Shore

Examination Number:

Set:

Year 12
Trial HSC Examination
August 2013

Mathematics

General Instructions

- Reading time − 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11−16, show relevant mathematical reasoning and/or calculations
- Start each of Questions 11−16 in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks − 100

Section I Pages 2–5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 7–15

90 marks

- Attempt questions 11–16
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1−10.

1 The land area of Earth is approximately 148 940 000 square kilometres. What is this measurement in scientific notation correct to two significant figures?

(A) 1.49×10^8

(B) 1.5×10^8

(C) 14×10^7

(D) 15×10^7

2 The cost of a lamp is \$203.50. This includes a 10% tax on the original price. What was original price of the lamp before the tax was included?

(A) \$18.50

(B) \$20.35

(C) \$183.15

(D) \$185.00

3 What is the locus of all points within three units of the x-axis?

(A) $|x| < 3$

(B) $|x| > 3$

(C) $|y| < 3$

(D) $|y| > 3$

4 A quadratic equation $ax^2 + bx + c = 0$ has $b^2 - 4ac = 81$. Which of the following best describes the roots of the equation?

(A) no real roots

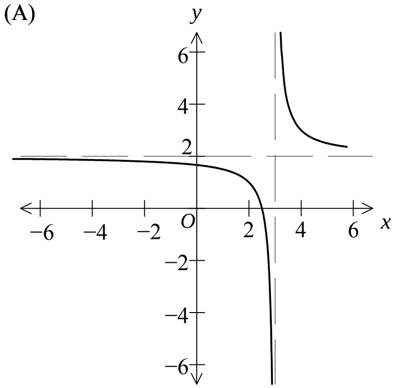
(B) one real root

(C) two rational real roots

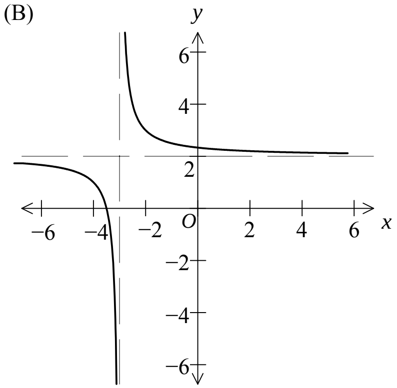
(D) two irrational real roots

5 Which graph correctly shows $y = \dfrac{1}{x-3} + 2$?

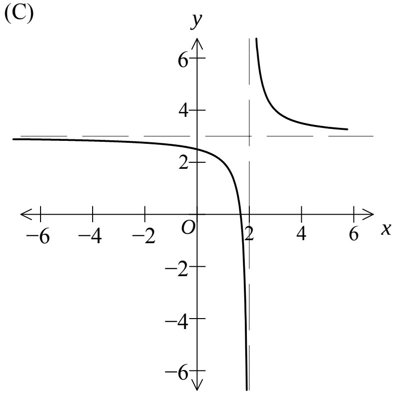
(A)



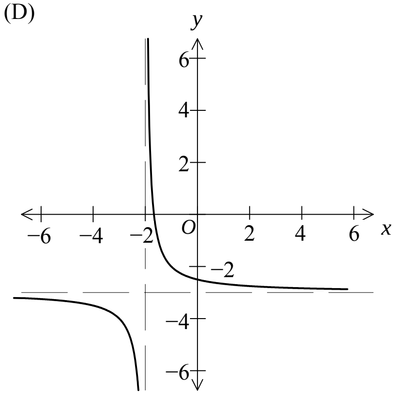
(B)



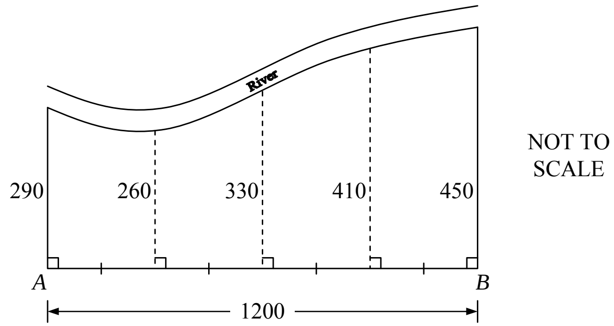
(C)



(D)



6 The diagram shows a field and its dimensions, in metres. The field is bounded on one side by a river. Measurements are taken at equal intervals perpendicular to the line AB, from AB to the river.



