Centre Number

Student Number

SCEGGS Darlinghurst

2002
Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Time allowed - 3 hours.
- An additional 5 minutes reading time is allowed
- Ensure that your student number is on this paper
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Approved scientific calculators should be used. Mathematical templates and geometrical instruments may be used.
- Begin each question on a new page - write your student number at the top of each page.
- The table of standard integrals is printed on the last page.

Questions 1 - 10

Pages 2 - 12

Total marks (120)

Assessment Weighting (40%)

- Attempt all **TEN** questions
- All questions are of equal value.

- Answer the questions on the pad paper provided
- Show ALL necessary working

Question 1 (12 marks) **Marks**

(a) Write down the exact value of $\cos\frac{\pi}{4}$. **1**

(b) Solve $|5-2x|<9$ and graph the solution on a number line. **2**

(c) Find the primitive function of $\frac{x}{3}+4$. **2**

(d) Simplify $\frac{3}{x+3} - \frac{1}{x-3}$. **2**

(e) By rationalising the denominator, express $\frac{7}{4-\sqrt{2}}$ in the form $a+b\sqrt{2}$. **3**

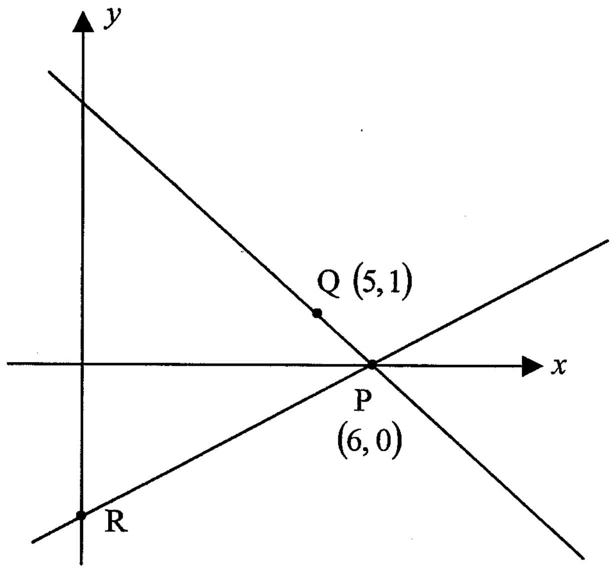
(f) Solve the pair of simultaneous equations: **2**

$$x-y=5$$

$$2x+7y+8=0$$

- Start a new page

Question 2 (12 marks) **Marks**



The diagram shows the point P $(6,0)$ and Q $(5,1)$.

The line PR cuts the y axis at R.

Draw this diagram onto your answer sheet.

(a) Find the gradient of PQ. **1**

(b) RQ is perpendicular to PQ. Find the equation of RQ. **2**

(c) Find the co-ordinates of R. **1**

(d) P and R are the endpoints of a diameter of a circle. Find the equation of the circle. **3**

(e) Show that Q lies on the circle. **1**

(f) RQTP is a parallelogram. Find the co-ordinates of T. **1**

(g) Find the area of RQTP. **3**

- Start a new page

Question 3 (12 marks) **Marks**

(a) If α and β are the roots of the equation $2x^2 - 3x - 1 = 0$, find the values of:

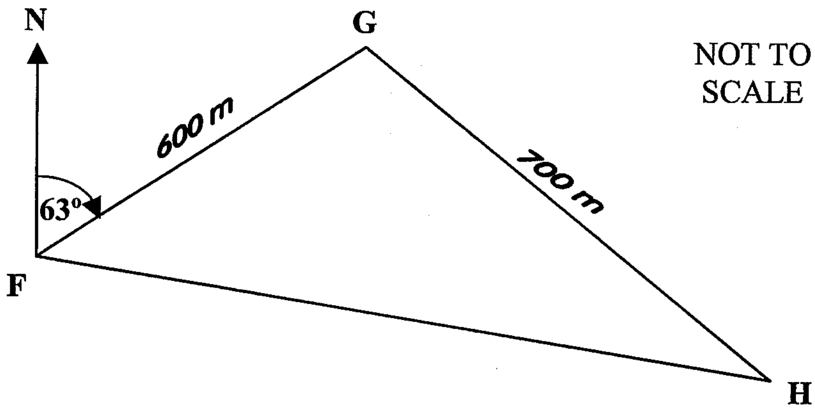
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ **3**

(b) Solve $3\tan^2 2x = 1$, where $0 \le x \le \pi$. **3**

(c) Jane is setting up part of an orienteering course.
She follows the course shown in the diagram below.



