Centre Number

Student Number

SCEGGS Darlinghurst

2003
Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

Total marks - 120

- Attempt Questions 1–10
- All questions are of equal value.

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Attempt Questions 1–10
All questions are of equal value

Answer each question on a NEW page.

Marks

Question 1 (12 marks)

(a) Evaluate, correct to two significant figures, **2**

$$\frac{195\cdot 32}{4\cdot 6^2+5\cdot 73}$$

(b) Solve $\frac{y}{4}-\frac{y-6}{8}=2$. **2**

(c) Solve $x^2+3x>10$ **2**

(d) State the range of: **1**

$$y=(x-1)^2+4$$

(e) Express $\frac{1}{\sqrt{2}}+\frac{3}{\sqrt{2}-1}$ as a single fraction with a rational denominator. **3**

(f) Simplify fully: **2**

$$\log_a a^2 - \log_a \frac{1}{a}$$

Question 2 (12 marks) Start a NEW page. **Marks**

$A(2,4)$ and $B(8,12)$ are the ends of a diameter of a circle.

(a) Find the co-ordinates of the centre, C, of the circle. **1**

(b) Find the radius of the circle. **1**

(c) State the equation of the circle. **1**

(d) Hence show that $D(5,13)$ lies on the circle. **1**

(e) Show that $AD \perp BD$. **2**

(f) The perpendicular bisector of AB meets the circle at X and Y. Find the equation of XY in general form. **2**

(g) Show that the area of ΔXDY is 20m^2 **2**

(h) Show that $3x+4y-22=0$ is a tangent to the circle. **2**

Question 3 (12 marks) Start a NEW page. **Marks**

(a) A yacht sails from Robinson Island on a bearing of $240°$ for 120km. It then turns and sails on a bearing of $110°$ until it reaches a destination due south of its original position. **3**

Calculate the distance of the yacht from Robinson Island to the nearest kilometre.

(b) Differentiate:

(i) $y=\ln(x^2+1)$ **1**

(ii) $y=\dfrac{e^{2x}}{\sin 3x}$ **2**

(c) Evaluate **1**

$$\lim_{x\to 3}\frac{x^2-9}{x^2-x-6}$$

(d) Solve $\sin^2\theta-\sin\theta-2=0,\ -\pi\le\theta\le\pi$. **3**

(e) α and β are the roots of the equation $3x^2-6x+2=0$ **2**

Find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$

Marks

Question 4 (12 marks) Start a NEW page.

(a) Evaluate $\sum_{r=1}^{4} r^2 - 1$ **1**

(b) Find:

(i) $\int 3\sqrt{x} + \frac{1}{x^4}\, dx$ **2**

(ii) $\int_0^2 \left(e^{3x} + e^{-3x}\right)^2 dx$ **3**

(c) Show that: **3**

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \cot\theta$$

(d) (i) Sketch the curve $y = 3\sin 2x$ in the domain $0 \le x \le 2\pi$ showing the main features of the graph. **2**

(ii) Hence use your graph to find the number of solutions to the equation $3\sin 2x - 1 = 0$ for $0 \le x \le 2\pi$. **1**

Marks

Question 5 (12 marks) Start a NEW page.

(a) A bottle of water is placed in the common room fridge where the temperature is maintained at 0°C. The rate at which the temperature of the water falls is proportional to its temperature at that time. ($\frac{dT}{dt} = -kT$ where T is its temperature.) When the water is placed in the fridge its temperature is 40°C and after 17 minutes its temperature is 24°C.

(i) Show that the function $T = Ce^{-kt}$ satisfies the equation $\frac{dT}{dt} = -kT$. **1**

(ii) Find the value of the constant C. **1**

(iii) Show the exact value of k is $\frac{1}{17}\ln\left(\frac{5}{3}\right)$. **2**

(iv) Find the temperature of the water after 43 minutes to the nearest degree. **1**

(b) A sheep, grazing in a paddock, is tethered to a stake by a rope 20m long. If the stake is 10m from a long fence, find the area over which the sheep can graze. **3**

(c) One set of cards contains the numbers 1, 2, 3, 4, 5 and another set contains the letters H, O, L, L, Y. One card is selected at random from each set. Find the probability of selecting:

(i) a 4 and an H. **1**

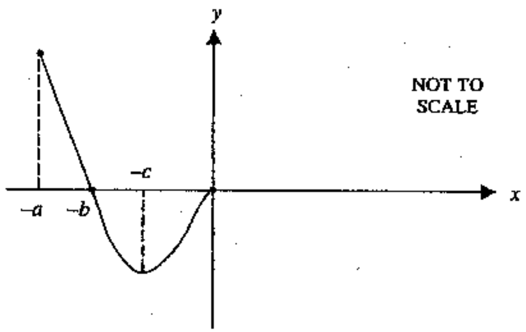
(ii) an odd number and a vowel. **1**

(iii) a number less than 3 or an L. **2**

Marks

Question 6 (12 marks) Start a NEW page.