Which of the following expressions correctly uses Simpson's rule with four subintervals to find an approximation for the area of the field?

(A) $100\left[290 + 4(260) + 2(330) + 4(410) + 450\right]$

(B) $200\left[290 + 4(260) + 2(330) + 4(410) + 450\right]$

(C) $300\left[290 + 4(260) + 2(330) + 4(410) + 450\right]$

(D) $600\left[290 + 4(260) + 2(330) + 4(410) + 450\right]$

7 What is the value of $\sum_{n=2}^{6} n^2$?

(A) 80

(B) 90

(C) 91

(D) 100

8 What is the period of the function $y = 4\cos\frac{x}{2}$?

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 4π

9 A particle moves in a straight line so that its displacement, in metres, is given by $x = 4t^3 - 12t^2$ where t is the time in seconds. At what time after $t = 0$ is the particle at rest?

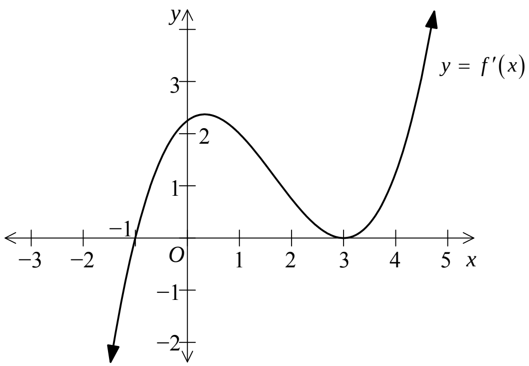
(A) $t = 1$

(B) $t = 2$

(C) $t = 3$

(D) $t = 4$

10 The diagram shows a sketch of the gradient function of the curve $y = f(x)$.



What feature must the graph of $y = f(x)$ have at $x = 3$?

(A) x-intercept

(B) minimum turning point

(C) maximum turning point

(D) horizontal point of inflection

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate writing booklet.

(a) Solve $|x - 5| \le 8$. **2**

(b) Find the limiting sum of the geometric series $3 + \frac{1}{2} + \frac{1}{12} + \ldots$. **2**

(c) Differentiate with respect to x.

(i) $\frac{\sin x}{x}$ **2**

(ii) $\left(1 + e^{3x}\right)^5$ **2**

(d) Find the gradient of the tangent to the curve $y = \log_e 2x$ at the point where $x = 3$ on the curve. **2**

(e) State the domain and range of the function $y = \sqrt{4 - x^2}$. **2**

(f) The quadratic equation $x^2 - 5x + 3 = 0$ has roots α and β.

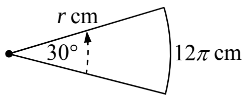
(i) Find $\alpha + \beta$. **1**

(ii) Find $\alpha\beta$. **1**

(iii) Find $\alpha^2\beta + \alpha\beta^2$. **1**

Question 12 (15 marks) Use a separate writing booklet.

(a) **2**

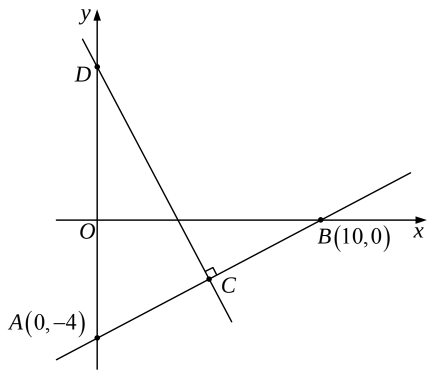


NOT TO SCALE

The diagram shows an arc of length 12π centimetres, subtending an angle of $30°$ at the centre of a circle of radius r centimetres. Find the value of r.

(b) Evaluate $\int_2^5 \frac{6}{2x-1}\,dx$. Leave your answer in simplest exact form. **3**

(c) The diagram shows the line joining the points $A(0,-4)$ and $B(10,0)$. The perpendicular bisector, CD, of the interval AB intersects the y-axis at D.



NOT TO SCALE

(i) Find the coordinates of C. **1**

(ii) Show that the equation of the line CD is $5x + 2y - 21 = 0$. **2**

(iii) Find the coordinates of D. **1**

(iv) Find the area of ΔACD. **2**

(d) In the Kurramurra Theatre, rows of seats are labelled alphabetically, starting with Row A. Row A has seats numbered 1 to 27, Row B has seats numbered 1 to 29, Row C has seats numbered 1 to 31, and so on until the last row of the theatre, Row M.

