THE SCOTS COLLEGE
Sydney

2004
TRIAL H.S.C.
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value
- Start a new booklet for every question

© The Scots College, Bellevue Hill

STUDENTS ARE ADVISED THAT THIS IS A TRIAL EXAMINATION ONLY AND CANNOT IN ANY WAY GUARANTEE THE CONTENT OR THE FORMAT OF THE HIGHER SCHOOL CERTIFICATE EXAMINATION.

QUESTION ONE **[12 MARKS]**

a. Evaluate correct to 1 decimal place $\dfrac{m^2+n^2}{mn}$, where $m=-4.3$ and $n=2.1$ **2**

b. A television is bought at a 20% discount sale for \$760. Calculate the original price of the television. **2**

c. Find the primitive function for $5-\sqrt{x}$ **2**

d. Solve $|x-7|>2$ **2**

e. Evaluate $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x-3}$ **2**

f. Simplify fully $\dfrac{m^2}{m^2+3m+2}-\dfrac{2m}{m+2}$ **2**

QUESTION TWO [12 MARKS]

a. Let α and β be the roots of the equation $x^2 - 7x + 3 = 0$. Calculate:

i. $\alpha + \beta$ **1**

ii. $\alpha\beta$ **1**

iii. $\alpha^2 + \beta^2$ **1**

iv. $\frac{1}{\alpha} + \frac{1}{\beta}$ **1**

b. P is the point (4,-8) and l is the line with equation $5x - 12y + 53 = 0$.

i. Calculate the gradient of the line l and the angle that it makes with the x axis. (Answer to the nearest degree.) **2**

ii. Show that the perpendicular to l through P has equation $12x + 5y - 8 = 0$. **2**

iii. Find the point of intersection, Q, of this perpendicular line with the line l. **2**

iv. Find the distance PQ. **2**

QUESTION THREE [12 MARKS]

a. Differentiate with respect to x:

i. $5\sin^2 x$ **2**

ii. $\ln(4x^3 + 3x)$ **2**

iii. $e^{(8x^3 - 5x)}$ **2**

b. Find:

i. $\int (3x+4)^5\, dx$ **1**

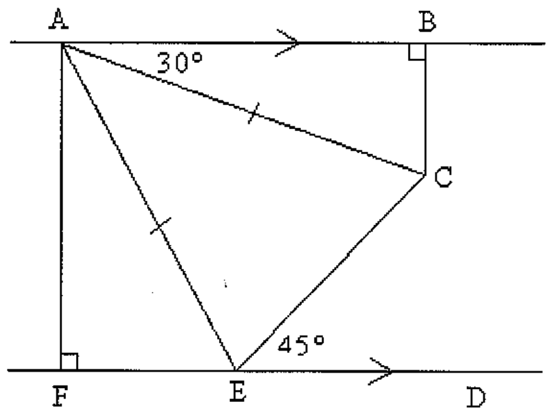
ii. $\int_1^{e^4} \frac{6}{x}\, dx$ **2**

c. **i.** Write down the discriminant of $5x^2 + 3x + k$. **1**

ii. For what values of k does $5x^2 + 3x + k = 0$ have real roots? **2**

QUESTION FOUR [12 MARKS]

a.



In the diagram AC = AE and AB is parallel to FD, angle CED = 45° and angle BAC = 30°, BC is perpendicular to AB and AF is perpendicular to FD.

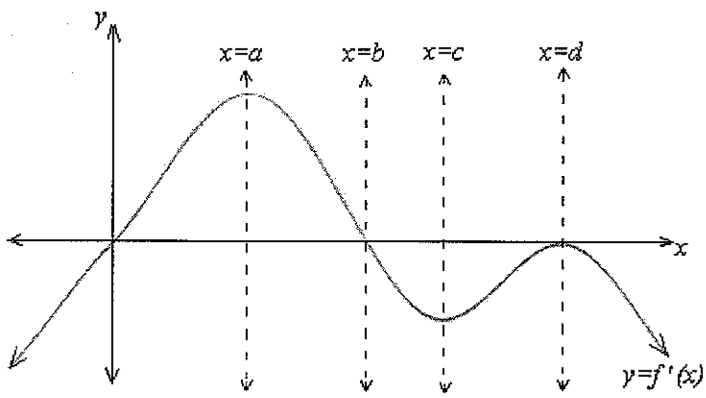
Copy this diagram into your answer booklet.