(i) If $\angle$FGH is $108°$, show that the distance FH to the nearest metre is 1053 metres **1**

(ii) Hence, or otherwise, calculate the size of $\angle$GFH to the nearest degree. **2**

(iii) If the bearing of G from F is $063°$ calculate the bearing of H from F to the nearest degree. **1**

- Start a new page

Question 4 (12 marks) **Marks**

(a) Evaluate: $$\lim_{x \to 3} \frac{3+2x-x^2}{x-3}$$ **2**

(b) Simplify: $$\frac{3^m \times 9^{m+1}}{27^{2m}}$$ **2**

(c) Explain why $x^2 + kx + k - 1 = 0$ has real roots for all values of k. **2**

(d) Differentiate: $$e^x (x+2)$$ **2**

(e) Evaluate: $$\int_{-2}^{1} \frac{dx}{\sqrt{2-x}}$$ **2**

(f) Describe the locus of a point moving so that it is equidistant from two fixed points on the number plane. **2**

- Start a new page

Question 5 (12 marks) **Marks**

(a) (i) Sketch the function $y = f(x)$ **2**

where $f(x) = \begin{cases} 1 - x^2 & \text{if } x > 1 \\ x^2 - 1 & \text{if } x \le 1 \end{cases}$

(ii) Find the value of $f(2) + f(1) - f(0)$ **1**

(b) (i) Show that $\dfrac{2x+7}{x+3} = 2 + \dfrac{1}{x+3}$ **1**

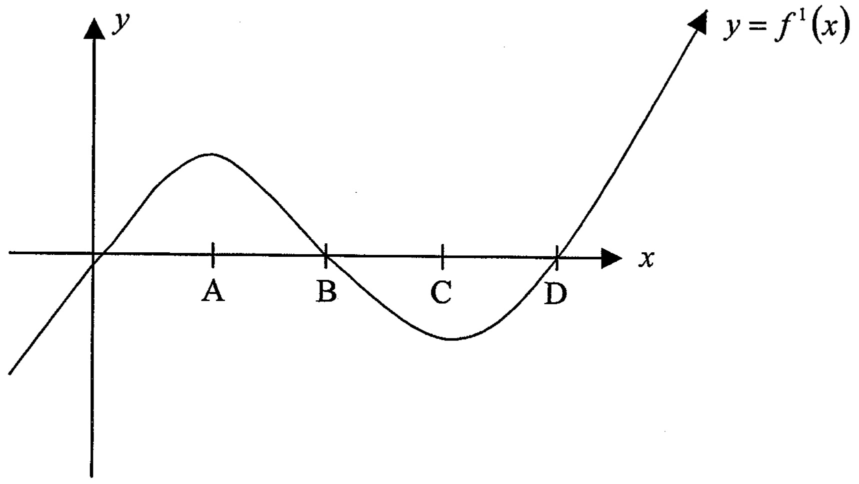
(ii) Find the domain and range of $y = \dfrac{2x+7}{x+3}$ **2**

(iii) Hence sketch $y = \dfrac{2x+7}{x+3}$ showing all important features. **2**

(iv) Evaluate $\displaystyle\int_0^1 \frac{2x+7}{x+3}\, dx$ **2**

Leave your answer in exact form.

(c)

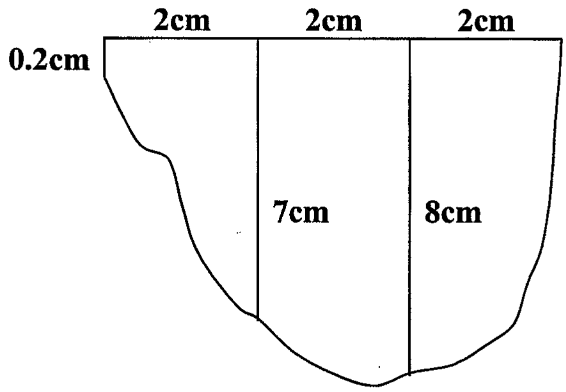


The above diagram shows a sketch of the gradient function of the curve $y = f(x)$. **2**
Draw a sketch of the function $y = f(x)$.

- Start a new page

Question 6 (12 marks) **Marks**

(a)



The diagram shows the cross-section of a puddle with the depth of the puddle shown in cm at 2cm intervals. The width of the puddle is 6cm.

