St Catherine's School
Waverley

2016 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Task Weighting – 40%

Total Marks – 100

Section I Pages 3 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II Pages 6 – 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section.

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Evaluate $\frac{2.48\times0.034}{\sqrt{0.081}-0.029}$, giving your answer correct to 2 decimal places.

(A) 0.32 (B) 0.33

(C) 0.36 (D) 0.37

2. Solve for x, $|4x+2|=6$

(A) $x=1,\ x=-2$ (B) $x=-1,\ x=2$

(C) $x=1,\ x=2$ (D) $x=-1,\ x=-2$

3. Express 215° in radian measure.

(A) $\frac{\pi}{215}$ (B) $\frac{215}{\pi}$

(C) $\frac{43\pi}{36}$ (D) $\frac{36\pi}{43}$

4. Which of the following is equal to $\frac{\cos(\frac{\pi}{2}-\alpha)}{\sin(2\pi-\alpha)}$?

(A) 1 (B) $\cot\alpha$

(C) -1 (D) $\cos(\frac{\pi}{2})$

5. Find $\int (2x+1)^5 dx$

(A) $\frac{(2x+1)^6}{6} + c$

(B) $5(2x+1)^4 + c$

(C) $10(2x+1)^4 + c$

(D) $\frac{(2x+1)^6}{12} + c$

6. $\frac{3\sqrt{3}}{\sqrt{7}-2}$ is equal to:

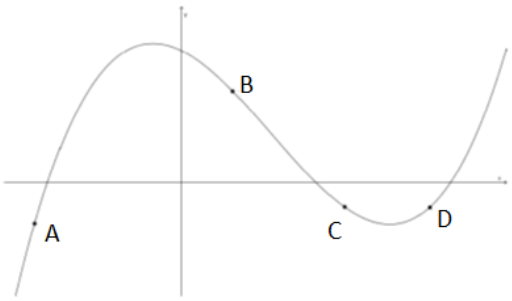
(A) $\frac{3\sqrt{3}(\sqrt{7}+2)}{5}$

(B) $\frac{3\sqrt{3}(\sqrt{7}-2)}{5}$

(C) $\sqrt{3}(\sqrt{7}+2)$

(D) $\sqrt{3}(\sqrt{7}-2)$

7. Which point on the graph satisfies the description: $y < 0$, $\frac{dy}{dx} > 0$, $\frac{d^2y}{dx^2} < 0$



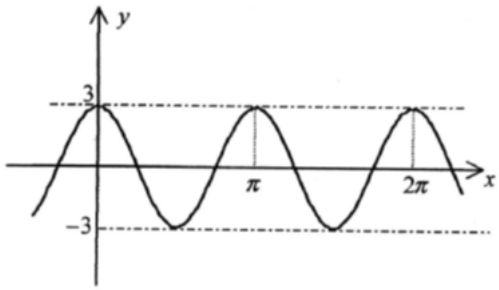
(A) Point A

(B) Point B

(C) Point C

(D) Point D

8. The graph below represents $y = a\cos mx$. Which statement is correct?



(A) $a = -3, m = 1$ (B) $a = 3, m = 2$

(C) $a = 3, m = 1$ (D) $a = 1, m = 3$

9. If $\log_a 7 = x$ and $\log_a 3 = y$, evaluate $\log_a 63$.

(A) $x + y^2$ (B) $x + 2y$

(C) xy^2 (D) $2x + y$

10. The common difference *d* of the arithmetic series $ln8 + ln16 + ln32 + \cdots$ is:

(A) 1 (B) 2

(C) $ln2$ (D) $ln8$

END OF SECTION I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a SEPARATE writing booklet**

(a) If $\sqrt{50} - 3\sqrt{75} + \sqrt{18} = a\sqrt{2} - b\sqrt{3}$, find the values of a and b. **2**

(b) State the domain and range of the function $y = \sqrt{2x-1} - 4$ **2**

(c) Find $\lim_{x\to 0} \frac{\sin 5x}{4x}$ **2**

(d) Differentiate with respect to t: **2**
$y = (5t^4 - 8)^9$

(e) Find a primitive function of $5x^3 + \sin 4x$ **2**

(f) A function $y = f(x)$ has $\frac{d^2y}{dx^2} = 6x - 2$ and a stationary point at (3,0). **3**
Find $f(x)$.