(i) How many seats are in Row M? **2**

(ii) What is the total number of seats in Kurramurra Theatre? **2**

Question 13 (15 marks) Use a separate writing booklet.

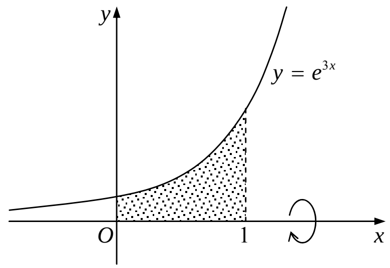
(a) A function is given by $f(x) = 4x^3 - 3x^4$.

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph of $y = f(x)$ showing the stationary points and any intercepts with the coordinate axes. **2**

(iii) For what values of x is the curve concave up? **1**

(b) **3**



NOT TO SCALE

The shaded region in the diagram is bounded by the curve $y = e^{3x}$, the x-axis, and the lines $x = 0$ and $x = 1$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis.

(c) A particle is moving in a straight line. Its velocity, v metres per second, at time t seconds is given by $v = 4\cos 2t$. Initially, $x = 3$ where x is the displacement of the particle in metres.

(i) What is the initial velocity of the particle? **1**

(ii) Find an expression for the displacement, x metres, of the particle at time t seconds. **2**

(iii) Sketch the graph of x as a function of t for $0 \le t \le 2\pi$. **2**

(iv) What is the total distance travelled by the particle between $t = 0$ and $t = 2\pi$? **1**

Question 14 (15 marks) Use a separate writing booklet.

(a) A total of 100 tickets are sold in a raffle which has two prizes. The first ticket drawn wins first prize and the second ticket wins second prize. At the drawing of the raffle, the winning ticket of first prize is NOT replaced before the winning ticket of second prize is drawn.

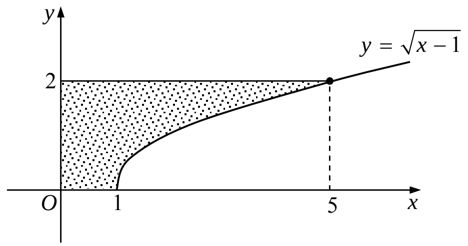
Sharon has five tickets in this raffle.

(i) What is the probability Sharon does not win any of the prizes? **2**

(ii) What is the probability that Sharon wins exactly one prize? **2**

(iii) What is the probability that Sharon wins at least one prize? **1**

(b) **3**

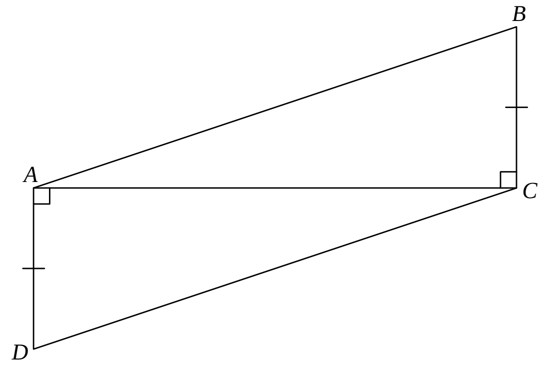


The shaded region in the diagram is bounded by the curve $y = \sqrt{x-1}$, the line $y = 2$, the x-axis and the y-axis.

Find the area of the shaded region.

Question 14 continues on page 11

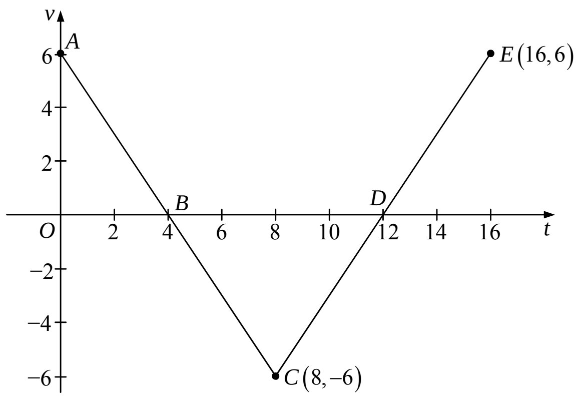
(c) Two triangles ABC and CDA are constructed from a common interval AC, such that $BC = DA$ and $\angle ACB = \angle CAD = 90°$ as shown.