(a) The diagram shows the graph of a function $y = f(x)$, for $-a \le x \le 0$.



It is known that $f(x)$ is an odd function and is stationary at $(0, 0)$.

(i) Sketch the graph $y = f(x)$, for $-a \le x \le a$. **1**

(ii) On a separate diagram, sketch $y = f'(x)$ **2**

(b) In a new quiz show called "Dopier than Ever", you win \$6 000 for answering the first question correctly, \$14 000 for answering the second question correctly and \$22 000 for answering the third question correctly and so on for the following questions. You finish when you answer a question incorrectly. Your total winnings for the contest is the sum of money you win on each question.

(i) What is the prize money for the 10th question only? **2**

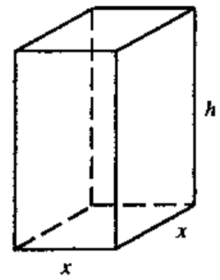
(ii) How many questions must you correctly answer to exceed \$1 000 000 in total winnings? **3**

Question 6 continues on page 8

Marks

Question 6 (continued)

(c) A box in the shape of a square prism has a volume of 32cm^3 and no lid. The square base has length x cm and the box is h cm high.



(i) Show that the surface area of the box is given by: $\text{SA} = x^2 + \dfrac{128}{x}$. **1**

(ii) Find the dimensions of the box that has the least surface area. **3**

Question 7 (12 marks) Start a NEW page. **Marks**

(a) Show that $x^3 - x^2 - x + 1 = (x+1)(x-1)^2$. **1**

(b) Consider the function, $f(x) = \frac{x-1}{x^2}$.

(i) Prove that $f'(x) = \frac{2-x}{x^3}$. **2**

(ii) Find the co-ordinates of the stationary point on $y = f(x)$ and determine its nature. **2**

(iii) Find the co-ordinates of P, the only point where $y = f(x)$ meets the x-axis. **1**

(iv) Sketch $y = f(x)$ showing all important features. **3**

(v) Show that the equation of the tangent at P is given by the equation $y = x - 1$. **1**

(vi) Using part (a) or otherwise, find the co-ordinates of the other point where this tangent meets the curve. **2**

Question 8 (12 marks) Start a NEW page. **Marks**

(a) The following is a table of values for the function $y = \frac{2}{x(x+1)}$.

x	1	2	3	4	5
y	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{15}$

(i) Using the information in the table and Simpson's rule with 5 function values, find an approximation for $\int_1^5 \frac{2}{x(x+1)}\,dx$ correct to 3 decimal places. **2**

(ii) It is also true that $\frac{2}{x(x+1)} = \frac{2}{x} - \frac{2}{x+1}$. Use direct integration to find the value of $\int_1^5 \frac{2}{x(x+1)}\,dx$ correct to 3 decimal places. **3**

(iii) Explain the difference between your answers in parts (i) and (ii). **1**

(b) A particle moves in a straight line so that its velocity v in metres per second at time t is given by $v = 4 - 2t$. At time $t = 0$ the particle is at $x = 1$.

(i) Find the displacement x of the particle as a function of t. **2**

(ii) When is the particle at rest and what is its acceleration at that time? **2**

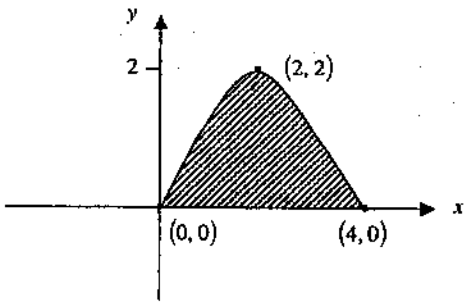
(iii) Find the distance the particle travels in the first 4 seconds. **2**

Marks

Question 9 (12 marks) Start a NEW page.

(a) Can there be a geometric series with a limiting sum of $\frac{2}{3}$ and a first term of 4? Justify your answer with appropriate calculations. **2**

(b) The producers of Play School are replacing the Arched Window. It will still have a base length of 4m and a height of 2m as shown in the diagram below.