(i) Student A uses 3 applications of the trapezoidal rule to find an approximate value for the area of the cross-section. Show the calculations Student A would do. **2**

(ii) Student B decides that the cross-section of the puddle approximates the parabolic function $y = x^2 - 6x$. She integrates to find the area of the cross-section between the curve and the x axis. Show the calculations for Student B. **3**

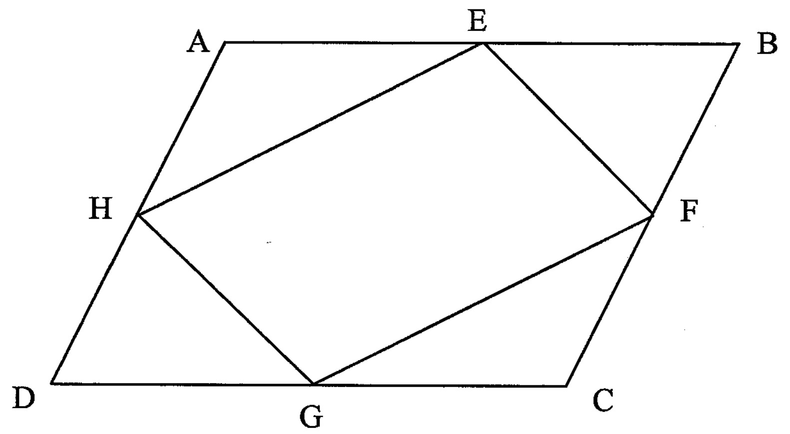
(b) In a class of thirty girls, 25 study Mathematics and 20 study History. Each girl studies at least one of these subjects. If a girl is picked at random from this class find the probability she studies:

(i) both Mathematics and History. **1**

(ii) History but not Mathematics. **1**

Question 6 continues on page 8

(c) In the diagram below ABCD is a parallelogram. Point E, F, G, H are the midpoints of AB, BC, CD and DA respectively.



(i) Copy diagram and mark on it the relevant information.

(ii) Prove that Δ AEH is congruent to Δ FGC. **3**

(iii) Hence prove that EFGH is a parallelogram. **2**

- Start a new page

Question 7 (12 marks) **Marks**

(a) (i) Factorise $u^2 - 6u - 16$ **1**

(ii) Hence, or otherwise, solve fox x **2**

$$[\log_2 x]^2 - 6[\log_2 x] - 16 = 0$$

(b) Sally deposits \$1700 into a superannuation fund on January 1st, 2001. She makes further deposits of \$1700 on the first of each month up to and including December 1st, 2010. The fund pays compound interest at a monthly rate of $0 \cdot 75\%$. In each of the following questions give your answer to the nearest dollar.

(i) Form a geometric series and hence determine the total amount in the fund on December 31st, 2010. **2**

(ii) If each deposit was increased to \$1800, what difference does it make to the total amount in the fund on December 31st, 2010. **2**

(c) The common ratio r of a geometric progression satisfies the quadratic equation $2r^2 - 3r - 2 = 0$.

(i) Solve for r: **1**

If the sum to infinity of the same progression is 6.

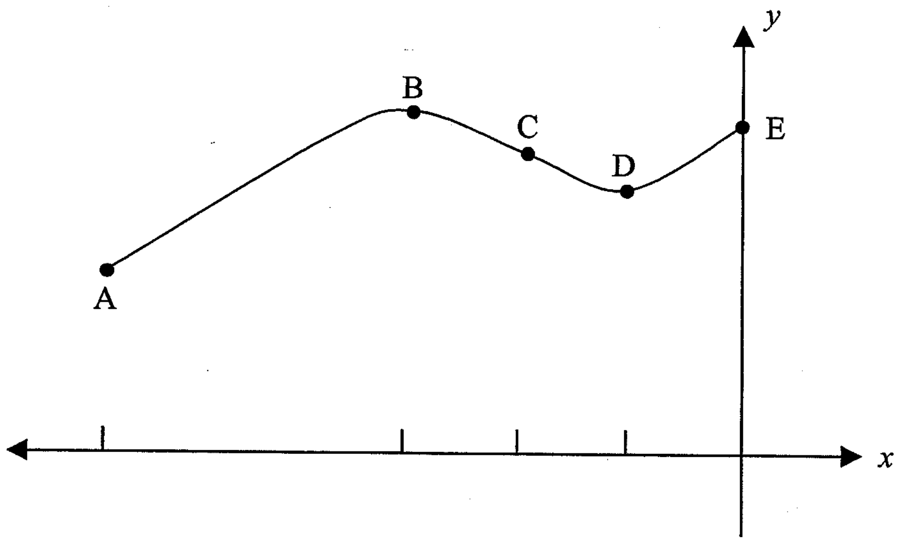
(ii) Explain why, in this case, r can only take on one value. Hence, state the common ratio r. **2**

(iii) Show that the first term a of this progression, is 9. **2**

- Start a new page

Question 8 (12 marks) **Marks**

(a) The curve $y = x^3 + 2x^2 + x + 7$ is sketched in the domain $-2 \le x \le 0$ with significant points labelled as below.



