St George Girls High School

Trial Higher School Certificate Examination

2010

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 120

- Attempt ALL questions.
- All questions are of equal value.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks) **Marks**

a) Given $g(x) = \begin{cases} 1 - x^2 & x \leq 2 \\ 2^x & x > 2 \end{cases}$ evaluate $g(3) + g(2)$ 2

b) Factorise fully $9x^3 - 9$ 2

c) Convert $108°$ to radians giving your answer in terms of π . 2

d) Solve $\frac{x-5}{3} - \frac{x+1}{4} = 5$ 2

e) Solve and graph $|5x - 3| < 7$ 2

f) Expand and simplify $\left(\sqrt{2} + \sqrt{6}\right)^2$ 2

Question 2 – (12 marks)

Marks

a) Differentiate

(i) $(1-5x)^5$ — 2

(ii) $3x\,e^{3x}$ — 2

(iii) $\log_e(\cos x)$ — 2

b) Evaluate $\int_0^{\frac{\pi}{6}} \cos 2x \; dx$ — 2

c) Find:

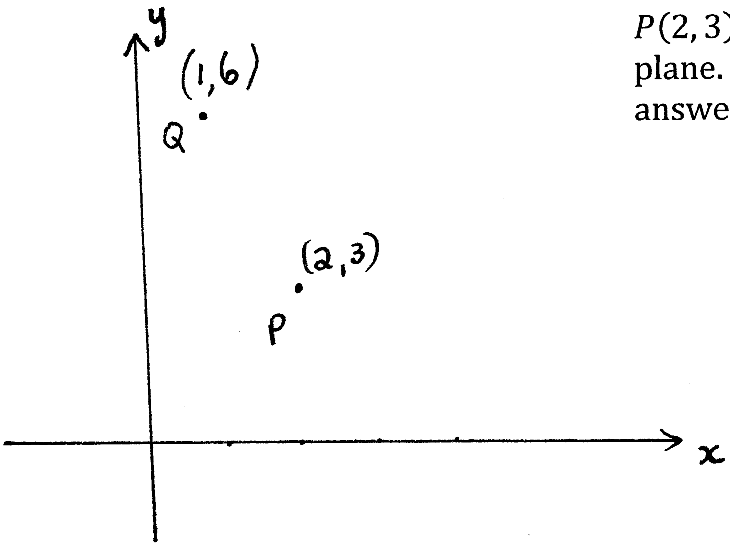
(i) $\int \sqrt{x} \; dx$ — 2

(ii) $\int 2x^2\, e^{x^3} \; dx$ — 2

Question 3 – (12 marks)

Marks

a) The diagram shows two points $P(2,3)$ and $Q(1,6)$ on the number plane. Copy the diagram into your answer booklet.



(i) Find the coordinates of M the midpoint of PQ. 1

(ii) Show that the equation of the perpendicular bisector of PQ is $x - 3y + 12 = 0$ 2

(iii) Find the coordinates of the point R that lies on the y-axis and is equidistant from P and Q. 1

(iv) The point S lies on the intersection of the line $y = 6$ and the perpendicular bisector, $x - 3y + 12 = 0$. Find the coordinates of S and mark the position of S on your diagram. 2

(v) Find the area of the triangle PQS. 2

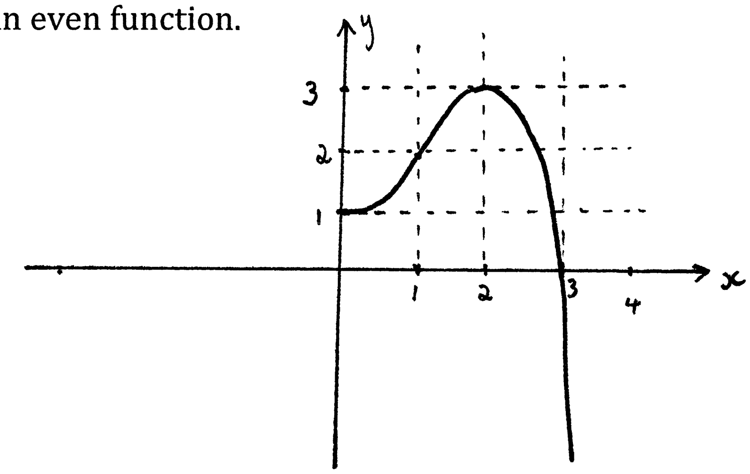
b) Differentiate the following:

(i) $y = x \sin 3x$ 2

(ii) $y = \dfrac{\ln x}{x^2}$ 2

Question 4 – (12 marks) **Marks**

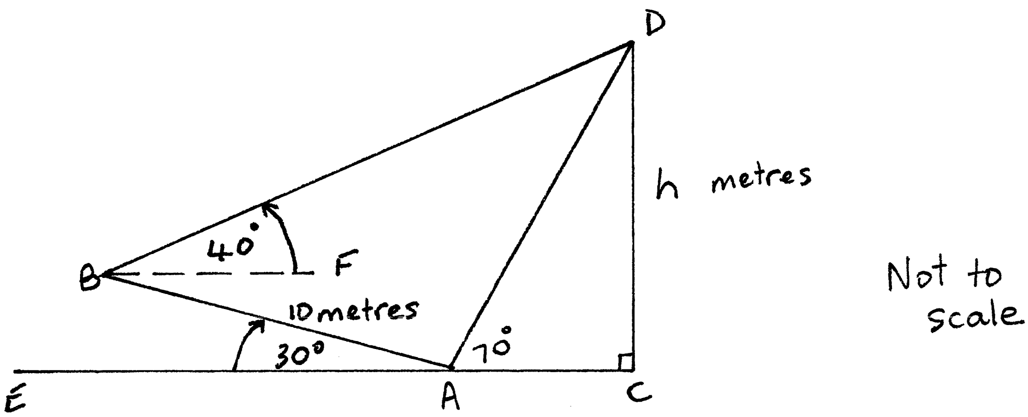
a) The following diagram shows the graph of $y = f(x)$ for $x \geq 0$. It is known that $f(x)$ is an even function.



(i) Copy the diagram into your answer booklet and complete the graph for $x < 0$. **2**

(ii) On a separate diagram sketch a graph of $y = f'(x)$ **2**

b)



The top of a vertical tower CD is observed at an angle of elevation 70° from A. After walking 10 metres up a ramp AB, inclined at an angle of 30° to the horizontal, the top of the tower is now observed at an angle of elevation of 40°.

(i) Find, with reasons the size of $\angle ABD$, and of $\angle BDA$. **4**

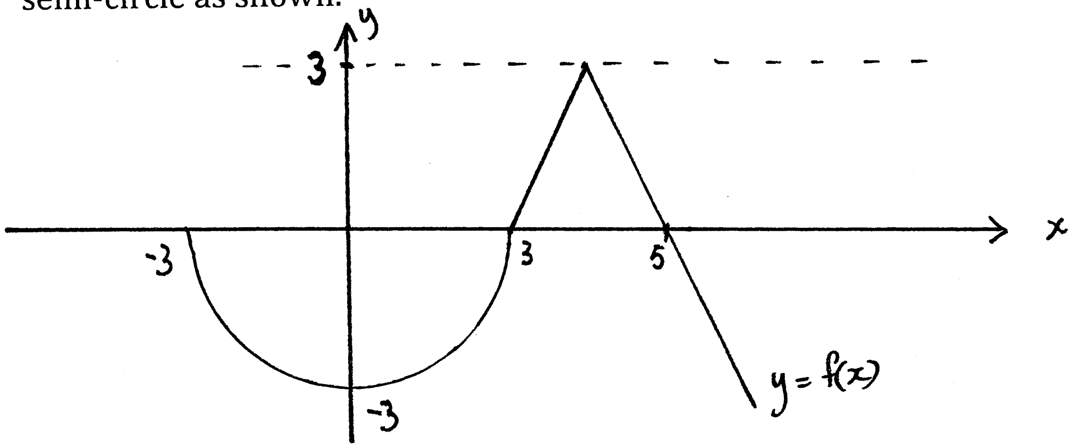
(ii) Find the length of AD. **2**

(iii) Hence find the height of the tower correct to one decimal place. **2**

Question 5 – (12 marks)

Marks

a) The graph of a function $y = f(x)$ consists of two straight line sections and a semi-circle as shown.



Find the exact value of $\int_0^5 f(x)\, dx$ 2

b) Let $\log_a 2 = x$ and $\log_a 3 = y$

Find an expression for $\log_a 24$ in terms of x and y. 2

c) Evaluate 2

$$\sum_{n=3}^{8} (2n^2 - 3)$$

d) Find:

(i) $\int_1^4 x \ \sqrt{x}\ dx$ 2

(ii) $\int \frac{5x^2}{x^3+1}\, dx$ 2

(iii) $\int_0^2 e^x + e^{-x} \ \ dx$ 2

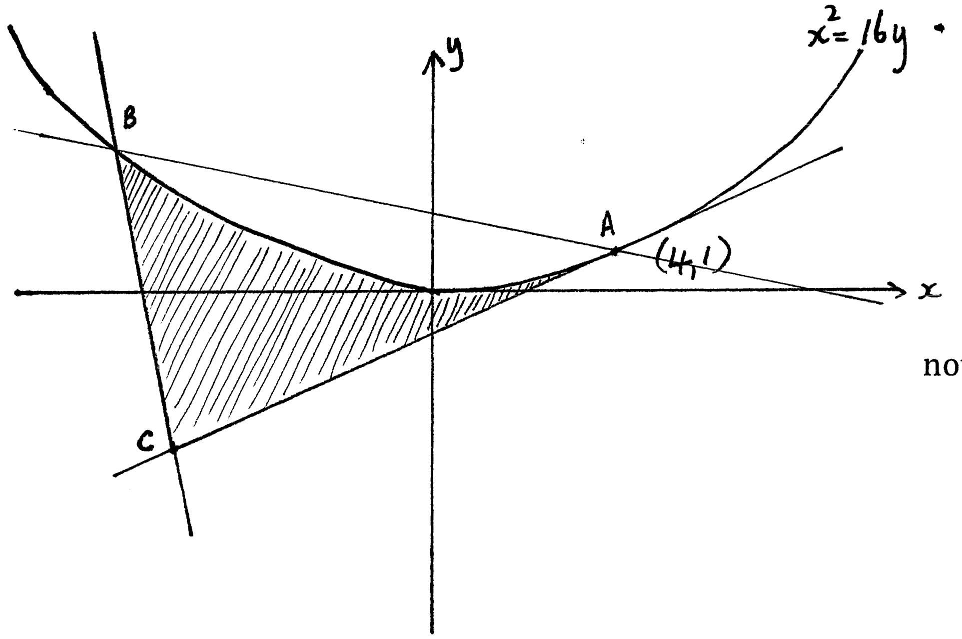
Question 6 – (12 marks)