(g) For the parabola $x^2 - 4x - 8y - 4 = 0$, find the coordinates of the vertex. **2**

End of Question 11

Question 12 (15 marks) **Use a SEPARATE writing booklet**

(a) Solve the equation for x: **3**

$3 \times 9^x + 2 \times 3^x - 1 = 0$

(b) α and β are the roots of the quadratic equation $3x^2 - 4x - 8 = 0$. Without calculating the roots, find the value of:

(i) $\alpha^2 + \beta^2$ **2**

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ **1**

(c) Differentiate the following with respect to x:

(i) $y = xe^{\sin x}$ **2**

(ii) $y = \frac{lnx}{x}$ **2**

(d) (i) Find $\int \frac{x^3+1}{x^2} dx$ **2**

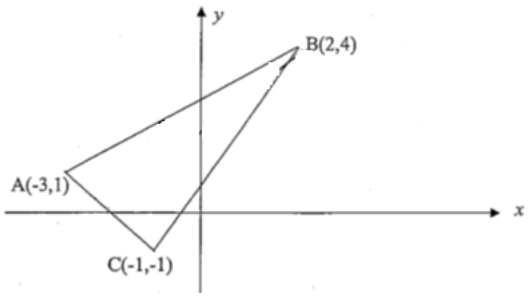
(ii) Show that $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x\right) = \cos^3 x$ **2**

Hence, find $\int 3\cos^3 x \, dx$ **1**

End of Question 12

Question 13 (15 marks) **Use a SEPARATE writing booklet**

(a)



In the diagram above A(-3,1), B(2,4) and C(-1,-1).

(i) Show that ΔABC is isosceles. **2**

(ii) Find the equation of AC. **2**

(iii) Find the perpendicular distance of point B from the line AC. **1**

(iv) Find the area of ΔABC. **1**

(b) Consider the function $y = 5xe^{-x}$.

(i) Copy and complete the table below, giving the value correct to 2 decimal places. **1**

x	0	1	2	3	4
y	0			0.75	0.37

(ii) Using Simpson's rule with 5 function values evaluate $\int_0^4 5xe^{-x}dx$ **2**

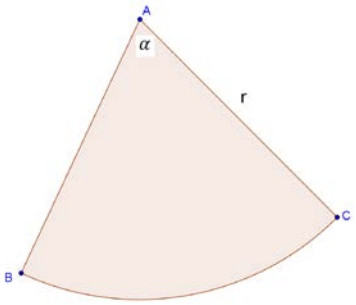
(iii) Show that $\frac{d^2y}{dx^2} = 5e^{-x}(x-2)$. **2**

(iv) Find the stationary points and the points of inflection. **4**

End of Question 13

Question 14 (15 marks) **Use a SEPARATE writing booklet**

(a)



The diagram above shows a sector with radius r and angle α. The area of the sector is $625m^2$.

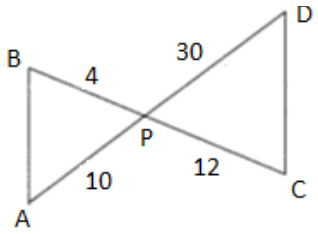
(i) Find the expression for α in terms of r. **1**

(ii) Show that the perimeter of the sector is $P = 2r + \frac{1250}{r}$ **1**

(iii) Find the value of r such that the sector has minimum perimeter. **3**

(iv) Find the value of α to the nearest degree such that the perimeter is minimum. **1**

b)



In the diagram above, AD and BC intersect at point P so that AP=10, BP=4, CP=12 and DP=30.

(i) Prove that ΔABP is similar to ΔCPD. **2**

(ii) Hence, prove that $AB \parallel CD$. **2**

Question 14 continues on page 10

(c) The population P of a town is changing exponentially $P = P_0 e^{kt}$, t is the time in years.

(i) At the start of 2000, the population was 15000 people and at the start of 2010 it was 35000. Show that the growth rate k is approximately 8.5%. **3**

(ii) Using $k = 0.085$, after how many years the population will reach 50000 people? **2**

End of Question 14

Question 15 (15 marks) **Use a SEPARATE writing booklet**

(a) The velocity of a particle is given by $v = \frac{4}{2t+1} ms^{-1}$. Initially the particle is $2m$ to the left of the origin.

(i) Find the expression for the displacement x. **2**

(ii) Find the exact time when the particle is at the origin. **2**

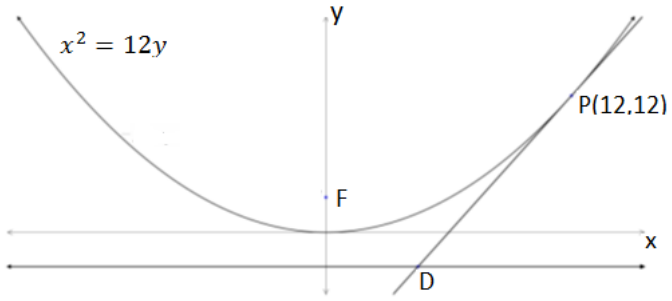
(iii) Show that the acceleration of the particle is always negative. **2**

(b) Two cards are chosen at random without replacement from six cards numbered: 1, 1, 3, 4, 4, 4. What is the probability that:

(i) First card is 1 and second card is 4? **1**

(ii) The sum of two numbers on the cards is less than 5? **2**

(c)



P(12,12) is a point on the parabola $x^2 = 12y$. F is the focus of the parabola. The tangent of the parabola at P intersects the directrix at point D.