(i) Find points B, C and D and using calculus determine their nature. **6**

(ii) Between which points is the function decreasing? **1**

(iii) What is the minimum value of the function? **1**

(iv) Describe the behaviour of the function for very large positive values of x. **1**

(b) Find the equation of the normal to the curve $y = 3\ln x$ at the point where $x = 1$. Give your answer in general form. **3**

- Start a new page

Question 9 (12 marks) **Marks**

(a) (i) Sketch the curve $y = x^2 - x - 2$ in the domain $0 \le x \le 4$ showing all important points. **2**

(ii) On your diagram for part (i) sketch the line $y = -x + 7$ in the domain $0 \le x \le 4$ clearly showing the point of intersection with the curve $y = x^2 - x - 2$ in this domain. **2**

(iii) Find the exact area of the region bounded by the curve, $y = x^2 - x - 2$ the line $y = -x + 7$ and the x axis between $x = 2$ and $x = 4$. **2**

(b) A particle moves on a horizontal line so that its displacement x cm to the right of the origin at time t seconds is $x = t\sin t$.

(i) Find expressions for the velocity and acceleration of the particle. **2**

(ii) Find the exact velocity of the particle at time $t = \frac{\pi}{4}$. **1**

(iii) What effect does the acceleration have on the velocity of the particle at $t = \frac{\pi}{4}$. **1**

(iv) After the particle leaves the origin is the particle ever at rest? Give reasons for your answer. **2**

- Start a new page

Question 10 (12 marks) **Marks**

(a) A bushfire is spreading through a forest and the amount of area burnt since the fire was reported is modelled by the exponential function

$$A = 2e^{kt}$$

where A is the area in hectares and t is the time in hours since the fire was reported. The amount burnt in the first three hours was double the amount burnt since the fire was reported.

(i) Find the amount of area burnt when the fire was reported? **1**

(ii) Find the exact value of k. **2**

(iii) If the fire was reported at 5am, at what time, to the nearest minute, will 15 hectares have been burnt? **2**

(iv) Explain why this model is unrealistic. **2**

(b) The rate at which babies are being born has been decreasing at an increasing rate for the last ten years. Draw a graph showing this information. **2**

(c) Find the volume of the solid of revolution formed when the curve $y = \ln 2x$ is rotated about the y axis between $y = 0$ and $y = 2$. Give your answer in exact form. **3**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2002 H.A.C. Trial Mathematics

Question 1

a) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ 1

b) $-9 < 5-2x < 9$

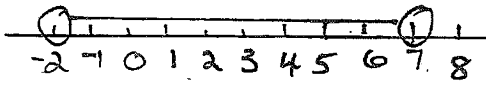
$-14 < -2x < 4$

$-14 < -2x \qquad -2x < 4$

$-2x > -14 \qquad x > -2$

$x < 7$

$-2 < x < 7$ 1



1

c) $\int \frac{x}{3} + 4\,dx$

$= \frac{x^2}{6} + 4x + C$ 1 1

d) $\frac{3x-9-x-3}{x^2-9} = \frac{2x-12}{x^2-9}$ 1 1

e) $\frac{7}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}} = \frac{28+7\sqrt{2}}{16-2}$ 1

$= \frac{28+7\sqrt{2}}{14} = \frac{4+\sqrt{2}}{2}$ 1 1

$= 2 + \frac{1}{2}\sqrt{2}$

f) $x - y = 5$ (1)

$2x + 7y = -8$ (2)

(1) $\times 7$ $\quad 7x - 7y = 35$ (3)

