SCEGGS Darlinghurst

2004
Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

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Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question on a NEW page

Marks

Question 1 (12 marks)

(a) Express $\frac{5\pi}{18}$ radians in degrees. **1**

(b) The line $2x + ky - 5 = 0$ passes through the point $(3, -1)$ **2**
Find the value of *k*.

(c) Express $\frac{6}{2 - \sqrt{3}}$ in the form $a + b\sqrt{3}$. **2**

(d) Factorise fully: **2**

$$a^2 - b^2 - 2a + 2b$$

(e) Solve the inequality and graph your solution on a number line. **2**

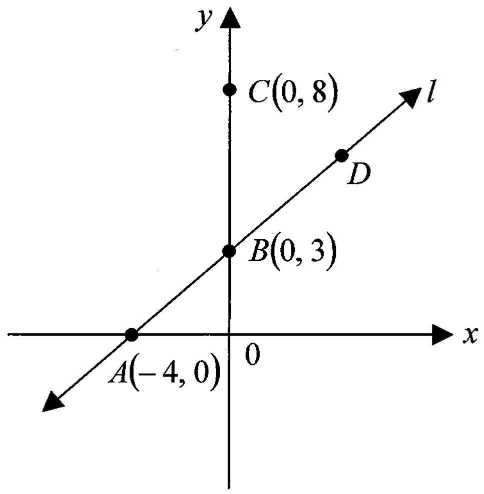
$$6 - \frac{x}{3} > 7$$

(f) If 7 apples and 2 oranges cost \$4, while 5 apples and 4 oranges cost \$4.40 find the cost of each apple and orange. **3**

Marks

Question 2 (12 marks) START A NEW PAGE

The line l has intercepts at $A(-4, 0)$ and $B(0, 3)$. D is a point on line l and C has co-ordinates $(0, 8)$.



Not to scale

(a) Show that the equation of line l is $3x - 4y + 12 = 0$. **2**

(b) Find the lengths of AB and BC. **2**

(c) What type of triangle is ΔABC? Give a clear reason. **1**

(d) If B is the midpoint of AD, find the co-ordinates of D. **2**

(e) Write down the equation of the circle through A, C and D with centre B. **1**

(f) Find the shortest distance from point C to the line l. **2**

(g) Hence, or otherwise, find the area of ΔABC. **2**

Marks

Question 3 (12 marks) START A NEW PAGE

(a) Differentiate:

(i) $y = (2-3x)^4$ **1**

(ii) $f(x) = \dfrac{e^x}{x+1}$ **2**

(b) Find:

(i) $\int \cos 2x \; dx$. **2**

(ii) $\displaystyle\int_1^2 \frac{2x^2+1}{x^2}\, dx$ **2**

(c) The roots of $2x^2 + 6x - 7 = 0$ are α and β.

Find:

(i) $\alpha + \beta$ **1**

(ii) $\dfrac{1}{\alpha^2 + \beta^2}$ **2**

(d) Find the value of k if **2**

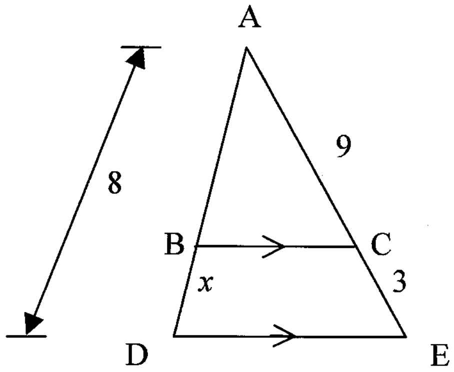
$$\int_1^k \frac{dx}{2x-1} = \log_e 3$$

Marks

Question 4 (12 marks) START A NEW PAGE

(a) Find the equation of the normal to the curve $y = \sqrt{1-x}$ at the point $x = -3$. **3**

(b) **2**



Not to scale

Find the value of x, giving reasons.

(c) (i) Find the locus of the point $P(x, y)$ which moves so that its distance from point $A(4, 0)$ is always twice its distance from point $B(1, 0)$, ie $PA = 2PB$. **3**

(ii) Describe this locus. **1**

(d) Find the exact value of the area bounded by the curve $y = x^2 - 7x + 10$, the x-axis and the lines $x = 0$ and $x = 3$. **3**

Marks

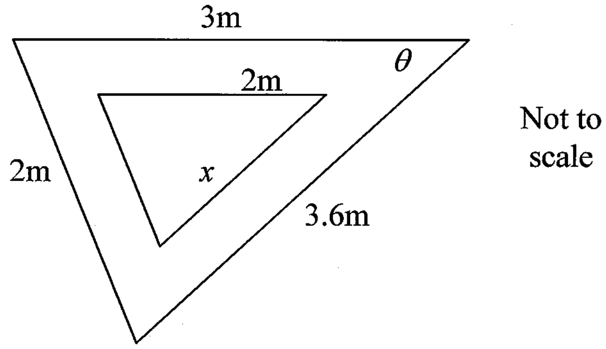
Question 5 (12 marks) START A NEW PAGE

(a) Solve for x: $3^{2x} - 10.3^x + 9 = 0$. **2**

(b) Two dates are selected at random for a school dance. One date is taken at random from those in October and another is taken from November. **1**

What is the probability that both are the twentieth of the month?