(i) Find the equation of the tangent at point P. **2**

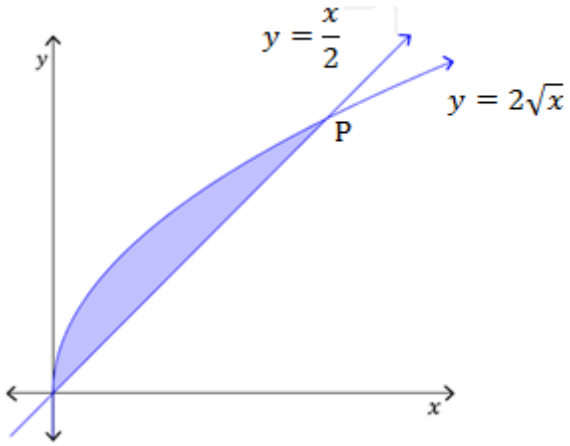
(ii) Show that the coordinates of point D are $(\frac{9}{2}, -3)$. **2**

(iii) Show that $\angle PFD = 90°$. **2**

End of Question 15

Question 16 (15 marks) **Use a SEPARATE writing booklet**

(a) The region between the curve $y = 2\sqrt{x}$ and $y = \frac{x}{2}$ is rotated about the x axis.



(i) Show that the coordinates of point P are 8 and 16. **1**

(ii) Find the volume of the solid in exact form. **2**

(b) Kate wants to save \$30000 for her holiday. She invests \$150 at the beginning of each month. Interest is paid at the rate of 12% per annum compounded monthly.

(i) How much money will Kate save after 5 years? **2**

(ii) How many months will it take Kate to reach her goal? **2**

(c) Consider the function $y = 1 - 2\sin x$.

(i) Find the exact values where the function y cuts the x-axis for $0 \leq x \leq 2\pi$. **2**

(ii) Find the values of y when $x = 0$ and $x = 2\pi$. **1**

(iii) Draw a neat sketch of the function y for $0 \leq x \leq 2\pi$. **2**

(iv) Find the area bounded by the curve $y = 1 - 2\sin x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{2}$, in exact form. **3**

End of paper

Student Number: Solutions

2016 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics

Multiple Choice Answer Sheet

Completely fill the response circle representing the most correct answer

	A	B	C	D
1.	○	●	○	○
2.	●	○	○	○
3.	○	○	●	○
4.	○	○	●	○
5.	○	○	○	●
6.	○	○	●	○
7.	●	○	○	○
8.	○	●	○	○
9.	○	●	○	○
10.	○	○	●	○

2016 Maths Trial (Solutions)

① 0.33 B

② $|4x+2|=6$

$4x+2=6$ $\quad -4x+2=-6$

$4x=4$ $\quad 4x=-8$ A

$x=1$ $\quad x=-2$

③ $215°$

$180° = \pi$

$215° = \frac{\pi}{180} \times 215$ C

$= \frac{43\pi}{36}$

④ $\frac{\cos(\frac{\pi}{2}-\alpha)}{\sin(2\pi-\alpha)} = \frac{\sin\alpha}{-\sin\alpha} = -1$ C

⑤ $\int (2x+1)^5 dx = \frac{(2x+1)^6}{6\times 2} + C$ D

$= \frac{(2x+1)^6}{12} + C$

⑥ $\frac{3\sqrt{3}}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{3\sqrt{3}(\sqrt{7}+2)}{7-4} = \sqrt{3}(\sqrt{7}+2)$ C

⑦ A

⑧ B

⑨ $\log_a 7 = x$ $\quad \log_a 3 = y$

$\log_a 63 = \log_a 7\times 9 = \log_a 7 + \log_a 3^2 = \log_a 7 + 2\log_a 3$

$= x + 2y$ B

⑩ $\ln 8 + \ln 16 + \ln 32 + \cdots$

$\ln 2^3 + \ln 2^4 + \ln 2^5 + \cdots$

$3\ln 2 + 4\ln 2 + 5\ln 2 + \cdots$ $\quad \therefore d = \ln 2$ C

Question 11

(11)

a) $\sqrt{50} - 3\sqrt{75} + \sqrt{18} = a\sqrt{2} - b\sqrt{3}$

LHS: $5\sqrt{2} - 15\sqrt{3} + 3\sqrt{2}$

$= 8\sqrt{2} - 15\sqrt{3}$ $\quad \therefore a = 8 \quad b = 15$

- 1 mark for SIMPLIFYING CORRECTLY
- 2 marks for CORRECT VALUES OF a AND b.

b) $y = \sqrt{2x-1} - 4$

D: $2x - 1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

domain: all real $x \geq \frac{1}{2}$

R: $\sqrt{2x-1} \geq 0$

$\sqrt{2x-1} - 4 \geq -4$

range: all real $y \geq -4$

- 1 mark for CORRECT DOMAIN OR RANGE
- 2 marks for CORRECT DOMAIN AND RANGE

c) $\lim_{x \to 0} \frac{\sin 5x}{4x} = \lim_{x \to 0} \frac{1}{4} \left(\frac{\sin 5x}{5x}\right)^{\to 1} \times 5 = \frac{5}{4}$