NOT TO SCALE

The new arch is to be either an arc of a parabola or a half-cycle of a sine curve.

(i) If the arch is the arc of a parabola, the equation of the curve is of the form: **1**

$$f(x) = ax(4 - x)$$

Show that the value of a is $\frac{1}{2}$.

(ii) If the arch is a sine curve, the equation of the curve is of the form, **1**

$$g(x) = A\sin\frac{\pi x}{4}$$

Find the value of A.

(iii) Calculate the area for each window design and hence decide which would be cheaper to build. **4**

(c) A rectangular lawn is 60 metres long and 30 metres wide. A pigeon wanders randomly around the lawn. Find the probability that the pigeon is:

(i) more than 10 metres from the edge of the lawn. **2**

(ii) not more than 10 metres from a corner of the lawn. **2**

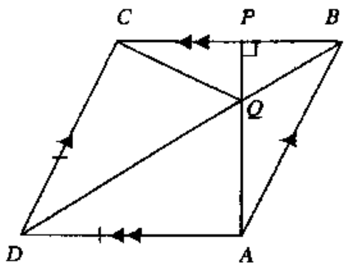
Marks

Question 10 (12 marks) Start a NEW page.

(a) Consider the function $y = xe^{-2x}$. **3**

Prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

(b) In the rhombus $ABCD$, AP is constructed perpendicular to BC and intersects the diagonal BD at Q.



NOT TO SCALE

(i) State why $\angle ADB = \angle CDB$. **1**

(ii) Prove that $\Delta AQD \equiv \Delta CQD$. **2**

(iii) Hence find $\angle QCD$. **1**

(c) After several mornings of horrendous traffic, Chris decides to move closer to work. She takes out a loan for \$500,000 at an interest rate of 12% p.a. compounded monthly for 20 years.

(i) Show that the amount she owes on the loan after n months, A_n, is given by the expression: **3**

$$A_n = 100M - 1.01^n(100M - 500{,}000)$$

where M is the size of her monthly repayments.

(ii) Her repayments are fixed at \$5505 per month. In which year does Chris still owe \$250,000? **2**

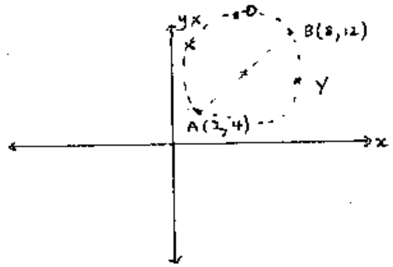
END OF PAPER

YR 12 : 2 UNIT

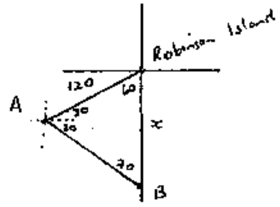
MATHEMATICS TRIAL 2003

SOLUTIONS + MARKING SCALE

QUESTION 1: MF	COMMENTS
(a) $7.26366\ldots$ ✓ $\doteq 7.3$ correct to 2 s.f. ✓	Rounding off to 2 s.f. <u>not</u> well done. Does not mean 2 decimal places.
(b) $\frac{y}{4} - \frac{y-6}{8} = 2$ $\therefore 2y - y + 6 = 16$ ✓ $\therefore y = 10$ ✓	this sign was a problem.
(c) $x^2 + 3x - 10 > 0$ $(x+5)(x-2) > 0$ ✓ $\therefore x < -5$ and $x > 2$ ✓ 	You <u>must</u> factorise, <u>sketch</u> and solve the quadratic inequality from your sketch. It is <u>incorrect to</u> solve this way $x+5>0 \quad x-2>0$ $x>-5 \quad x>2$ Please <u>don't do it.</u>
(d) $y = (x-1)^2 + 4$ Range: $y \geq 4$ ✓	Students who sketched the parabola were the most successful in finding the range.
(e) $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓ $\frac{3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{3\sqrt{2}+3}{1}$ ✓ $\therefore \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}-1} = \frac{\sqrt{2}}{2} + \frac{2(3\sqrt{2}+3)}{2}$ $= \frac{7\sqrt{2}+6}{2}$ ✓	
(f) $\log_a a^2 - \log_a a^{-1} = 2 - -1$ ✓ $= 3$ ✓	1 mark for correct use of a log rule.

