St George Girls High School

Trial Higher School Certificate Examination

2007

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Write your student number on every booklet
- Begin each question in a new booklet
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks (120)

- Attempt Questions 1-10
- All questions are of equal value

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 (12 marks) – **Start a new booklet** **Marks**

a) Find the value of $\dfrac{27.1}{\sqrt{25.04} \times 57.2}$ correct to 3 significant figures 2

b) By rationalizing the denominator, express $\dfrac{\sqrt{5}}{4-\sqrt{5}}$ in the form $a + b\sqrt{5}$ 2

c) Differentiate $\tan\dfrac{x}{2}$ with respect to x

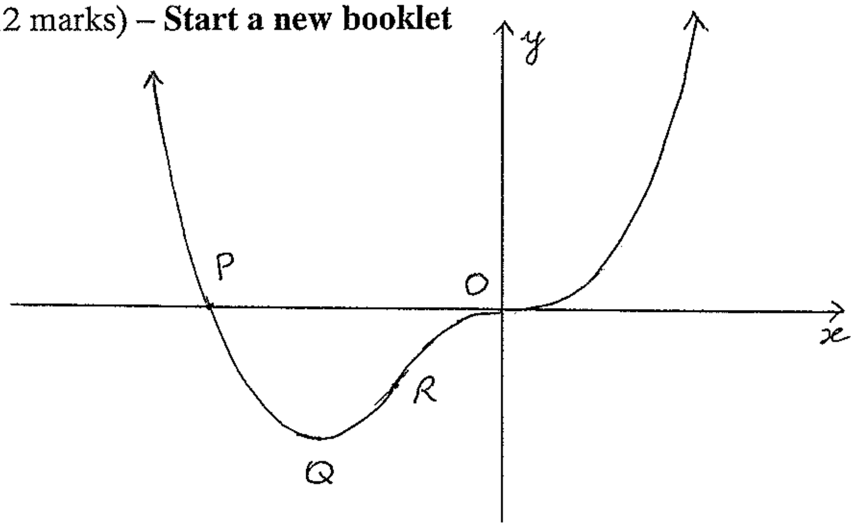
d) Solve for x: $\dfrac{x-2}{2} + \dfrac{x+1}{5} = 2$ 3

e) Find the primitive function of $3 - \dfrac{3}{x^2}$ 2

f) Express $\dfrac{1-x^{-2}}{1-x^2}$ in its simplest form 2

Question 2 (12 marks) – **Start a new booklet** **Marks**

a)



The diagram shows a sketch of the curve $y = 2x^3 + x^4$. The curve cuts the x-axis at P, has a minimum at Q and a point of inflection at R. 6

(i) Find the coordinates of P.

(ii) Find the coordinates of Q.

(iii) Find the coordinates of R.

b) Solve $|x-1| = 2x-1$ 3

c) Consider the parabola with equation $y^2 = 12(1-x)$ 3

(i) Find coordinates of the vertex of the parabola.

(ii) Find the coordinates of the focus of the parabola.

Question 3 (12 marks) – **Start a new booklet** **Marks**

a) Differentiate the following functions with respect to x 6

(i) $(2-3x)^5$

(ii) $x\ln x$

(iii) $\dfrac{2x}{x^2-1}$

b) On the number plane, the points $A(-1, 1)$, $B(4, 3)$, $C(3, 6)$ and $D(-2, 4)$ form a parallelogram. 6

(i) Show that the equation of AB is $2x-5y+7=0$

(ii) Find the perpendicular distance from the point C to the line AB.

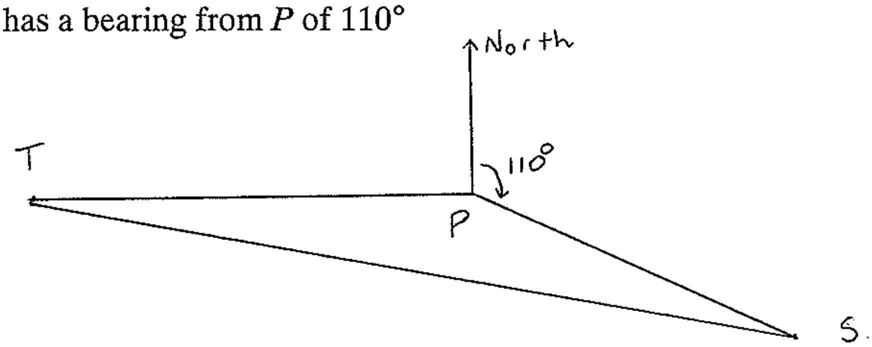
(iii) Hence find the area of the parallelogram $ABCD$ 2