- 1 mark for EXPRESS LIMIT AS A FUNCTION OF $\lim_{x \to 0} \frac{\sin 5x}{5x}$
- 2 marks for CORRECT ANSWER

d) $y = (5t^4 - 8)^9$

$y' = 9(5t^4 - 8)^8 \times 20t^3$

$= 180t^3(5t^4 - 8)^8$

- 1 mark for SUBSTANTIAL EFFORT TO DIFFERENTIATE CORRECTLY
- 2 marks for CORRECT ANSWER

e) $\int (5x^3 + \sin 4x)\,dx = 5 \times \frac{x^4}{4} - \frac{\cos 4x}{4} + c$

$= \frac{5x^4 - \cos 4x}{4} + c$

- 1 mark for INTEGRATING A TERM CORRECTLY
- 2 marks for: CORRECT ANSWER

Question 11 (cont.)

f) $y'' = 6x - 2$ S.P. at $(3, 0)$

$y' = \int (6x - 2)\, dx$

$y' = \frac{6x^2}{2} - 2x + c$

$= 3x^2 - 2x + c$

S.P. at $(3,0)$ $\therefore y' = 0$

$27 - 6 + c = 0 \quad \therefore c = -21$

$y' = 3x^2 - 2x - 21$

$y = \int (3x^2 - 2x - 21)\, dx$

$y = \frac{3x^3}{3} - \frac{2x^2}{2} - 21x + c_1$

$y = x^3 - x^2 - 21x + c_1$

$x = 3,\ y = 0$

$27 - 9 - 63 + c_1 = 0 \quad \therefore c_1 = 45$

$f(x) = x^3 - x^2 - 21x + 45$

- 1 mark for CORRECT PRIMITIVE OF y''
- 2 marks for CORRECT PRIMITIVE OF y'
- 3 marks for CORRECT ANSWER

g)(i) $x^2 - 4x - 8y - 4 = 0$

$x^2 - 4x = 8y + 4$

$x^2 - 4x + 4 = 8y + 4 + 4$

$(x - 2)^2 = 8(y + 1)$

$V(2, -1)$

- 1 mark for PUTTING EQUATION IN THE FORM $(x-h)^2 = 4a(y-k)$ OR SUBSTANTIAL EFFORT USING A CORRECT TECHIQUE TO FIND THE VERTEX
- 2 marks for CORRECT ANSWER

Question 12

a) $3 \times 9^x + 2 \times 3^x - 1 = 0$

$3 \times (3^x)^2 + 2 \times 3^x - 1 = 0$

Let $3^x = t$

$3t^2 + 2t - 1 = 0$

$t = \frac{-2 \pm \sqrt{4+12}}{6} = \frac{-2 \pm 4}{6}$

$t = -1, \quad t = \frac{1}{3}$

$t = -1$ or $t = \frac{1}{3}$

$3^x = -1$ $\qquad$ $3^x = \frac{1}{3}$

no solutions $\qquad$ $3^x = 3^{-1}$ $\qquad$ $\therefore \boxed{x = -1}$

- 1 mark for correct simplification to a quadratic equation
- 2 marks for correct solving of the quadratic equation
- 3 marks for correct solution of x and dismissing one solution

b)

$3x^2 - 4x - 8 = 0$

(i) $\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

$\alpha + \beta = \frac{4}{3}$ $\qquad$ $\alpha\beta = -\frac{8}{3}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{16}{9} + \frac{16}{3}$

$= \frac{16+48}{9}$

$= \frac{64}{9}$

- 1 mark for correct sum and product
- 2 marks for recognising $(\alpha+\beta)^2 - 2\alpha\beta$ and correct answer

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{\frac{64}{9}}{-\frac{8}{3}} = -\frac{8}{3}$

- 1 mark for correct answer

Question 12 (cont.)

c)(i) $y = x e^{\sin x}$

$y' = e^{\sin x} + x e^{\sin x} \times \cos x$

$= e^{\sin x}(1 + x\cos x)$

- 1 mark for correctly using the product rule
- 2 marks for correct answer

(ii) $y = \frac{\ln x}{x}$

$y' = \frac{\frac{1}{x} \times x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

- 1 mark for correctly using either the quotient or product rule
- 2 marks for correct answer

d)

(i) $\int \frac{x^3+1}{x^2}\,dx = \int \frac{x^3}{x^2}\,dx + \int \frac{dx}{x^2}$

$= \int x\,dx + \int x^{-2}\,dx$

$= \frac{x^2}{2} + \frac{x^{-1}}{-1} + C$

$= \frac{x^2}{2} - \frac{1}{x} + C$

- 1 mark for splitting and simplifying the fraction
- 2 marks for correct integration + C

ii) $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x\right) = \cos x - \frac{3}{3}\sin^2 x \cos x$

$= \cos x(1 - \sin^2 x)$

$= \cos x \times \cos^2 x$

$= \cos^3 x$

- 1 mark for correct differentiation
- 2 marks for substituting trig. identities to show the RHS.

