Roseville College

2005 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

(Marks)

Question 1 (12 marks) **Start a new page**

(a) Work out the gradient of the line $2x+3y-1=0$ (1)

(b) Evaluate $e^{1.4}$ (to 4 significant figures) (1)

(c) Solve $\dfrac{3x+1}{4}-\dfrac{x+1}{3}=1$ (2)

(d) Differentiate $e^{3x}+\ln x$ (2)

(e) Solve $2^x=7$ correct to 3 significant figures (2)

(f) Evaluate $(\tan 60^\circ)^2+(\tan 30^\circ)^2$ (2)

(g) Asafa Powell of Jamaica was clocked at 9.77s for 100m on June 14, 2005.

Convert this to an average speed in km/h giving your answer to the nearest tenth of km/h.

(2)

(Marks)

Question 2 (12 marks) **Start a new page**

(a) Draw a large clear diagram showing the points A(3, 5), B(5, -2) and C(-7, -2)

(i) Determine the length of the interval AC (1)

(ii) Show that AC has equation $7x-10y+29=0$ (1)

(iii) Find the perpendicular distance from B to AC (1)

(iv) Hence or otherwise find the area of ΔABC (1)

(v) Find the co-ordinates of the point D such that ABCD is a parallelogram (1)

(vi) Find the area of ABCD (1)

(vii) Find the angle of inclination of AC to the nearest whole degree (1)

(b) Find a primitive function for

(i) $\sin 2x$ (1)

(ii) $\dfrac{x^2+1}{2x}$ (2)

(iii) $\dfrac{2x}{x^2+1}$ (1)

(iv) $(2x+1)^4$ (1)

(Marks)

Question 3 (12 marks) **Start a new page**

(a) A parabola has equation $(x-2)^2 = 12(y+1)$

Determine:

(i) The co-ordinates of its vertex (1)

(ii) The equation of its axis (1)

(iii) The co-ordinates of its focus (1)

(iv) The equation of its directrix (1)

(b) Solve the equation $2\sin x + 1 = 0$ for $0 \le x \le 2\pi$ (3)

(c) The series which begins 12 + 17 + 22 … has 16 terms.

Find:

(i) The 16^{th} term (2)

(ii) The sum of the first 16 terms (2)

(d) Evaluate exactly $\sqrt{1\frac{32}{49}}$ (1)

(Marks)

Question 4 (12 marks) **Start a new page**

(a) Find the area bounded by $y = \sqrt{x}$, the x axis, $x = 1$ and $x = 4$ in the first quadrant. (3)

(b) Sam sets out from a marker on a bearing of 140°T. Having walked for 12km, she then changes direction to a bearing of 067°T. She walks on this bearing until she is due East of the original marker. Calculate the distance covered by Sam on the second leg of her journey. (3)

(c) Sketch the region on a number plane which simultaneously satisfies the inequalities

$$y \geq x^2 \text{ and } x^2 + y^2 < 1$$ (3)

(d) Differentiate the following (3)

(i) $(4x^3 - 1)^2$

(ii) $x\cos(3x)$

(Marks)

Question 5 (12 marks) **Start a new page**

(a) Evaluate $\sum_{K=1}^{5}(K+1)^2$ (2)

(b) A function is defined as $f(x)=9x(x-2)^2$

(i) Find the stationary points (3)

(ii) Find the intercepts on the x and y axes (1)

(iii) Sketch the graph of $y=f(x)$ for $-1\le x\le 3$ (3)

(iv) Find the range of $f(x)$ (1)

(c) Find $\int(\frac{2}{3}x^2-\frac{4}{x})dx$ (2)

(Marks)

Question 6 (12 marks) **Start a new page**

(a) Find the exact value of $\int_{\frac{\pi}{3}}^{\pi} \cos(\frac{x}{2})dx$ (3)

(b) Jenny buys 4 tickets in a raffle in which 50 tickets are sold. Three different prizes are drawn out for first, second and third prizes. Find the probability that:

(i) Jenny wins all three prizes (1)

(ii) Jenny does not win a prize (1)

(iii) Jenny wins at least one prize (1)

(iv) Jenny wins exactly one prize (1)

(c) Evaluate the geometric series (4)

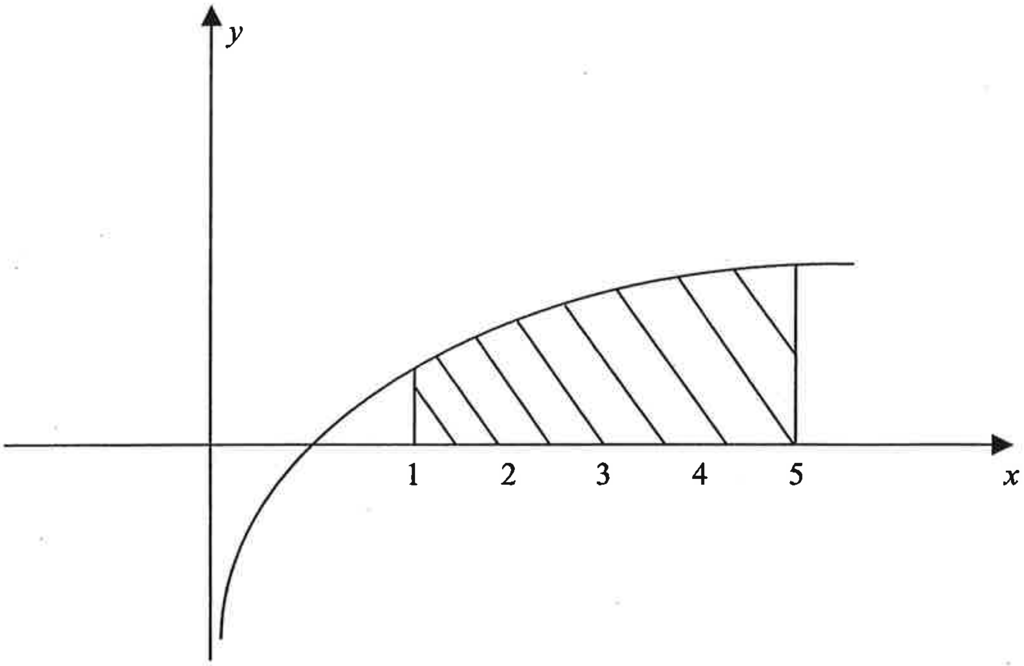
$$\frac{1}{1024}+\frac{1}{512}+\frac{1}{256}+\ldots\ldots\ldots\ldots+4096$$

