 100\cos\theta + 50\cos^2\theta}{4\sin^2\theta}$$

$$= \frac{50(\sin^2\theta + \cos^2\theta) - 100\cos\theta}{4\sin^2\theta}$$

$$= \frac{50 - 100\cos\theta}{4\sin^2\theta}$$

$$= \frac{25 - 50\cos\theta}{2\sin^2\theta}$$

$$T'' = \frac{50\sin\theta . 2\sin^2\theta - 4\sin\theta\cos\theta(25 - 50\cos\theta)}{4\sin^4\theta}$$

Min. when $T_1 = 0$

$25 - 50\cos\theta = 0$ $\quad \sin\theta \neq 0$

$25(1 - 2\cos\theta) = 0$

$\cos\theta = \frac{1}{2}$

$\theta = 60° \rightarrow (1)$

when $\theta = 60°$

$$\left.\begin{array}{l} T'' = \dfrac{811.8988 - 0}{4\sin^4\theta} > 0 \\ \therefore \text{min at } \theta = 60°. \end{array}\right\} \rightarrow (1)$$

when $\theta = 60°$

$$T = \left| \frac{50 - 12\frac{1}{2}}{\sqrt{3}} \right|$$

$\doteqdot 22$ secs. $\rightarrow (1)$

4