Question 4 (12 marks) – **Start a new booklet** **Marks**

a) (i) Evaluate $\int_1^3 e^{2x}\,dx$ 2

(ii) Find $\int \frac{x^2}{x^3-4}\,dx$ 2

b) The diagram shows a point P which is 20km due east of the point T. The point S is 10km from P and has a bearing from P of 110° 4



(i) Find the length of TS.

(ii) Find the bearing of T from S.

c) In a card competition the first prize is \$100, the second prize is \$90, the third prize is \$80 and so on. 4

(i) Write down the value of the nth prize.

(ii) If the total prize money is \$550 how many prizes will there be?

Question 5 (12 marks) – **Start a new booklet** **Marks**

a) The table shows the values of a function $f(x)$, for 5 values of x.

x	0	2	4	6	8
$f(x)$	12	25	20	18	8

Use Simpson's rule with these 5 values to estimate $\int_0^8 f(x)\,dx$ 3

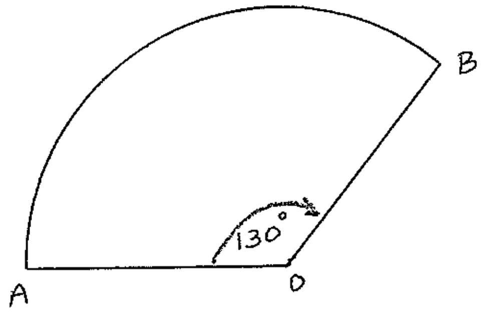
b) (i) Sketch $y = x^2 + 6$ and $y = 12 - x$ on the same axes. 5

(ii) Find the area in the first quadrant bounded by the y-axis, $y = x^2 + 6$ and $y = 12 - x$

c) Matthea wishes to invest \$A at the beginning of each month at a compound interest rate of 0.7% per month. How much does she invest each month in order to have \$15 000 saved at the end of the first year. 4

Question 6 (12 marks) – **Start a new booklet** **Marks**

a)



A windscreen wiper 25cm long traces out the above pattern on a wet day. 3

(i) Express 130° in radians

(ii) Find the length of the arc AB.

(iii) Calculate the area of the sector swept out by the wiper.

b) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 0$ 3

c) A particle moves in a straight line such that its distance, x metres, from a fixed point O at any time t seconds is given by: 6

$$x = 4 + 6t - t^3$$

(i) Find an equation for its velocity after t seconds.

(ii) At what time does the particle stop?

(iii) Where is the particle initially?

(iv) Find the velocity after 2 seconds.

(v) How far has the particle travelled in the first 2 seconds.

Question 7 (12 marks) – **Start a new booklet** **Marks**

a) Simplify $\dfrac{\sin(90-\theta)}{\cos(180-\theta)}$ 2

b) A company donated \$25 000 to charity during 2000. Each subsequent year after 2000 it donated 80% of its previous years donation to charity. 4

(i) What was the amount donated to charity during 2005?

(ii) If this arrangement continued indefinitely, what was the maximum total amount the company would donate to charity?

c) The size of an insect colony is given by the equation $P = 1000e^{kt}$ where P is the population after t days. 6

(i) What is the initial population?

(ii) If there are 1200 insects after one day, find the value of k, correct to 2 decimal places.

(iii) When will the colony double its initial population (answer correct to the nearest day)

(iv) At what rate is the population growing after 2 days?

Question 8 (12 marks) – **Start a new booklet** **Marks**

a) (i) Write down the discriminant of $x^2 - 3kx + 9k$ 3

(ii) For what values of k is $x^2 - 3kx + 9k$ always positive?

b) If α and β are the roots of $\frac{1}{x} = x + p$, find in terms of p 4

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

c) Catherine had a full drink bottle containing 500ml of Gatorade. She drank from it so that the volume, V millilitres, of Gatorade in the bottle changed at a rate given by $\frac{dV}{dt} = \left(\frac{2}{5}t - 20\right)$ millilitres per second. 5

(i) Find a formula for V.