i. Find the size of angle ACE giving reasons. **2**

ii. Hence find the size of angle CAE giving reasons. **2**

iii. Prove $\Delta ABC \equiv \Delta AFE$. **3**

b. Below is the graph of $y = f'(x)$



i. For the graph of $y = f(x)$, what would be the feature that would occur when:

α) $x = b$ **1**

β) $x = c$ **1**

γ) $x = d$ **1**

ii. Transfer this sketch into your answer booklet and sketch a graph of $y = f''(x)$ onto the same axes. **2**

QUESTION FIVE [12 MARKS]

a. A company making shoes makes 200 pairs in the first month of operation. They intend to increase their output by 25 pairs a month. How many pairs of shoes do they intend to make:

i. in their 24^{th} month of operation; **2**

ii. in total over the two year period. **2**

b. Describe the graph of $x^2 + y^2 + 4x - 6y - 3 = 0$ as accurately as possible. **2**

c. A bag contains 10 blue marbles, 5 red marbles and 1 green marble. Tim selects one marble and does not replace it before drawing a second marble.

i. Draw a probability tree to show all of the possible outcomes. **2**

ii. Find the probability that both of the marbles are blue. **1**

iii. Find the probability that at least one of the marbles is blue. **2**

iv. Find the probability that the two marbles are the same colour. **1**

QUESTION SIX [12 MARKS]

a. Consider the two curves $y + x^2 - 6 = 0$ and $y + 2x - 3 = 0$.

i. Find the points of intersection. **2**

ii. Calculate the area between the two curves. **2**

b. A geometric series has the seventh term 640 and fourth term 80. Find the first term and common ratio. **3**

c. Solve $2\cos x = \sqrt{3}$ for $0 \le x \le 2\pi$. **2**

d. Sketch the graph of the function $y = 10 - 5\sin 2x$ for $0 \le x \le \pi$. **3**

QUESTION SEVEN [12 MARKS]

a. Consider the curve $y = 4x^3 - 6x^2 - 24x + 1$.

i.	Find the stationary points and determine their nature.	3
ii.	Find any points of inflexion.	2
iii.	Sketch the curve for the domain $-2 \le x \le 3$.	2
iv.	For what values of x is the curve increasing?	1

b. Consider the parabola $x^2 - 14x - 8y + 41 = 0$.

i.	By first rewriting the function in the form $(x-b)^2 = 4a(y-c)$, find the coordinates of the vertex.	2
ii.	Find the focal length and the coordinates of the focus of this parabola.	2

QUESTION EIGHT [12 MARKS]

a. The population of Scotsville at the beginning of 1980 was 12000 and at the beginning of 1990 it was 13080. Assume the population of Scotsville is governed by the equation $P = P_0 e^{kt}$ where t is in years, P_0 and k are both constants and P is the population at time t.

i.	Find k correct to 4 decimal places.	2
ii.	What would the population be at the start of 2010?	2
iii.	In which year would the population reach 20000?	2

b. The velocity of a particle moving along the x axis is given by $v = 8t - t^2$.

i.	When will the particle be at rest?	1
ii.	If the particle was originally 3 metres to the right of the origin, find the displacement of the particle (x) as a function of time (t).	2
iii.	When will the particle have maximum velocity?	1
iv.	What is the total distance travelled by the particle in the first 9 seconds?	2