Question 12 (cont.)

$$\int 3\cos^3 x\,dx = 3\int \cos^3 x\,dx$$

$$= 3\left(\sin x - \frac{1}{3}\sin^3 x\right) + C$$

$$= 3\sin x - \sin^3 x + C$$

- 1 mark for correct integration

Question 13

a) A $(-3,1)$ B $(2,4)$ C $(-1,-1)$

i) $d(A,B) = \sqrt{5^2+3^2} = \sqrt{34}$

$d(A,C) = \sqrt{4^2+2^2} = \sqrt{20}$

$d(B,C) = \sqrt{9+25} = \sqrt{34}$ ✓

$AB = BC \therefore \Delta ABC$ is isosceles.

- 1 mark for 1 correct distance
- 2 marks for 2 correct dist. and making a statement

(ii) AC: $m_{AC} = \frac{2}{-2} = -1$

$$y+1 = -1(x+1)$$

$$y+1 = -x-1$$

$$x+y+2 = 0$$

- 1 mark for gradient
- 2 marks for correct application of $y-y_1 = m(x-x_1)$

(iii) B $(2,4)$ $x+y+2=0$

$$d = \frac{|2+4+2|}{\sqrt{2}} = \frac{8}{\sqrt{2}} \text{ or } \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

- 1 mark for correct subst. and simplification

Question 13 (cont.)

(iv) $A = \frac{1}{2} AC \times d = \frac{1}{2} \times \sqrt{20} \times 4\sqrt{2} = 2\sqrt{40}$

$= 4\sqrt{10}\ u^2$

$= 8u^2$

- 1 mark for correct application of area

b)

(i)

	x_0	x_1	x_2	x_3	x_4
x	0	1	2	3	4
y	0	1.84	1.35	0.75	0.37

- 1 mark for y value

(ii) $\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4 \times \text{odds} + 2 \times \text{evens}\right]$

$h = \frac{4-0}{4} = 1$

$\int_0^4 5xe^{-x}\,dx = \frac{1}{3}\left[0 + 0.37 + 4(1.84 + 0.75) + 2 \times 1.35\right]$

$= 4.48$

- 1 mark for h and substitution
- 2 marks for correct application and answer

(iii) $y' = 5e^{-x} + 5xe^{-x} \times (-1)$

$= 5e^{-x} - 5xe^{-x}$

$y'' = -5e^{-x} - 5e^{-x} + 5xe^{-x}$

$= 5e^{-x}(x-2)$

- 1 mark for y'
- 2 marks for y''

Question 13 (cont.)

(iv) $y' = 5e^{-x}(1-x)$

$y' = 0$ if $1-x = 0$ $\therefore x = 1$

		1	
$1-x$	+	−	$1-x \geq 0$ $\quad e^{-x} > 0$
y'	+	−	$x \leq 1$
y	↗	↘	

for $x = 1$ $\quad y = 1.84$

$(1, 1.84)$ is a max

$y'' = 5e^{-x}(x-2)$

$y'' = 0$ if $x-2 = 0$ $\therefore x = 2$

$(2, 1.35)$ is a possible point of inflection

$5e^{-x} > 0$

		2	
$x-2$	−	+	$x-2 > 0$
y''	−	+	$x > 2$
y	∩	∪	concavity changes

$(2, 1.35)$ is a point of inflection

- 1 mark for x-coord. of SP.
- 2 marks for SP
- 3 marks for SP and a point of inflexion
- 4 marks for testing the point of inflex. + all above

Question 14

a) i) $A = \frac{1}{2} r^2 \alpha$

$625 = \frac{1}{2} r^2 \alpha \quad \therefore \quad \alpha = \frac{2 \times 625}{r^2}$

$\alpha = \frac{1250}{r^2}$

- 1 mark for $\alpha = \frac{1250}{r^2}$

(ii) $P = r + r + r\alpha$

$= 2r + \frac{1250}{r}$

- 1 mark for correct answer

(iii) $P = 2r + 1250 r^{-1}$

$\frac{dP}{dr} = 2 + 1250 r^{-2} \times (-1)$

$= 2 - \frac{1250}{r^2}$

$= \frac{2r^2 - 1250}{r^2}$

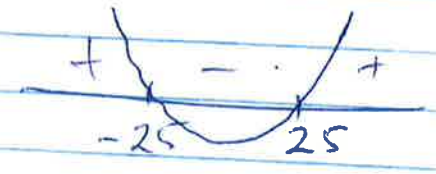
$\frac{dP}{dr} = 0$ if $2r^2 - 1250 = 0$

$r^2 - 625 = 0$

$r^2 = 625$

$r = 25$

$(r-25)(r+25) = 0$



		25	
$\frac{dP}{dr}$	−		+
P	↘		↗

at 25 → min.

- 1 mark for $2r^2 - 1250 = 0$
- 2 marks for $r = 25$
- 3 marks for checking $r = 25$ is min.

Question 14 (cont.)