(ii) Show that it took Catherine 50 seconds to drink the entire contents of the bottle.

(iii) How long to the nearest second, did it take Catherine to drink half the contents of the bottle?

Question 9 (12 marks) – **Start a new booklet** **Marks**

a) A and B are the points $(-2, 0)$ and $(2, -1)$ respectively on the number plane. The point P has coordinates (x, y) and given that $\angle APB = 90^\circ$ show that the locus of P is the curve whose equation is $x^2 + y^2 + y = 4$ 4

b) Consider the function $y = \sin x + \cos x$ in the domain $0 \le x \le 2\pi$ 8

(i) Find $\dfrac{dy}{dx}$

(ii) Find the maximum and minimum values of $\sin x + \cos x$ in the given domain.

(iii) Show that the curve cuts the x-axis at $x = \dfrac{3\pi}{4}$ and $x = \dfrac{7\pi}{4}$

(iv) Hence sketch the curve of $y = \sin x + \cos x$ in the domain $0 \le x \le 2\pi$

Question 10 (12 marks) – **Start a new booklet** **Marks**

a) (i) Sketch the curve $y = \tan x, 0 \leq x \leq \dfrac{\pi}{2}$ 4

(ii) The area under the curve $y = \tan x$ between $x = 0$ and $x = \dfrac{\pi}{3}$ is rotated about the x-axis. Find the volume of the solid formed.

b) Emma inherited one million dollars on the 1st of January 2000. She decided to deposit the whole amount into an account which paid 6% p.a. The interest was calculated and paid half-yearly. Emma decided that she would make an annual withdrawal of \$75 000, starting on the 1st of January 2001. 8

(i) Write down an expression for the amount in Emma's account immediately after the first withdrawal.

(ii) Show that the amount is Emma's account immediately after her third withdrawal is given by $10^6 \times (1.03)^6 - 75000(1 + 1.03^2 + 1.03^4)$

(iii) At this rate of withdrawal how many years will the money last?

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Solutions T.A.S.C. - Mathematics 2007

Q1

(a) 0.0947 (3 sig fig)

$(0.09467\ldots)$

(b)
$$\frac{\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$= \frac{4\sqrt{5}+5}{16-5}$$
$$= \frac{5}{11} + \frac{4}{11}\sqrt{5}$$

(c)
$$\frac{d}{dx}\left(\tan\frac{x}{2}\right) = \sec^2\frac{x}{2} \times \frac{1}{2}$$
$$= \frac{1}{2}\sec^2\frac{x}{2}$$

(d)
$$(5x-10) + (2x+2) = 20$$
$$7x = 28$$
$$x = 4$$

(e)
$$\int (3 - 3x^{-2})\,dx$$
$$= 3x - \frac{3x^{-1}}{-1} + c$$
$$= 3x + \frac{3}{x} + c$$

(f)
$$\frac{1-\frac{1}{x^2}}{1-x^2} = \frac{\;\frac{x^2-1}{x^2}\;}{1-x^2}$$
$$= \frac{\;-\frac{\cancel{(1-x^2)}}{x^2}\;}{\cancel{(1-x^2)}}$$
$$= -\frac{1}{x^2}$$

Q 2 (a) (i) let $x^3(2+x)=0$

then $x=0, -2$

so $P(-2,0)$ & $O(0,0)$

(ii) $\frac{dy}{dx} = 6x^2+4x^3$

let $6x^2+4x^3=0$

$2x^2(3+2x)=0$

Stat. at $x=-\frac{3}{2}$, $x=0$

x	-2	$-\frac{3}{2}$	-1	0	1
y'	-8	0	2	0	10

Min at $x=-\frac{3}{2}$

$\therefore Q\left(-\frac{3}{2}, \left(-\frac{54}{8}+\frac{81}{16}\right)\right)$

$Q\left(-\frac{3}{2}, -\frac{27}{16}\right)$

(iii) let $\frac{d^2y}{dx^2}=0$

then $12x+12x^2=0$

$12x(1+x)=0$

at $x=-1$ & $x=0$

x	-2	-1	$-\frac{1}{2}$	0	1
y	24	0	-3	0	24

Inflexion at $x=-1$

$\therefore R(-1,-1)$

(b) (i) $x-1=2x-1$

$x=0$

Check $|0-1| \neq 1$

not a soln

$-(x-1)=2x-1$

$3x=2$

$x=\frac{2}{3}$

Check $\left|\frac{2}{3}-1\right| = \frac{1}{3}$

$= \frac{4}{3}-1$ ✓

$x=\frac{2}{3}$ is a soln.