(c)



The diagram shows a small triangular garden bed surrounded by a concrete path. The sides of the path are parallel to the sides of the garden and the two triangles are similar.

(i) Find the length of x. **1**

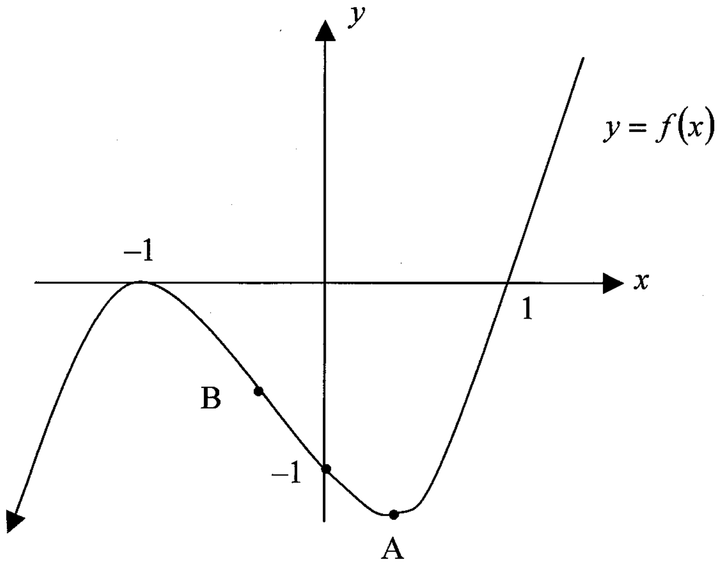
(ii) Prove that the angle θ is $34°$ correct to the nearest degree. **1**

(iii) Hence find the area of the concrete path correct to 2 significant figures. **2**

Question 5 continues on page 7

Question 5 (continued)

(d)



The curve is $y = f(x)$ where $f(x) = x^3 + x^2 - x - 1$.

It has a maximum stationary point at $(-1, 0)$ and a minimum stationary point at A.
It has an inflexion point at B.

(i) Find the co-ordinates of the point A. **2**

(ii) Show that the co-ordinates of B are $\left(-\frac{1}{3}, -\frac{16}{27}\right)$ **2**

(iii) Copy the sketch of $y = f(x)$ and, on the same diagram, sketch $y = f'(x)$. **1**

Marks

Question 6 (12 marks) START A NEW PAGE

(a) Calvin and Hobbes are making a large snowman. To collect the snow Calvin walks 1m away from the snowman, fills a bucket with snow and takes it back to the snowman. Then he walks 1.5m away, fills the bucket and returns. Each time he goes to fill the bucket he walks 0.5m further away from the snowman and returns.

(i) How far from the snowman does Calvin go to fill the 5th bucket? **1**

(ii) In total he uses 20 buckets of snow. Find the total distance Calvin walks collecting all of the snow. **2**

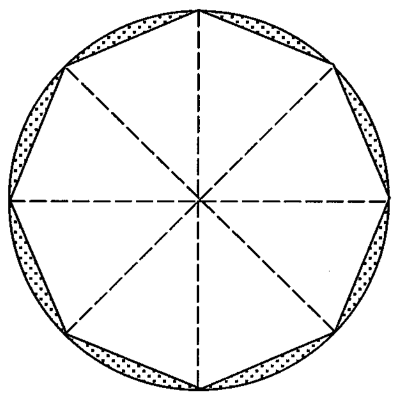
(b) Consider the geometric series

$$m + 2m^2 + 4m^3 + 8m^4 \ldots$$

(i) For which values of m will the series have a limiting sum? **1**

(ii) If $m = \frac{1}{4}$ find the limiting sum. **2**

(c) **3**



Not to scale

A table top is made in the shape of a regular octagon by removing the shaded segments from a circular piece of wood as shown.
The radius of the original circle was 1.5m.
Prove that the total area of the shaded segments is:

$$9\frac{\left(\pi - 2\sqrt{2}\right)}{4}\text{m}^2$$

Question 6 continues on page 9

Marks

Question 6 (continued)

(d) For the equation $4x^2 + (m-4)x - m = 0$:

(i) write an expression for the discriminant in fully factorised form. **2**

(ii) Explain why the equation has rational roots for all rational values of m. **1**