(i) Prove that ΔABC is congruent to ΔCDA. **2**

(ii) Prove that $ABCD$ is a parallelogram. **2**

(d) The graph shows the velocity, v, of a particle as a function of time, t.



The graph consists of two straight line segments, AC and CE, intersecting the x-axis at the points B and D respectively.

Initially the particle is at the origin.

(i) How far does the particle travel between $t = 0$ and $t = 4$? **1**

(ii) At what time does the particle return to the origin? Justify your answer. **2**

End of Question 14

Question 15 (15 marks) Use a separate writing booklet.

(a) Find the coordinates of the focus of the parabola $12y = x^2 - 8x - 8$. **2**

(b) A radioactive isotope decays at a rate proportional to the amount of isotope present. That is, if $A(t)$ is the amount of isotope at time t, then $A = A_0 e^{-kt}$, where k is a positive constant and A_0 is the amount present at $t = 0$.

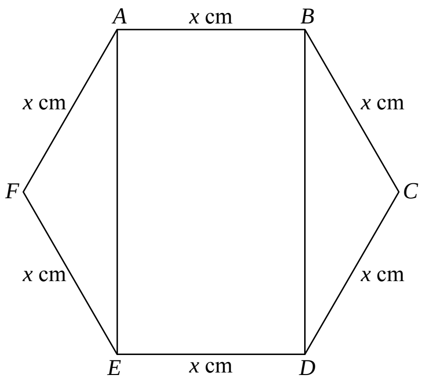
It takes 138 days for an amount of isotope to reduce by half.

(i) Find the value of k. **2**

(ii) A scientist obtains 25 micrograms of the isotope for an experiment. **1**

What mass of isotope will remain after 1000 days? Give your answer in micrograms, correct to three significant figures.

(c) *ABCDEF* is a regular hexagon with side length x cm. *AE* and *BD* are drawn such that *ABDE* is a rectangle.



NOT TO SCALE

(i) Find the size of $\angle AFE$. **1**

(ii) Use the cosine rule in ΔAFE to show that $AE = \sqrt{3}x$. **2**

(iii) Find the exact area of the hexagon *ABCDEF* in terms of x. **2**

Question 15 continues on page 13

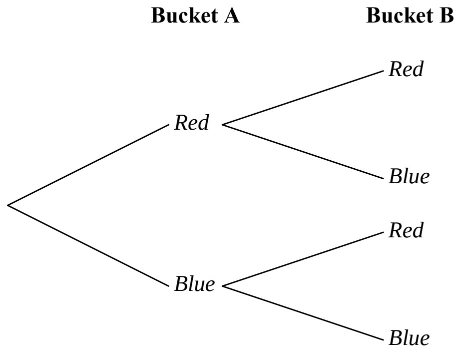
(d) Two buckets each contain red balls and blue balls.

Bucket A contains 10 balls in total: 4 red and 6 blue balls.

Bucket B contains n balls in total: 16 red and the rest blue balls.

A single ball is randomly chosen from each bucket.

(i) A tree diagram for the selection of balls is shown. **2**



Copy and complete the tree diagram showing the correct probability, in terms of n where necessary, on each branch.

(ii) The probability that both balls are the same colour is 0.55. **3**

How many blue balls are there in Bucket B?

End of Question 15

Question 16 (15 marks) Use a separate writing booklet.

(a) Solve for x: $2\cos^2 x - 7\cos x - 4 = 0$ for $0 \le x \le 2\pi$. **2**

(b) Anton takes out a loan of \$800 000 to buy a house. The first six months of the loan are an 'introductory period' in which the interest rate is 4.2% per annum, calculated monthly, and no repayments are made.

(i) Show that the amount owing at the end of the introductory period is \$816 947.69. **1**

After the introductory period, the interest rate switches to 5.4% per annum, calculated monthly, with interest calculated and charged just before each repayment. The loan is to be repaid in equal monthly repayments of $\$M$ at the end of each month over the remaining 24.5 years (294 months).

Let $\$A_n$ be the amount owing n months after the end of the introductory period.