(c) (i) Vertex $(1,0)$

(ii) $(y-0)^2 = -12(x-1)$

$= -4\times 3(x-1)$

$a=3$

Focus $(-2,0)$

Q3

(a) (i) $\frac{d}{dx}\left[(2-3x)^5\right] = 5(2-3x)^4 \times -3$

$= -15(2-3x)^4$

(ii) $\frac{d}{dx}\left[x\ln x\right] = \ln x \times 1 + x \times \frac{1}{x}$

$= \ln x + 1$

(iii) $\frac{d}{dx}\left[\frac{2x}{x^2-1}\right] = \frac{(x^2-1)\times 2 - 2x \times 2x}{(x^2-1)^2}$

$= \frac{2x^2-2-4x^2}{(x^2-1)^2}$

$= \frac{-2(1+x^2)}{(x^2-1)^2}$

(b) (i) $m_{AB} = \frac{3-1}{4--1}$

$= \frac{2}{5}$

$\therefore$ Eqn. $(y-1) = \frac{2}{5}(x--1)$

$5y-5 = 2x+2$

$2x - 5y + 7 = 0$

(ii) $d = \frac{|2\times 3 + -5\times 6 + 7|}{\sqrt{2^2+(-5)^2}}$

$= \frac{|-17|}{\sqrt{29}}$

$= \frac{17}{\sqrt{29}}$

(iii) $d_{AB} = \sqrt{(4-1)^2 + (3-1)^2}$

$= \sqrt{29}$

So, area $= bh$

$= \sqrt{29} \times \frac{17}{\sqrt{29}}$

$= 17$ sq units.

Q4

(a) (i) $\int_1^3 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big]_1^3$

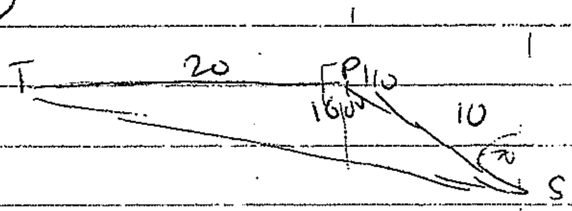
$= \frac{1}{2}(e^6 - e^2)$

$= \frac{e^2}{2}(e^4 - 1)$

(ii) $\int \frac{x^2}{x^3-4}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3-4}\,dx$

$= \frac{1}{3}\ln(x^3-4) + C$

(b)



(i) $TS^2 = 20^2 + 10^2 - 2\times20\times10\times\cos 160$

$TS^2 = 875.877$

$TS = 29.6$ km (1 dec pl)

(ii) $\frac{\sin S}{20} = \frac{\sin 160}{\sqrt{875.877}}$

$\therefore S = 13^\circ 35'$

So Bearing of T from S

$= (360 - [70 + 13^\circ 35'])$

$= 276^\circ 25'$

(c) (i) 100, 90, 80, .. $(110 - 10n)$ $[100 + (n-1)\times -10]$

(ii) $S_n = 550$

$\therefore 550 = \frac{n}{2}[2a + (n-1)\times -10]$

$550 = \frac{n}{2}[200 - 10n + 10]$

$= \frac{n}{2}[210 - 10n]$

$1100 = 210n - 10n^2$

So $n^2 - 21n + 110 = 0$

$(n-10)(n-11) = 0$

$n = 10,\ n = 11$

There are two solutions mathematically

But, practically we do not award a prize of \$0.

$\therefore$ Ten prizes given.