(iv) for $r = 25$

$$\alpha = \frac{1250}{25^2} = 2 \text{ rad.}$$

$$\alpha \approx 115^\circ$$

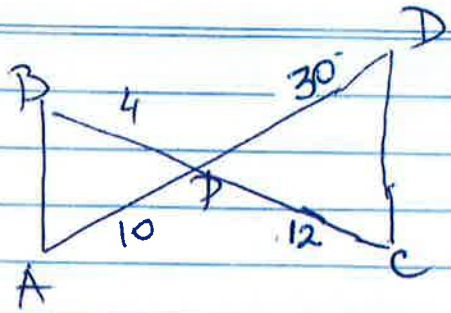
$\pi = 180^\circ$

$2 \text{ rad} = \frac{180^\circ}{\pi} \times 2$

- 1 mark for α in degrees

b)

(i)



$\triangle APB$ and $\triangle CPD$

$\angle APB = \angle CPD$ (vertically opposite angles are equal.)

$$\frac{AP}{PB} = \frac{10}{4} = \frac{5}{2}$$

$$\frac{PD}{PC} = \frac{30}{12} = \frac{5}{2}$$

2 pairs of matching sides are in the same ratio and included angles are equal

$\therefore \triangle APB \parallel\parallel \triangle CPD$

- 1 mark for ratio of sides
- 2 marks for correct reasoning (including vertically opp angles)

(ii) All matching angles in similar triangles are equal $\therefore \angle ABP = \angle PCD$

These are alternate angles $\therefore AB \parallel CD$

- 1 mark for $\angle ABP = \angle PCD$ with correct reason
- 2 marks for alternate angles with reason

Question 14 (cont.)

c) $P = P_0 e^{kt}$

(i) $t = 0 \quad P = 15000$

$15000 = P_0 e^{k \times 0} \quad \therefore P_0 = 15000$

$P = 15000 e^{kt}$

2010!, $t = 10 \quad P = 35000$

$35000 = 15000 e^{10k}$

$e^{10k} = \frac{35}{15} \qquad e^{10k} = \frac{7}{3}$

$10k = \ln \frac{7}{3}$

$k = \frac{1}{10} \ln \frac{7}{3} \qquad k = 0.0847247\ldots$

$k \approx 8.5\%$

- 1 mark for $P_0 = 15000$ or $P = 15000e^{kt}$
- 2 marks for correct substitution $35000 = 15000e^{10k}$
- 3 marks for $k = 0.8472\ldots$ or $k = 8.5\%$

(ii) $P = 50000$

$P = 15000 e^{0.085t}$

$50000 = 15000 e^{0.085t}$

$e^{0.085t} = \frac{50}{15} = \frac{10}{3}$

$0.085t = \ln \frac{10}{3}$

$t = \frac{1}{0.085} \ln \frac{10}{3}$

$t = 14.16438\ldots$

after 15 years

- 1 mark for $50000 = 15000 e^{0.085t}$
- 2 marks for $t = 15$ years or $t = 14.2$ years

Question 15

(15) a) $v = \dfrac{4}{2t+1}$ m/s

(i) initially: $t=0 \quad x=-2$

$$x = \int \frac{4\,dt}{2t+1} = \frac{4}{2}\int \frac{2\,dt}{2t+1} = 2\ln(2t+1) + c$$

$t=0: \quad x = 2\ln 1 + c = -2$

$$c = -2$$

$$x = 2\ln(2t+1) - 2$$

- 1 mark for correct integration + c
- 2 marks for correct value of c

(ii) $x = 0$

$$2\ln(2t+1) - 2 = 0$$
$$\ln(2t+1) = 1$$
$$2t+1 = e$$
$$2t = e-1$$
$$t = \frac{e-1}{2} \text{ sec.}$$

- 1 mark for solving to $2t+1=e$
- 2 marks for correct value of t.

Question 15 (cont.)

(iii) $V = \frac{4}{2t+1} = 4(2t+1)^{-1}$

$a = -4(2t+1)^{-2} \times (2)$

$= -\frac{8}{(2t+1)^2}$

$(2t+1)^2 > 0 \quad -8 < 0$

$\therefore a < 0$

- 1 mark for correct differentiation
- 2 marks for correct reasoning to prove $\ddot{x} < 0$

b) (i) $P(1,4) = \frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$

- 1 mark for correct answer

(ii) $P(\text{sum less than } 5) = P(1,1) + P(1,3) + P(3,1)$

$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5}$

$= \frac{2}{15} + \frac{1}{15}$

$= \frac{3}{15}$

$= \frac{1}{5}$

- 1 mark for correct combinations and one correct prob.
- 2 marks for two other correct probabilities

Question 15 (cont.)

c) $P(12, 12)$ $\quad x^2 = 12y$

(i) $y = \frac{x^2}{12}$

$\frac{dy}{dx} = \frac{2x}{12} = \frac{x}{6}$

at $x = 12$ $\quad m_t = \frac{12}{6} = 2$

$y - 12 = 2(x - 12)$

$y - 12 = 2x - 24$

$2x - y - 12 = 0$

- 1 mark for gradient
- 2 marks for equation of the tangent

(ii) $4a = 12 \quad \therefore a = 3$

directr. $y = -3$

$2x - y - 12 = 0$

$y = -3$

$2x + 3 - 12 = 0$

$2x = 9$

$x = \frac{9}{2}$

$D\left(\frac{9}{2}, -3\right)$

- 1 mark for clearly showing the y-value of D
- 2 marks for x-value of D

Question 15 (cont.)