QUESTION 2: KB	COMMENTS
	
(a) Centre: $\left(\frac{2+8}{2}, \frac{4+12}{2}\right) = (5, 8)$ ✓	Good
(b) $d = \sqrt{(12-8)^2 + (8-5)^2}$ $= \sqrt{16+9}$ ✓ $= 5$	Good
(c) Equation: $(x-5)^2 + (y-8)^2 = 5^2$ ✓	Good
(d) When $x = 5$: $(5-5)^2 + (y-8)^2 = 5^2$ $0 + (y-8)^2 = 25$ $y - 8 = \pm 5$ ✓ $\therefore y = 13$ or 3 $\therefore (5, 13)$ does lie on the circle	Good
(e) $m_{AD} = \frac{13-4}{5-2} = \frac{9}{3} = 3$ $m_{BD} = \frac{13-12}{5-8} = -\frac{1}{3}$ ✓ $\therefore m_{AD} \times m_{BD} = 3 \times -\frac{1}{3} = -1$ $\therefore AD \perp BD$ ✓	Both gradients correct for 1 mark.
(f) $m_{AB} = \frac{12-4}{8-2} = \frac{8}{6} = \frac{4}{3}$ $\therefore$ Grad XY : $-\frac{3}{4}$ ✓ $\therefore y - 8 = -\frac{3}{4}(x-5)$ $\therefore 3x + 4y - 47 = 0$ ✓	not well done. A diagram would assist in the next sections <u>must</u> be in general form.

Question 2 (cont.)	
(g) $XY = 10$ units (diameter of circle). ✓	
$\perp$ height is dist of D from XY:	
$d = \dfrac{\lvert 3\times 5 + 4\times 13 - 47\rvert}{\sqrt{9+16}}$	
$= \dfrac{20}{5} = 4$ ✓	
$\therefore$ Area $= \frac{1}{2}\times 10\times 4$	
$= 20u^2$	
(h) $\perp$ distance of line from centre:	
Centre: $(5, 8)$	
$d = \dfrac{\lvert 3\times 5 + 4\times 8 - 22\rvert}{\sqrt{3^2+4^2}}$ ✓	
$= \dfrac{25}{5}$	
$= 5$	
$\therefore$ Since the line is 5 units from the centre of the circle of radius 5, it is a tangent to the circle. ✓	

QUESTION 3: Com 1 Calc 3 CB	COMMENTS
(a)  ✓	Com 1 – diagram only
$\angle RBA = 70°$	
$\dfrac{x}{\sin 50} = \dfrac{120}{\sin 70}$ ✓	
$\therefore x = 97.824\ldots$ ✓	must round to nearest km.
$\doteqdot 98$ km	
(b) (i) $y = \ln(x^2+1)$	(b) Calc /3
$\dfrac{dy}{dx} = \dfrac{2x}{x^2+1}$ ✓	
(ii) $y = \dfrac{e^{2x}}{\sin 3x}$ $u = e^{2x}$ $v = \sin 3x$ $u' = 2e^{2x}$ $v' = 3\cos 3x$ ✓	
$\dfrac{dy}{dx} = \dfrac{2e^{2x}\sin 3x - 3e^{2x}\cos 3x}{(\sin 3x)^2}$ ✓	Several students failed to put v^2 in the denominator (further simplification is not necessary) ignore subsequent errors
$= \dfrac{e^{2x}(2\sin 3x - 3\cos 3x)}{(\sin 3x)^2}$	
(c) $\lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)} = \dfrac{6}{5}$ ✓	
(d) $\sin^2\theta - \sin\theta - 2 = 0$	
$(\sin\theta - 2)(\sin\theta + 1) = 0$ ✓	
$\therefore \sin\theta = 2$ or $\sin\theta = -1$	
no solution ✓ $\theta = -\dfrac{\pi}{2}$ ✓	(must <u>not</u> just ignore no sol^n case).
(e) $\alpha+\beta = 2$, $\alpha\beta = \dfrac{2}{3}$ ✓ $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = 2 \div \dfrac{2}{3} = 3$ ✓	Well done.

QUESTION 4:

Com /2
Calc /5
Reas /3
HG

COMMENTS

(a) $\sum_{r=1}^{4} r^2 - 1 = 0 + 3 + 8 + 15 = 26$ ✓

· Some students forget to add!

(b) (i) $\int 3\sqrt{x} + x^{-4}\,dx$

· Learn index rules carefully.

$= \frac{3x^{3/2}}{3/2} + \frac{x^{-3}}{-3} + C$ ✓✓+C

further simplification not required
+C = 2nd mark.