$T_{10} = 100 + (10-1)\times -10$

$= 10$

$T_{11} = 100 + (10 \times -10)$

$= 0$

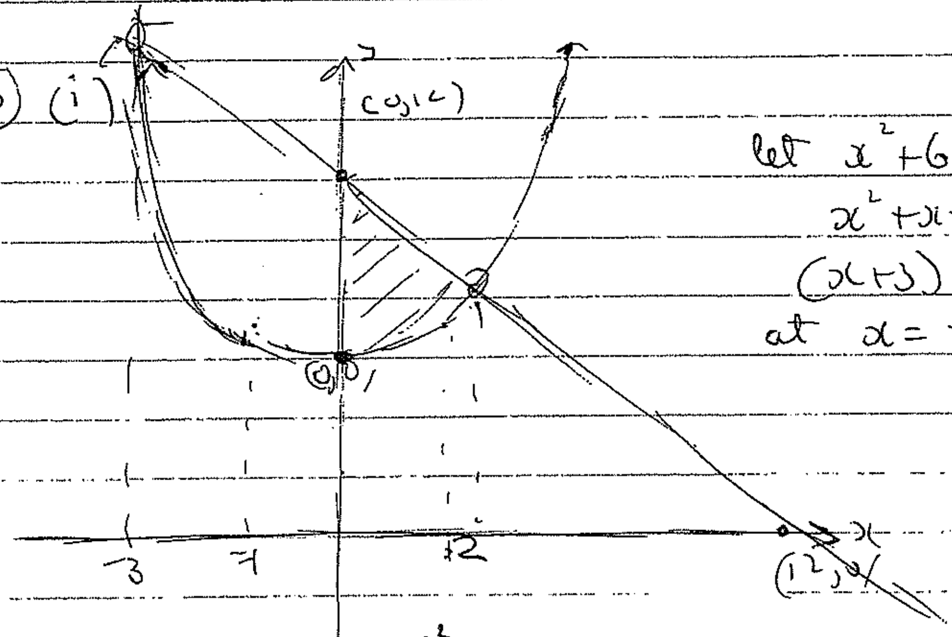
Q5

(a) $A \doteqdot \frac{2}{3}\left\{12 + 4\times 25 + 20\right\} + \frac{2}{3}\left\{20 + 4\times 18 + 8\right\}$

$\doteqdot \frac{2}{3}\left\{232\right\}$

$\doteqdot \frac{464}{3}$ sq units. $\left[154\frac{2}{3}\right] \doteqdot 154.7$

(b) (i)



let $x^2+6 = 12-x$

$x^2+x-6=0$

$(x+3)(x-2)=0$

at $x=-3, 2$

(ii) Area $= \int_0^2 \left[(12-x)-(x^2+6)\right]dx$

$= \int_0^2 (6-x-x^2)\,dx$

$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^2$

$= \left(12-2-\frac{8}{3}\right) - 0$

$= 7\frac{1}{3}$ sq units

(c) Total $= A(1+0.007)^{12} + A(1+0.007)^{11}$

$\quad + \dots + A(1+0.007)$

$15000 = A(1.007)\left[1 + 1.007^2 + \dots + 1.007^{11}\right]$

$= A(1.007)\frac{(1.007^{12}-1)}{1.007-1}$ G.S.

$A = \frac{15000\times .007}{1.007(1.007^{12}-1)}$

$A = \$1194.24$. needs to be invested

Q6

(a) (i) $130 \times \frac{\pi}{180} = \frac{13\pi}{18}$

(ii) $l = 25 \times \frac{13\pi}{18}$

$= \frac{325\pi}{18}$ cm [56.7 cm]

(iii) $A = \frac{1}{2} \times 25^2 \times \frac{13\pi}{18}$

$= \frac{8125\pi}{36}$ cm² [709.0 cm²]

(b) $\frac{dy}{dx} = e^{2x} \times 2$
$= 2e^{2x}$

Gradient of tangent at $x = 0$

$m = 2 \times e^0$
$= 2$

at $x = 0$, $y = e^0$
$y = 1$
$(0, 1)$

Equation: $y - 1 = 2(x - 0)$
$y = 2x + 1$

(c) (i) $v = \frac{dx}{dt} = 6 - 3t^2$

(ii) Stationary when $v = 0$, $6 - 3t^2 = 0$
$3(2 - t^2) = 0$
$t = \sqrt{2}$ [$t > 0$]

Stops after $t = \sqrt{2}$ seconds

(iii) Initially $t = 0$, $x = 4 + 6 \times 0 - 0^2$
$x = 4$

Initially 4 m to right of origin

(iv) Let $t = 2$, $v = 6 - 3 \times 2^2$
$v = -6$ m/sec.