(iii) $\angle PFD = 90°$?

$F(0, 3)$ $D(\frac{9}{2}, -3)$ $P(12, 12)$

$$m_{FD} = \frac{6}{-\frac{9}{2}} = -\frac{12}{9} = -\frac{4}{3}$$

$$m_{FP} = \frac{12-3}{12-0} = \frac{9}{12} = \frac{3}{4}$$

$$m_{FD} \times m_{FP} = -\frac{4}{3} \times \frac{3}{4} = -1$$

$\therefore$ $FD \perp FP$

$\therefore \angle PFD = 90°$

- 1 mark for gradients of FP and FD
- 2 marks for showing FP ⊥ FD clearly

Question 16

a) (i) $2\sqrt{x} = \frac{x}{2}$ $\therefore$ $4x = \frac{x^2}{4}$ $/ \times 4$

$x^2 - 16x = 0$

$x(x-16) = 0$

$x = 0$, $x = 16$

for $x = 16$ $y = \frac{16}{2} = 8$

$P(16, 8)$

- 1 mark for CORRECT ANSWER

(ii) $V = \pi \int_0^{16} \left((2\sqrt{x})^2 - \left(\frac{x}{2}\right)^2 \right) dx$

$= \pi \int_0^{16} \left(4x - \frac{x^2}{4} \right) dx$

$= \pi \left[\frac{4x^2}{2} - \frac{x^3}{12} \right]_0^{16}$

$= \pi \left[2x^2 - \frac{x^3}{12} \right]_0^{16}$

$= \pi \left[2 \times 16^2 - \frac{16^3}{12} \right] =$

$= \pi \left(512 - \frac{4096}{12} \right)$

$= \pi \left(512 - \frac{1024}{3} \right)$

$= \frac{512\pi}{3}$

- 1 mark for CORRECT SIMPLIFIED EXPRESSION FOR THE VOLUME INTEGRAL
- 2 marks for CORRECT ANSWER

Question 16 (cont.)

b)

(i) $r = 12\%$ p.a $= 0.12$

$r = 0.01$ per month

$n = 5 \times 12 = 60$

First annuity will earn the interest for 60 months

$A_1 = 150(1+0.01)^{60}$

$= 150 \times 1.01^{60}$

Second annuity will earn the interest for 59 months

$A_2 = 150 \times 1.01^{59}$

⋮

60th annuity will earn the interest for 1 month

$A_{60} = 150 \times 1.01$

$\text{Total} = A_1 + A_2 + \cdots + A_{60}$

$= 150 \times 1.01^{60} + 150 \times 1.01^{59} + \cdots + 150 \times 1.01$

$= 150\,(1.01^{60} + 1.01^{59} + \cdots + 1.01)$

$= 150\,(1.01 + 1.01^2 + \cdots + 1.01^{60})$

$1.01 + 1.01^2 + \cdots + 1.01^{60}$ is a geometric series with $a = 1.01$ $r = 1.01$

$$S_{60} = \frac{a(r^n - 1)}{r - 1} = \frac{1.01(1.01^{60} - 1)}{1.01 - 1}$$

$$T = 150 \times \frac{1.01(1.01^{60} - 1)}{1.01 - 1} = \$12372.95$$

Kate will save \$12372.95 after 5 years.

- 1 mark for CORRECT EXPRESSION FOR THE TOTAL INVESTMENT
- 2 marks for CORRECT ANSWER.

Question 16 (cont.)

(ii)

$$30000 = 150 \times \frac{1.01(1.01^n - 1)}{1.01 - 1}$$

$$1.01^n - 1 = \frac{30000(1.01-1)}{150 \times 1.01}$$

$$1.01^n = \frac{30000(1.01-1)}{150 \times 1.01} + 1$$

$$n = \log_{1.01}\left(\frac{30000(1.01-1)}{150 \times 1.01} + 1\right)$$

$$n = \frac{\ln \frac{30000(1.01-1)}{150 \times 1.01} + 1}{\ln 1.01}$$

$$n = 109.744\ldots$$

$$n = 110 \text{ months}$$

- 1 mark for CORRECT EXPRESSION FOR 1.01^n
- 2 marks for CORRECT ANSWER.

c) (i) $1 - 2\sin x = 0$ $\quad 0 \le x < 2\pi$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ, 180^\circ - 30^\circ$$

$$x = 30^\circ, 150^\circ$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

- 1 mark for CORRECT ACUTE SOLUTION $x = \frac{\pi}{6}$
- 2 marks for CORRECT ANSWER