Marks

a) Solve the following equation for x 3

$$e^{2x} + 5e^{x} - 14 = 0$$

b)



not to scale

The diagram shows the graph of the parabola $x^2 = 16y$. The points $A(4, 1)$ and $B(-8, 4)$ are on the parabola, and C is the point where the tangent to the parabola at A intersects the directrix.

(i) Write down the equation of the directrix of the parabola $x^2 = 16y$. 1

(ii) Find the equation of the tangent to the parabola of A. 2

(iii) Show that C is the point $(-6, -4)$. 1

(iv) Given that the equation of AB is $y = 2 - \frac{x}{4}$

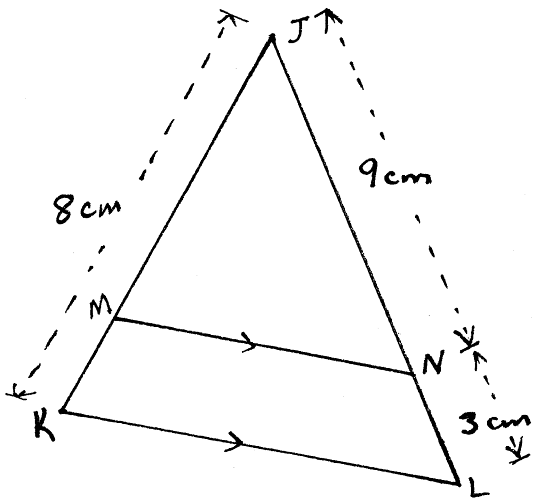
(α) Find the perpendicular distance of C to the line AB. 1

(β) Find the area bounded by the line AB and the parabola $x^2 = 16y$. 2

(v) Hence or otherwise find the shaded area bounded by the parabola, the tangent at A and the line BC. 2

Question 7 – (12 marks) **Marks**

a)



In the diagram triangle JKL is shown $MN \parallel KL$, $JK = 8$ cm, $JN = 9$ cm and $NL = 3$ cm.

(i) Prove that ΔJMN is similar to ΔJKL. 2

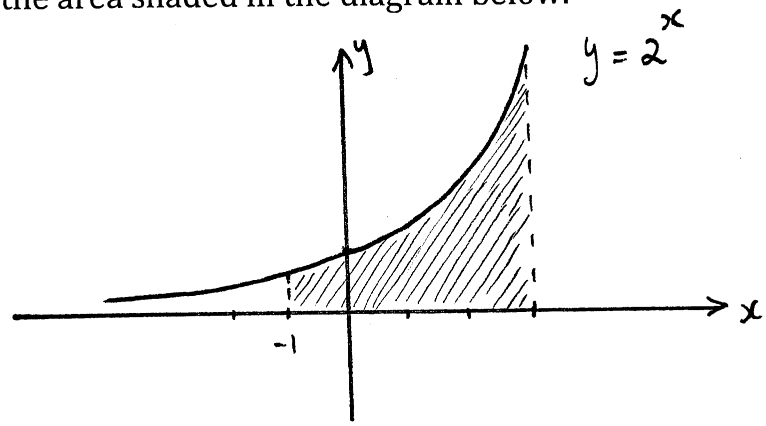
(ii) Find the length of MK. 2

b) Consider the function $y = 2^x$.

x	-1	0	1	2	3
2^x					

(i) Copy and complete the above table in your writing booklet. 1

(ii) Using Simpson's Rule with these five function values, find an estimate for the area shaded in the diagram below. 2



Question 7 (cont'd) **Marks**

c) A particle is moving in a straight line according to the formula $x = t^2 - 2t + 7$, where t is time in seconds, and x is the displacement from the origin O in metres. Find:

(i) the initial displacement of the particle. 1

(ii) the time when the particle is stationary. 2

(iii) the total distance travelled by the particle in the first 4 seconds. 2

Question 8 – (12 marks)

Marks

a) (i) Write down the discriminant of $3x^2 + 2x + k$. 1

(ii) For what values of k does $3x^2 + 2x + k = 0$ have real roots? 1

b) A ball is dropped from a height of 3 metres onto a hard floor and bounces. After each bounce, the maximum height reached by the ball is 80% of the previous maximum height. Thus, after it first hits the floor it reaches a height of 2.4 metres before falling again and after the second bounce, it reaches a height of 1.92 metres before falling again.