(2) + (3) $\quad 9x = 27$

$x = 3$ 1

sub in (1) $\quad 3 - y = 5$

$y = -2$ 1

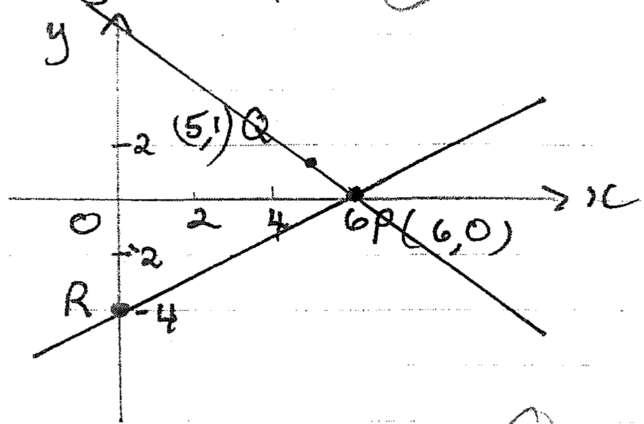
Question 2

a) $m_{PQ} = \frac{1}{5-6} = -1$ 1

b) $m_{RQ} = 1$ 1

$y - 1 = 1(x-5)$

$y = x - 4$ 1



c) $(0,-4) = R$ 1

d) centre $= (3,-2)$ 1

$r = \sqrt{3^2 + (-2+4)^2}$

$= \sqrt{13}$ 1

$(x-3)^2 + (y+2)^2 = 13$ 1

e) $(5-3)^2 + (1+2)^2 = 13$

$4 + 9 = 13$ ✓ 1

$\therefore$ Q lies on the circle.

f) mid pt. of $PQ = (5\tfrac{1}{2}, \tfrac{1}{2})$

Let $T = (x_1, y_1)$

$\therefore \frac{x_1+0}{2} = \frac{11}{2}, \quad \frac{y_1 + -4}{2} = \frac{1}{2}$

$x_1 = 11 \qquad y_1 = 5$

$\therefore T = (11, 5)$ 1

g) eqn of RP

$\frac{y+4}{x-0} = \frac{0+4}{6-0} = \frac{2}{3}$

$3y + 12 = 2x$

$2x - 3y - 12 = 0$ 1

$\perp$ dist. of Q to RP

$d = \left|\frac{10-3-12}{\sqrt{4+9}}\right|$

$= \frac{5}{\sqrt{13}}$ 1

$\therefore$ Area RQTP

$= 2\sqrt{13} \times \frac{5}{\sqrt{13}} = 10\ u^2$ 1

Question 3

a) i) $\alpha + \beta = \frac{3}{2}$ (1)

ii) $\alpha\beta = -\frac{1}{2}$ (1)

iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$ (1)

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ (1)

$= \frac{\frac{9}{4} + 1}{-\frac{1}{2}} = -6\frac{1}{2}$ (1)

b) $\tan 2x = \pm \frac{1}{\sqrt{3}}$ (1)

$\therefore 2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ (1)

$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ (1)

c) i) $FH^2 = 600^2 + 700^2 - 2 \times 600 \times 700 \times \cos 108^\circ$

$\therefore FH \doteqdot 1053$ (1)

ii) $\frac{\sin \angle GFH}{700} = \frac{\sin 108^\circ}{1053}$ (1)

$\therefore \angle GFH \doteqdot 39^\circ$ (1)

iii) The bearing of H from F is 102°T (1)

Question 4

a) $\lim_{x \to 3} \frac{(3-x)(1+x)}{x-3}$ (1)

$= \lim_{x \to 3} \frac{(3-x)(1+x)}{-(3-x)}$

$= \lim_{x \to 3} -(1+x)$

$= -4$ (1)

b) $\frac{3^m \times 9^{m+1}}{27^{2m}} = \frac{3^m \times 3^{2m+2}}{3^{6m}}$ (1)

$= \frac{3^{3m+2}}{3^{6m}} = 3^{-3m+2}$ (1)

c) $\Delta = k^2 - 4k - 4$

$\Delta = (k-2)^2$ (1)

which is a perfect square ∴ Δ is always +ve or equal to 0 (1)

∴ the quadratic eqn. has real roots for all values of k.

d) Let $y = e^x(x+2)$

$\frac{dy}{dx} = e^x(x+2) + e^x$ (1)

$= e^x(x+3)$ (1)

e) $\int_{-2}^{1} \frac{dx}{\sqrt{2-x}} = \left[-2(2-x)^{\frac{1}{2}}\right]_{-2}^{1}$ (1)