(d) Find the value of $\ln e^{2.6}$ (1)

(Marks)

Question 7 (12 marks) **Start a new page**

(a) The diagram shows the function $y = 1 + \ln x$



(i) Use 5 functions and the trapezoidal rule to find an approximation for the shaded area (3)

(ii) Use 2 applications (i.e. 5 function values) of Simpson's rule to find an approximation for the shaded area. (3)

(iii) α) Differentiate $x\ln x$ (2)

β) Hence or otherwise find the exact value of the shaded area (2)

γ) Decide which of the approximations (i) or (ii) is better and explain your reasoning (2)

(Marks)

Question 8 (12 marks) **Start a new page**

(a) At the beginning of 1990, the population of a small city called Mathsville was 12 000. At the beginning of 1995, the Mathsville population was 8000. The bank head office has decided to close the bank branch in any town with a population of 3000 or less.

Assume that the population (P) is decreasing exponentially and that P satisfies an equation of the form $P = P_0 e^{kt}$ where P_0 and k are constants, and t is measured in years from the beginning of 1990.

(i) Show that $P = P_0 e^{kt}$ satisfies $\frac{dP}{dt} = kP$ (1)

(ii) What is the value of P_0? (1)

(iii) Find the value of k (2)

(iv) Find the population at the beginning of 2000 (1)

(v) Predict the year the Mathsville bank will close (2)

(b) (i) Find the equation of the tangent to the curve $y = \sin 2x$, in exact form, at the point where $x = \frac{5\pi}{6}$ (3)

(ii) Using a sketch or otherwise, determine the number of solutions of the equation $\sin 2x = \frac{1}{2}(x+1)$ in the domain $0 \le x \le \pi$ (2)

(Marks)

Question 9 (12 marks) **Start a new page**

(a) Jennifer and Ben need to borrow \$480 000 to buy an apartment. They decide to pay it off in equal monthly instalments over 15 years. They are able to negotiate an interest rate fixed at 6% p.a. for the life of the loan. Interest is charged monthly.

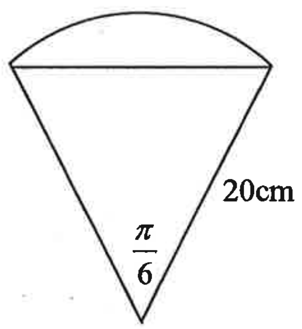
(i) Calculate their outstanding balance before they make their first instalment of \$**I**. (1)

(ii) Show that the balance owing immediately after they make their second instalment is given by (1)

$$B_2 = \$480\,000 \times 1.005^2 - \$\mathbf{I} - \$\mathbf{I} \times 1.005$$

(iii) Calculate the value of \$**I** to the nearest whole dollar (4)

(b) Angelina is skillful at playing a game of modified darts called "Hit the segment". The target in this game is shown. It is a sector of a circle with a chord drawn to produce a segment and a triangle.



(i) Calculate the area of the sector (1)

(ii) Calculate the area of the segment (2)

(iii) Angelina's friend Brad plays "Hit the segment" for the first time and just throws his darts randomly. If he is good enough to hit the target, what is the probability that his first dart will land in the segment? Give your answer as a percentage (1)

(iv) If Brad throws five darts in a row, find the probability that he will "Hit the segment" at least once. (2)

(Marks)

Question 10 (12 marks) **Start a new page**

(a) Solve $2\ln x = \ln 2 + \ln(x+4)$ (3)

(b) ABCD is a trapezium in which AB||DC. EGH is any line which cuts AB in E, DB in G and DC in H. (4)

Prove that GB = $\dfrac{GD \times EB}{DC - HC}$

(c) Given the quadratic equation

$$(k+3)n^2 + (6-2k)n + k - 1 = 0$$

(i) Find the value or values of k for which this equation will have real roots. (2)

(ii) Find the values of k for which this equation will have one root six times the other. (3)

Suggested Solutions	Comments
Question 1.	

a) $2x + 3y - 1 = 0$

$3y = -2x + 1$

$y = -\frac{2}{3}x + \frac{1}{3}$ $\quad \therefore \quad m = -\frac{2}{3}$ $\quad \boxed{1}$

b) $e^{1.4} = 4.0551999.....$

$= 4.055$ to 4 sig. figs. $\quad \boxed{1}$

c) $\cancel{12}^{3} \times \left(\frac{3x+1}{\cancel{4}}\right) - \cancel{12}^{4} \times \left(\frac{x+1}{\cancel{3}}\right) = 1 \times 12$