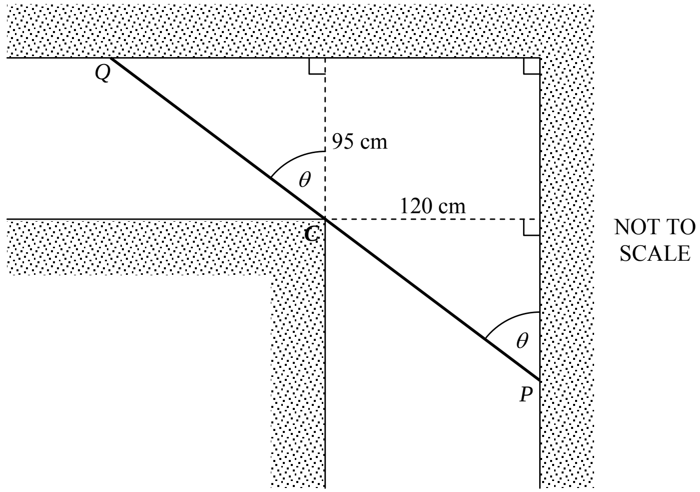
(ii) Show that the minimum monthly repayment required to repay the loan in full over the remaining 294 months is approximately \$5016.24. **2**

(iii) Instead of the minimum monthly repayment, Anton decides to repay \$5600 each month after the introductory period. After how many repayments will he have repaid the loan in full? **3**

(iv) How much will Anton save over the term of the loan by repaying \$5600 each month instead of the minimum monthly repayment? **1**

Question 16 continues on page 15

(c) A hallway is 120 centimetres wide. At the end of the hallway there is a right-angled turn and the hallway narrows to 95 centimetres wide. A piece of pipe is to be carried down the hallway so that the pipe stays in a horizontal position. At the turn, the ends of the pipe will be touching the outer wall of the hallway at P and Q and the inner corner at C, forming angle θ $\left(\text{where } 0 \le \theta \le \frac{\pi}{2}\right)$ with the outer wall at the point P as shown.



The length of the pipe, L centimetres, from P to Q, is given by

$$L = 95\sec\theta + 120\operatorname{cosec}\theta.$$

(i) Show that $\dfrac{dL}{d\theta} = \dfrac{95\sin\theta}{\cos^2\theta} - \dfrac{120\cos\theta}{\sin^2\theta}$. **2**

(ii) Find the largest length of pipe that will fit around the turn in the hallway, correct to the nearest centimetre. Justify your answer. **4**

End of paper

YEAR 12 MATHEMATICS (2U) TRIAL EXAM 2013

(1) B	(2) D	(3) C	(4) C	(5) A
(6) A	(7) B	(8) D	(9) B	(10) D

QUESTION 11

a) $|x-5| \leq 8$

critical points

$|x-5| = 8$

$x-5 = 8$ or $x-5=-8$

$x = 13$ $\quad x=-3$

$\therefore \underline{-3 \leq x \leq 13}$

b) $S_\infty = \dfrac{3}{1-\frac{1}{6}}$

$= 3\frac{3}{5}$

c) i) $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right) = \dfrac{x\cos x - \sin x.1}{x^2}$

$= \dfrac{x\cos x - \sin x}{x^2}$

ii) $\dfrac{d}{dx}(1+e^{3x})^5 = 5(1+e^{3x})^4 . 3e^{3x}$

$= 15e^{3x}(1+e^{3x})^4$

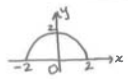
d) $y = \log_e 2x$

$\dfrac{dy}{dx} = \dfrac{1}{2x} \times 2$

$= \dfrac{1}{x}$

When $x = 3$

$m_{tangent} = \dfrac{1}{3}$

e) 