Marks

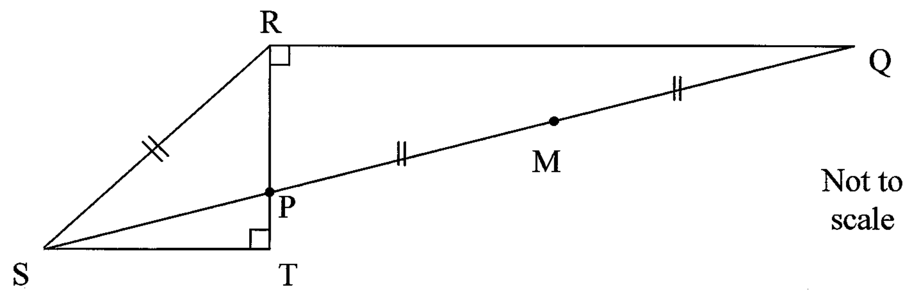
Question 7 (12 marks) START A NEW PAGE

(a) (i) Sketch the curve $y = 2\sin x - 1$ in the domain $0 \le x \le 2\pi$, showing all important features. **2**

(ii) Shade the region for which $y \le 2\sin x - 1$ and $y \ge 0$ in this domain. **1**

(iii) Find the exact value of the area of this shaded region. **3**

(b)



The triangles PRQ and RTS are right angled triangles and PM = MQ = SR as shown in the diagram.

Copy the diagram onto your answer page.

(i) Explain why $\angle$PST = $\angle$PQR. **1**

(ii) The point A is positioned so that PRQA is a rectangle. Label point A on your diagram. **1**

(iii) Explain why M is the midpoint of RA. **1**

(iv) Explain why PM = RM. **1**

(v) Hence prove that $\angle$PSR = 2 × $\angle$PST. **2**

Marks

Question 8 (12 marks) START A NEW PAGE

(a) (i) Copy and complete the table given for the function $y = x + \log_e x$. Values may be calculated correct to 2 decimal places. **1**

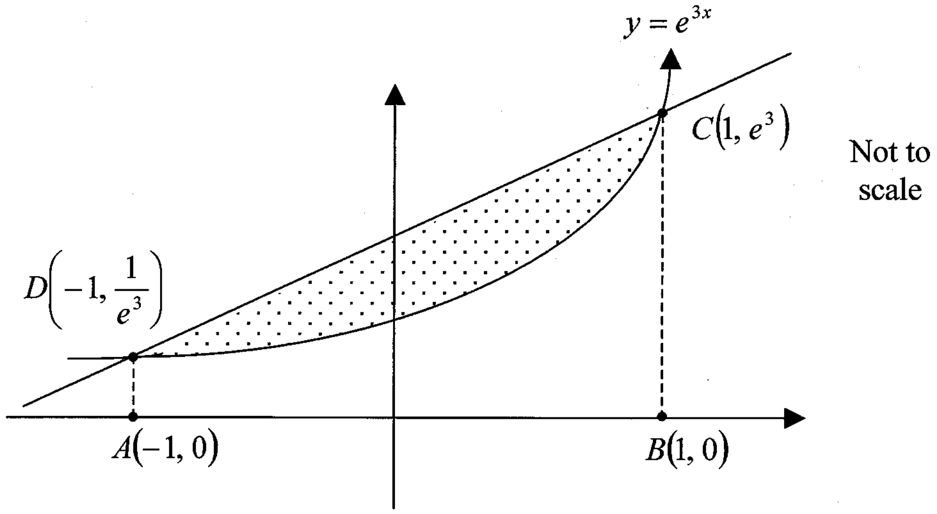
x	1	1.5	2	2.5	3
y	1				4.10

(ii) Hence find an approximation to $\int_1^3 x + \log_e x \, dx$ using Simpson's Rule with 5 function values. **2**

(b) Find the co-ordinates of the focus of the parabola **2**

$$x^2 - 8x - 12y - 8 = 0$$

(c)



In the diagram above, a straight line has been drawn intersecting $y = e^{3x}$ at the points $C(1, e^3)$ and $D\left(-1, \frac{1}{e^3}\right)$. A has co-ordinates $(-1, 0)$ and B has co-ordinates $(1, 0)$. The area between the line and the curve is shaded.

(i) Show that the area of the trapezium $ABCD$ is given by $\frac{e^6 + 1}{e^3}$. **1**

(ii) Hence, or otherwise, find the exact value of the shaded area. **3**

(d) If $\tan\theta = \frac{1}{\sqrt{2}}$ and $180° \le \theta \le 270°$, prove that $\frac{1}{\cot\theta - \operatorname{cosec}\theta} = \sqrt{3} - \sqrt{2}$. **3**

Marks

Question 9 (12 marks) START A NEW PAGE

(a) In a lottery, the probability of the jackpot prize being won in any draw is $\frac{1}{60}$.

(i) What is the probability that the jackpot prize will be won in each of four consecutive draws? **1**

(ii) How many consecutive draws need to be made for there to be a greater than 98% chance that at least one jackpot prize will have been won? **3**

(b) A bowl is formed by rotating the region bounded by the curve $y = x^3$, the line $x = 3$ and the x-axis about the y-axis. **4**

Find the volume of the bowl, giving your answer correct to 3 significant figures.