$3(3x+1) - 4(x+1) = 12$ — 1

$9x + 3 - 4x - 4 = 12$

$5x - 1 = 12$

$5x = 13$

$x = \frac{13}{5}$ — 1 $\quad \boxed{2}$

d) $\frac{d}{dx}\left(e^{3x} + \ln x\right)$

$= 3e^{3x} + \frac{1}{x}$ $\quad \boxed{2}$

Comments: 1 each

e) $2^x = 7$

$\log_2 7 = x$

$x = \frac{\ln 7}{\ln 2}$ — 1

$= 2.80735....$

$= 2.81$ (3 sig figs) — 1 $\quad \boxed{2}$

Comments: $-\frac{1}{2}$ sig figs

f) $3 + \frac{1}{3} = 3\frac{1}{3}$ $\quad \boxed{2}$

Comments: 1 each

g) 100m : 9.77s.

$\frac{100}{9.77}$ m/s — 1

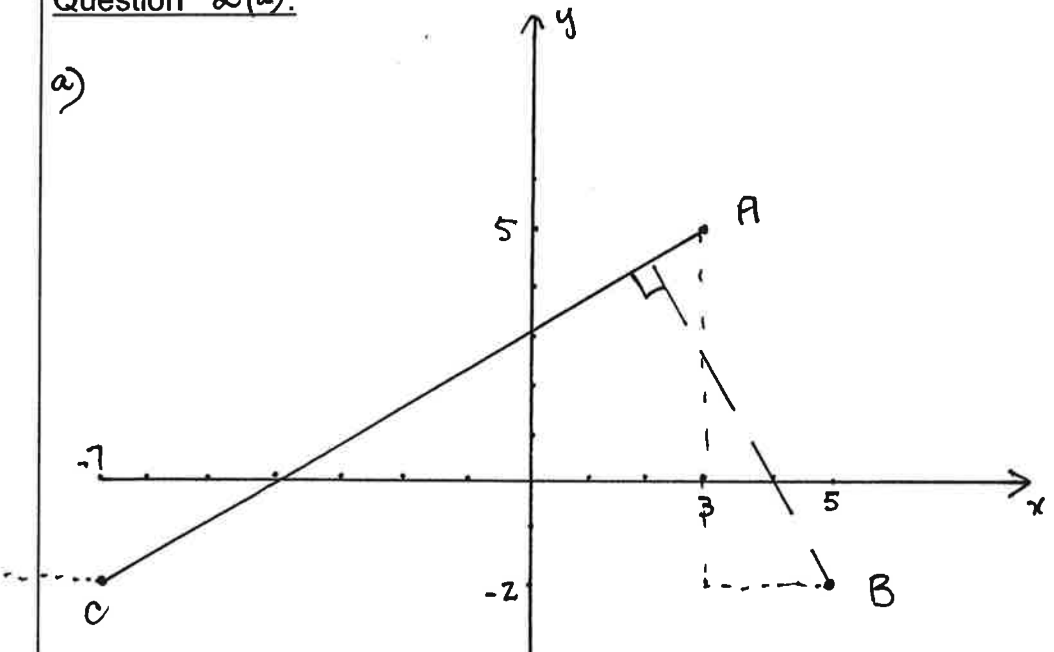
$\frac{100}{9.77} \times \frac{60 \times 60}{1000}$ km/h $= 36.847...$ km/h — 1

ie 36.8 km/h [nearest tenth] $\quad \boxed{2}$

Suggested Solutions	Comments

Question 2(a).

a)



i) $AC = \sqrt{(3+7)^2 + (5+2)^2}$ $\boxed{1}$

$= \sqrt{149}$ units

ii) $AC: \; m = \frac{5+2}{3+7}$ through $(3, 5)$

$m = \frac{7}{10}$

$y - 5 = \frac{7}{10}(x-3)$

$10y - 50 = 7x - 21$ $\boxed{1}$

$7x - 10y + 29 = 0$

iii) $(5, -2)$ $7x - 10y + 29 = 0$

$d = \frac{|7\times5 - 10\times-2 + 29|}{\sqrt{7^2+10^2}}$

$= \frac{|84|}{\sqrt{149}}$

$= \frac{84}{\sqrt{149}}$ units $\boxed{1}$

iii) Area $= \frac{1}{2} b\times h$

$= \frac{1}{2} \times \cancel{\sqrt{149}} \times \frac{84}{\cancel{\sqrt{149}}}$

$= \frac{42\sqrt{83}}{\sqrt{149}}$ units^2 $\boxed{1}$

v) left 2 and up 7 from C ie $D(-9, 5)$ $\boxed{1}$ | $\frac{1}{2}$ $(-5, -9)$

vi) Area $= b\times h = 12 \times 7 = 84$ units^2 $\boxed{1}$

Suggested Solutions	Comments

Question 2 (cont)

vii) $m = \frac{7}{10}$ $\therefore$ angle of inclination $= \tan^{-1}\left(\frac{7}{10}\right)$

$= 34.99\ldots^\circ$

$= 35^\circ$ nearest degree. [1]