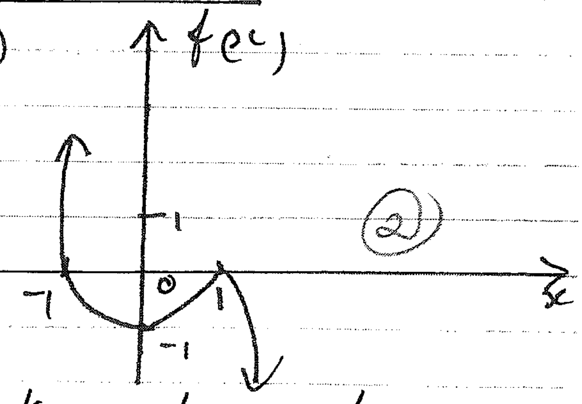
$= (-2) - (-2\sqrt{3})$

$= -2 + 2\sqrt{3}$ (1)

f) The locus is the perpendicular bisector (2) of the interval joining the 2 fixed points

Question 5

a) i)



(2)

ii) $f(2) + f(1) - f(0)$

$= 1 - 4 + 1 - 1 - -1$

$= -3 + 1 = -2$ (1)

Q5 2002

b) i) L.H.S. $= \frac{2x+7}{x+3}$

$= \frac{2x+6+1}{x+3}$

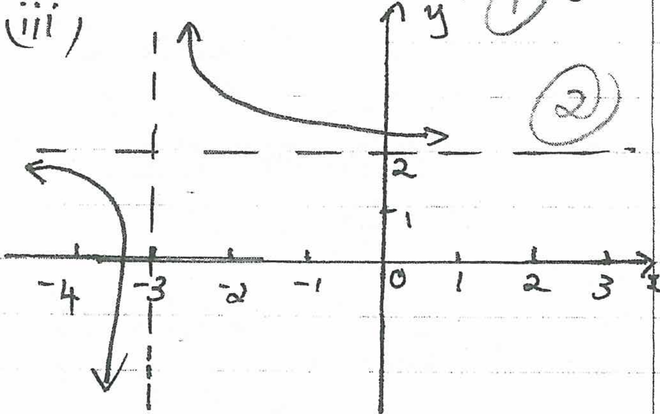
$= \frac{2(x+3)+1}{x+3}$ (1)

$= 2 + \frac{1}{x+3} = R.H.S.$

ii) D: all real x except $x = -3$ (1)

R: all real y except $y = 2$ (1)

iii)

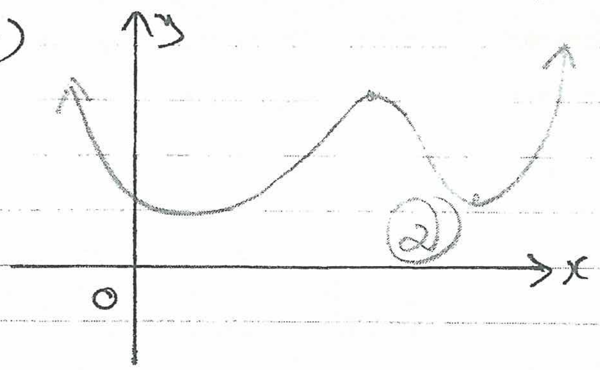


iv) $\int_0^1 2 + \frac{1}{x+3}\,dx$

$= \left[2x + \ln(x+3)\right]_0^1$ (1)

$= 2 + \ln 4 - \ln 3$ (1)

c)



Question 6

a) i) $h = 2$

x	$f(x)$	w	$w \times f(x)$
0	0·2	1	0·2
2	7	2	14
4	8	2	16
6	0	1	0

(1)

30·2 (1)

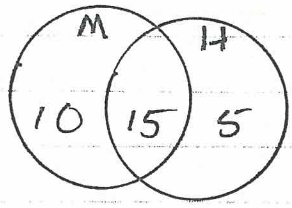
Area $\doteqdot \frac{2}{2} \times 30{\cdot}2 = 30{\cdot}2\ cm^2$

ii) $A \doteqdot \left| \int_0^6 x^2 - 6x\,dx \right|$ (1)

$= \left| \left[\frac{x^3}{3} - 3x^2 \right]_0^6 \right|$ (1)

$= |72 - 36| = |36| = 36$ (1)