$= 2\sqrt{x^3} - \frac{1}{3x^3} + C$

(ii) $\int_0^2 \left(e^{5x} + e^{-5x}\right)^2 dx$

(b) Calc /5

(ii) Poorly done
· Expansion poor
· Many integration errors
eg $\int e^{10x}dx \neq \frac{1}{10}e^{10x} + C$

$= \int_0^2 e^{10x} + 2 + e^{-10x}\,dx$ ✓

$= \left[\frac{1}{10}e^{10x} + 2x + \frac{1}{10}e^{-10x}\right]_0^2$ ✓

$= \left(\frac{1}{10}e^{20} + 4 - \frac{1}{10}e^{-20}\right) - \left(\frac{1}{10} + 0 - \frac{1}{10}\right)$

← must show evidence of substituting x = 0!

$= \frac{1}{10}e^{20} + 4 - \frac{1}{10}e^{-20}$ ✓

(c) LHS : $\frac{\cos\theta\,(2\cos^2\theta - 1)}{\sin\theta\,(\cos^2\theta - \sin^2\theta)}$ ✓

(c) Reas /3
factorise
· Many students gave up after factoring!

$= \frac{\cos\theta\,(2\cos^2\theta - 1)}{\sin\theta\,(\cos^2\theta - (1 - \cos^2\theta))}$ ✓

substitute trig expression.
(Ext students may use $\cos 2\theta$)

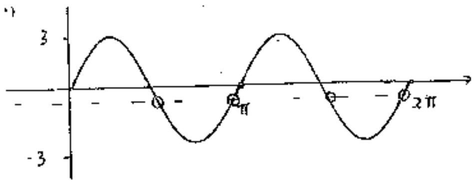
$= \frac{\cos\theta\,\cancel{(2\cos^2\theta - 1)}}{\sin\theta\,\cancel{(2\cos^2\theta - 1)}}$

$= \cot\theta$ ✓

simplify.

$= RHS$

(d) (i)



✓ period
✓ amplitude

(i) Com /2
· Well done!

(ii) ∴ 4 solutions to the equation. ✓

(or cfpe).

QUESTION 5:

Calc /5
Reas /3
MF

COMMENTS

(a) (i) $T = Ce^{-kt}$

(a) Calc /5

$\frac{dT}{dt} = -k \times Ce^{-kt}$ ✓

$= -kT$

(ii) when $t = 0$, $T = 40$

$\therefore 40 = Ce^0$ ✓

$\therefore C = 40$

(iii) when $t = 17$, $T = 24$

$\therefore 24 = 40e^{-k \times 17}$

$\frac{3}{5} = e^{-17k}$ ✓

First mark given for knowing to take logs both sides and using a log law.

$\therefore \ln\left(\frac{3}{5}\right) = -17k$

$\therefore k = -\frac{1}{17}\ln\left(\frac{3}{5}\right)$

$= \frac{1}{17}\ln\left(\frac{3}{5}\right)^{-1}$ ←

The log law used here should really be shown in working. No students did this so the mark was awarded for having the required answer.

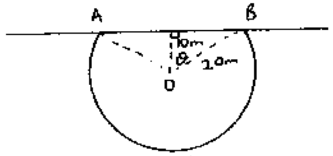
$= \frac{1}{17}\ln\left(\frac{5}{3}\right)$ $(\doteq 0.03)$ ✓

(iv) $T = 40 \times e^{-\frac{1}{17}\ln\left(\frac{5}{3}\right) \times 43}$

$= 10.982\ldots$ ✓

$\doteq 11°$.

(b)



(b) Reas /3

Note Very important
Angle must be in radians.

Required Area
= Circle − Minor Segment
= Circle − (sector − triangle)

$\cos\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore \angle AOB = \frac{2\pi}{3}$ ✓

$\therefore$ Area minor seg $= \frac{1}{2} \times 20^2 \times \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ $(\doteq 245.7)$ ✓

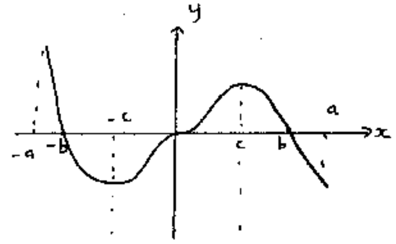
$\therefore$ Area of major seg $= \pi \times 20^2 - 200\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{800\pi}{3} + 100\sqrt{3}$ ✓