(c) On 1st January, 1994, the population of Baileyville was 25000 and the population of Brimfield Park was 12500. Fabio noticed that the population of the towns t years later could be found using the following equations:

Baileyville: $\quad \mathrm{A} = 25000e^{-pt}$

Brimfield Park: $\quad \mathrm{B} = 12500e^{qt}$

(i) Ten years later, on 1st January, 2004, the population of Baileyville and Brimfield Park are 20000 and 15000 respectively. **2**
Find the exact values of p and q.

(ii) In what year will the populations of Baileyville and Brimfield Park be equal? **2**

Marks

Question 10 (12 marks) START A NEW PAGE

(a) Every year, Beryl deposits \$1500 into a superannuation fund that earns 4% p.a. interest, compounded annually. How much will Beryl have in her superannuation fund after 20 years? **2**

(b) The velocity of a particle in metres per second, moving in a straight line is given by the equation:

$$\dot{x} = 1 - \frac{2}{t+3}.$$

The particle is initially at the origin.

(i) Prove that the particle is never at rest. **1**

(ii) Find an expression for the displacement after time t seconds. **2**

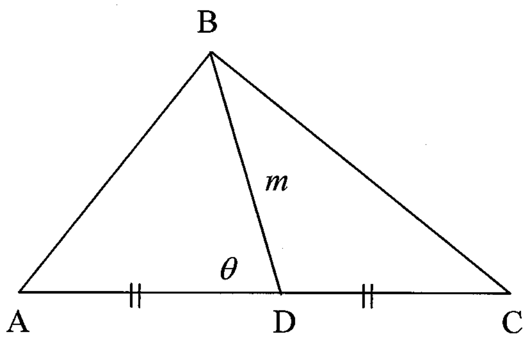
(iii) Find the distance travelled in the first second. **1**

Question 10 continues on page 14

Marks

Question 10 (continued)

(c) ABC is a triangle and D is the midpoint of AC.



Also BC = a, AC = b, AB = c and BD = m.

(i) Simplify $\cos(180-\theta)$ **1**

(ii) Show that $\cos\theta = \dfrac{4m^2 + b^2 - 4c^2}{4mb}$. **2**

(iii) Hence show that: **3**

$$a^2 + c^2 = 2m^2 + \frac{1}{2}b^2$$

End of Paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Trial

Question	Comm	Reas	Calc
1	–	–	–
2	c) (1)	g) (2)	
3		d) (2)	a) b) (7)
4	c (ii) (1)	b) (2)	d) (3)
5		c) (4)	d) (5)
6	d)(ii) (1)	c) (3)	
7	a) (i) (2) b) (i)(ii)(iii) (iv) (4)	b) (v) (2)	
8		d) (3)	
9		c) (ii) (2)	b) (4)
10	b) (i) 1	c) (ii) (iii) 5	b) (ii) (iii) (3)
	~~9~~ 10.	25	22

2004.

1) a) $\frac{5\pi}{18} = \frac{5 \times 180}{18}$ ✓

$= 50^\circ$

b) $2 \times 3 - k - 5 = 0$ ✓

$-k + 1 = 0$

$k = 1$ ✓

c) $\frac{6}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 12 + 6\sqrt{3}$ ✓✓

d) $(a-b)(a+b) - 2(a-b)$ ✓

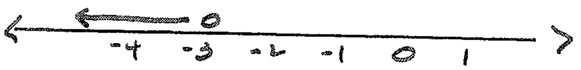
$(a-b)(a+b-2)$ ✓

e) $6 - \frac{x}{3} > 7$

$18 - x > 21$

$-x > 3$

$x < -3$ ✓



✓

f) $7A + 2G = 4$ ✓

$5A + 4G = 4.4$

$14A + 4G = 8$

$9A = 3.6$

$A = 0.4$ ✓

$2.8 + 2G = 4$

$G = 0.6$

$\therefore$ apples cost 40 cents

oranges cost 60 cents. ✓

(2) a) gradient $= \frac{3}{4}$ ✓

equation : $y - 3 = \frac{3}{4}x$

$4y - 12 = 3x$

$3x - 4y + 12 = 0$ ✓

b) $AB^2 = 4^2 + 3^2$

$AB = 5u$ ✓

$BC = 5u$ ✓

c) Isosceles since 2 sides are equal ✓ (Comm)

d) $0 = \frac{a-4}{2}$

$a = 4$ ✓

$3 = \frac{b+0}{2}$

$b = 6$ ✓

$\therefore$ D is $(4, 6)$

e) $x^2 + (y-3)^2 = 25$ ✓

f) $d = \left| \frac{0 \times 3 - 4 \times 8 + 12}{\sqrt{3^2 + 4^2}} \right|$ ✓

$= \left| \frac{-32 + 12}{5} \right|$

dist $= 4u.$ ✓

g) Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 4$ ✓

✓

$= 10u^2.$ (Reas)

Comm /1 Reas /2.

③ a) (i) $y' = -12(2-3x)^3$ (Calc) | Well done.

(ii) $f'(x) = \dfrac{(x+1)e^x - e^x}{(x+1)^2}$ (Calc) | Use Quotient Rule and make sure it is learnt correctly.