(i) What is the maximum height reached on the third bounce? 1

(ii) What kind of sequence is formed by the successive maximum heights? 1

(iii) What is the total distance travelled by the ball from the time it was first dropped until it eventually comes to rest on the floor? 2

c) (i) Sketch the graph of $y = x^2 - 6$ and label all intercepts with the axes. 2

(ii) On the same set of axes, carefully sketch the graph of $y = |x|$ 1

(iii) Find the x coordinates of the two points where the graphs intersect. 2

(iv) Hence solve the inequality $x^2 - 6 \leq |x|$ 1

Question 9 – (12 marks) **Marks**

a) Penny is saving for an overseas trip. At the beginning of each month she deposits \$400 into a bank account which pays 6% p.a. calculated monthly.

(i) How much will she hve after 2 years? 2

(ii) Penny actually needed \$16 000 for her trip. How much should she have deposited each month into this account for her to have reached her goal in 2 years? 2

b) Given $y = 2\cos 3x$ write down

(i) the amplitude of this graph. 1

(ii) the period. 1

(iii) sketch the graph for $0 \leq x \leq 2\pi$. 2

c) (i) Show that the volume of the solid formed when $y = \tan 2x$ is rotated about the x-axis between $x = 0$ and $x = \frac{\pi}{6}$ is given by 2

$$V = \pi \int_0^{\frac{\pi}{6}} (\sec^2 2x - 1)\ dx$$

(ii) Find the exact volume of the solid. 2

Question 10 – (12 marks) **Marks**

a) For the parabola $(x+3)^2 = 4y - 14$ find the following:

(i) the vertex. 1

(ii) the focus. 2

(iii) the equation of the directrix. 1

b) A cam is formed with a cross-section as shown in the figure.

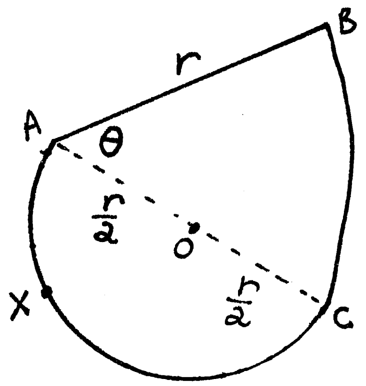


figure not to scale

The cross-section consists of a semicircle AXC, centre O and radius $\frac{r}{2}$, and a sector ABC of radius r and centre A and angle θ.

(i) What is the perimeter (P) of the cam $ABCX$ in terms of r and θ? 2

(ii) If the area of the cross section of the cam is 1 square unit, by first finding θ show that the perimeter P is given by 3

$$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right)$$

(iii) Show that the least perimeter occurs when $r^2 = \frac{8}{\pi+4}$ and calculate the value of θ when the perimeter is least. 3

End of Paper

Solutions - Mathematics Trial 2010

Question 1.

(a) $g(3) + g(2) = 2^3 + 1 - 2^2$

$= 5$

(b) $9x^3 - 9 = 9(x^3 - 1)$

$= 9(x-1)(x^2 + x + 1)$

(c) $108^\circ = \frac{3\pi}{5}$ radians

(d) $\frac{x-5}{3} \times 12 - \frac{x+1}{4} \times 12 = 5 \times 12$

$4(x-5) - 3(x+1) = 60$

$4x - 20 - 3x - 3 = 60$

$x - 23 = 60$

$x = 83$

(e) $|5x - 3| < 7$

$-7 < 5x - 3 < 7$

$-4 < 5x < 10$

$-\frac{4}{5} < x < 2$

(f) $(\sqrt{2} + \sqrt{6})^2 = 2 + 2\sqrt{12} + 6$

$= 8 + 2 \times 2\sqrt{3}$

$= 8 + 4\sqrt{3}$

Question 2.

(a) (i) $y = (1 - 5x)^5$

$y' = 5(1-5x)^4 \times -5$

$= -25(1-5x)^4$ 2

(ii) $y = 3x e^{3x}$

$y' = 3e^{3x} + 3x \times 3e^{3x}$

$= 3e^{3x} + 9x e^{3x}$

$= e^{3x}(3 + 9x)$ 2

$= 3e^{3x}(1 + 3x)$

(iii) $y = \log(\cos x)$

$y' = \frac{1}{\cos x} \times -\sin x$

$= -\frac{\sin x}{\cos x}$ 2

$= -\tan x$

(b) $\int_0^{\frac{\pi}{6}} \cos 2x \, dx = \frac{1}{2}\sin 2x \Big]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\sin 2 \times \frac{\pi}{6} - \frac{1}{2}\sin 2$

$= \frac{1}{2}\sin\frac{\pi}{3} - \frac{1}{2}\sin 0$

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - 0$ 2

$= \frac{\sqrt{3}}{4}$ = 0.433

(c) (i) $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx$

$= \frac{x^{1\frac{1}{2}}}{1\frac{1}{2}} + c$ 2

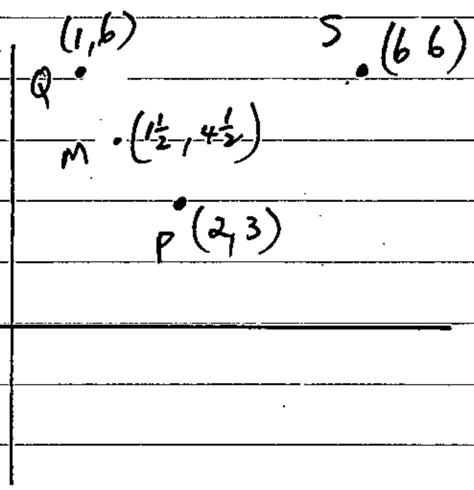
$= \frac{2}{3} x^{\frac{3}{2}} + c$

(ii) $\int 2x^2 e^{x^3} dx = \frac{2}{3}\int 3x^2 e^{x^3} dx$

$= \frac{2}{3} e^{x^3} + c$ 2

Question 3.