$(\doteq 1010.96)$

Question 5 (cont.)	COMMENTS
(c) (i) $P(4,H) = \frac{1}{5}\times\frac{1}{5} = \frac{1}{25}$ ✓	Some students added the fractions. You must multiply the successive events probabilities together.
(ii) $P(\text{odd, vowel}) = \frac{3}{5}\times\frac{1}{5} = \frac{3}{25}$ ✓	
(iii) $P(<3 \text{ or } L) = \frac{2}{5} + \frac{3}{5}\times\frac{2}{5}$ ✓ $= \frac{16}{25}$ ✓	(iii) Reas /2 These events in iii) are <u>not</u> mutually exclusive. They have something in common $P(<3 \text{ or } L) = P(<3) + P(L) - P(<3 \text{ and } L)$ $= \frac{2}{5} + \frac{2}{5} - \frac{2}{5}\times\frac{2}{5}$ $= \frac{10}{25} + \frac{10}{25} - \frac{4}{25}$ $= \frac{16}{25}$

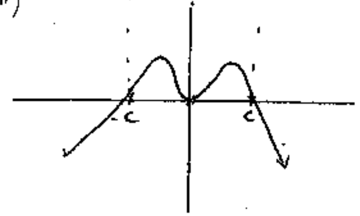
QUESTION 6 Com /3 Calc /4 Reas /3 MF	COMMENTS
(a) (i)	(a) Com /3



✓✓✓ odd fn + correct derivative showing horiz. P.O.I.

✓✓ odd fn + correct derivative with no indication of horiz. P.O.I.

(ii)



✓ int. at ±c

✓ t.p at (0,0)

✓ max. for even fn. + correct derivative.

(b)

(i) 6000, 14000, 22000, ...

$a = 6000$
$d = 8000$ ✓
$n = 10$

$T_{10} = 6000 + 9\times 8000$
$= \$78000$ ✓

Show <u>clear formulae</u>, <u>setting out</u> + working.

(ii) $S_n = \frac{n}{2}\left(12000 + (n-1)\,8000\right)$ (ii) Reas /3

$= 6000n + 4000n^2 - 4000n$

$= 4000n^2 + 2000n$ ✓

But $S_n \geq 1000000$

$4000n^2 + 2000n - 1000000 \geq 0$

$2n^2 + n - 500 \geq 0$ ✓

$\therefore n \leq -16.1$ or $n \geq 15.6$

∴ Must answer 16 questions correctly to exceed one million. ✓

Question 6 (cont.)	COMMENTS
(c) (i) Volume = 32 $\therefore x^2h = 32$ $\therefore h = \frac{32}{x^2}$ $S.A = x^2 + 4xh$ $= x^2 + 4.x.\frac{32}{x^2}$ ✓ $= x^2 + \frac{128}{x}$	(c) Calc /4 Substitution of h must be clearly shown.
(ii) $S = x^2 + 128x^{-1}$ $\frac{dS}{dx} = 2x - 128x^{-2}$ ✓ min SA $\Rightarrow \frac{dS}{dx} = 0$ $2x - \frac{128}{x^2} = 0$ $2x = \frac{128}{x^2}$ $\therefore x^3 = 64$ $\therefore x = 4$ ✓ $\therefore$ Dimensions are $4 \times 4 \times 2$ ✓	Students found it difficult to solve this sort of equation. Must give both x + h values.

QUESTION 7: Com /3, Reas /2, HG	COMMENTS
(a) RHS: $(x+1)(x-1)^2$ $= (x^2-1)(x-1)$ $= x^3 - x^2 - x + 1$ ✓ $=$ LHS {or by factorising LHS}	· Many students tried to fudge it!
(b) (i) $f(x) = \frac{x-1}{x^2}$ $f'(x) = \frac{x^2.1 - 2x.(x-1)}{x^4}$ ✓ $= \frac{x^2 - 2x^2 + 2x}{x^4}$ $= \frac{2x - x^2}{x^4}$ ✓ $= \frac{2-x}{x^3}$	· (i), (ii), (iii) well done by most students.
(ii) Stat pts $\Rightarrow f'(x) = 0$ $\frac{2-x}{x^3} = 0$ $2 - x = 0$ $\therefore x = 2$ ✓	(or by 2^{nd} derivative) $f''(x) = \frac{2x-6}{x^4}$

x	2^-	2	2^+
$f'(x)$	+ve	0	−ve

$\therefore (2, \frac{1}{4})$ is a max t.p. ✓

(iii) $f(x) = 0$

$\frac{x-1}{x^2} = 0$

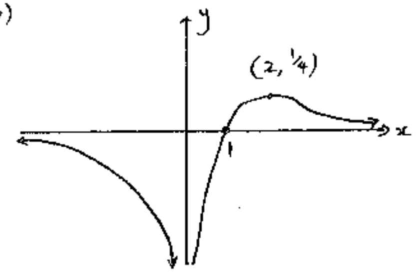
$\therefore x - 1 = 0$

$\therefore x = 1$ ✓

$\therefore (1, 0)$

QUESTION 7 (cont.)	COMMENTS

(iv)



✓ - max
✓ - asymptote $x = 0$
✓ $\lim x \to \pm\infty$

Com /3

· Many students didn't sketch the curve for $x < 0$.