Comments: $\frac{1}{2}$ $\tan\alpha = \frac{7}{10}$

b) i) $\int \sin 2x \, dx = -\frac{1}{2}\cos 2x + c$ [1]

ii) $\int \frac{x^2+1}{2x} dx = \int \frac{x^2}{2x} + \frac{1}{2x} \, dx$ (1)

$= \frac{x^2}{4} + \frac{1}{2}\ln x + c$ (1) [2]

iii) $\int \frac{2x}{x^2+1} dx = \ln(x^2+1) + c$ [1]

Comments: $\frac{1}{2}$ - $\ln x^2 + 1 + c$

iv) $\int (2x+1)^4 dx = \frac{(2x+1)^5}{5\times 2} + c$

$= \frac{(2x+1)^5}{10} + c$ [1]

Comments: $\frac{1}{2}$ $\frac{(2x+1)^5}{5} + c$

Question 3

a) $(x-2)^2 = 12(y+1)$

of the form $(x-h)^2 = 4a(y-k)$

i) vertex $(2,-1)$ [1]

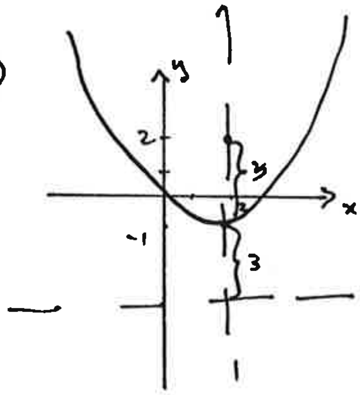
$4a = 12$

$a = 3$

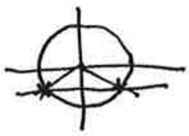
ii) axis : $x = 2$ [1]

iii) focus : $S(2,2)$ [1]

iv) directrix : $y = -4$. [1]



b) $2\sin x + 1 = 0$ $\quad 0 \leq x \leq 2\pi$



$2\sin x = -1$

$\sin x = -\frac{1}{2}$

Comments: $\frac{1}{2}$ $\sin x = -\frac{1}{2}$.

acute $x = \frac{\pi}{6}$. [1]

$\therefore x = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ [2]

(1) (1)

Suggested Solutions	Comments

Question 3 (cont)

c) $12 + 17 + 22 + \ldots$

arithmetic series $a = 12$ $d = 5$

$\therefore T_n = a + (n-1)d$

$= 12 + (n-1) \times 5$

$T_n = 7 + 5n$ — 1

i) $\therefore T_{16} = 7 + 5 \times 16$

$= 87$ — 1 $\boxed{2}$

ii) $S_n = \frac{n}{2}\{a + l\}$

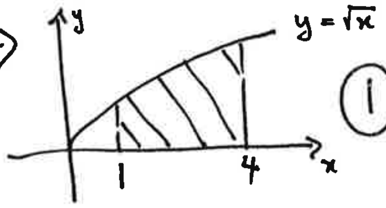
$= \frac{16}{2}\{12 + 87\}$

$= 792$ $\boxed{2}$

d) $\sqrt{1\frac{32}{49}} = \sqrt{\frac{81}{49}}$

$= \frac{9}{7}$ $\boxed{1}$

Question 4

a) 

1

$$\text{Area} = \int_1^4 \sqrt{x}\, dx$$

1

$$= \int_1^4 x^{1/2}\, dx$$

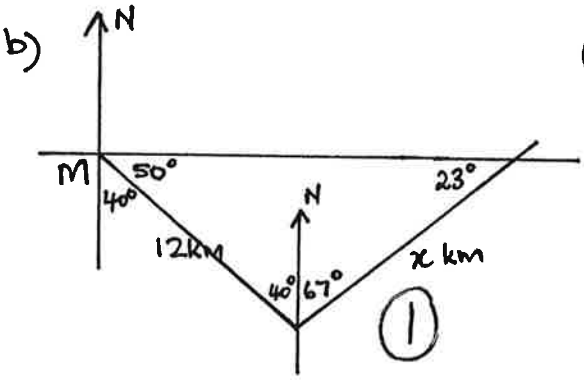
$$= \left[\frac{2}{3}x^{3/2}\right]_1^4$$

$$= \frac{2}{3} \times 4^{3/2} - \frac{2}{3} \times 1^{3/2}$$

$$= \frac{14}{3} \text{ units}^2. \text{ — 1}$$

$\boxed{3}$ $-\frac{1}{2}$ no units

(or $4\frac{2}{3}\,u^2$)

b) 

1

1

$$\frac{x}{\sin 50°} = \frac{12}{\sin 23°}$$

$$x = \frac{12}{\sin 23°} \times \sin 50°$$

$$= 23.5264\ldots$$

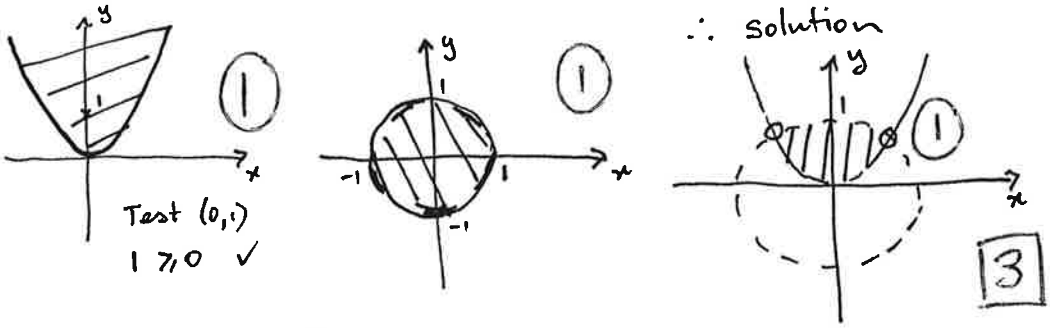
$$\doteqdot 24\text{ km}$$ 1

$\boxed{3}$

Suggested Solutions	Comments

Question 4 (cont)