(a)



(i) $M = \left(\frac{1+2}{2}, \frac{3+6}{2}\right)$

$= \left(1\frac{1}{2}, 4\frac{1}{2}\right)$

(ii) $m_{PQ} = \frac{3-6}{2-1}$

$= \frac{-3}{1}$

$\therefore m_1 = -3 \qquad m_2 = \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 4\frac{1}{2} = \frac{1}{3}\left(x - 1\frac{1}{2}\right)$

$3y - 13\frac{1}{2} = x - 1\frac{1}{2}$

$x - 3y + 12 = 0$

(iii) $x = 0 \quad \therefore \quad -3y + 12 = 0$

$3y = 12$

$y = 4$

(iv) $y = 6 \qquad x - 3y + 12 = 0$

$x - 18 + 12 = 0$

$x = 6$

(v) $PQ = \sqrt{(2-1)^2 + (3-6)^2}$

$= \sqrt{1^2 + (-3)^2}$

$= \sqrt{10}$

$MS = \sqrt{(6-1\frac{1}{2})^2 + (6-4\frac{1}{2})^2}$

$= \sqrt{(4\frac{1}{2})^2 + (1\frac{1}{2})^2}$

$= \sqrt{22.5}$

$\therefore \text{Area} = \frac{\sqrt{10} \times \sqrt{22.5}}{2}$

$= \frac{\sqrt{225}}{2}$

$= \frac{15}{2}$ sq. units.

(b)

(i) $y = x \sin x$

$y' = 1 \times \sin x + x \times \cos x$

$= \sin x + x \cos x$

(ii) $y = \frac{\ln x}{x^2}$

$y' = \frac{x^2 \times \frac{1}{x} - 2x \ln x}{x^4}$

$= \frac{x - 2x \ln x}{x^4}$

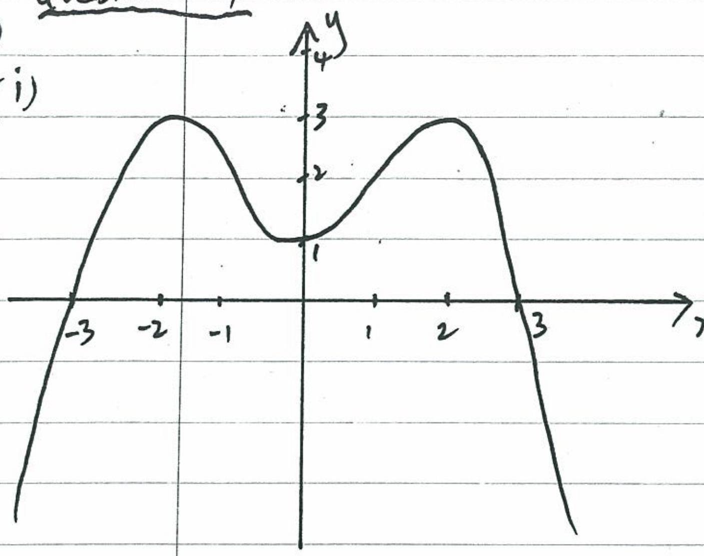
$= \frac{\cancel{x}(1 - 2\ln x)}{\cancel{x^4}\, x^3}$

$= \frac{1 - 2\ln x}{x^3}$

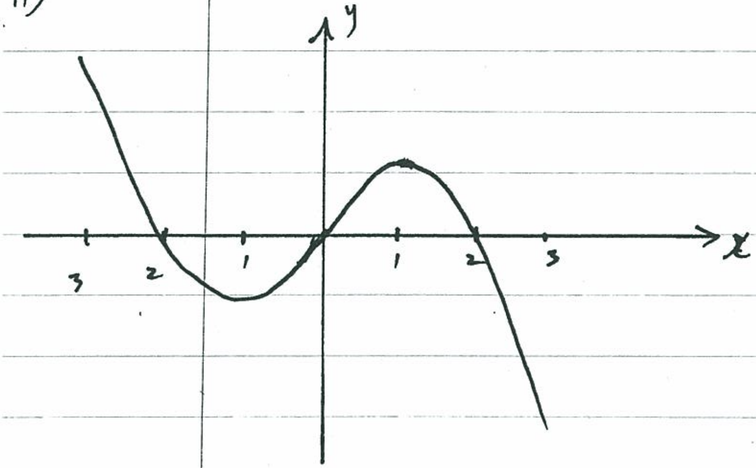
Question 4

(a)