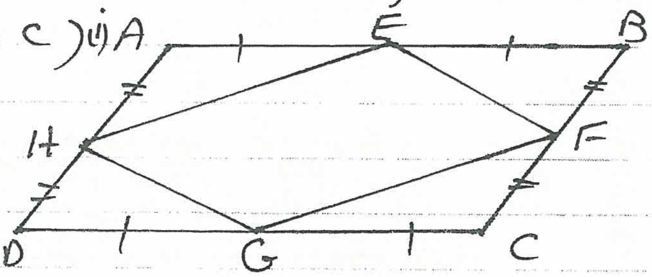
b)



i) $P(\text{both}) = \frac{15}{30} = \frac{1}{2}$ (1)

ii) $P(H \text{ no } M) = \frac{5}{30} = \frac{1}{6}$ (1)

c) i)



ii) In Δ's AEH and FGC

AB = DC (opp. sides of ∥'gram)

∴ AE = GC (E & G mid pts. of opposite sides of ∥'gram)

AD = BC (opp. sides of ∥'gram)

∴ AH = FC (H F mid pts. of opp. sides of ∥'gram)

∠HAE = ∠FGC (opp. ∠'s of ∥'gram)

∴ Δ AEH ≡ Δ FGC (SAS)

(3)

(iii) Similarly it can be proved $\Delta HDG \equiv \Delta EBF$

$\therefore HE = GF$ (matching sides of $\equiv \Delta$'s) and

$HG = FE$ (matching sides of $\equiv \Delta$'s) (2)

$\therefore EFGH$ is a $\parallel$'gram

Question 7

a) (i) $(u-8)(u+2)$ (1)

(ii) Let $\log_2 x = u$

$u^2 - 6u - 16 = 0$

$(u-8)(u+2) = 0$

$\therefore u = 8, -2$

$\log_2 x = 8$

$\therefore x = 2^8 = 256$ (1)

$\log_2 x = -2$

$\therefore x = 2^{-2} = \frac{1}{4}$ (1)

b) (i) $A_1 = \$17000 \times 1.0075^{120}$

$A_2 = \$17000 \times 1.0075^{119}$

and so on

$A_{120} = \$17000 \times 1.0075$

$\therefore$ at the end of 10 years the investment is worth

$\$1700(1.0075^{120} + 1.0075^{119} + \ldots + 1.0075)$ (1)

$= 1700 \times \frac{1.0075(1.0075^{120} - 1)}{1.0075 - 1}$

$= 1700 \times 194.96563$

$\therefore$ total amount $=$

$\$331641.58$ (1)

(ii) New amount $=$

$\$1800 \times 194.96563$

$= \$350938.14$ (1)

$\therefore$ Diff $= \$19496.56$ (1)

error

c) (i) $(2r+1)(r-2) = 0$

$\therefore r = -\frac{1}{2}, 2$ (1)

(ii) For the series to have a S_∞ $-1 < r < 1$ (1)

$\therefore r = -\frac{1}{2}$ (1)

(iii) $6 = \frac{a}{1 - -\frac{1}{2}}$ (1)

$6 = \frac{a}{3/2}$ (1)

$\therefore a = 6 \times \frac{3}{2} = 9$

Question 8

a) (i) $\frac{dy}{dx} = 3x^2 + 4x + 1$

$3x^2 + 4x + 1 = 0$ (1)

$(3x+1)(x+1) = 0$

$\therefore x = -\frac{1}{3}, -1$

$y = 6\frac{23}{27}, 7$

$\frac{d^2y}{dx^2} = 6x + 4$ (1)

when $x = -1$, $= -ve$

$\therefore$ max. at $(-1, 7)$ (1) B

when $x = -\frac{1}{3}$, $= +ve$

$\therefore$ min at $(-\frac{1}{3}, 6\frac{23}{27})$ (1) D

$6x + 4 = 0$

$\therefore x = -\frac{2}{3}$

x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$
$\frac{d^2y}{dx^2}$	$-ve$	0	$+ve$

(1)

$\therefore$ pt inflex $(-\frac{2}{3}, 6\frac{25}{27})$ (1) C

(ii) B and D (1)

(iii) when $x = -2$

$y = (-2)^3 + 2 \times (-2)^2 - 2$

$\therefore$ min value is 5 (1)

(iv) as $x \to \infty$
$y \to \infty$ (1)

b) $y = 3\ln x$

$\frac{dy}{dx} = \frac{3}{x}$ (1)

when $x = 1$

$m_{tan} = 3 \quad \therefore m_{norm} = -\frac{1}{3}$ (1)

$y - 0 = -\frac{1}{3}(x-1)$

$3y = -x + 1$

$x + 3y - 1 = 0$ (1)