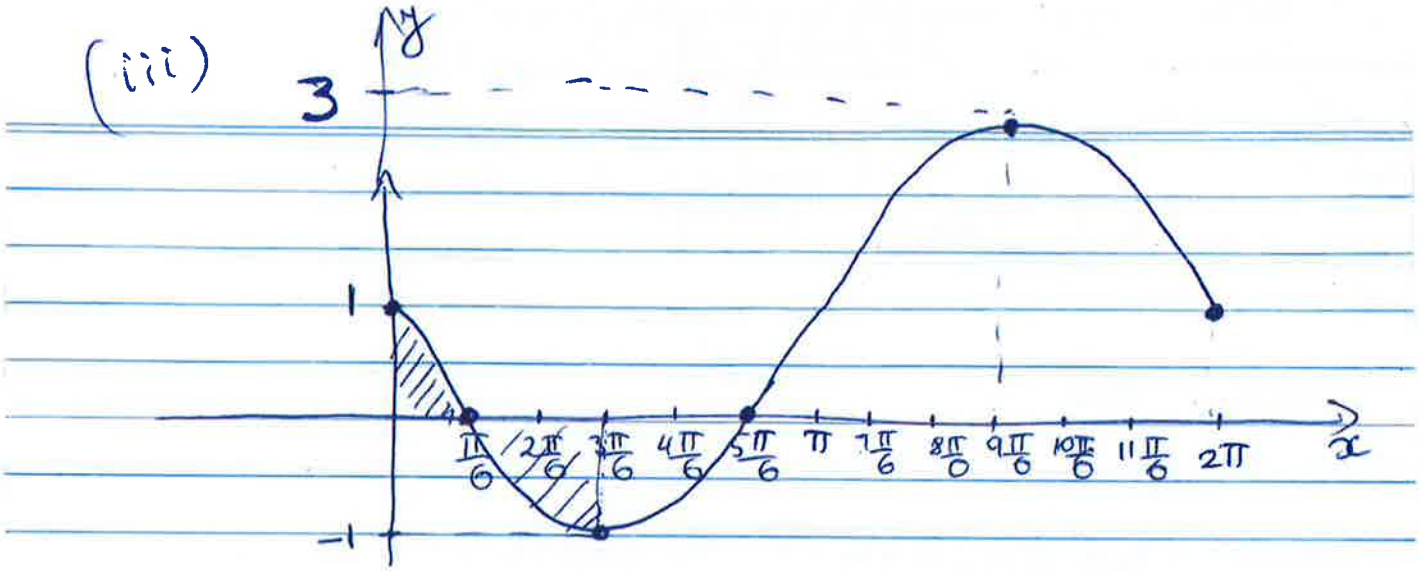
Question 16 (cont.)

(ii) $x = 0 \quad y = 1 - 2\sin 0 = 1$

$x = 2\pi \quad y = 1 - 2\sin 2\pi = 1$

- 1 mark for CORRECT ANSWERS

(iii)



$-1 \leq \sin x \leq 1$

$-2 \leq 2\sin x \leq 2$

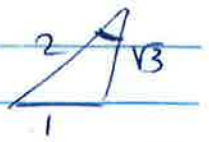
$-2 \leq -2\sin x \leq 2$

$-1 \leq 1 - 2\sin x \leq 3$

- 1 mark for CORRECT SHAPE AND RANGE
- 2 marks for CORRECT CURVE, INCLUDING CORRECT INTERCEPTS AND ENDPOINTS.

Question 16 (cont.)

(iv) $A = \int_0^{\frac{\pi}{6}} (1 - 2\sin x)\,dx - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - 2\sin x)\,dx$



$A = \left[x + 2\cos x\right]_0^{\frac{\pi}{6}} - \left[x + 2\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \left[\frac{\pi}{6} + 2\cos\frac{\pi}{6} - 2\cos 0\right] - \left[\frac{\pi}{2} + 2\cos\frac{\pi}{2} - \frac{\pi}{6} - 2\cos\frac{\pi}{6}\right]$

$= \left[\frac{\pi}{6} + 2 \times \frac{\sqrt{3}}{2} - 2\right] - \left[\frac{\pi}{2} - \frac{\pi}{6} - 2\frac{\sqrt{3}}{2}\right]$

$\frac{\pi}{6} + \sqrt{3} - 2 - \frac{\pi}{2} + \frac{\pi}{6} + \sqrt{3}$

$\frac{\pi}{3} - \frac{\pi}{2} + 2\sqrt{3} - 2$

$2\sqrt{3} - 2 - \frac{\pi}{6} \simeq 0.94\ u^2$

- 1 mark for CORRECT INTEGRAL SPLIT INTO TWO AREAS BETWEEN $x = 0$ TO $x = \frac{\pi}{6}$, AND $x = \frac{\pi}{6}$ TO $x = \frac{\pi}{2}$
- 2 marks for CORRECT INTEGRATION
- 3 marks for CORRECT ANSWER

– End of solutions –