(v) $f'(x) = \frac{2-x}{x^3}$

At $P(1,0)$, $f'(x) = 1$

$\therefore\ y - 0 = 1(x-1)$ ✓

$\therefore\ y = x - 1$

(vi) Tangent meets curve:

$$\begin{cases} y = x - 1 \\ y = \frac{x-1}{x^2} \end{cases}$$

$\therefore\ x - 1 = \frac{x-1}{x^2}$

$\therefore\ x^3 - x^2 = x - 1$ ✓

$x^3 - x^2 - x + 1 = 0$

using (a):

$(x+1)(x-1)^2 = 0$

$\therefore\ x = \pm 1$

$\therefore$ The tangent also meet the curve at $(-1, -2)$ ✓

Reas /2

(vi) Poorly done. many students did not use the hint to use part (a).

QUESTION 8 Calc /6 Com /1 CB	COMMENTS

(a) $\int_1^5 \frac{2}{x(x+1)}\,dx$

$\doteq \frac{1}{3}\left(1 + 4\left(\frac{1}{3} + \frac{1}{10}\right) + 2\left(\frac{1}{6}\right) + \frac{1}{15}\right)$ ✓

$\doteq \frac{1}{3} \times 3\frac{2}{15}$

$\doteq 1.044$ ✓

Several students chose $n=5$ rather than $n=4$ and several confused the formula.

(ii) $\int_1^5 \frac{2}{x(x+1)}\,dx$

$= \int_1^5 \frac{2}{x} - \frac{2}{x+1}\,dx$ ✓

$= \left[2\ln x - 2\ln(x+1)\right]_1^5$

$= 2\ln 5 - 2\ln 6 - \cancel{2\ln 1} + 2\ln 2$ ✓

$= 2\ln\frac{5}{3}$

$\doteq 1.022$ ✓

Generally well done by those who used the appropriate substitution.

(iii) Simpson's rule is an approximation for the integral (using parabolic arcs) ✓ whereas (ii) calculated the exact value of the integral.

Com /1

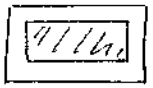
Mostly well understood.

Question 8 (cont.)	COMMENTS
(b) $v = 4 - 2t$	(b) Calc /6
(i) $x = 4t - t^2 + C$ ✓	
when $t = 0$, $x = 1$	
$\therefore 1 = 0 - 0 + C$	
$\therefore C = 1$	
$\therefore x = 4t - t^2 + 1$ ✓	
(ii) Rest $\Rightarrow v = 0$	
$0 = 4 - 2t$	
$\therefore t = 2$ ✓	
$a = -2$ at all time, t. ✓	
(iii) At $t = 0$, $x = 1$	Only a few students were able to manage this question.
$t = 2$, $x = 5$ ✓	
$t = 4$, $x = 1$	
$\therefore$ Distance travelled $= 4 + 4$	
$= 8$ metres ✓	

QUESTION 9 Reas /8 CB	COMMENTS
(a) If $a = 4$ and $S_\infty = \frac{2}{3}$	Students did not generally know the condition for a limiting sum with several incorrectly stating that $\|r\| \le 1$.
then $\frac{2}{3} = \frac{4}{1-r}$	Other students did not interpret/read the question and substituted $r = \frac{2}{3}$ instead of $S_\infty = \frac{2}{3}$
$2 - 2r = 12$	
$\therefore r = -5$ ✓	
But $\|r\| < 1$ for S_∞ to exist.	
$\therefore$ No series exists. ✓	
(b) (i) The parabola must pass through $(2,2)$	Several students failed to recognize that the problem could be solved by mere substitution of a point into the equation
$\therefore 2 = a \times 2(4-2)$ ✓	
$2 = 4a$	
$\therefore a = \frac{1}{2}$	
(ii) A is the amplitude	
$\therefore A = 2$ ✓	
(iii) Parabola: $A = \int_0^4 \frac{1}{2}x(4-x)\,dx$	(iii) Reas /4
$= \int_0^4 2x - \frac{1}{2}x^2\,dx$	Product rule does not apply to integration!
$= \left[x^2 - \frac{1}{6}x^3\right]_0^4$ ✓	
$= 16 - \frac{32}{3}$	
$= \frac{16}{3}$ units^2 ✓	
Sine curve $A = \int_0^4 2\sin\frac{\pi x}{4}\,dx$	
$= \left[-\frac{8}{\pi}\cos\frac{\pi x}{4}\right]_0^4$ ✓	Few students were able to handle this integration
$= -\frac{8}{\pi}\cos\pi + \frac{8}{\pi}\cos 0$	
$= \frac{16}{\pi}$ units^2 ✓	
$\therefore$ Sine curve is smaller & hence cheaper to build	