(v) Total distance $= \int_0^{\sqrt{2}} (6 - 3t^2)\,dt + \left| \int_{\sqrt{2}}^{2} (6 - 3t^2)\,dt \right|$

$= \left[6t - t^3 \right]_0^{\sqrt{2}} + \left| \left[6t - t^3 \right]_{\sqrt{2}}^{2} \right|$

$= (6\sqrt{2} - 2\sqrt{2}) + \left| (12 - 8) - 6\sqrt{2} - 2\sqrt{2} \right|$

$= 4\sqrt{2} + 4\sqrt{2} - 4$

$= (8\sqrt{2} - 4)$ m.

* 7.3 m travelled.

Q7

(a) $\frac{\sin(90-\theta)}{\cos(180-\theta)} = \frac{\cos\theta}{-\cos\theta}$

$= -1$

(b) (i) Geometric series, $a = 25000$ $r = .8$ $n = 5$

$T_5 = ar^{n-1}$

$= 25000 \times .8^5$

$= 8192$

\$8192 donated in 2005

(ii) $|r| < 1$ gives infinite sum, $|.8| < 1$

so $S_\infty = \frac{25000}{1 - .8}$

$= \$125000$ is max. total

(c) (i) let $t=0$, $P = 1000\, e^{k \times 0}$

$P = 1000$ initial population

(ii) let $t=1$, $P = 1200$

$1200 = 1000\, e^{k \times 1}$

$1.2 = e^k$

$\ln(1.2) = k$ $\therefore k = 0.18$ [2 dec pl

(iii) Find t when $P = 2000$

$2000 = 1000\, e^{kt}$

$2 = e^{kt}$

$\ln 2 = kt$ $\Rightarrow t = \frac{\ln 2}{\ln(1.2)}$

$= 3.8$

approx 4 days.

(iv) $\frac{dP}{dt} = 1000 k e^{kt}$

$= k \times 1000 e^{kt}$

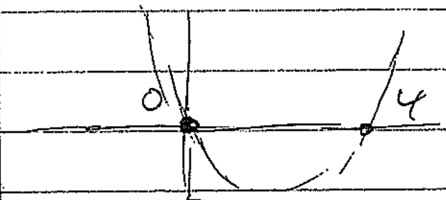
When $t = 2$, $\frac{dP}{dt} = 262.5 \approx$ approx 262 insects per day.

Q8 a) (i) $\Delta = (-3k)^2 - 4\times1\times9k$

$= 9k^2 - 36k$

(ii) Positive definite $a>0$, $1>0$ concave up

$\Delta < 0$, so $9k(k-4) < 0$



From graph.