Question Nine [12 marks]

a. Find the exact volume of revolution when the area bounded by the curve $y = 3\sec x$ and the x axis between $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x axis. **2**

b. Consider the function $f(x) = 6\log_e\left(\frac{x-3}{5}\right)$

i. What is the domain of the function $y = f(x)$ **1**

ii. Sketch the curve $y = f(x)$ showing all important features. **2**

iii. Use Simpson's Rule with 5 ordinate values to find an approximation for the area between the curve, the x axis and the ordinates $x = 4$ and $x = 8$. (Answer to 3 d.p.) **4**

c. Solve the equation $\ln(2x+3) + \ln 4 = 2\ln 2x$ for x. **3**

Question Ten [12 marks]

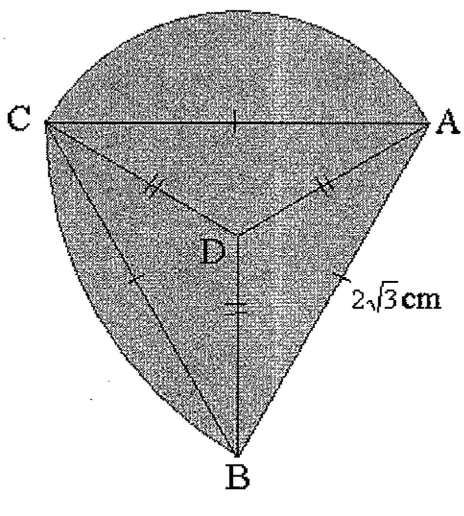
a. A 12m long piece of string is cut into two pieces to form a circle of radius r metres and a square of side x metres.

i. Show that the total area of the circle and square is given by **3**

$$A = \pi r^2 + 9 - 3\pi r + \frac{\pi^2 r^2}{4}$$

ii. Find the exact radius r of the circle such that the total area will be a minimum. **4**

b.



The 2 dimensional shape shown has the arc BC formed from centre A and the arc CA formed from centre D. AC, BC and AB are all equal. A, C and B are all equidistant from D. AB is $2\sqrt{3}$ cm. Calculate the exact area of this shape. **5**

END OF EXAMINATION

THE SCOTS COLLEGE
2 Unit Trial HSC 2004

QUESTION ONE [12 MARKS]

a. $\dfrac{m^2+n^2}{mn} = \dfrac{(-4.3)^2+(2.1)^2}{-4.3\times 2.1}$

$= -2.5$ (to 1 d.p.)

b. $80\% = \$760$

$1\% = \$9.50$

$100\% = \$950$

c. primitive of $5 - x^{1/2}$

$= 5x - \dfrac{2x^{3/2}}{3} + c$

$= 5x - \dfrac{2\sqrt{x^3}}{3} + c$

d. $|x-7| > 2$

$-2 > x-7$ and $x-7 > 2$

$5 > x$ and $x > 9$

e. $\lim_{x\to 3} \dfrac{x^2-2x-3}{x-3}$

$= \lim_{x\to 3} \dfrac{(x-3)(x+1)}{x-3}$

$= \lim_{x\to 3} (x+1)$

$= 4$

f. $\dfrac{m^2}{m^2+3m+2} - \dfrac{2m}{m+2}$

$= \dfrac{m^2}{(m+2)(m+1)} - \dfrac{2m}{m+2}$

$= \dfrac{m^2-2m(m+1)}{(m+2)(m+1)}$

$= \dfrac{m^2-2m^2-2m}{(m+2)(m+1)}$

$= \dfrac{-m^2-2m}{(m+2)(m+1)}$

$= \dfrac{-m(m+2)}{(m+2)(m+1)}$

$= \dfrac{-m}{m+1}$

QUESTION TWO [12 MARKS]

a. i. $\alpha+\beta = 7$

ii. $\alpha\beta = 3$

iii. $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 49 - 2(3)$

$= 43$

iv. $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta}$

$= \dfrac{-7}{3}$

b.i. $5x-12y+53=0$

$12y = 5x+53$

$y = \dfrac{5}{12}x + \dfrac{53}{12}$

Gradient = 5/12

Angle $\tan\theta = \dfrac{5}{12}$

$\theta = 23°$

ii. $m = -12/5$ thru $(4,-8)$

$y-(-8) = -\dfrac{12}{5}(x-4)$

$5y+40 = -12x+48$

$12x+5y-8=0$.