QUESTION 9 (cont.)	COMMENTS

(c) (i)



Lawn = 1800

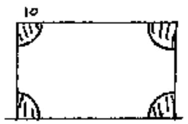
Area = 40 × 10

= 400

$\therefore P = \frac{400}{1800}$

$= \frac{2}{9}$ ✓

(ii)



Lawn = 1800

Area = $\pi . 10^2$

$= 100\pi$ ✓

$\therefore P = \frac{100\pi}{1800}$

$= \frac{\pi}{18}$ ✓

Comments: (c) Reas /4

Well done.

QUESTION 10 Calc /3 Reas /9 HG	COMMENTS

(a) $y = x e^{-2x}$

$\frac{dy}{dx} = e^{-2x} . 1 + x . (-2e^{-2x})$

$= e^{-2x} - 2xe^{-2x}$ ✓

$\frac{d^2y}{dx^2} = -2e^{-2x} - 2(e^{-2x} - 2xe^{-2x})$

$= -4e^{-2x} + 4xe^{-2x}$ ✓

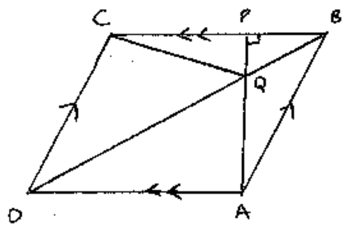
$\therefore \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y$ ✓

$= \cancel{-4e^{-2x}} + 4xe^{-2x} + \cancel{4e^{-2x}} - 8xe^{-2x} + 4xe^{-2x}$

$= 0$

Comments: (a) Calc /3   • Many students forgot to do the product rule!

(b)



(i) ∠APB = ∠CDB (diagonals of a rhombus bisect the angles they pass through). ✓

(ii) ∠ADB = ∠CDB (as above)

DQ is common ✓

CD = AD (given)

$\therefore \triangle AQD \equiv \triangle CQD$ (SAS) ✓

(iii) ∠QAD = 90° (alt ∠ = as CB ∥ AD)

$\therefore$ ∠QCD = 90° (corr. ∠ in ≡ Δ are =) ✓

Comments: (b) Reas /4

• Reasons – poor.

QUESTION 10 (cont.)	COMMENTS
(c) (i) $A_1 = 500000(1.01) - M$ $A_2 = A_1 \times 1.01 - M$ $= 500,000(1.01)^2 - M(1.01) - M$ $A_3 = A_2 \times 1.01 - M$ $= 500000(1.01)^3 - M(1.01)^2 - M(1.01) - M$ $\vdots$ $\therefore A_n = 500000(1.01)^n - M\left[1.01^{n-1} + 1.01^{n-2} + \ldots + 1\right]$ ✓ GP $a = 1$ $r = 1.01$ $n = n$ $\therefore S_n = \dfrac{1((1.01)^n - 1)}{1.01 - 1}$ $= \dfrac{1.01^n - 1}{0.01}$ ✓ $\therefore A_n = 500000(1.01)^n - M \times \left(\dfrac{1.01^n - 1}{0.01}\right)$ $= 500000(1.01)^n - 100M(1.01^n - 1)$ ✓ $= 100M - 1.01^n(100M - 500000)$	~~Less~~ /5 · Many attempt at fudging - not very successful!
(ii) $250000 = 100 \times 5505 - 1.01^n(100 \times 5505 - 500000)$ $250000 = 550500 - 1.01^n(50500)$ $\therefore 1.01^n = \dfrac{550500 - 250000}{50500}$ ✓ $\doteq 5.95$ $\therefore n = \dfrac{\ln 5.95}{\ln 1.01}$ $\doteq 179.23$ months ✓ $\therefore$ In the 15^{th} year.	· Many students still got 2 marks in (ii) despite being stuck in (i).