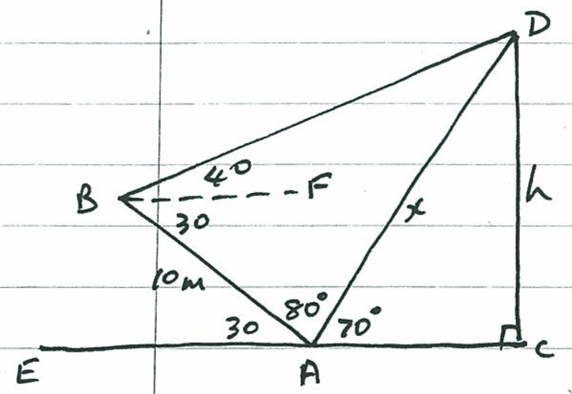
(i)



(ii)



(b)



(i) BF // EA horizontal lines

$\therefore F\hat{B}A = E\hat{A}B$ (alternate angles on parallel lines)

$\therefore F\hat{B}A = 30^\circ$

$\therefore A\hat{B}D = 70^\circ$

(ii) $\dfrac{x}{\sin 70} = \dfrac{10}{\sin 30}$

$\therefore x = \dfrac{10 \sin 70}{\sin 30}$

$= 18.79$

(iii) $\dfrac{h}{x} = \sin 70$

$h = x \cdot \sin 70$

$= \dfrac{10 \sin 70}{\sin 30} \times \sin 70$

$= 17.66$

$= 17.7$

Question 5

(a) $\displaystyle\int_0^5 f(x)\,dx =$ 

$= \dfrac{bh}{2} - \dfrac{\pi r^2}{4}$

$= \dfrac{2 \times 3}{2} - \dfrac{\pi 3^2}{4}$

$= 3 - \dfrac{9\pi}{4}$ 2

(b) $\log_a 24 = \log_a 3 \times 8$

$= \log_a 3 + \log_a 8$

$= \log_a 3 + \log_a 2^3$

$= \log_a 3 + 3 \log_a 2$

$= y + 3x$ 2

(c) $\displaystyle\sum_{n=3}^{8} 2n^2 - 3 = 15 + 29 + 47 + 69 + 95 + 1$

$= 380$

2

Question 5 Cont.

(d) (i) $\int_1^4 x\sqrt{x}\,dx = \int_1^4 x \cdot x^{\frac{1}{2}}\,dx$

$= \int_1^4 x^{1\frac{1}{2}}\,dx$

$= \left.\frac{2x^{2\frac{1}{2}}}{5}\right]_1^4$

$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1\right)$

$= \frac{2}{5}(32 - 1)$ 2

$= \frac{62}{5}$

(ii) $\int \frac{5x^2}{x^3+1}\,dx = \frac{5}{3}\int \frac{3x^2}{x^3+1}\,dx$

$= \frac{5}{3}\ln(x^3+1) + C$ 2

iii) $\int_0^2 e^x + e^{-x}\,dx = \left. e^x - e^{-x}\right]_0^2$

$= (e^2 - e^{-2}) - (e^0 - e^{-0})$

$= e^2 - \frac{1}{e^2} - (1-1)$

$= e^2 - \frac{1}{e^2}$ 2

Question 6.

a) $e^{2x} + 5e^x - 14 = 0$

let $m = e^x$

$\therefore m^2 + 5m - 14 = 0$

$(m+7)(m-2) = 0$

$m = -7, 2$

$\therefore e^x = -7$ no solution $\quad e^x = 2$, $x = \log 2$

(b) (i) $x^2 = 16y$

$4a = 16$

$a = 4$

$\therefore$ directrix is $y = -4$

(ii) $y = \frac{x^2}{16}$

$y' = \frac{2x}{16}$

$= \frac{1}{8}x$

$\therefore x = 4 \quad m = \frac{4}{8}$

$= \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x - 4)$

$2y - 2 = x - 4$

$x - 2y - 2 = 0$

(iii) $y = -4$

$x - 2y - 2 = 0$

$x - 2\times -4 - 2 = 0$

$x + 8 - 2 = 0$

$x = -6$

$\therefore C = (-6, -4)$

(iv) (α) $y = 2 - \frac{x}{4}$

$\frac{1}{4}x + y - 2 = 0$

$d = \frac{\left|\frac{1}{4}x_1 + y_1 - 2\right|}{\sqrt{(\frac{1}{4})^2 + 1^2}}$

$= \frac{\left|\frac{1}{4}\times -6 - 4 - 2\right|}{\sqrt{1\frac{1}{16}}}$

$= \frac{7\frac{1}{2}}{\sqrt{\frac{17}{16}}}$

$= \frac{15}{2} \times \frac{4}{\sqrt{17}}$

$= \frac{30}{\sqrt{17}}$

Question 6 cont.

(B)

$$\int_{-8}^{4} 2 - \frac{x}{4} - \frac{x^2}{16}\,dx$$

$$= \left[2x - \frac{x^2}{8} - \frac{x^3}{48}\right]_{-8}^{4}$$

$$= \left(2\times4 - \frac{4^2}{8} - \frac{4^3}{48}\right) - \left(-16 - \frac{(-8)^2}{8} - \frac{(-8)^3}{48}\right)$$

$$= 29\tfrac{1}{3}$$

(V) $AB = \sqrt{(-12)^2 + 3^2}$

$= \sqrt{144+9}$

$= \sqrt{153}$

Area of ABC $= \dfrac{\sqrt{153} \times \frac{30}{\sqrt{17}}}{2}$

$= 45$

Area of shaded region

$= 45 - 29\tfrac{1}{3}$

$= 15\tfrac{2}{3}$ sq. units.