iii. $12x+5y-8=0$ ---(1)

$5x-12y+53=0$ --(2)

(1) x 5 and (2) x 12

$60x+25y-40=0$ ---(3)

$60x-144y+636=0$ --(4)

(4)-(3)

$-169y+676=0$

So $y=4$

Subst. into (1) $12x+20-8=0$

$x=-1$ therefore intersection (-1,4)

iv. $PQ = \sqrt{(4--8)^2+(-1-4)^2}$

$PQ = \sqrt{144+25}$

$PQ = 13$

© The Scots College, Bellevue Hill.

QUESTION THREE [12 MARKS]

a.i. $\dfrac{d5\sin^2 x}{dx}$

$= \dfrac{d5(\sin x)^2}{dx}$

$= 10\cos x.\sin x$

ii. $\dfrac{d\log_e(4x^3+3x)}{dx}$

$= \dfrac{12x^2+3}{4x^3+3x}$

iii. $\dfrac{de^{(8x^3-5x)}}{dx} = (24x^2-5)e^{(8x^3-5x)}$

b. i. $\int (3x+4)^5\,dx$

$= \dfrac{1}{18}(3x+4)^6 + c$

ii. $\int_1^{e^4} \dfrac{6}{x}dx = [6\ln x]_1^{e^4}$

$= [6\ln e^4 - 6\ln 1]$

$= 24$

c.i. $5x^2+3x+k$.

$\Delta = 9 - 4.5.k$

$\Delta = 9 - 20k$

ii. real roots $\Delta \geq 0$

$9 - 20k \geq 0$

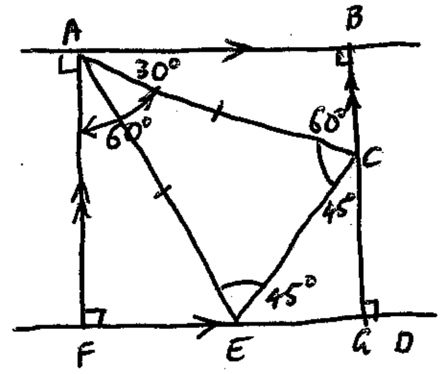
$9 \geq 20k$

$\dfrac{9}{20} \geq k$

© The Scots College, Bellevue Hill.

Question Four

a)



i) Produce BC to the line FD call the point G.

$\angle ACB + \angle ACE + \angle EGG = 180$

$60° + \angle ACE + 45 = 180$

$\angle ACE = 180 - 60 - 45$

$= 75°$

ii) ΔACE is isosceles

$\therefore \angle AEC = 75°$

$\therefore \angle CAE = 30°$ anglesum of Δ

iii) In Δ's ABC and AFE

AC = AE (given). S

$\angle ABC = \angle AFE$ (90° given perpendicular) A

$\angle FAE = 90 - \angle BAC - \angle CAE$

so $\angle FAE = 30°$ (angles adding to 90°).

Now $\angle BAC = \angle FAE$ A.

$\therefore \Delta ABC \equiv \Delta AFE$ (AAS)

b i) α) $f'(b) = 0 \therefore$ turn pt.

$f'(a^-) = +ve$ $f'(a^+) = -ve$.

$\therefore$ max. turn point.

β) $f'(c)$ minimum

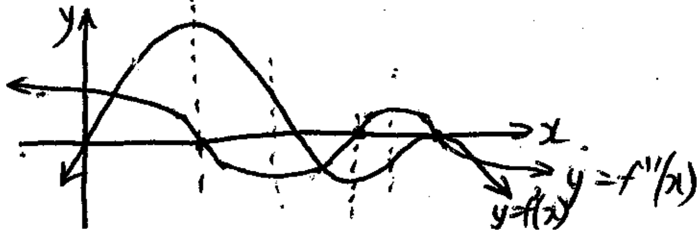
$\therefore$ pt. of inflexion

γ) $f'(d) = 0$ also gradient of $f'(x) = 0$

so $f'(d) = 0$ and $f''(d) = 0$

so horizontal point of inflexion.