c) $y \geq x^2 \qquad x^2 + y^2 < 1$



no shading

d) i) $\frac{d}{dx}(4x^3-1)^2$

$= 2(4x^3-1) \times 12x^2$

$= 24x^2(4x^3-1) \quad \text{or} \quad 96x^5 - 24x^2$ $\boxed{1}$

ii) $\frac{d}{dx}(x\cos 3x)$ 1 1

$= x \times -3\sin 3x + \cos 3x \times 1$

$= -3x\sin 3x + \cos 3x$ $\boxed{2}$

Question 5.

a) $\sum_{K=1}^{5} (K+1)^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2$ 1

$= 90$ 1 $\boxed{2}$

b) $f(x) = 9x(x-2)^2 \quad \text{or} \quad f(x) = 9x(x^2-4x+4)$

$= 9x^3 - 36x^2 + 36x$

i) $f'(x) = 27x^2 - 72x + 36$

let $f'(x) = 0$ for stationary pts.

$27x^2 - 72x + 36 = 0$

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

$\therefore x = 2 \text{ or } \frac{2}{3}$ 1

$x = 2, \ f(2) = 9 \times 2(2-2)^2 = 0$

$x = \frac{2}{3}, \ f(\frac{2}{3}) = 9 \times \frac{2}{3}(\frac{2}{3} - 2)^2 = 10\frac{2}{3}$

$(2,0)$ $\quad (\frac{2}{3}, 10\frac{2}{3})$ 1

are the stationary pts.

Suggested Solutions	Comments

Question 5(cont)

$f''(x) = 54x - 72$

$f''(2) = 54 \times 2 - 72$

+ve.

$\therefore (2,0)$ is a minimum turning pt

$f''(\frac{2}{3}) = 54 \times \frac{2}{3} - 72$

$= -36$ (−ve) 1

$\therefore (\frac{2}{3}, 10\frac{2}{3})$ is a maximum turning pt.

$\boxed{3}$

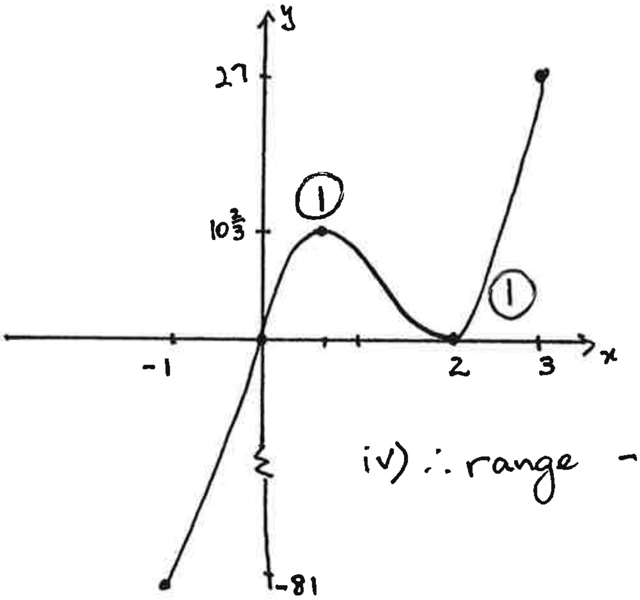
ii) intercepts. $x = 0$ $y = 0$

$y = 0$ $9x(x-2)^2 = 0$

$\therefore x = 0$ or 2.

$\boxed{1}$

iii)



$f(-1) = 9 \times -1(-1-2)^2$

$= -81$. 1

$f(3) = 9 \times 3(3-2)^2$

$= 27$

$\boxed{3}$

iv) $\therefore$ range $-81 \leq y \leq 27$

$\boxed{1}$

c) $\int \left(\frac{2}{3}x^2 - \frac{4}{x}\right) dx$

$= \frac{2x^3}{9} - 4\ln x + c$ 1 1

$\boxed{2}$

Question 6.

a) $\int_{\frac{\pi}{3}}^{\pi} \cos\left(\frac{x}{2}\right) dx = \left[2\sin\frac{x}{2}\right]_{\frac{\pi}{3}}^{\pi}$ 1

$= 2\sin\frac{\pi}{2} - 2\sin\frac{\pi}{6}$ 1

$= 2 \times 1 - 2 \times \frac{1}{2}$

$= 1$ 1

$\boxed{3}$

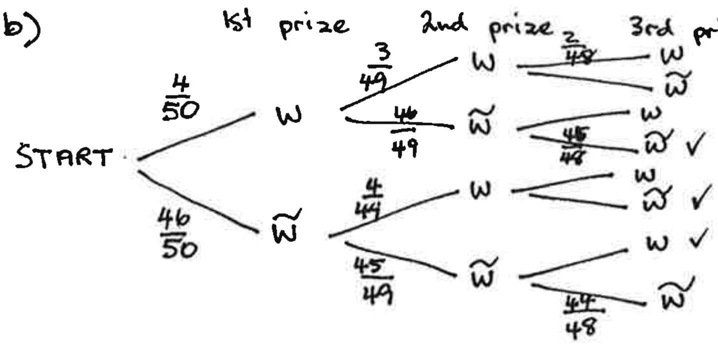
Comments:

−1 $[-2\sin\frac{x}{2}]$

−1 [$\frac{1}{2}$ not 2]

Question 6 (Cont)

b)



$P(w\tilde{w}\tilde{w}) = \frac{4}{50} \times \frac{46}{49} \times \frac{45}{48}$

i) $P(www) = \frac{4}{50} \times \frac{3}{49} \times \frac{2}{48}$

$= \frac{1}{4900}$ (1)

ii) $P(\tilde{w}\tilde{w}\tilde{w}) = \frac{46}{50} \times \frac{45}{49} \times \frac{44}{48}$

$= \frac{759}{980}$ (1)

iii) $P(\text{at least 1 prize}) = 1 - P(\text{no prizes})$

$= 1 - \frac{759}{980}$ (1)

$= \frac{221}{980}$

iv) $P(\text{exactly 1 prize}) = P(w\tilde{w}\tilde{w}) + P(\tilde{w}w\tilde{w}) + P(\tilde{w}\tilde{w}w)$

$= \frac{4}{50} \times \frac{46}{49} \times \frac{45}{48} + \frac{46}{50} \times \frac{4}{49} \times \frac{45}{48} + \frac{46}{50} \times \frac{45}{49} \times \frac{4}{48}$

$= \frac{207}{980}$ (1) $\boxed{4}$

c) geometric series $a = \frac{1}{1024}$ $r = 2$ (1)

$T_n = ar^{n-1}$

$T_n = \frac{1}{1024} \times 2^{n-1}$

$4096 = \frac{1}{1024} \times 2^{n-1}$ (1)

$4\,194\,304 = 2^{n-1}$

$n - 1 = \frac{\ln 4194304}{\ln 2}$

$n = 1 + 22 = 23.$ (1)

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ $\boxed{4}$

$S_{23} = \frac{1}{1024}\frac{(1-2^{23})}{(1-2)}$

$= 8191.999023\ldots$ (1)

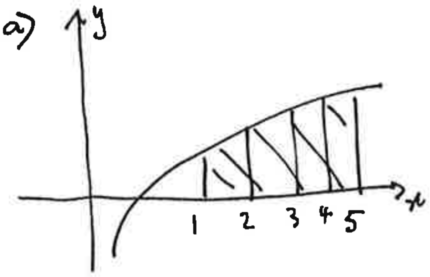
d) $\ln e^{2.6} = 2.6$

(1) $\boxed{1}$

Suggested Solutions	Comments

Question 7 .

a)



x	1	2	3	4	5
y	1	1 + ln 2	1 + ln 3	1 + ln 4	1 + ln 5

i) Trapezoidal rule

$$\int_a^b f(x)\,dx \doteqdot \frac{(b-a)}{2}\left\{f(a)+f(b)\right\}$$

$$\int_1^5 f(x)\,dx \doteqdot \frac{(2-1)}{2}\left\{f(1)+2f(2)+2f(3)+2f(4)+f(5)\right\}$$

$$\doteqdot \frac{1}{2}\left\{1+2(1+\ln 2+1+\ln 3+1+\ln 4) + 1+\ln 5\right\}$$

$$\doteqdot 7.982772787\ldots \text{units}^2 \qquad \boxed{3}$$

ii) Simpson's Rule $\int_a^b f(x)\,dx \doteqdot \frac{(b-a)}{6}\left\{f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right\}$

$$\int_1^5 f(x)\,dx \doteqdot \frac{(3-1)}{6}\left\{f(1)+4f(2)+f(3)\right\} + \frac{(5-3)}{6}\left\{f(3)+4f(4)+f(5)\right\}$$

1

$$\doteqdot \frac{1}{3}\left\{1+4(1+\ln 2)+1+\ln 3\right\} + \frac{1}{3}\left\{1+\ln 3+4(1+\ln 4)+1+\ln 5\right\}$$

$\boxed{3}$

$$\doteqdot 8.041476219\ldots \text{units}^2$$

iii) α) $\frac{d}{dx}x\ln x = x \times \frac{1}{x} + \ln x \times 1$

$$= 1+\ln x \qquad \boxed{2}$$

β) $\int_1^5 1+\ln x\,dx = \left[x\ln x\right]_1^5$

$$= 5\ln 5 - 1\ln 1 \qquad 1\tfrac{1}{2}$$

$$= 5\ln 5 \text{ or } \ln 5^5 \qquad \boxed{2}$$

$$= \ln(3125)$$

Comments: 1 ln 1 had to be simplified to 0 for full marks. Needed to be <u>exact</u>

γ) $\ln 3125 \doteqdot 8.0471\ldots$

$\ln 3125 - 7.982772787$	$\ln 3125 - 8.041476219$
$= 0.064416775 17\ldots$	$= 0.005713343\ldots$

$\boxed{2}$

∴ ii) is a better approximation as the error is smaller.