$= \dfrac{xe^x}{(x+1)^2}$

b) (i) $\int \cos 2x \, dx = \frac{1}{2}\sin 2x + c$ (Calc) | Use Standard Integral Sheet.

(ii) $\int_1^2 \dfrac{2x^2+1}{x^2} \, dx = \int_1^2 2 + x^{-2} . dx$ | Divide First.

$= \left[2x - \frac{1}{x}\right]_1^2$

$= (4 - \frac{1}{2}) - (2 - 1)$

$2\frac{1}{2}$ (Calc)

c) (i) $\alpha + \beta = -\frac{6}{2} = -3$ | Well done.

(ii) $\alpha\beta = -\frac{7}{2}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ | This was not well known.

$= 9 + 7$

$= 16$

$\therefore \dfrac{1}{\alpha^2+\beta^2} = \dfrac{1}{16}$

d) $\int_1^k \dfrac{dx}{2x-1} = \frac{1}{2}\left[\log_e(2x-1)\right]_1^k$ | Log Integration needs more work.

$= \frac{1}{2}\left[\log_e(2k-1) - \log_e 1\right]$

$= \frac{1}{2}\log_e(2k-1)$

$\therefore \log_e(2k-1) = 2\log_e 3$

$2k - 1 = 9$ | Log Law: $2\log_e 3 = \log_e 3^2$

$k = 5$ (Reas)

④ a) $y = (1-x)^{\frac{1}{2}}$

$y' = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$

$= \dfrac{-1}{2\sqrt{1-x}}$

if $x=-3$, $y' = \dfrac{-1}{2\times\sqrt{4}} = -\frac{1}{4}$

$y = \sqrt{4} = 2$

$\therefore$ normal: $y-2 = 4(x+3)$

$4x - y + 14 = 0$

b) $\dfrac{x}{8-x} = \dfrac{3}{9}$ (‖ lines cut off proportional intercepts on trans.)

$9x = 24 - 3x$

$12x = 24$ (Reas)

$x = 2.$

c) (i) $PA^2 = (x-4)^2 + y^2$

$PB^2 = (x-1)^2 + y^2.$

$PA^2 = 4PB^2$

$x^2 - 8x + 16 + y^2 = 4(x^2 - 2x + 1 + y^2)$

$= 4x^2 - 8x + 4 + 4y^2.$

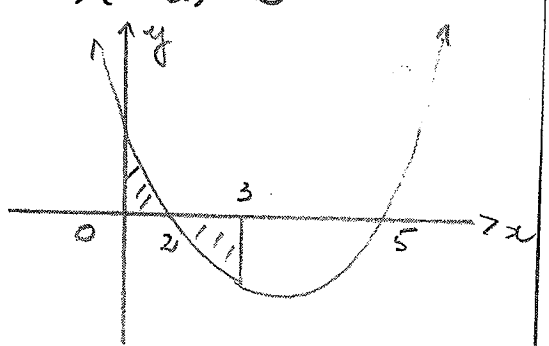
$\therefore 3x^2 + 3y^2 = 12$

$x^2 + y^2 = 4$

(ii) circle centre (0,0) radius 2 u (Comm)

d) $x^2 - 7x + 10 = 0$ (Calc)

$(x-5)(x-2) = 0$



$$\text{Area}_1 = \int_0^2 x^2 - 7x + 10 \, dx$$

$$= \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x \right]_0^2$$

$$= \frac{8}{3} - 14 + 20$$

$$= 8\frac{2}{3} u^2$$

$$\text{Area}_2 = \left| \int_2^3 x^2 - 7x + 10 \, dx \right|$$

$$= \left| \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x \right]_2^3 \right|$$

$$= \left| \left(9 - \frac{63}{2} + 30\right) - \left(\frac{8}{3} - 14 + 20\right) \right|$$

$$= 1\frac{1}{6} u^2$$

$$\therefore \text{ Area required } = 9\frac{5}{6} u^2$$

(5) a) Let $y = 3^x$

$y^2 - 10y + 9 = 0$

$(y-9)(y-1) = 0$

$y = 9$ or 1

$\therefore x = 2$ or 0

not well known.

$3^x = 9$ $\quad$ $3^x = 1$

$\therefore x = 2$ $\quad$ $\therefore x = 0$

errors here more care!

b) $\text{Prob} = \frac{1}{30} \times \frac{1}{31} = \frac{1}{930}$

Well done.

c) (i) $\frac{x}{3 \cdot 6} = \frac{2}{3}$

$x = 2 \cdot 4$ m

(ii) $\cos\theta = \frac{3^2 + 3 \cdot 6^2 - 2^2}{2 \times 3 \times 3 \cdot 6}$

$= 0 \cdot 83148 \ldots$ (Reas)

$\theta = 33 \cdot 7 \ldots$

$= 34^\circ$ (nearest degree)

(iii) $\text{Area}_1 = \frac{1}{2} \times 2 \times 2 \cdot 4 \times \sin 34^\circ$

$= 1 \cdot 342 \ldots$

$\text{Area}_2 = \frac{1}{2} \times 3 \times 3 \cdot 6 \times \sin 34^\circ$

$= 3 \cdot 0196$

$\therefore$ Area path $= 1 \cdot 7 \text{ m}^2$ (2 s.f.)