Question 9

a)(i) $y = x^2 - x - 2$

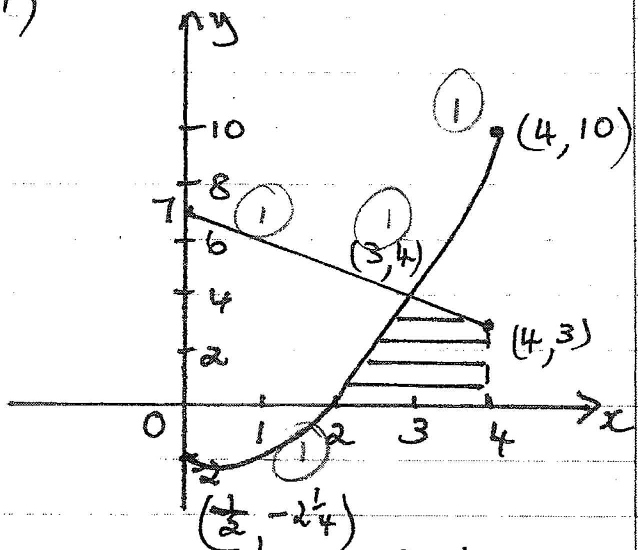
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, -1$

$(4, 10)$

(ii)



(iii) $A = \int_2^3 x^2 - x - 2\,dx + \int_3^4 -x + 7\,dx$ (1)

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3 + \left[-\frac{x^2}{2} + 7x\right]_3^4$

$= (9 - \frac{9}{2} - 6) - (\frac{8}{3} - 2 - 4) + (-8 + 28) - (-\frac{9}{2} + 21)$

$= 5\frac{1}{3}\ u^2$ (1)

b)(i) $x = t\sin t$

$\dot{x} = \sin t + t\cos t$ (1)

$\ddot{x} = \cos t + \cos t - t\sin t$

$\ddot{x} = 2\cos t - t\sin t$ (1)

(ii) when $t = \frac{\pi}{4}$

$\dot{x} = \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$ cm/sec (1)

(iii) when $t = \frac{\pi}{4}$

$\ddot{x} = \frac{1}{\sqrt{2}} \times 2 - \frac{\pi}{4} \times \frac{1}{\sqrt{2}}$ (1)

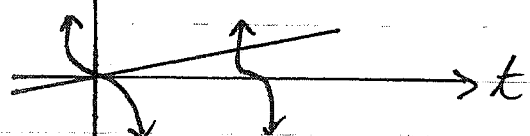
$= \frac{2}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}$ cm/s^2 = +ve

$\therefore$ acceleration is (1) speeding up the particle

(iv) $0 = t\cos t + \sin t$

$-\sin t = t\cos t$

$-\tan t = t$ (1)



Yes from sketch the particle is at rest again after the origin (1)

Question 10

a)(i) $A = 2e^{kt}$

$t = 0,\ A = 2$ ha. (1)

(ii) $4 = 2e^{3k}$ (1)

$2 = e^{3k}$

$\ln 2 = 3k$

$k = \frac{1}{3}\ln 2$ (1)

(iii) $15 = 2e^{\frac{1}{3}\ln 2\, t}$

$\ln\frac{15}{2} = \frac{1}{3}\ln 2\, t$

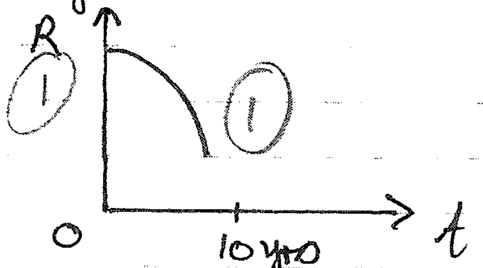
$t = \ln\frac{15}{2} \div \frac{1}{3}\ln 2$ (1)

$= 8.7$ hrs $= 8$ hrs 43 mins

$\therefore$ 1.43 pm (1)

(iv) this model assumes the bushfire's will continue burning at the same rate under the same conditions for ever which does not happen due to weather and different type of vegetation (2)

b)



decreasing $y' < 0$
increasing rate $y'' > 0$

c) $y = \ln 2x$

$2x = e^y$ (1)

$x = \frac{1}{2} e^y$

$x^2 = \frac{1}{4} e^{2y}$

$V = \pi \int_0^2 \frac{1}{4} e^{2y} \, dy$ (1)

$= \pi \left[\frac{1}{8} e^{2y} \right]_0^2$

$= \pi \left(\frac{1}{8} e^4 - \frac{1}{8} \right) = \frac{\pi}{8} (e^4 - 1)$ (1) cu units