b ii)



Question Five

AP.

a i) $a = 200 \quad d = 25.$

$T_{24} = 200 + 23 \times 25.$

$= 200 + 575.$

$= 775.$

ii) $S_{24} = \frac{n}{2}(a + l)$

$= \frac{24}{2}(200 + 775)$

$= 12(975)$

$= 11700$

b) $x^2 + y^2 + 4x - 6y - 3 = 0$

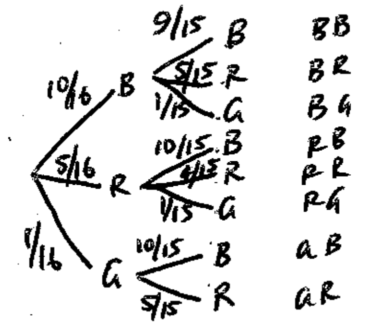
$x^2 + 4x \quad + y^2 - 6y \quad = 3$

$(x+2)^2 - 4 \quad + (y-3)^2 - 9 = 3$

$(x+2)^2 \quad + (y-3)^2 \quad = 16$

CIRCLE Centre $(-2, 3)$ radius 4

c) i)



ii) $P(BB) = \frac{10}{16} \times \frac{9}{15}$

$= 3/8$

iii) $P(\text{at least 1 blue})$

$= P(B\bar{B}) + P(\bar{B}\&B) + P(BB)$

$= \left(\frac{10}{16} \times \frac{6}{15}\right) + \left(\frac{6}{16} \times \frac{10}{15}\right) + 3/8$

$= \frac{1}{4} + \frac{1}{4} + 3/8$

$= 7/8$

iv) $= P(BB) + P(RR) + \sout{P(GG)}$

$= \left(\frac{10}{16} \times \frac{9}{15}\right) + \left(\frac{5}{16} \times \frac{4}{15}\right)$

$= 3/8 + 1/12$

$= 11/24$

QUESTION SIX

a i) $y = 6 - x^2$ — ①

$y = -2x + 3$ — ②

from ① and ②

$6 - x^2 = -2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ and $x = -1.$

when $x = 3 \quad x = 1$

$y = 6 - 9 \quad y = 6 - 1$

$= -3 \quad = 5.$

ii) $A = \int_{-1}^{3} (6 - x^2) - (-2x + 3)\, dx$

$= \int_{1}^{3} 6 - x^2 + 2x - 3 \; dx$

$= \int_{1}^{3} -x^2 + 2x + 3 \; dx.$

$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{1}^{3}$

$= (-9 + 9 + 9) - (+1/3 + 1 - 3)$

$= 10\,2/3 \text{ units}^2$

b) G.P. $T_7 = 640 \quad T_4 = 80$

$ar^6 = 640$ — ①

$ar^3 = 80$ — ②.

① ÷ ②

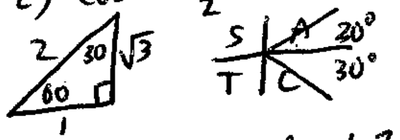
$r^3 = 8 \quad$ so $r = 2$

subst. into ②

$a\,2^3 = 80$

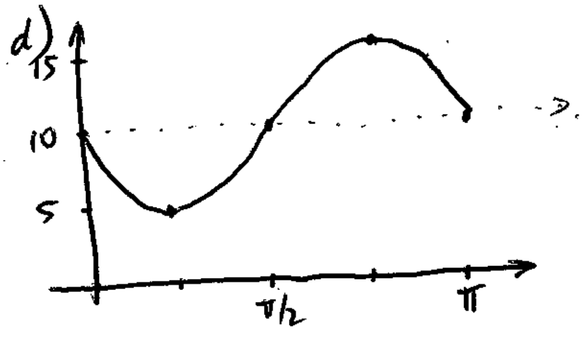
$a = 10$

c) $\cos x = \frac{\sqrt{3}}{2} \qquad x = 30°.$



$x = 30°$ and $330°$

d)