Question 7

a)

(i) $MN \parallel KL$ given

$J\hat{M}N = J\hat{K}L$ (corresponding angles on parallel lines)

similarly $J\hat{N}M = J\hat{L}K$ (")

$\hat{J}$ is common

$\therefore \triangle JMN \parallel\!| \triangle JKL$ (equiangular)

(ii) $\dfrac{9}{MJ} = \dfrac{12}{8}$

$72 = 12\,MJ$

$MJ = \dfrac{72}{12}$

$= 6$

$\therefore MK = 8 - 6$

$= 2$

(b)

(i)

x	-1	0	1	2	3
2^x	$\frac{1}{2}$	1	2	4	8

(ii) $h = 1$

$A \approx \frac{h}{3}\left\{\frac{1}{2} + 4\times1 + 2\right\} + \frac{h}{3}\left\{2 + 4\times4 + \ldots\right.$

$= \frac{1}{3}\left(6\tfrac{1}{2}\right) + \frac{1}{3}(26)$

$= 10.83$

(c)

(i) $x = t^2 - 2t + 7$

$t = 0 \quad x = 7$

(ii) $\dot{x} = 2t - 2$

$\dot{x} = 0 \quad 2t - 2 = 0$

$2t = 2$

$t = 1$

(iii) $t = 1 \quad x = 1 - 2 + 7$

$= 6$

$t = 4 \quad x = 16 - 8 + 7$

$= 15$

in 1st second dist = 1 m

in next 3 seconds 9 m

$\therefore$ total dist. travelled = 10 m.

Question 8

(a) $3x^2 + 2x + k$

(i) $b^2 - 4ac = 2^2 - 4 \times 3 \times k$

$= 4 - 12k$

(ii) real roots $b^2 - 4ac > 0$

$\therefore 4 - 12k > 0$

$-12k > -4$

$k < \frac{-4}{-12}$

$k < \frac{1}{3}$

(b) 3, 2·4, 1·92

(i) $r = 0.8$

$1.92 \times 0.8 = 1.536$ m

(ii) Geometric sequence

(iii) $S_\infty = \frac{a}{1-r}$

$= \frac{2.4}{1-0.8}$

$= 12$

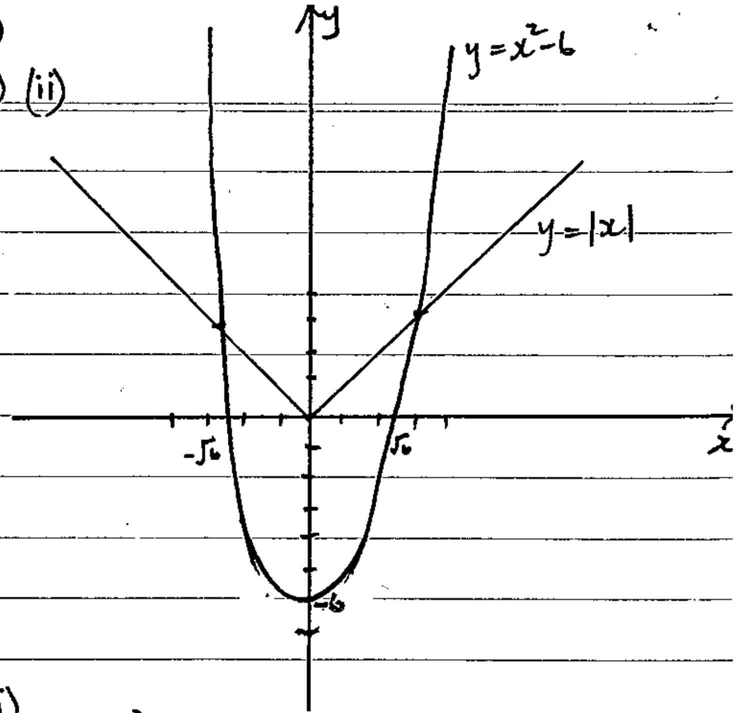
Total dist $= 3 + 2 \times S_\infty$

$= 3 + 24$

$= 27$ m.

(c)

(i) (ii)



(iii) $x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2, 3$

$x = 3$ but by symmetry from gra

$x = -3$

(iv) $-3 < x < 3$

Question 9

(a)

(i) $P = 400$ $\qquad A_1 = 400(1+0.005)^{24}$

$r = \frac{6}{12}$ $\qquad A_2 = 400(1.005)^{23}$

$= .5\%$ $\qquad A_{24} = 400(1.005)$

$= 0.005$

$a = 400(1.005) \quad r = 1.005 \quad n = 24$

$$S_{24} = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{400(1.005)(1.005^{24}-1)}{1.005-1}$$