Many calculation errors. Show all working for marks to be allocated.

d) (i) $y = x^3 + x^2 - x - 1$

$y' = 3x^2 + 2x - 1$

$= (3x - 1)(x + 1)$

if $y' = 0$, $x = \frac{1}{3}$ or -1

if $x = \frac{1}{3}$, $y = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} - 1$

$= -1\frac{5}{27}$ (calc)

A is $\left(\frac{1}{3}, -1\frac{5}{27}\right)$

This question was well done by most.

(ii) $y'' = 6x + 2$

if $y'' = 0$ $x = -\frac{1}{3}$

$y = \left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)^2 + \frac{1}{3} - 1$

$= -\frac{16}{27}$

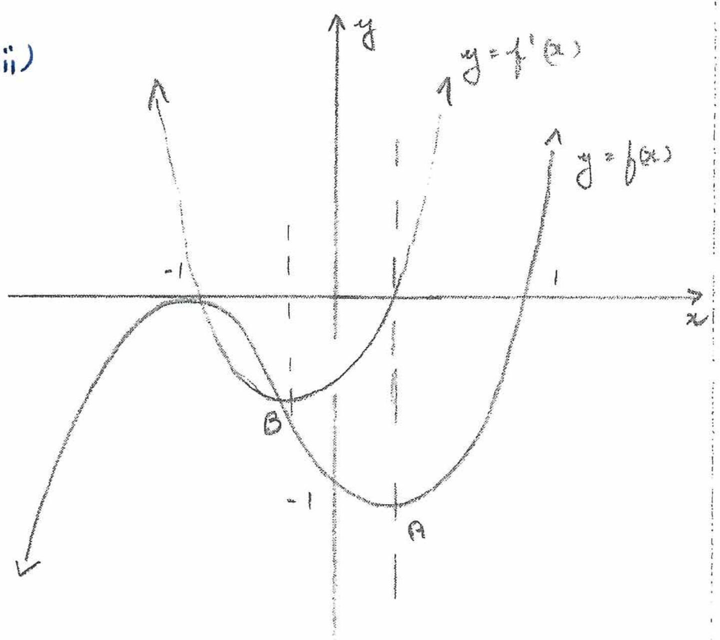
if $x < -\frac{1}{3}$, $y'' < 0$

$x > -\frac{1}{3}$, $y'' > 0$

since concavity has changed

$(-\frac{1}{3}, -\frac{16}{27})$ is inflexion pt B.

(iii)



Two sketches on the same axes.

The minimum point of $y = f'(x)$ should line up with B.

6

(6) a) (i) $T_1 = 1$, $T_2 = 1\cdot5$, $T_3 = 2$

A.P., $a = 1$, $d = 0\cdot5$

$T_5 = a + 4d$

$= 1 + 2$

distance $= 3m$

(ii) $D = \frac{20}{2} \times 2(2 + 19 \times 0\cdot5)$

distance $= 230m$

b) (i) $r = 2m$

$\therefore -1 < 2m < 1$

$-\frac{1}{2} < m < \frac{1}{2}$ for limiting sum

(ii) $S = \dfrac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$

c) Area one triangle

$= \frac{1}{2} \times \frac{3}{2} \times \frac{3}{2} \times \sin\frac{\pi}{4}$

$= \dfrac{9}{8\sqrt{2}}\ m^2$

Area eight triangles $= \dfrac{9}{\sqrt{2}}\ m^2$

$= \dfrac{9\sqrt{2}}{2}\ m^2.$

Area circle $= \pi \times \frac{3}{2} \times \frac{3}{2}$

$= \dfrac{9\pi}{4}\ m^2.$

$\therefore$ Shaded area $= \dfrac{9\pi}{4} - \dfrac{9\sqrt{2}}{2}$

$= \dfrac{9}{4}(\pi - 2\sqrt{2})\ m^2.$

(Reas)

d) (i) $\Delta = (m-4)^2 + 16m$

$= m^2 - 8m + 16 + 16m$

$= m^2 + 8m + 16$

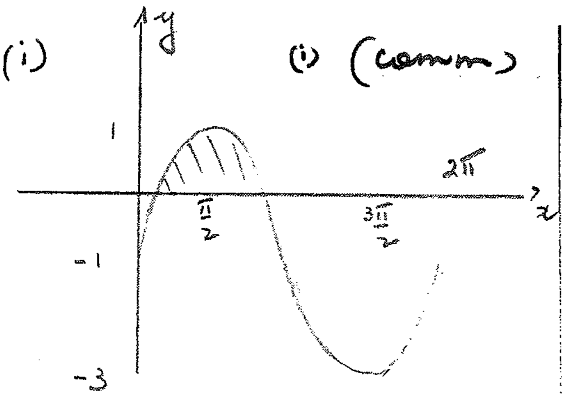
$= (m+4)^2.$

(ii) for the roots to be rational, Δ must be a perfect square.