QUESTION SEVEN

$y = 4x^3 - 6x^2 - 24x + 1.$
$y' = 12x^2 - 12x - 24$
$y'' = 24x - 12.$

i) stat pts $y' = 0$
$12x^2 - 12x - 24 = 0$
$12(x^2 - x - 2) = 0$
$(x-2)(x+1) = 0$
so $x = 2$ and $x = -1$.

when $x = -1$ — $y = 15$, $y'' = -ve$, so max turn pt. at $(-1, 15)$
$x = 2$ — $y = -39$, $y'' = +ve$, so min turn pt. at $(2, -39)$

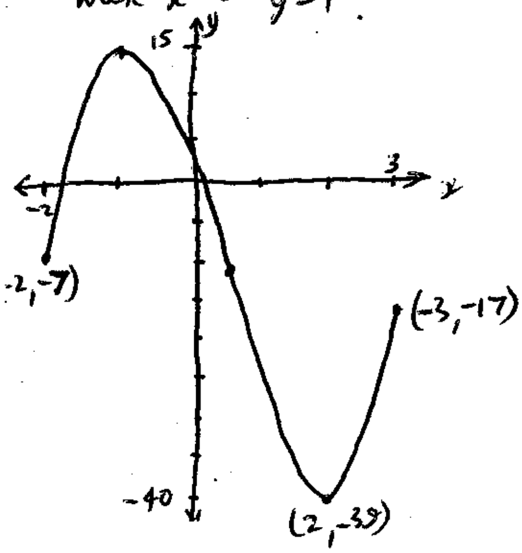
ii) Inflexion when $y'' = 0$
$24x - 12 = 0$
$x = 1/2$

x	$1/2^-$	$1/2$	$1/2^+$
$f''(x)$	$-$	0	$+$

when $x = 1/2$ $y = -12$

iii) end points $x = -2$ $y = -7$
$x = 3$ $y = -17$

when $x = 0$ $y = 1$.



b) i) $x^2 - 14x - 8y = -41$
$(x-7)^2 - 49 - 8y = -41$
$(x-7)^2 = 8y + 8$
$(x-7)^2 = 8(y+1)$
$V(7,-1)$ $a = 2$

ii) focal length $= 2$
Focus $= (7, 1)$

QUESTION EIGHT

a i) let 1980 be $t = 0$
so $P_0 = 12000$
when $t = 10$ $P = 13080$
so $13080 = 12000e^{10k}$
$e^{10k} = \frac{13080}{12000}$
$k = \left(\ln \frac{1308}{1200}\right) \div 10$
$= 0.0086$ to 4 d.p.

ii) when $t = 30$
$P = 12000e^{30k}$
$= 15540$

iii) find t when $P = 20000$
$20000 = 12000e^{0.0086t}$
$e^{0.0086t} = \frac{20000}{12000}$
$t = \ln(5/3) \div 0.0086$
$= 59.3$ years
so in 60th year ie 2040

b) $v = 8t - t^2$

i) rest means $v = 0$
$8t - t^2 = 0$
$t(8 - t) = 0$
so $t = 0$ and $t = 8$

ii) $x = 4t^2 - \frac{t^3}{3} + c$
when $t = 0$ $x = 3$
so $c = 3$
$\therefore x = 4t^2 - \frac{t^3}{3} + 3$

iii) Max vel. when $\ddot{x} = 0$
$\frac{dv}{dt} = 8 - 2t$
so when $t = 4$ max. will occur as $8t - t^2$ is concave down

iv)

no change of direction when $t = 8$.
when $t = 0$ $x = 3$
$t = 8$ $x = 88\frac{1}{3}$
$t = 9$ $x = 84$
First 8 seconds distance $= 85\frac{1}{3}$ M
last second $= 4\frac{1}{3}$ M
TOTAL $= 89\frac{2}{3}$ M