$$= \$10223.65$$

(ii) $$16000 = \frac{P(1.005)(1.005^{24}-1)}{0.005}$$

$$P = \frac{16000 \times 0.005}{(1.005)(1.005^{24}-1)}$$

$$= 625.99$$

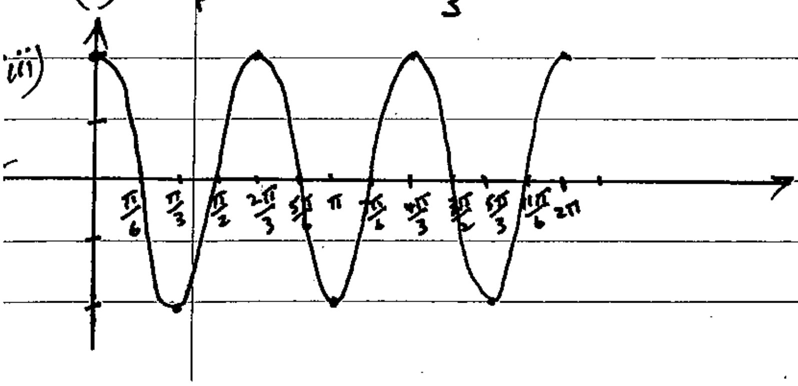
$\therefore$ Penny needs to save $\approx \$626$

(b) $y = 2\cos 3x$

(i) amp $= 2$

(ii) period $= \frac{2\pi}{3}$

(iii)



(c) (i) $$Vol = \pi\int_0^{\frac{\pi}{6}} (\tan 2x)^2\,dx$$

$$= \pi\int_0^{\frac{\pi}{6}} \tan^2 2x\,dx.$$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$$\therefore \pi\int_0^{\frac{\pi}{6}} \tan^2 2x\,dx = \pi\int_0^{\frac{\pi}{6}} (\sec^2 2x - 1)\,dx$$

(ii) $$= \pi\left(\tan 2x - x\right)\Big]_0^{\frac{\pi}{6}}$$

$$= \pi\left[\left(\tan\frac{\pi}{3} - \frac{\pi}{6}\right) - (\tan 0 - 0)\right]$$

$$= \pi\left(\sqrt{3} - \frac{\pi}{6}\right)$$

Question 10

(a) $(x+3)^2 = 4y - 14$

$(x+3)^2 = 4\left(y - 3\tfrac{1}{2}\right)$

(i) vertex $= \left(-3, 3\tfrac{1}{2}\right)$

(ii) focal length $4a = 4$

$a = 1$

$\therefore$ focus $= \left(-3, 4\tfrac{1}{2}\right)$

(iii) directrix $y = 2\tfrac{1}{2}$

(b) (i) arc length $= r\theta$

semi circle $= \frac{1}{2} \cdot 2\pi r$

but $r = \frac{r}{2}$ $\quad \therefore = \frac{\pi r}{2}$

$\therefore P = r + r\theta + \frac{\pi r}{2}$

(ii) Area $= \frac{1}{2} r^2\theta + \frac{1}{2}\pi\left(\frac{r}{2}\right)^2$

$= \frac{1}{2} r^2 \theta + \frac{\pi r^2}{8}$

$A = 1$

$\therefore \frac{1}{2} r^2\theta + \frac{\pi r^2}{8} = 1$

$\frac{1}{2} r^2 \theta = 1 - \frac{\pi r^2}{8}$

$\frac{1}{2} r^2 \theta = \frac{8 - \pi r^2}{8}$

$\theta = \frac{8 - \pi r^2}{8} \times \frac{2}{r^2}$

$= \frac{8 - \pi r^2}{4r^2}$

$\therefore \theta = \frac{8}{4r^2} - \frac{\pi r^2}{4r^2}$

$\theta = \frac{2}{r^2} - \frac{\pi}{4}$

but $P = r + r\theta + \frac{\pi r}{2}$

$\therefore P = r + r\left(\frac{2}{r^2} - \frac{\pi}{4}\right) + \frac{\pi r}{2}$

$= r + \frac{2}{r} - \frac{\pi}{4} r + \frac{\pi}{2} r$

$= r + \frac{2}{r} + \frac{\pi}{4} r$

$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right)$

(iii) $P = 2r^{-1} + \left(1 + \frac{\pi}{4}\right) r$

$P' = -2r^{-2} + 1 + \frac{\pi}{4}$

$= \frac{-2}{r^2} + 1 + \frac{\pi}{4}$

$P' = 0$ for min $\qquad P'' = -4r^{-3}$

$= \frac{-4}{r^3}$

$\therefore \frac{-2}{r^2} + 1 + \frac{\pi}{4} = 0$

$\therefore$ for min

$\frac{2}{r^2} = 1 + \frac{\pi}{4}$ $\qquad r = +\sqrt{\frac{8}{4+\pi}}$

$2 = \left(1 + \frac{\pi}{4}\right) r^2$

$r^2 = \frac{2}{1 + \frac{\pi}{4}}$

$r^2 = \frac{2}{\frac{4+\pi}{4}}$

$= \frac{8}{4+\pi}$

$r = \pm\sqrt{\frac{8}{4+\pi}}$

Question 10 cont.

$\therefore$

$\theta = \frac{2}{r^2} - \frac{\pi}{4}$

$= \frac{2}{\frac{8}{4+\pi}} - \frac{\pi}{4}$

$= \frac{2(4+\pi)}{8} - \frac{\pi}{4}$

$= \frac{4}{4} + \frac{\pi}{4} - \frac{\pi}{4}$

$= 1$ radian