$(m+4)^2$ is a perfect square provided m is rational. (comm)

(7) a)(i)

(i) (comm)



Graphs very poor. Confusion with $y = \sin 2x - 1$ etc.
Must show maximum, minimum values and where curve crosses x axis.

(ii) Very poor shading.

$$\text{Area} = \int_a^b 2\sin x - 1 \,.\, dx$$

if $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\therefore \text{Area} = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin x - 1 \,.\, dx$$

$$= \left[-2\cos x - x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= -2\cos\frac{5\pi}{6} - \frac{5\pi}{6} + 2\cos\frac{\pi}{6} + \frac{\pi}{6}$$

$$= 2\times\frac{\sqrt{3}}{2} - \frac{5\pi}{6} + 2\times\frac{\sqrt{3}}{2} + \frac{\pi}{6}$$

$$= 2\sqrt{3} - \frac{2\pi}{3} \ u^2 \qquad 3.$$

(iii) Integration usually correct. Many students found the evaluation of $\cos\frac{5\pi}{6}$ etc difficult.

b) (i) $\angle RTS = \angle TRQ$ (given)

$\therefore RQ \parallel ST$ (alt. $\angle$'s $=$)

$\therefore \angle PST = \angle PQR$ (alt $\angle$'s $=$ $RQ \parallel ST$)

(iii) M is midpoint PQ (given)

$\therefore$ M is also midpoint RA
(diagonals of a rectangle bisect each other)

(iv) RA = PQ (diagonals of a rectangle are equal)

comm (i) → (iv)

(i) RQ was not given parallel to ST. This must be proved

(ii) to (iv) well done.

$\therefore PM = RM$ (M is midpoint of equal intervals)

(iv) Let $\angle MRP = \alpha$ (Reas)

$PM = RM$ (proved in iii)

$\therefore \angle MRP = \angle RPM = \alpha$ (opp equal sides in Δ MRP)

$\therefore \angle RMP = 180^\circ - 2\alpha$ (angle sum of Δ RPM $= 180^\circ$)

$\therefore RS = RM$ ($RS = PM = RM$)

$\therefore \angle RMS = \angle PSR$ (opp equal sides in Δ RSM)

$\therefore \angle PSR = 180^\circ - 2\alpha$.

But $\angle SPT = \angle MPR = \alpha$ (vert. opp $\angle$'s $=$)

$\therefore \angle PST = 180^\circ - 90^\circ - \alpha$ (angle sum of Δ 180°)

$= 90^\circ - \alpha$.

$\therefore \angle PSR = 2\angle PST$.

(v) Well done to the students who succeeded in this question.

(8) a)(i)

x	1	1.5	2	2.5	3
y	1	1.91	2.69	3.42	4.10

(ii) $\int_1^3 x + \log_e x \, dx$

$\doteqdot \frac{0.5}{3}\left[1 + 4.1 + 4(1.91 + 3.42) + 2 \times 2.69\right]$

$\doteqdot 5.3$

b) $12y + 8 = x^2 - 8x$

$12y + 8 + 16 = x^2 - 8x + 16$

$12(y+2) = (x-4)^2.$

vertex $(4, -2)$

focal length 3 units.

focus $(4, 1)$

c) (i) Area $= \frac{1}{2}.AB.(AD + BC)$

$= \frac{1}{2} \times 2 \times \left(\frac{1}{e^3} + e^3\right)$

Area $= \frac{e^6 + 1}{e^3} u^2$

(ii) Area under curve

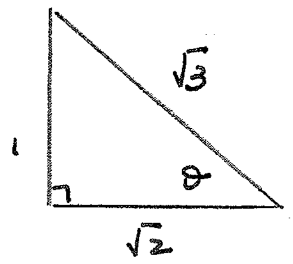
$= \int_{-1}^{1} e^{3x} dx$

$= \frac{1}{3}\left[e^{3x}\right]_{-1}^{1}$

$= \frac{1}{3}(e^3 - e^{-3}) u^2$

Area $= \frac{1}{e^3} + e^3 - \frac{e^3}{3} + \frac{1}{3e^3}$

$= \frac{2}{3}e^3 + \frac{4}{3e^3} \quad u^2$

d) 

(Reas)

θ in 3rd quadrant.

$\cot\theta = \sqrt{2}$

$\sin\theta = -\dfrac{1}{\sqrt{3}}$

$\therefore \operatorname{cosec}\theta = -\sqrt{3}.$

$$\frac{1}{\cot\theta - \operatorname{cosec}\theta} = \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{2-3}$$

$$= \sqrt{3}-\sqrt{2}$$

9) a) (i) $P(4 \text{ wins}) = \left(\frac{1}{60}\right)^4$

(ii) P(at least 1 win)

$= 1 - P(\text{no wins})$

Let no. of draws be n.