Domain: real x such that $-2 \leq x \leq 2$

Range: real y such that $0 \leq y \leq 2$

f) i) $\alpha+\beta = \dfrac{--5}{1} = 5$

ii) $\alpha\beta = \dfrac{3}{1} = 3$

iii) $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$

$= 3 \times 5$

$= 15$

QUESTION 12

a) $30° = \dfrac{\pi}{6}$ radians

$l = r\theta$

$12\pi = r \times \dfrac{\pi}{6}$

$r = 12\pi \div \dfrac{\pi}{6}$

radius $= 72$ cm

b) $\displaystyle\int_2^5 \frac{6}{2x-1}\,dx = 3\int_2^5 \frac{2}{2x-1}\,dx$

$= 3\left[\ln(2x-1)\right]_2^5$

$= 3\left(\ln(2(\ )-1) - \ln(2(2)-1)\right)$

$= 3(\ln 9 - \ln 3)$

$= 3\ln\left(\frac{9}{3}\right)$

$= 3\ln 3$

c) i) $C = \left(\dfrac{0+10}{2}, \dfrac{-4+0}{2}\right)$

$= (5,-2)$

ii) $m_{AB} = \dfrac{4}{10}$

$= \dfrac{2}{5}$

$\therefore m_{CD} = -\dfrac{5}{2}$ & use $(5,-2)$

$y--2 = -\frac{5}{2}(x-5)$

$2y+4 = -5(x-5)$

$2y+4 = -5x+25$

$5x+2y-21=0$

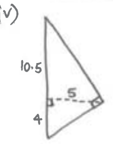
c) iii) y-int $\Rightarrow x=0$

$0+2y-21=0$

$y = \dfrac{21}{2}$

$= 10\frac{1}{2}$

$\therefore$ D is $(0, 10\frac{1}{2})$

iv) 

Area $= \frac{1}{2} \times 14.5 \times 5$

$= 36\frac{1}{4}$ units^2

d) i) $27+29+31+\ldots.$

Row M = 13th row

$T_{13} = 27 + (13-1)\times 2$

$= 51$

ii) $S_{13} = \dfrac{13}{2}(27+51)$

$= 507$

QUESTION 13

a) i) $f(x) = 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1-x)$

Stat pts when $f'(x)=0$

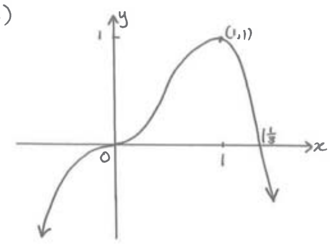
$12x^2(1-x)=0$

$\therefore x=0$ or $x=1$.

x	-1	0	$\frac{1}{2}$	1	2
$f'(x)$	24	0	1.5	0	-48

$\therefore$ Horizontal point of inflection at $(0,0)$

& Maximum turning point at $(1,1)$

a) ii) 

iii) $f''(x) = 24x - 36x^2$

$= 12x(2-3x)$

Pts of inflection when $f''(x)=0$

$12x(2-3x)=0$

$x=0$ or $x = \frac{2}{3}$

$\therefore$ Concave up for $0<x<\frac{2}{3}$ from graph.

b) Volume $= \pi\displaystyle\int_0^1 (e^{3x})^2\,dx$

$= \pi\displaystyle\int_0^1 e^{6x}\,dx$

$= \pi\left[\frac{1}{6}e^{6x}\right]_0^1$

$= \pi\left(\frac{1}{6}e^{6(1)} - \frac{1}{6}e^{6(0)}\right)$

$= \pi\left(\frac{1}{6}e^6 - \frac{1}{6}\right)$

$= \dfrac{\pi}{6}(e^6-1)$ units^3 (or 210.711... units^3)

c) i) when $t=0$

$v = 4\cos 2(0)$

$= 4$ m/s

ii) $x = \int 4\cos 2t\,dt$

$x = 2\sin 2t + C$

see over→

13 cont'd...

c) ii) cont'd...

when $t=0$, $x=3$

$3 = 2\sin 2(0) + c$

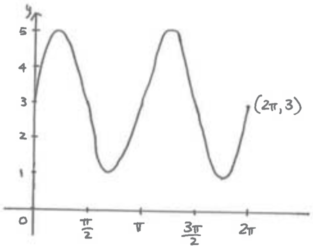
$c = 3$

$\therefore\ x = 2\sin 2t + 3$

iii) period $= 2\pi \div 2 = \pi$

amp. $= 2$

Shift up by 3.



iv) total dist $= 2+4+4+4+2$

$= 16$ metres.

QUESTION 14

a) i) $P(LL) = \frac{95}{100} \times \frac{94}{99}$

$= \frac{893}{990}$ (or $0.9\dot{0}\dot{2}$ or $90.\dot{2}\dot{0}\%$)

ii) $P(\text{exactly } 1\ W) = P(WL) + P(LW)$

$= \left(\frac{5}{100} \times \frac{95}{99}\right) + \left(\frac{95}{100} \times \frac{5}{99}\right)$

$= 2 \times \left(\frac{5}{100} \times \frac{95}{99}\right)$

$= \frac{19}{198}$ (or $0.0\dot{9}\dot{5}$ or $9.\dot{5}\dot{9}\%$)

iii) $P(\text{at least one } W) = 1 - P(LL)$

$= 1 - \frac{893}{990}$

$= \frac{97}{990}$ (or $0.0\dot{9}\dot{7}$ or $9.\dot{7}\dot{9}\%$)