QUESTION NINE

a) $y = 3\sec x$

$y^2 = 9\sec^2 x$

so $V = \pi\int_0^{\pi/4} 9\sec^2 x\, dx$

$= 9\pi\left[\tan x\right]_0^{\pi/4}$

$= 9\pi \text{ units}^3$

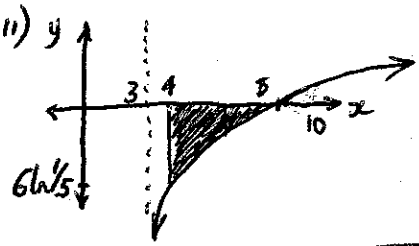
b) $f(x) = 6\log_e\left(\frac{x-3}{5}\right)$

i) domain $\frac{x-3}{5} > 0$

$x - 3 > 0$

$x > 3$

ii)



iii)

x	4	5	6	7	8
$f(x)$	-9.66	6ln0.4	6ln0.6	6ln0.8	0

$A \doteqdot \left|\frac{1}{3}\left\{f(4)+f(8)+4\{f(5)+f(7)\}+2\{6\}\right\}\right|$

$\doteqdot \left|\frac{1}{3}\left[-9.66+0+4(6\ln 0.4+6\ln 0.8)+2(6\ln 0.6)\right]\right|$

note all below axis.

$\doteqdot 14.378$

c) $\ln(2x+3)+\ln 4 = 2\ln 2x$

$\ln[(2x+3)\times 4] = \ln(2x)^2$

$8x + 12 = 4x^2$

$4x^2 - 8x - 12 = 0$

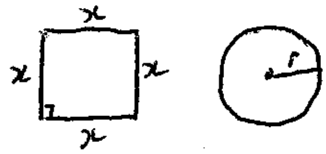
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ and $x = -1$

QUESTION TEN

a)



$P = 4x$ $\quad C = 2\pi r$

$A = x^2$ $\quad A = \pi r^2$

total Perimeter $= 4x + 2\pi r$

$= 12$

$4x + 2\pi r = 12$

$4x = 12 - 2\pi r$

$x = 3 - \frac{\pi r}{2}$

So $A = x^2 + \pi r^2$

$= \left(3 - \frac{\pi r}{2}\right)^2 + \pi r^2$

$= 9 - 3\pi r + \frac{\pi^2 r^2}{4} + \pi r^2$

$= \pi r^2 + 9 - 3\pi r + \frac{\pi^2 r^2}{4}$

ii) $A = \pi r^2 + 9 - 3\pi r + \frac{\pi^2 r^2}{4}$

$A' = 2\pi r - 3\pi + \frac{\pi^2 r}{2}$

$A'' = 2\pi + \frac{\pi^2}{2}$

Max and min area when $A' = 0$

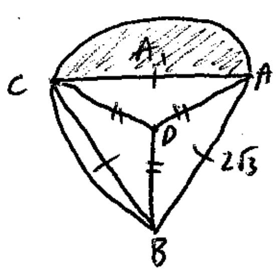
$r\left(2\pi + \frac{\pi^2}{2}\right) - 3\pi = 0$

$r = \frac{3\pi}{2\pi + \pi^2/2}$

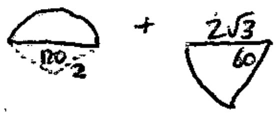
$= \frac{6\pi}{4\pi + \pi^2}$

$r = \frac{6}{4 + \pi}$

this is a minimum at A'' is +ve.



Area =



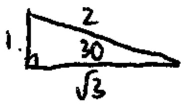
$= \frac{1}{3}\times\pi\times 2^2 - \frac{1}{2}\times 2\times 2\sin 120 + \frac{1}{6}\times\pi\times(2\sqrt{3})^2$

$= \frac{4\pi}{3} - \sqrt{3} + 2\pi$

$= \frac{10\pi - 3\sqrt{3}}{3}\ \text{cm}^2$

$\angle ADB = 120°$

$\angle DAB = 30°$



$\therefore AD = 2$