$0 < k < 4$ is

inequality soln

(b) (i) $\alpha + \beta = \frac{-b}{a}$ for $1 = x^2 + px$

$= -\frac{p}{1}$ $\quad x^2 + px - 1 = 0$

(ii) $\alpha\beta = \frac{c}{a}$

$= -1$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{-p}{-1}$

$= p.$

(c) (i) $V = \int \left(\frac{2}{5}t - 20\right) dt$

$= \frac{2}{5}\times\frac{1}{2}t^2 - 20t + C$

$V = \frac{1}{5}t^2 - 20t + C$ at $t=0$, $V = 500$

$\therefore V = \frac{1}{5}t^2 - 20t + 500$

(ii) Let $V=0$, $\frac{1}{5}t^2 - 20t + 500 = 0$

$t^2 - 100t + 2500 = 0$

$(t-50)^2 = 0$

Empty at $t = 50$, zero of quadratic

(iii) Let $V = 250$, $\frac{1}{5}t^2 - 20t + 500 = 250$

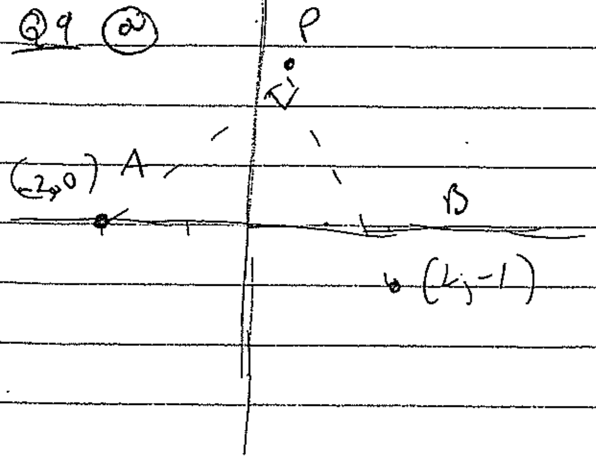
$t^2 - 100t + 1250 = 0$

$t = \frac{100 \pm \sqrt{100^2 - 4\times1\times1250}}{2}$

$= 14.6$ sec. or 85.4

Since empty after 50 seconds it took appr 15 seconds.

Q9 (a)



$m_{AP} \times m_{BP} = -1$

[Perpendicular lines $m_1 \times m_2 = -1$]

$\therefore \dfrac{y-0}{x--2} \times \dfrac{y--1}{x-2} = -1$

$y(y+1) = -(x+2)(x-2)$

$y^2 + y = -[x^2 - 4]$

$\therefore x^2 - 4 + y^2 + y = 0$

or $x^2 + y^2 + y = 4$

(b) (i) $\dfrac{dy}{dx} = \cos x - \sin x$

(ii) let $\dfrac{dy}{dx} = 0$, $\cos x - \sin x = 0$

$\sin x = \cos x$ [Check $x = \frac{\pi}{2}, \frac{3\pi}{2}$ not sol]

$\tan x = 1$

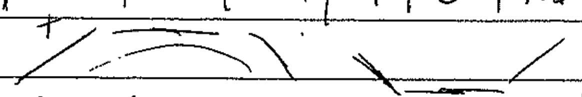
1st & 3rd Qd

$\frac{\pi}{4}$, $\pi + \frac{\pi}{4}$

$\frac{5\pi}{4}$

OR

x	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	π	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$
y'	0.5	0	−0.4	−1	0	+0.4



Local max at $\left(\frac{\pi}{4}, \frac{2}{\sqrt{2}}\right)$ Local Min at $\left(\frac{5\pi}{4}, -\frac{2}{\sqrt{2}}\right)$

or $\dfrac{d^2y}{dx^2} = -\sin x - \cos x$

$= -(\sin x + \cos x)$

at $x = \frac{\pi}{4}$, $\dfrac{d^2y}{dx^2} = -\left(\frac{2}{\sqrt{2}}\right) < 0$ $x = \frac{5\pi}{4}$, $\dfrac{d^2y}{dx^2} = -\left(\frac{-2}{\sqrt{2}}\right) > 0$

Concave down   Concave up.

Max value $\frac{2}{\sqrt{2}}$ $\left(\frac{\sqrt{2}}{}\right)$; Min value $-\sqrt{2}$

(iii) Solve $\sin x + \cos x = 0$

$\sin x = -\cos x$

$\tan x = -1$

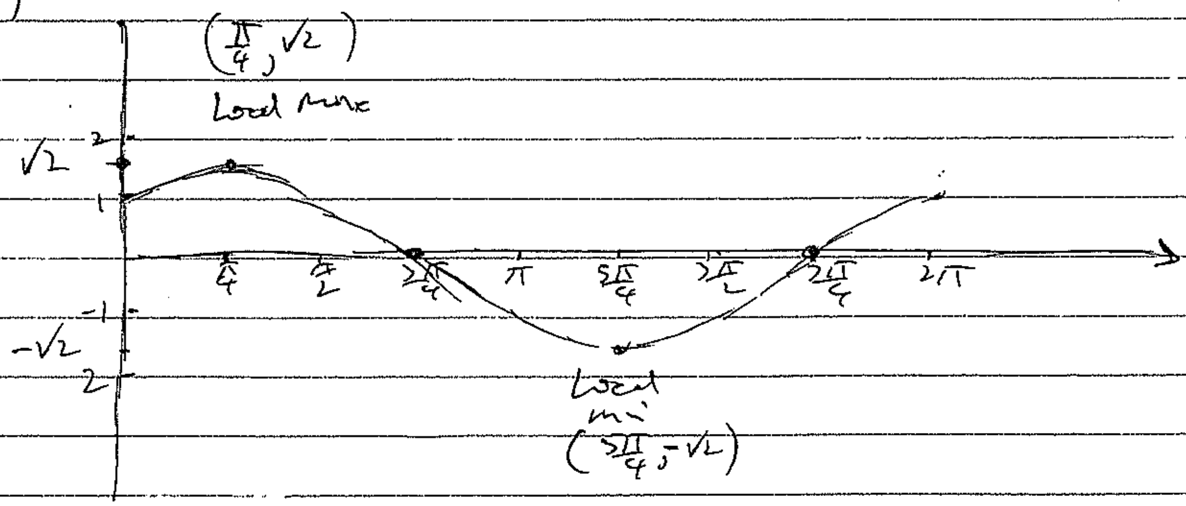
[Check $\frac{\pi}{2}$, $\frac{3\pi}{2}$ not soln]