14 cont'd...

b) $\text{Area} = \int_0^2 (y^2+1)\,dy$

$= \left[\frac{y^3}{3} + y\right]_0^2$

$= \left(\frac{2^3}{3} + 2\right) - (0)$

$= 4\frac{2}{3}$ units^2

c) i) In Δ's ABC and CDA:

- AC is common side
- $\angle ACB = \angle CAD = 90°$ (given)
- $AD = CB$ (given)

$\therefore\ \Delta ABC \equiv \Delta CDA$ (SAS)

ii) $AB = CD$ (matching sides of congruent triangles ABC & CDA)

and $AD = BC$ (given)

$\therefore$ ABCD is a parallelogram (2 pairs of opposite sides equal)

d) i) Distance travelled = area under vel-time graph

$= \frac{1}{2} \times 4 \times 6$

$= 12$ units

ii) Between $t=0$ and $t=4$, particle moves 12 units with positive velocity from origin. From $t=4$ to $t=8$, particle moves 12 units with negative velocity velocity. $\therefore$ particle returns to origin at $t=8$.

QUESTION 15

a) $12y + 8 = x^2 - 8x$ complete square on x.

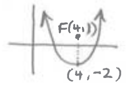
$12y + 8 + 16 = x^2 - 8x + 16$

$12y + 24 = (x-4)^2$

$12(y+2) = (x-4)^2$

parabola with vertex $(4,-2)$ and $4a = 12$

$a = 3$.



$\therefore$ Focus is $(4,1)$

15 cont'd...

b) i) when $t = 138$, $P = \frac{P_0}{2}$

$\therefore\ \frac{P_0}{2} = P_0 e^{-138k}$

$\frac{1}{2} = e^{-138k}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-138k}$

$\ln\left(\frac{1}{2}\right) = -138k$

$k = \frac{\ln\left(\frac{1}{2}\right)}{-138}$

$k = 0.005022805$

ii) $P = 25e^{-0.005022\ldots \times 1000}$

$= 0.16465\ldots$

$\therefore$ 0.165 micrograms will remain.

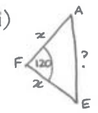
c) i) Angle sum hexagon $= 180 \times (6-2)$

$= 720$

$\therefore\ \angle AFE = 720 \div 6$

$= 120°$

ii)



$AE^2 = x^2 + x^2 - 2xx\cos 120°$

$= 2x^2 - 2x^2 \times -\frac{1}{2}$

$= 2x^2 + x^2$

$= 3x^2$

$\therefore\ AE = \sqrt{3}x$ units

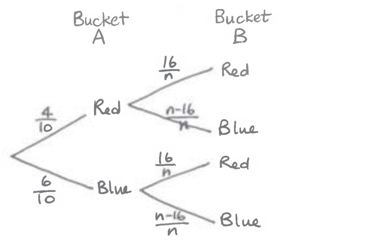
iii) Area hexagon $= 2 \times \left(\frac{1}{2} \times x \times x \times \sin 120\right) + x \times \sqrt{3}x$

$= \frac{\sqrt{3}x^2}{2} + \sqrt{3}x^2$

$= \frac{3\sqrt{3}x^2}{2}$ units^2

15 cont'd...

d) i) There are $n-16$ blue balls in Bucket B.



ii) $P(\text{same colour}) = P(RR) + P(BB)$

$0.55 = \frac{4}{10} \times \frac{16}{n} + \frac{6}{10} \times \frac{n-16}{n}$

$0.55 = \frac{6.4}{n} + \frac{0.6(n-16)}{n}$

$0.55 = \frac{6.4}{n} + \frac{0.6n - 9.6}{n}$

$0.55n = 6.4 + 0.6n - 9.6$

$3.2 = 0.05n$

$n = 64$

$\therefore$ 64 balls in total in 2nd urn.

$\therefore$ $64 - 16 = 48$ blue balls.