$\therefore 1 - \left(\frac{59}{60}\right)^n > 0.98$

$-\left(\frac{59}{60}\right)^n > -0.02$

$\left(\frac{59}{60}\right)^n < 0.02$

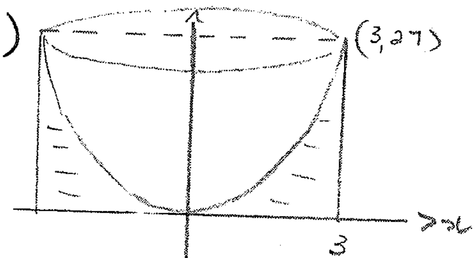
$n \log_e \frac{59}{60} < \log_e 0.02$

$n > \dfrac{\log_e 0.02}{\log_e \frac{59}{60}}$

n is 233 draws.

(ii) Be careful of inequality signs.

b)



(calc)

$V_1 = \pi \int_0^{27} x^2 \, dy$

$= \pi \int_0^{27} y^{2/3} \, dy$

$= \pi \left[\frac{3}{5} y^{5/3} \right]_0^{27}$

$= \pi \times \frac{3}{5} \times 243$

$= \frac{729\pi}{5} u^3$

$= (458 \text{ (3 s.f.)})$

b) Poor execution.

- most students found the incorrect volume.
- did not find the y value (27)
- did not find x^2 correctly
- forgot π.

$V_2 = \pi R^2 H$

$= \pi \times 9 \times 27$

$= 243\pi$

$\therefore$ Volume of bowl

$= \left(243 - \frac{729}{5}\right)\pi \; u^3$

$= \underline{48\frac{3}{5}\pi} \; u^3$

$= 305 \; u^3$

e) (i) $A = 25000 e^{-pt}$

$20000 = 25000 e^{-10p}$

$e^{-10p} = \frac{4}{5}$

$p = \frac{-1}{10} \log_e \frac{4}{5}$

$B = 12500 e^{qt}$

$15000 = 12500 e^{10q}$

$\therefore e^{10q} = \frac{6}{5}$

$q = \frac{1}{10} \log_e \frac{6}{5}$

i) $25000 e^{-pt} = 12500 e^{qt}$

$2 e^{-pt} = e^{qt}$

$\log_e 2e^{-pt} = \log_e e^{qt}$

$\log_e 2 + \log_e e^{-pt} = \log_e e^{qt}$

$\log_e 2 - pt = qt$

$pt + qt = \log_e 2.$

$t = \frac{\log_e 2}{p+q}$ (Reas)

$= \frac{\log_e 2}{0.04054651}$

$= 17.095\ldots$ years

in 201 equal populations.

well done.

Difficult question.

Must take logs of both sides carefully.

10) a) First \$1500 becomes

$1500(1.04)^{20}$

Second \$1500 becomes $1500(1.04)^{19}$

Last \$1500 becomes $1500(1.04)$

Total :

$1500\left[1.04 + 1.04^2 + \cdots + 1.04^{20}\right]$

$= 1500 \times 1.04 \left(\dfrac{1.04^{20} - 1}{1.04 - 1}\right)$

$= \$46453.80$ (nearest cent)

b) (i) if $\dot{x} = 0$ at rest.

$\therefore \dfrac{2}{t+3} = 1$

$2 = t + 3$ (Comm)

$t = -1$

since $t \geq 0$, never at rest.

(ii) $x = t - 2\log_e(t+3) + c$

if $x = 0$, $t = 0$

$0 = 0 - 2\log_e 3 + c$

(Calc)

$c = 2\log_e 3$

$\therefore x = t - 2\log_e(t+3) + 2\log_e 3.$

(iii) distance travelled

$= 1 - 2\log_e 4 + 2\log_e 3.$ m

(Calc)

c) (i) $\cos(180° - \theta) = -\cos\theta$

(ii) $\cos\theta = \dfrac{BD^2 + AD^2 - AB^2}{2\,BD.AD}$ (Cosine Rule in ΔBAD)

$= \dfrac{m^2 + \frac{b^2}{4} - c^2}{2 \times m \times \frac{b}{2}} \times \dfrac{4}{4}$

$= \dfrac{4m^2 + b^2 - 4c^2}{4mb.}$ (Reas)

(iii) Using Cosine Rule in ΔBDC

$$\cos(180^\circ-\theta) = \frac{BD^2+DC^2-BC^2}{2\times BD\times DC}$$

$$-\cos\theta = \frac{m^2+\frac{b^2}{4}-a^2}{2\times m\times \frac{b}{2}} \times \frac{4}{4}$$

$$= \frac{4m^2+b^2-4a^2}{4mb}$$

$$\therefore \frac{4m^2+b^2-4c^2}{4mb} = \frac{4a^2-b^2-4m^2}{4mb}$$

$$\therefore 4m^2+b^2-4c^2 = 4a^2-b^2-4m^2$$

$$\therefore 4a^2+4c^2 = 8m^2+2b^2$$

$$a^2+c^2 = 2m^2+\frac{b^2}{2}$$

(Reas)