Comments: Depended on answers to (i), (ii), (iii β). Needed some 'words' in explain

Suggested Solutions	Comments

Question 8

a) $P = P_0 e^{kt}$

i) $\frac{dP}{dt} = P_0 e^{kt} \times k$

$= k \times P_0 e^{kt}$

$\frac{dP}{dt} = kP$ as $P = P_0 e^{kt}$ $\boxed{1}$

ii) $P_0 = 12000$ (population start of 1990, $t=0$) $\boxed{1}$

$\therefore P = 12000 e^{kt}$

iii) $\left.\begin{array}{l} t = 5 \\ P = 8000 \end{array}\right\}$ $\frac{8000}{12000} = \frac{12000 e^{5k}}{12000}$ —1

$\ln\left(\frac{8}{12}\right) = 5k$

$\therefore k = \frac{1}{5}\ln\left(\frac{8}{12}\right)$

$= -0.0810930\ldots$ $\boxed{2}$

iv) $\left.\begin{array}{l} t = 10 \\ P = ? \end{array}\right\}$ $P = 12000 e^{10k}$ $\boxed{1}$ — Round for 'people'

$\doteqdot 5333$ people

v) $\left.\begin{array}{l} P = 3000 \\ t = ? \end{array}\right\}$ $\frac{3000}{12000} = \frac{12000 e^{kt}}{12000}$

$\frac{1}{4} = e^{kt}$

$kt = \ln\left(\frac{1}{4}\right)$

$t = \frac{\ln\left(\frac{1}{4}\right)}{k}$

$t = 17.095\ldots$ —2 $\boxed{2}$

ie during 2007 the bank will close.

b) i) $y = \sin 2x$

$y' = 2\cos 2x$

$x = \frac{5\pi}{6}$, $y' = 2\times\cos\left(2\times\frac{5\pi}{6}\right)$

$= 2\times\cos\left(\frac{5\pi}{3}\right)$

$= 2\times 0.5$

$= 1$ 1

$x = \frac{5\pi}{6}$, $y = \sin\left(2\times\frac{5\pi}{6}\right)$

$y = -\frac{\sqrt{3}}{2}$ 1 $\boxed{3}$

$\therefore$ tangent

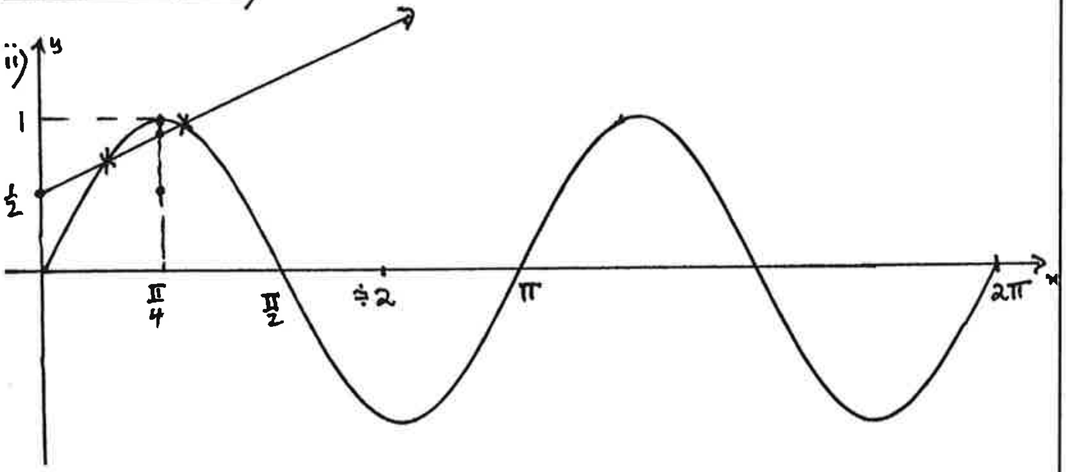
$y + \frac{\sqrt{3}}{2} = 1\left(x - \frac{5\pi}{6}\right)$ 1

$x - y - \frac{5\pi}{6} - \frac{\sqrt{3}}{2} = 0$

Suggested Solutions	Comments

Question 8 (cont)

ii)



$y = \frac{1}{2}(x+1)$ so if $x = \frac{\pi}{4}$, $y = \frac{1}{2} \times \frac{\pi}{4} + \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}$ $\doteqdot 0.89\ldots$ [2]

$\therefore$ 2 solutions.

Question 9

6% p.a = 0.5%/m.

15×12 = 180

a) i) Balance $= \$480\,000 \times 1.005$

$= \$482\,400$ [1]

ii) B_1 after instalment $= \$480\,000 \times 1.005 - \$I$

$\therefore B_2 = (\$480\,000 \times 1.005 - \$I) \times 1.005 - \$I$ [1]

$= \$480\,000 \times 1.005^2 - \$I - \$I \times 1.005$

iii) $B_3 = (\$480\,000 \times 1.005^2 - \$I - \$I \times 1.005)1.005 - \$I$

$= \$480\,000 \times 1.005^3 - \$I \times 1.005^2 - \$I \times 1.005 - \$I$

$= \$480\,000 \times 1.005^3 - \$I[1 + 1.005 + 1.005^2]$

$\vdots$

$B_{180} = \$480\,000 \times 1.005^{180} - \$I[1 + 1.005 + 1.005^2 + \ldots 1.005^{179}]$

$\Downarrow$ geometric series. $a = 1$, $r = 1.005$, $n = 180$

minus 1 if months wrong.

But $B_{180} = 0$ as the loan is repaid.