QUESTION 16

a) $2\cos^2 x - 7\cos x - 4 = 0$

$2\cos x \quad +1$

$\cos x \quad -4$

$(2\cos x + 1)(\cos x - 4) = 0$

$\cos x = -\frac{1}{2}$ or $\underbrace{\cos x = 4}_{\text{no soln as } -1 \le \cos x \le 1.}$

$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$\therefore\ x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

(16) cont'd...

b) i) 4.2% pa $= 0.042 \div 12$
$= 0.0035$ per month

Owing after intro $= 800\,000(1+0.0035)^6$
$= \$816\,947.6878$
$\doteq \$816\,947.69$

ii) 5.4% pa $= 0.054 \div 12$
$= 0.0045$ per month

$$A_1 = 816\,947.69(1.0045) - M$$
$$A_2 = 816\,947.69(1.0045)^2 - 1.0045M - M$$
$$A_3 = 816\,947.69(1.0045)^3 - 1.0045^2M - 1.0045M - M$$
$$\vdots$$
$$A_{294} = 816\,947.69(1.0045)^{294} - M\underbrace{\left(1 + 1.0045 + 1.0045^2 + \ldots + 1.0045^{293}\right)}_{\text{GP with } a=1,\ r=1.0045,\ n=294}$$
$$= 816\,947.69(1.0045)^{294} - M \times \frac{1\left(1.0045^{294}-1\right)}{1.0045-1}$$

But, when repaid, $A_{294} = 0$

$$\therefore\ 0 = 816\,947.69(1.0045)^{294} - M \times 609.6759\ldots$$
$$M \times 609.6759\ldots = 816\,947.69(1.0045)^{294}$$
$$M = \frac{816\,947.69(1.0045)^{294}}{609.6759\ldots}$$
$$= 5016.2349\ldots$$

$\therefore$ Minimum monthly payment $= \$5016.24$

iii) $$0 = 816\,947.69(1.0045)^n - 5600 \times \frac{1\left(1.0045^n - 1\right)}{1.0045-1}$$
$$\frac{5600\left(1.0045^n - 1\right)}{0.0045} = 816\,947.69(1.0045)^n$$

(16) cont'd...

iii) cont'd...

$$5600(1.0045^n - 1) = 3676.264605(1.0045)^n$$
$$5600(1.0045)^n - 5600 = 3676.264605(1.0045)^n$$
$$5600(1.0045)^n - 3676.264605(1.0045)^n = 5600$$
$$(1.0045)^n\left[5600 - 3676.264605\right] = 5600$$
$$(1.0045)^n\left[1923.735395\right] = 5600$$
$$1.0045^n = \frac{5600}{1923.735395}$$
$$1.0045^n = 2.911003257$$
$$\log_e 1.0045^n = \log_e 2.911003257$$
$$n \log_e 1.0045 = \log_e 2.911003257$$
$$n = \frac{\log_e 2.911003257}{\log_e 1.0045}$$
$$n = 237.977\ldots$$

$\therefore$ He needs to make 238 repayments of $5600.

iv) Saving $= (294 \times 5016.24) - (238 \times 5600)$
$= \$141\,974.56$

(16) cont'd...

c) i) $$L = \frac{95}{\cos\theta} + \frac{120}{\sin\theta}$$
$$= 95(\cos\theta)^{-1} + 120(\sin\theta)^{-1}$$
$$\frac{dL}{d\theta} = -95(\cos\theta)^{-2} \times -\sin\theta + -120(\sin\theta)^{-2} \times \cos\theta$$
$$= \frac{95\sin\theta}{\cos^2\theta} - \frac{120\cos\theta}{\sin^2\theta}$$

ii) max/min when $\frac{dL}{d\theta} = 0$

$$\frac{95\sin\theta}{\cos^2\theta} - \frac{120\cos\theta}{\sin^2\theta} = 0$$
$$95\sin^3\theta - 120\cos^3\theta = 0$$
$$95\sin^3\theta = 120\cos^3\theta$$
$$\frac{\sin^3\theta}{\cos^3\theta} = \frac{120}{95}$$
$$\tan^3\theta = \frac{120}{95}$$
$$\tan\theta = \sqrt[3]{\frac{120}{95}}$$
$$\theta = 0.82429468 \text{ radians.}$$

test

θ	0.81	0.82	0.83
$\frac{dL}{d\theta}$	−12.9...	0	5.19

$\therefore$ minimum when $\theta = 0.82\ldots$

and $L = \frac{95}{\cos 0.82\ldots} + \frac{120}{\sin 0.82\ldots}$
$= 303.368\ldots$

$\therefore$ 303 cm is largest piece of pipe to fit around corner.

In the context, the length L will be longer for angles $< 0.82\ldots$ and angles $> 0.82\ldots$. The smallest value of L is required to answer the question.