Related angle, $\frac{\pi}{4}$ 2nd & 4th quad

$\frac{3\pi}{4}$ $\frac{7\pi}{4}$

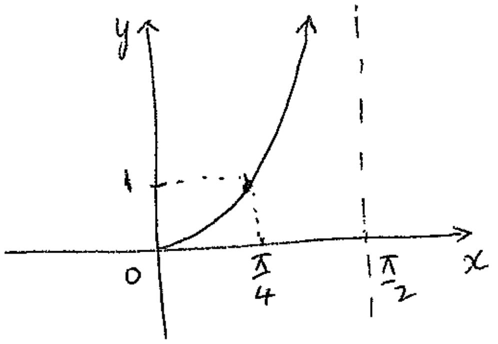
$y = 0$ when $x = \frac{3\pi}{4}$, $x = \frac{7\pi}{4}$

(iv)



Q 10.

a) i)



ii) $V = \pi \int_0^{\frac{\pi}{3}} \tan^2 x \, dx$ $\qquad \frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{1}{c^2}$

$= \pi \int_0^{\pi/3} (\sec^2 x - 1)\, dx$

$= \pi \left[\tan x - x \right]_0^{\pi/3}$

$= \pi \left[\left(\tan \frac{\pi}{3} - \frac{\pi}{3} \right) - (\tan 0 - 0) \right]$

Vol. $= \pi \left(\sqrt{3} - \frac{\pi}{3} \right) u^3$

b) i) after 6 mths she has

$10^6 \times 1.03$

after 12 mths she has

$10^6 \times 1.03^2$

Let A_1 be amt after 1st withdrawal.

$A_1 = 10^6 \times 1.03^2 - 75000$

ii) After another 6 mths she has

$(10^6 \times 1.03^2 - 75000)\, 1.03$

$(10^6 \times 1.03^2 - 75000) \times 1.03^2 - 75000$

$\therefore A_2 = (10^6 \times 1.03^2 - 75000) \times 1.03^2 - 75000$

$= 10^6 \times 1.03^4 - 75000 \times 1.03^2 - 75000$

and $A_3 = A_2 \times 1.03^2 - 75000$

$= (10^6 \times 1.03^4 - 75000 \times 1.03^2 - 75000)\, 1.03^2 - 75000$

$= 10^6 \times 1.03^6 - 75000 (1.03^4 + 1.03^2 + 1)$

iii) Following this pattern A_n = amt in account after n withdrawals.

$A_n = 10^6 \times 1.03^{2n} - 75000 (1 + 1.03^2 + \ldots + 1.03^{2n-2})$

$a = 1, \quad r = 1.03^2, \quad n$ terms

$A_n = 10^6 \times 1.03^{2n} - 75000 \left(\frac{1.03^{2n} - 1}{1.03^2 - 1} \right)$

Money will run out when $A_n = 0$.

$$0 = 10^6 \times 1.03^{2n}(1.03^2-1) - 75000 \times 1.03^{2n} + 75000$$

$$1.03^{2n}\left(75000 - 10^6 \times (1.03^2-1)\right) = 75000$$

$$2n \log 1.03 = \log \frac{75000}{75000 - 10^6(1.03^2-1)}$$

$$n = 28.27 \text{ yrs}$$

or 28 yrs 3 mths.

$\therefore$ It will take about 28 yrs to run out.