$0 = \$480\,000 \times 1.005^{180} - \$I \times S_{180}$

$\$I = \dfrac{\$480\,000 \times 1.005^{180}}{S_{180}}$

$= \$4050.512775\ldots$

or $4051 (nearest dollar)

$S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{1(1-1.005^{180})}{(1-1.005)}$

$= 290.818\ldots$

[4]

Suggested Solutions	Comments

Question 9 (cont)

b) i) Area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times \frac{\pi}{6} \times 20^2$

$= 104.719 \ldots \text{ cm}^2$ $\boxed{1}$

ii) Area segment $= 104.719\ldots - \frac{1}{2} \times 20^2 \times \sin\frac{\pi}{6}$

$= 4.719755\ldots$ $\boxed{2}$

iii) $P(\text{segment}) = \frac{4.719755}{104.719\ldots} \times 100\%$

$\doteqdot 4.507\ldots\%$ $\boxed{1}$

iv) $P(\text{at least once}) = 1 - P(5 \text{ misses})$

$= 1 - (0.95)^5$ $\boxed{2}$

$= 0.2262\ldots$

ie probability about 23% of hitting the segment at least once.

1 mark for the idea of $(\ )^5$

Question 10

a) Solve $2\ln x = \ln 2 + \ln(x+4)$

$\frac{1}{2} \rightarrow \ln x^2 = \ln 2(x+4)$

$\ln x^2 = \ln 2x + 8$

$\therefore x^2 = 2x + 8$ -1 $\boxed{3}$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\therefore x = -2$ or 4

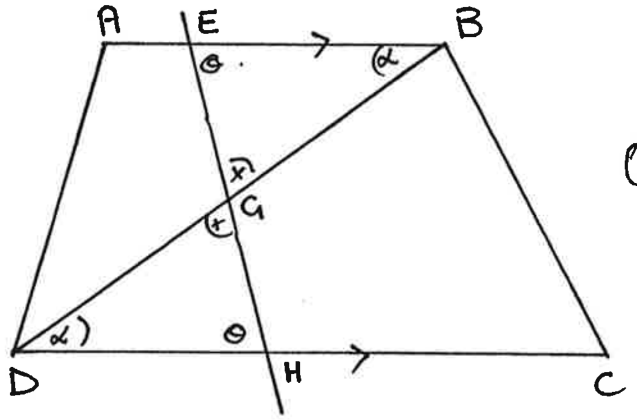
but $x \neq -2$ as its negative. -1

$\boxed{\therefore x = 4}$

Suggested Solutions	Comments

Question 10 (cont)

b)



① for diagram

In ΔEBG & ΔDHG

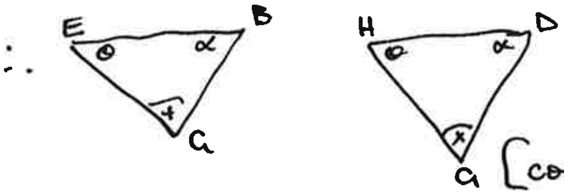
$\angle EBG = \angle HDG$ [alternate angles, $AB \parallel DC$]

$\angle EGB = \angle HGD$ [vertically opposite] ②

$\boxed{4}$

$\therefore \Delta EBG \,|||\, \Delta HDG$ [2 angles of 1 triangle equal 2 angles of the other]

$\therefore$



$\therefore \dfrac{GB}{GD} = \dfrac{EB}{HD}$ [corresponding sides are in the same proportion]

$\therefore GB = \dfrac{GD \times EB}{HD}$ as $HD = DC - HC$

$\therefore GB = \dfrac{GD \times EB}{DC - HC}$ ①

c) $(k+3)n^2 + (6-2k)n + k - 1 = 0$

i) real roots $\Delta \geq 0$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (6-2k)^2 - 4\times(k+3)(k-1) \\ &= 36 - 24k + 4k^2 - 4(k^2+2k-3) \\ &= 36 - 24k + \cancel{4k^2} - \cancel{4k^2} - 8k + 12 \\ &= -32k + 48\end{aligned}$$

$\boxed{2}$

$\Delta \geq 0$ $32k \leq 48$

$k \leq \frac{3}{2}$.

ii) let α and β be α, 6α

$\therefore \alpha + \beta = -\frac{b}{a}$

$\alpha + 6\alpha = \dfrac{-(6-2k)}{k+3}$

Question 10 (cont)

$$7\alpha = \frac{2k-6}{k+3} \quad — (1)$$

$$\alpha\beta = \frac{c}{a}$$

$$\therefore \alpha \times 6\alpha = \frac{k-1}{k+3}$$

$$6\alpha^2 = \frac{k-1}{k+3} \quad — (2)$$

from (1) $\alpha = \dfrac{2k-6}{7(k+3)}$

into (2) $\dfrac{6 \times (2k-6)^2}{49(k+3)^2} = \dfrac{(k-1)}{k+3}$ 1 mark $[\times 49(k+3)^2]$

$$6(2k-6)^2 = 49(k-1)(k+3)$$

$$6(4k^2-24k+36) = 49(k^2+2k-3)$$

$$24k^2-144k+216 = 49k^2+98k-147$$

$$\therefore 25k^2+242k-363 = 0$$

$$k = \frac{-242 \pm \sqrt{242^2+4 \times 25 \times 363}}{2 \times 25}$$

$$= \frac{-242 \pm 308}{50}$$

$$\therefore k = \frac{33}{25} \text{ or } -11 \qquad \boxed{3}$$